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PROJECTIONS—HOW TO MAKE THEM AND HOW TO USE THEM

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INTRODUCTION

One of the problems facing the pension actuary is how to make his findings intelligible to persons not well acquainted with actuarial terminology. Such phrases as present value, past-service liability, level cost, reserve, and the like, have a somewhat mysterious connotation to the layman. It is particularly difficult for him to appreciate the relationships among the more important items which appear in the usual valuation report. Yet, it is the management and labor officials, not the actuaries, who are charged with the responsibility for making decisions upon which the fate of the pension plan ultimately rests. The actuary should therefore make an effort to present his findings in such a manner that the layman will have no special difficulty in comprehending the real significance of the valuation or cost estimate, as the case may be.

Two problems appear to be particularly troublesome. One is the question of reserves and the other is the past-service liability. In a large self-insured pension plan, financed by a level percentage of payroll, the reserves are bound, in a relatively short time, to come up to figures which stagger the imagination of the ordinary layman. When such a layman examines a financial statement for a relatively recent plan, he soon notices that the funds on hand taken by themselves would be sufficient to pay benefits at the current rate for a good number of years. He furthermore notices that the current rate of contributions far exceeds the current rate of disbursements. He therefore begins to wonder why such excess sums are necessary and why the reserves on hand cannot be drawn upon either to increase the rate of benefits or to lower the rate of contributions. Unless the actuary will find a method of explaining in a way understandable to the layman the necessity for accumulating and keeping reserves, the latter will begin to make his own "calculations" and draw his own conclusions.

The second troublesome problem arises in connection with the past-service liability. Let us again consider a large self-insured plan financed in such a manner that the past-service liability is not amortized but is kept

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constant through the payment of annual interest charges. The actuary explains that this past-service liability is due mainly to service performed before the introduction of the pension plan and is attributable to former and present employees. The valuation also quotes a normal rate for new entrants which is considerably below the over-all rate applicable to the plan as a whole. The layman begins to figure that the past-service liability is in some way bodily attached to the present and former employee population. The logical conclusion from his point of view is that at the time when the last present member will have left the picture, there should be no more past-service liability and the over-all rate of contributions should then come down to the normal rate level. How can the fallacy of such reasoning be explained without resorting to some more obscure concepts and arguments which will even add to the confusion?

One way of dealing with problems like the two described above, and many others, is to present a year by year projection of numbers of beneficiaries on the rolls, benefit disbursements, and progress of reserves. A projection of this type would not only provide a method of presentation, which is from the layman's point of view far superior to that given by the valuation balance sheet, but would also enable the actuary to answer a variety of questions which could not be readily answered otherwise.

The value of projections has been stressed before. Mr. Robert J. Myers in a paper entitled "Some Considerations in Pension Fund Valuation" (*TASA* XLVI, 51–58) even considers the possibility of dispensing with the traditional valuations altogether in favor of projections. In Mr. Myers' own words:

> . . . it would seem highly desirable to supplement, if not replace, the traditional exhibits of accrued liability and cost as a percentage of payroll (after bringing out the range in such figures) by year-by-year projections of income and outgo to the fund, again on a range basis.

The types of questions that will come up obviously depend on the type of the pension plan and the method of its financing. This writer has been chiefly concerned with a large governmental plan which contains many features similar to those found in self-insured industrial pension plans. The plan is financed by an essentially level payroll tax and no provision is made for the amortization of the past-service liability over a definite period of time. The methods of making a projection which will be discussed in this paper have been tested with respect to this rather complex particular plan. It is believed that with proper simplifications or modifications these methods will be equally applicable to the more usual types of pension plans.

As related to a pension plan of a type mentioned in the preceding paragraph, a projection will provide an answer to the following questions:

1. At what time will current contributions begin to run behind current disbursements?

2. At what time will the system reach maturity? What will be the ultimate number of beneficiaries on the rolls?

3. What will the ultimate benefit load be in absolute amount and in terms of a percentage of the then current payroll?

4. What will be the ultimate reserve if the present financing structure is maintained?

5. In what way and by how much will the reserve reduce the ultimate rate of contribution which would otherwise be required on a strict pay-as-you-go basis?

6. Assuming that the financial arrangements call for only interest charges on the past-service liability, why cannot the rate of contributions be reduced to the normal level after all employees with past service have left the picture?

7. In what ways can the financing schedule be changed? What will be the effect of such changes on the financial soundness of the plan?

8. What will happen if benefits are increased by a specified percentage? At what time would the reserve under such conditions begin to go down? When would the reserve become exhausted?

9. What financing schedule can be instituted in order to keep the reserve within predetermined limits?

10. What would be the effect of experiencing interest rates higher or lower than assumed in the valuation?

The above list of questions is by no means exhaustive. It has been drawn up as an illustration in order to show the tremendous possibilities which a projection has to offer. The nature of these questions seems to indicate that a projection is a study well worth the additional work connected with its preparation.

There are many ways in which a projection can be prepared. The simplest is to assume a single average age at entry, a constant number of retirements each year all at the same age, flat wage scales, level annual payrolls whenever these enter the picture, and the like. With such assumptions, a projection would require relatively little work and would involve no special technical difficulties. The trouble, however, with a projection of this type is that it is not sufficiently realistic and that it is very likely to be inconsistent with the valuation. If a projection is to be more than just a very crude illustration, it should be prepared with great care, and should be as far as possible realistic. The purpose of this paper is to

outline a method by which a reasonably good projection can be made without an excessive amount of work. To make the discussion more complete, we will also show how to project widows' monthly benefits, even though such benefits are not now included in the typical industrial plans. An attempt will be made not only to present working formulae with brief descriptions of the rationale for the methods suggested, but also to give in the appendix a fuller description of certain stages of the work in worksheet form.

GENERAL CONSIDERATIONS

There are a number of general rules which should be followed in the preparation of a projection which is intended to serve as a supplement to a valuation. It would be difficult, if not altogether impossible, to formulate a set of rules upon which all actuaries could agree. After all, a projection involves so much individual judgment that uniformity is perhaps even not desirable. As far as this writer is concerned, the following six rules seem important. Here again, the criteria are geared to a particular type of plan, but can easily be modified for other types.

1. The annual payroll estimates or number of man-hours worked used in the projection should be the same or very nearly the same as in the valuation.

2. Although it is not necessary to insist on a one-to-one correspondence between the age distribution of a group of new retirements in the projection and the retirement rates used in the valuation, a certain degree of conformity should be maintained. The same holds true with respect to other types of decrements like disability, withdrawal, and death in active service. This implies that the projection should not be based on single average ages, since such a procedure would generally produce results substantially different from the general trends indicated by the valuation.

3. When new entrants are considered separately, their number and age distribution as used in the projection should be consistent with the payroll estimates, wage scales, and present employee census of the valuation.

4. The ultimate numbers of beneficiaries on the rolls and the ultimate level of benefit disbursements in the projection should reasonably agree with corresponding figures derived on the basis of the service tables and benefit schedules used in the valuation.

5. The main projection should check out with the valuation or with a predetermined variation thereof in regard to the present value of all benefits and also in regard to the several components of the total present value.

6. In existing pension plans, the projection figures for the first couple

of years at least should fit in with the actual past and current experience. In new pension plans, such figures for the first few years should be checked against expectations based on general common-sense reasoning.

It is obvious that when a projection is made directly on the basis of the valuation, a high degree of agreement between the two can be attained. But, even then, some rather unpleasant problems may arise. This writer came across situations where it was difficult to reconcile future wage scales and decrement factors used in the valuation on one hand with the yearly payroll estimates on the other. As a result of these inconsistencies, the projection came up with negative numbers of new entrants for the first several years. The algebra eventually straightened itself out, so that the projection as a whole still made sense. The instance is mentioned only to point out the difficulties that even a straightforward projection based on the valuation itself may run into. The problems encountered in a projection not directly based on the valuation material are, as a rule, more complex and require far more individual judgment. The important saving feature is that in general this type of projection requires considerably less work.

The amount of time and effort which the actuary will want to spend on a projection is dependent upon factors which may differ from case to case. In some instances, a very simple projection requiring only a minimum amount of work will suffice; in others, the projection should strive to have a degree of reliability comparable to that claimed for the valuation itself. This writer had occasions to work on projections which were intended to serve as a guide for answering rather important and difficult questions. It has been his experience that a more realistic, or shall we say plausible, projection can be obtained if it is run without the direct use of the service tables and the benefit schedules assumed in the valuation. One reason for this opinion is that the service tables of the valuation do not even propose to recognize temporary fluctuations in economic conditions which affect the progress of the pension fund when such progress is viewed as a time series. In a realistic projection, such fluctuations should be given at least limited recognition. Another advantage already mentioned is that a projection run without the direct use of the valuation requires less computational work.

There are a number of important questions which the actuary should consider in the advance planning of a projection. We shall enumerate only a few of them here.

1. Is it advisable to make two separate projections, one according to high and the other according to low cost assumptions, or will a single intermediate projection suffice?

2. If a single intermediate projection is decided upon, what degree of agreement should be maintained between the projection and the valuation?

3. What approximations will be considered tolerable? With respect to age distribution, for instance, the question is whether single average ages for new retirements, withdrawals, deaths, and so on, will be sufficient or is it necessary to introduce finer breakdowns for the groups to be projected?

4. How is the projection to deal with the problem of improving mortality?

5. If the projection is to extend to the point of maturity, how are the ultimate numbers of beneficiaries and the ultimate load to be determined in advance?

There is obviously no single answer to these and similar questions. The best that this writer can offer is to present his own notions in the hope that the discussion of this paper will bring out other ideas and opinions. For the present, our ideas as to how to deal with the problems stated above are as follows:

1. We favor a single projection corresponding to intermediate cost assumptions. Aside from the saving in work, our main reason for the single projection is that it can be made to agree with the valuation. Admittedly, a single projection, or a single valuation for that matter, does not in itself indicate the range of costs, and this is a serious shortcoming. However, the necessity for presenting cost ranges is not as yet universally recognized and the whole problem is still in the discussion stage. An illuminating exposition of the pros and cons may be found in Mr. Robert J. Myers' paper previously referred to and in the discussion (*TASA* XLVI).

2. In the paper here presented, we are not concerned with projections which are made for the purposes of estimating costs. We are discussing a projection which is intended to serve as a supplement to a single valuation, and we feel that, for such purposes, a close correspondence between the projection and the valuation appears most logical. The agreement between the projection and the valuation should undoubtedly extend to the total present values of future disbursements and ultimate figures. It is felt, however, that a complete agreement between the two is not absolutely necessary. Thus, for instance, the projection may proceed according to retirement rates somewhat lower than those used in the valuation provided, of course, that the rates of the valuation are considered more than moderately conservative. The deficiency in the present value of future pensions can either be disregarded or adjusted by introducing in the projection somewhat higher ultimate numbers of pensioners on the rolls. The

last suggestion is based on the proposition that a projection can hope to be more or less realistic for, at best, a short period of time. From a practical point of view, it therefore matters relatively little what figures are obtained for the later years so long as the projection conforms with the valuation in a general way.

3. A projection run without the use of broad groupings would be extremely laborious. Meticulous computational work will generally not improve the reliability or even plausibility of the projection. What really matters is the soundness of the general assumptions, not the precision of the computations. It is therefore advisable to strike a happy medium between the maximum and minimum computational work that may be performed. It is our opinion that quinquennial age groupings will give good results and that it is advisable to maintain constant age distributions for new accessions even though their number is varied.

4. The question of improving mortality falls in the category of problems which were considered in the paragraph immediately above. If we were to allow in the projection for improving levels of mortality, we would be forced to make separate projections for the accessions at particular points of time. In our opinion, the extra amount of labor would not be justified. It appears that it is best to decide upon an average level of mortality that can be assumed to exist for, say, the next decade or so and to use mortality rates corresponding to that level throughout the projection. Such an approach appears particularly appropriate for a projection of pensions, since, for the time being at least, no tremendous changes in mortality levels are anticipated for the older ages. It is probably best to utilize the same mortality rates which were adopted in the valuation, since in this way conformity between present values can be achieved.

5. If the projection is to run to the point of maturity, an advance estimate of the ultimate numbers of beneficiaries on the rolls and the ultimate benefit load seems to be absolutely necessary. Such ultimate figures provide the first check for the reasonability of the time series that are produced by the projection. Here again, it is not necessary to have the ultimate figures of the projection identical with those estimated on the basis of the valuation. This is particularly true if, for a certain period of time, the assumptions underlying the projection are not strictly the same as those underlying the valuation. In any event, advance ultimate figures are a very valuable guide in the preparation of a projection.

MECHANICS OF A PROJECTION

We shall now present a brief sketch of the mechanics involved in making a projection which is run without the direct use of the material de-

veloped in the course of the valuation. For convenience, we shall consider a projection starting with 1950.

Throughout this section we shall use the following notations:

$(PR)_x$ = number of retired employees on the rolls for whom x equals 1949 minus the year of birth;

$(PB)_x$ = actual total yearly pension for the pensioners in the $(PR)_x$ group;

r_x = number of new immediate retirements at attained age x in a typical group of r new immediate retirements at all ages;

$r^{(n)}$ = estimated total number of immediate retirements in the calendar year $1949 + n$;

$B_r^{(n)}$ = estimated average annual pension for the $r^{(n)}$ new retirements;

w_x = number of active service withdrawals at attained age x eligible for a deferred pension in a typical group of w such withdrawals at all ages;

$w^{(n)}$ = estimated total number of withdrawals in calendar year $1949 + n$ eligible for a deferred pension;

$B_w^{(n)}$ = estimated average annual pension for the $w^{(n)}$ withdrawals;

l_x, d_x, etc. = usual functions from aggregate mortality tables which are considered appropriate.

The projection figures for the calendar year $1949 + n$ ($n = 1, 2, \ldots$) can then be obtained from the following formulae.

1. *Projection of Pensioners on the Rolls on December 31, 1949*

$$\text{Number of present pensioners surviving to the middle of the year } 1949 + n = \sum_x \frac{(PR)_x}{l_{x+1/2}} \cdot l_{x+n}. \tag{1}$$

$$\text{Disbursements during the year } 1949 + n \text{ with respect to present pensioners} = \sum_x \frac{(PB)_x}{l_{x+1/2}} \cdot l_{x+n}. \tag{2}$$

2. *Simple Projections*

If a projection of the simplest type is considered sufficient we can assume a single average age at retirement u, a single average age at withdrawal z, a constant number of new immediate retirements $r^{(0)}$ in each year, a constant number of withdrawals $w^{(0)}$ in each year, and constant average benefits $B_r^{(0)}$ and $B_w^{(0)}$, respectively. We will then have:

Number of pensioners in the middle of the year 1949 + n coming from immediate retirements after 1949 $= \frac{r^{(0)}}{l_{u+1/2}} \cdot \sum_{j=1}^{n-1} l_{u+1/2+j} + \frac{r^{(0)}}{2}$. (3)

Pension disbursements during the year 1949 + n with respect to immediate retirements after 1949 $= B_r^{(0)} \times \left(\begin{matrix}\text{Figure from} \\ \text{formula 3}\end{matrix}\right)$. (4)

Number of pensioners in the middle of the year 1949 + n coming from withdrawals after 1949 ($n > 65 - z$) $= \frac{w^{(0)}}{l_{z+1/2}} \cdot \sum_{j=1}^{n-66+z} l_{65+j+1/2} + \frac{w^{(0)}}{2} \cdot {}_{65-z-1/2}p_{z+1/2}$. (5)

Pension disbursements during the year 1949 + n with respect to deferred retirements after 1949 $= B_w^{(0)} \times \left(\begin{matrix}\text{Figure from} \\ \text{formula 5}\end{matrix}\right)$. (6)

It might be well to point out that the time series obtained from formulae (5) and (6) should be adjusted so as to have figures corresponding to calendar years 1951 to 1950 + 65 − z. The adjustment which is designed to offset the effect of using a single average age at withdrawal may be made at the expense of the figures originally computed for 1950 + 65 − z and, say, 10 or 15 subsequent years so as to obtain for the adjusted series the same totals as for the original ones. The adjusted columns should begin with relatively small figures and slowly increase for a selected period of time, and then level off. It is also necessary to watch out for consistency between the adjusted columns showing numbers and amounts.

3. *More Elaborate Projections*

If a more adequate projection is considered advisable, the work may proceed according to the following formulae:

(a) Projection of a typical group of immediate retirements in a single year, by duration t after retirement.

$$\text{Number in typical group} \equiv r = \sum_x r_x. \tag{7}$$

Number of pensioners surviving to the middle of duration t ($t \geqslant 2$) $\equiv R^{(t)} = \sum_x \frac{r_x}{l_{x+1/2}} \cdot l_{x+t-1/2}$. (8)

For the year of retirement, that is, $t = 1$, we shall use

$$R^{(1)} = \frac{r}{2}. \tag{8a}$$

(b) Projection of immediate retirements after 1949 based on the typical group r and on estimated numbers of new retirements $r^{(n)}$ in calendar year 1949 + n.

Number of pensioners in the middle of year 1949 + n coming from immediate retirements after 1949 $= \sum_{j=1}^{n-1} \frac{r^{(j)}}{r} \cdot R^{(n+1-j)} + \frac{r^{(n)}}{2}$. (9)

Disbursements during the year 1949 + n with respect to immediate retirements after 1949 $= \sum_{j=1}^{n-1} \frac{B_r^{(j)} \cdot r^{(j)}}{r} \cdot R^{(n+1-j)} + \frac{B_r^{(n)} \cdot r^{(n)}}{2}$. (9a)

(c) Projection of a typical group of withdrawals eligible for deferred pensions, by duration after withdrawal.

Number in typical group $\equiv w = \sum_x w_x$. (10)

Number of accessions in the kth year after withdrawal $\equiv W^{(k)} = w_{65-k} \cdot {}_{k-1/2}p_{65-k+1/2}$. (11)

Number of deferred pensions on the rolls exactly t years after withdrawal $\equiv (\mathrm{DR})^{(t)} = \frac{1}{l_{65}} \sum_{k=1}^{t} W^{(k)} \cdot l_{65-k+1/2+t}$. (12)

It might be mentioned that the step indicated by formula (11) could be omitted. We believe, though, that it is useful to have the accessions in a separate column, especially since very little extra work is involved.

(d) Projection of deferred retirements coming from withdrawals after 1949 based on the typical group w and on estimated numbers of withdrawals $w^{(n)}$ in calendar year 1949 + n.

Number of pensioners in the middle of year 1949 + n coming from withdrawals after 1949 $= \sum_{j=1}^{n-1} \frac{w^{(j)}}{w} \cdot (\mathrm{DR})^{(n-j)}$. (13)

Disbursements during the year 1949 + n with respect to deferred pensions coming from withdrawals after 1949 $= \sum_{j=1}^{n-1} B_w^{(j)} \cdot \frac{w^{(j)}}{w} \cdot (\mathrm{DR})^{(n-j)}$. (13a)

4. *Derivation of Ultimate Figures for Checking Purposes*

The checking of ultimate figures is particularly pertinent if the projection runs to the point at which maturity is assumed to be reached. The

first question is whether the ultimate number of pensions coming from future immediate retirements as developed in the projection agrees with the valuation. We shall develop the proposed checking method on the assumption that a single average age at entry was used in the valuation. The procedure can easily be adapted to a more elaborate valuation which considers several central ages at entry.

Let

m = the average age at entry;

l_x^a = the tabular number of employees in active service at age x;

r_x^a = the tabular number of immediate age retirements between the ages x and $x + 1$;

S_x^a = the yearly salary earned between ages x and $x + 1$; and

P = the ultimate annual payroll.

The constant number of new entrants in a year in the ultimate period can be computed from the formula

$$(\mathrm{NE}) = l_m^a \cdot \frac{P}{\sum_x l_{x+1/2}^a \cdot S_x^a}. \tag{14}$$

The ultimate number of pensioners on the rolls corresponding to l_m^a new entrants in a year is $\sum_x r_x^a \cdot \mathring{e}_{x+1/2}$. As related to (NE) new entrants, the ultimate number of pensioners coming from immediate age retirements becomes

$$\frac{(\mathrm{NE})}{l_m^a} \sum_x r_x^a \cdot \mathring{e}_{x+1/2} \equiv P \cdot \frac{\sum_x r_x^a \cdot \mathring{e}_{x+1/2}}{\sum_x l_{x+1/2}^a \cdot S_x^a}. \tag{15}$$

Similar checks can easily be developed for ultimate numbers of deferred pensions as well as for ultimate annual disbursements.

5. *Treatment of Discrepancies in Ultimate Figures*

When the ultimate number of immediate pensioners in the projection is too far off from that obtained by means of formula (15), we should re-examine our estimates of the numbers of new retirements in a year, that is, the $r^{(n)}$ numbers. It is true that the figure obtained from formula (15) cannot be considered more than a general indication of where the ultimate number of pensioners coming from immediate retirements should lie. It

is doubtful, though, whether there are other means by which a better estimate of the ultimate number of this type of pensioners can be obtained. We therefore think that too great discrepancies between the projection and the valuation with regard to ultimate figures should not be permitted. Perhaps the upper limit for such discrepancies should be set somewhere between 5 and 10 percent. An additional check should be based on comparing present values. For the projection as a whole, present values should agree rather closely with the valuation. Otherwise, the actuary may be called upon to explain the discrepancies. However, for particular sections of the projection which are not likely to be published, such an agreement need not be insisted upon. What are the limits for discrepancies between present values is again a matter of individual judgment.

6. *Projection of Widows' Benefits*

We now come to what is probably the most difficult type of a projection, that is, to a projection of survivor benefits. We shall limit ourselves to widows' annuities which become payable at a certain stated age, say, 65. We shall assume that such benefits are available to widows of employees who died in service as well as to widows of retired employees. The method by which the amount of benefit is computed is irrelevant to the present discussion.

For the purposes of such a projection, the following data should be available.

a) Age distributions of employees in active service, of retired employees on the rolls, and of typical groups of new retirements.
b) Percentages of employees dying at a given age who leave widows, with a breakdown between widows who are under 65 and those who are 65 or over.
c) The average age of the widows in each group as related to the employees' age at death.
d) A mortality table for widows or a combined mortality and remarriage table if widows' benefits terminate on remarriage as well as on death.

Family composition factors of the type described in paragraphs (*b*) and (*c*) above are available from various studies, including the studies described in Mr. Joseph Musher's paper which appears in this volume of the *Transactions* (Tables 8, 8a, and 9, pp. 28–29). The author believes that a breakdown of widows into two groups, namely under and over 65, is preferable to a single group with a single average age. It has been found that the use of a single average age practically always tends to overstate the liabilities. A projection might even be affected more than a cost calculation, since the point in time at which the younger widows first come on the benefit rolls is also important.

Data on mortality and remarriage of widows are also available. Here we might mention this writer's Revised American Remarriage Table which appeared in *RAIA* XXXVIII and a paper by Mr. Robert J. Myers in *PCAS*, November 1949. It goes without saying that reliable statistics developed from the operation of a particular pension plan are always preferable to data obtained from other sources. It is with this qualification that the references cited above are given.

There is no need to discuss the methods which are to be employed in a projection of aged widows' benefits in force. This is no different from a projection of pensions in force. It is the projection of future widows' benefits that causes some difficulties. Consider, first, deaths in active service. It will be convenient to introduce the following notations.

E_x = number of employees aged x in active service;

h_x = the percentage of employees dying between the ages x and $x + 1$ who leave widows under the age of 65;

g_x = percentage of employees dying between the ages x and $x + 1$ who leave widows aged 65 or over;

y_x = average age of the widows under 65 left by employees dying between x and $x + 1$;

z_x = average age of the widows aged 65 or over left by employees dying between x and $x + 1$;

l''_y = the l functions from a mortality and remarriage table applicable to widows.

It will be convenient to deal with age groupings instead of single ages. Considering, for instance, the group of employees aged 50 to 54, inclusive, we compute the following:

$$\text{Number of widows under age 65} = \sum_{x=50}^{54} E_x q_x h_x . \tag{16}$$

$$\text{Average age of these widows at the time of the employees' death} = \frac{\sum_{x=50}^{54} E_x q_x h_x y_x}{\sum_{x=50}^{54} E_x q_x h_x} . \tag{17}$$

For widows age 65 or over on the date of the employees' death, the formulae will be similar. In actual practice, for employees dying at ages younger than 65, the age incidence of widows over 65 may be disregarded, since

wives of such employees are not likely to be that old. It is only with respect to retired employees that the distinction between deferred and immediate widows' benefits becomes of some importance.

Formulae (16) and (17) provide a census of widows under age 65 left by employees who die in a single year. This census will be considered typical with regard to the age distribution of the widows. The number of widows may vary from year to year depending on the assumptions made with respect to the variations in the annual numbers of employees whose death will give rise to potential widows' benefits.

The projection for a typical group of widows under 65 will follow the same type of procedure as suggested for the projection of active service withdrawals. We first develop a column of accessions according to the duration after the employees' death. Because of the age groupings, the column of accessions will generally not be smooth. It is therefore advisable to first redistribute the accessions so as to retain the same total. After that the work will proceed by means of formulae similar to (12), (13), and (13a).

The projection of potential widows' benefits with respect to retired employees involves greater difficulties. Here, separate calculations have to be made for deaths occurring at particular durations after retirement, since the numbers and age distribution of deaths coming from closed groups of retired employees cannot be assumed constant. This part of the work is facilitated by the use of factors which combine the tabular number of deaths d_{x+t} with the percentage survived by widows h_{x+t} or g_{x+t} as the case may be. For example, when we consider new retirements at age 65, the number of widows age 65 or over who will be left by the pensioners dying at attained age $65 + t$ will be $(r_{65}/l_{65}) \cdot d_{65+t} \cdot g_{65+t}$, while the average age of these widows will be z_{65+t}. The actual work involves considerable detail which will not be discussed here, since widows' benefits are not found in the usual pension plan and are therefore of limited interest. It is believed that the reader who is interested in projections for survivor benefits should have no great difficulties in developing procedures which follow the general train of thought that runs throughout the whole paper.

PRESENTATION AND INTERPRETATION OF RESULTS

Once the basic time series of the projection are developed, the presentation of its results would not involve any particular difficulties. It should be remembered, however, that the usefulness of the projection will be greatly enhanced by a presentation which will best fit the needs at hand. This writer is of the opinion that a projection does not appear complete unless it also shows estimated numbers of beneficiaries on the rolls. It is there-

fore recommended that the set of projection tables, in whatever way they are arranged, include a separate table showing numbers of pensioners with a possible breakdown by type, such as present pensioners, future immediate retirements, future deferred retirements, and the like.

If the benefit provisions have already been settled, the main object of the projection, aside from showing the benefit curve, is to show the effect of different financing arrangements and to emphasize the importance of reserves. In such a case, the actuary might present only the following items:

a) Total annual disbursements.
b) Income other than interest with a breakdown between employee and employer contributions.
c) Income from interest on investments.
d) Balance at end of year.

A set of supplementary tables may then be prepared to show the effect of variations in the amount and timing of employer contributions.

Another set of tables will be presented in cases where the contributions have been determined but the benefit provisions are still open for discussion. Here, it may be worth while to present a breakdown of disbursements so as to show separately benefits to present pensioners, future immediate retirements, future deferred retirements, etc. Such a presentation would permit the lay executive to examine the effect of changing the benefits to a particular group or of a flat increase or decrease in all benefits. In addition to the disbursement columns, we would also show the annual income other than interest, income from interest on investments, and the reserves.

At this point, a few words may still be in order in connection with a joint presentation of the valuation and the projection. If the projection is presented in sufficient detail, the actuary will do well to point out the disbursement time series which correspond to particular present value figures of the valuation. The cost figures of the valuation, in whatever form they are expressed, may be translated in terms of required annual contributions under different possible schedules. Finally, the required contributions may be compared with those resulting from the actual schedule.

As far as estimating the effect of changes in the interest rate, a projection is the best basis for speedy calculations of that type. As a matter of fact, the projection will supply us with present values of benefits according to different interest rates, and also with corresponding present values of unit contributions if annual employment or payroll estimates are available. Since the interest rate which is to be assumed is most frequently

open to discussion, the actuary would do well to illustrate the progress of the fund under different interest assumptions. These supplementary projections should obviously be interpreted in terms of cost figures as shown by the valuation.

An interesting supplement would be a table showing the liabilities existing at particular points of time with respect to retirements and withdrawals which took place before that date. These can be calculated projectionwise by assigning in formulae (9a) and (13a) zeros for all average pensions with respect to exits taking place after the specified calendar year, and then computing present values corresponding to the modified disbursement series. These accrued liabilities could be compared with the reserves on the assumed termination date. By a similar method, accrued liabilities could be computed for employees who will be already on the pension rolls on the termination date.

Other interesting points can be brought out in connection with the handling of the past-service liability. Here again, several contribution schedules may be prepared to show the progress of the fund under different amortization schedules as contrasted with the frozen initial liability method. The amount of the past-service liability would have to be available from the valuation, since it would be rather difficult to develop it by projection methods.

An illustration of a strict pay-as-you-go method of financing is also very useful. Here, the table may show the following columns:

a) Total annual disbursements.
b) Total annual payroll.
c) Total man-hours paid for.
d) Total number of active employees.
e) Cost as a percentage of payroll.
f) Cost per man-hour.
g) Cost per employee.

The flow of the cost figures in columns (*e*), (*f*), and (*g*) will be quite a revelation to the layman and will help him to grasp the importance of advance funding.

Whatever method of presenting the projection is selected, the tables should be accompanied by a brief explanatory note outlining the questions which the tables are supposed to answer and the way in which the projection figures should be interpreted. There is one danger in presenting projections which should not be overlooked. Unless properly qualified, the projection will be looked upon as a forecast and the actuary will be called to task if actual experience does not conform with the projection. It is therefore necessary that the projection be described as a useful illustra-

tion which merely indicates the progress of the fund over a long period of time without any pretense to accuracy of particular figures.

It is believed that each new valuation would not require the preparation of a new projection, so long as the benefit provisions of the plan remain the same. The projection already available can generally be adjusted by some rough and ready method which would not require much work. In some cases, it will be possible to make adjustments even for changes in the benefit provisions themselves. This "durability" is another argument in favor of undertaking the job of preparing a reasonably good projection.

In concluding, the author would like to point out that he is not underestimating the difficult problems of a nonactuarial nature which are involved in the preparation of a reasonably good projection or valuation. Problems of that type involve estimates of payrolls, employment, wage trends, and the like. The very nature of these factors would seem to indicate that the actuary might well consult with specialists in the appropriate fields. Perhaps in the not too distant future another paper will be presented before this Society which will discuss these special aspects of a projection.

APPENDIX

The paper contains several formulae which do not give a clear indication of how best to arrange the computational work. This is particularly true with respect to formulae (9), (9a), (12), (13), and (13a). Since (9) and (9a) are of the same type as (13) and (13a), we shall limit ourselves to an illustration covering deferred retirements which are discussed in the text in conjunction with formulae (10) to (13a). The first part of the worksheet would be entitled:

PROJECTION OF A TYPICAL GROUP OF DEFERRED RETIREMENTS COMING FROM WITHDRAWALS IN A SINGLE YEAR

Age at Withdrawal x	Number Withdrawing at Age x	$_{65-x-\frac{1}{2}}p_{x+\frac{1}{2}}$ from 1937 S.A.	Duration after Withdrawal t	Number of Accessions at Duration t $W^{(t)}=(2)\times(3)$	Cumulative Number of Pensioners at Duration t, $(DR)^{(t)}$ of Formula 12
(1)	(2)	(3)	(4)	(5)	(6)
64	100	.9865	1	98.6	97.2
63	125	.9611	2	120.1	212.7
62	150	.9382	3	140.7	344.8
61	175	.9175	4	160.6	492.1
60	200	.8986	5	179.7	653.0
	750			699.7	

For purposes of column 6, it is best to prepare a strip of $l_{x+1/2}/l_{65}$ running as follows:

x	$l_{x+1/2} \div l_{65}$
. . . . .	.
. . . . .	.
. . . . .	.
69	.8600
68	.8934
67	.9254
66	.9562
65	.9856

Thus, for instance,

$$344.8 = 98.6 \times .9254 + 120.1 \times .9562 + 140.7 \times .9856 .$$

The arrangement of the strip in reverse order permits proper matching for purposes of cumulative multiplication.

The second part of the worksheet would be entitled:

NUMBERS OF DEFERRED PENSIONS ON THE ROLLS AND AMOUNT OF DISBURSEMENTS BY CALENDAR YEAR

Calendar Year $1949 + n$ (7)	Number of Withdrawals in the Year $w^{(n)}$ (8)	Withdrawal Index (8) ÷ Total of (2) (9)	Average Pension for the Year $B_w^{(n)}$ (10)	Number of Pensions in Middle of Year (11)	Disbursements during Year (thousands) (12)
1950	800	1.067	$400	. . . . .	. . . . .
1951	750	1.000	425	104	$ 42
1952	825	1.100	450	324	132
1953	700	.933	475	688	286
1954	650	.867	475	1,195	505

Here again, we make use of two reverse strips as shown below:

Calendar Year	(a) Factor for Column 11	(b) Factor for Column 12
.	.	.
.	.	.
.	.	.
1954	.867	412
1953	.933	443
1952	1.100	495
1951	1.000	425
1950	1.067	427

Strip (a) comes from column 9, while strip (b) contains the products (9) × (10). Both strips are applied against column 6. Thus, for instance,

$$688 \text{ (column 11)} = 97.2 \times 1.100 + 212.7 \times 1.000 + 344.8 \times 1.067 .$$

Similarly,

$$286 \text{ (column 12)} = 97.2 \times 495 + 212.7 \times 425 + 344.8 \times 427 .$$

The techniques can easily be explained to the computers and the operations proceed rapidly. It is recommended that figures involving cumulative multiplications which are of steadily increasing length be computed at every fifth year only. It is not necessary to have yearly figures for each group of beneficiaries separately. Only when the several components of the projection are brought together are year by year figures required, particularly for charting the progress of the reserves. The missing disbursement and beneficiary figures can be obtained by interpolation. In order to obtain smoother final time series, it may be advisable to use a modified formula such as Jenkins' five-point formula. Coefficients for this formula in linear compound form, as well as for several other commonly used formulae, are given by John Boyer in "Osculatory Interpolation in Practice," which appeared in *RAIA* XXXI.

DISCUSSION OF PRECEDING PAPER

ROBERT J. MYERS:

The first sentence of Mr. Niessen's paper in my opinion presents a major problem facing all pension actuaries, namely, making the results understandable to nonactuaries. I am afraid that in actuality most laymen look upon our cost reports with complete mystification and perhaps even skepticism; in fact, too often is heard the comment that the figures quoted are "only 'actuarial' costs and do not represent real costs" because they are so high in relation to the benefits that are actually going to be paid in the next year or so. It seems to me that such criticism would be greatly lessened if we were to follow Mr. Niessen's suggestion of at least having an auxiliary projection in combination with the usual type of pension cost presentation.

Not only has Mr. Niessen very well brought out the problem involved, but he has indicated how to solve it both from a presentation standpoint and also as far as the technical procedures are involved. As he indicates, if the usual balance sheet valuation is made, then the projection should tie in as closely as possible, making use of suitable approximations and short cuts within the limits of available time.

It would seem particularly appropriate nowadays that a projection type cost estimate should be available. It is my understanding that for some of the major pension plans recently adopted by large industries the cost presentation has swung to a picayune statement of so many cents per hour without any indication of just what this means. In this respect I might call your attention to a very interesting article by Mr. Edward L. Schwartz, entitled, "Employer Initiative in Pension Programs" (*Harvard Business Review*, May 1950). There it is recommended that cost implications should be given a very thorough study and that there should not be considered merely a single figure of so many cents per hour. As Mr. Schwartz states, "The emphasis should rather be on the growth of the pension benefit roll," and he then goes on to argue vigorously for the presentation of the other statistics usually shown in projection type estimates.

I think, and hope, that Mr. Niessen's excellent paper will tend to awaken actuaries to the desirability, and even the necessity, of producing and presenting a better product for those nonactuaries who are interested in and concerned with pension plans of all sorts. At any rate, it should be

of considerable interest and value to all actuaries and especially to students who are concerned with pension fund calculations, even though not intimately connected therewith. Further, the nontechnical portions of the paper, when read by laymen interested in pension plans, will certainly whet their appetites for this type of cost presentation. Moreover, Mr. Niessen's paper will possibly strengthen the position of those who favor the projection method as the fundamental one because of its indicated versatility, flexibility, and general usefulness.

DORRANCE C. BRONSON:

In my paper on Pensions last fall, I mentioned briefly my interest in the matter of projections and their growing importance. Now, only a year later, we are indebted to Mr. Niessen for a whole paper on the subject.

In discussing his work let me first say that I have not had time to delve into his procedural formulae. Offhand, however, I would have a preference for assigning values and rates which are as consonant with those of the valuation as possible, making any practical adjustments empirically. For instance, a conservative valuation might assume all retirements to occur at age 65, as expressing the "commitment" of the pension plan (and also giving *pro forma* allowance for "early retirement" cases). Actually, however, a projection which "retired" everyone reaching that age during the next few years would be patently contrary to the probabilities in the existing milieu of high employment. Therefore, an adjustment on a judgment basis could be introduced. As the pension roll grew, the relative effect of delay in retirement would grow less and less on the total pension roll in the projection, but, even so, to regain the conservatism of the valuation, the assumption for later years could again adopt age 65 for purposes of the projection.

Mr. Niessen does not mention any distinction of the actuary's attitudes and problems between those pertaining to large systems and those for small plans. Obviously, the latter are so troublesome and would be so unmeaningful to Mr. Niessen's "layman"—involving, as they frequently do, a fractional death or separation, or a part of a person retiring (which actually may not be without merit)—that they are best avoided. Another nice distinction not mentioned is a projection under a "going system"—such as his Railroad Retirement—and one, *ab initio*, for a new case. Under the former, the projection can at least look at what *has* happened and permit the advantages of extrapolation. Under the latter, one lacks entirely this assistance, and even if terminations of service before 65 and terminations, presumably for age, after 65 are statistically available,

how much good are they in the light of no pension plan having then been in force compared to the conditioning influence of having a plan in force hereafter? Most projections of time series—economic, biologic, insurance, etc.—have a nexus with the past but this is not true for a new pension plan.

It is interesting that Mr. Niessen's layman is still disturbed by the large reserve concept and by the past service liability of the usual valuations. I think this used to be more true than is the case today. I have, on occasion in the past, only with some trepidation and soothing words, introduced the so-called "accrued liability" to an employer. I got rather more callous, however, after a couple of instances where we had merged "past service" in with "future" for the purpose of level funding to retirement age. In these instances the employer was perplexed and hurt that we had not brought out a big shiny accrued liability for his Board to admire. To perform the actuarial duties apparently expected of us, we went back and produced an alternate with the desired feature.

In the two preceding paragraphs I have spoken of the "layman" from my hastily drawn concept of this ubiquitous character. It was not until hearing Mr. Niessen's preface to his paper that it suddenly occurred to me that Mr. Niessen's "layman" was not my "layman." This is a very interesting example of how easy it is to run off the track of the other fellow's trend of thinking. I have been thinking of a layman who is an employer, or a Board of Directors, or a trustee of a private pension plan. It appears that Mr. Niessen, in his paper, was thinking of the layman as the employee, or an employee group, or the labor leader, or the politician or the legislator. When one goes back and rereads the applicable portions of Mr. Niessen's paper with his concept of "layman" in mind, the connotations are very interestingly different—being focused on size of *benefit*—from those of the reader who has in mind the other type of "layman," for whom the focus is on *costs*. A little clarification of definition—as is usually the case in pension subjects—might have helped in understanding the references to "layman" in Mr. Niessen's paper.

In the matter of single versus dual (or multiple) projections, I agree with Mr. Niessen from the practical standpoint that for industrial plans a single projection usually has to suffice. As mentioned earlier, I think this should tie in pretty well to the valuation. While, theoretically, each of a "family" of projections could tie in to the valuation, the one I prefer would be that which develops reasonably closely to the valuation assumptions. But I agree with Mr. Myers, whom Mr. Niessen liberally quotes, that a "range" is more honest when we have the time for it and the right

audience. And this last is very important; many employers or employee groups (or legislators) want *the* answer and are impatient with multiple tables. They are willing, initially at least, to dispense with the actuary in his fallible role. In large public plans—OASI, Railroad Retirement, Civil Service—I believe the honesty of the range techniques should be revealed and I am sorry that, as far as I know, it has only been followed as yet by the first of these, OASI. It would be interesting and enlightening to see the penumbra, especially on the "up" side, of costs for the other two systems.

Concerning the range method, Mr. Niessen does concede the advisability of projecting the *fund* at two or more interest rates. I have noticed this particularism before—indeed, I have indulged in it myself. Why is it we pick on the interest rate? Mr. Myers' paper, already referred to, illustrated a shift in interest rate by a 19% effect on costs. But his mortality effect was 30% and his pay scale incidence brought 31%. Our concern with the interest rate, it seems to me, must lie in the psychology that here is something tangible that will respond to our managerial perspicacity. We can't direct mortality this way or that, or salary scales, or withdrawal rates. But, somehow, the feeling is that if you will only show the investors —insurance company or trustee—of a pension fund what can be done with 3% interest as against 2½%, they will be enabled to transmute the knowledge into action or assurance.

I had a few quibbles with Mr. Niessen's paper. He seems somewhat inconsistent in spots. He says, for instance, that a projection should be prepared with great care and should be realistic, with a reliability comparable to that of the valuation itself. These—realistic and reliable—are rather vague and overextended criteria, it seems to me. In another instance, he states that a projection is only good for a short time and yet elsewhere he claims the projection rests greatly on the soundness of the general assumptions, and toward the close of the paper, the projection has become a very "durable" entity.

My general feeling is that Mr. Niessen envisions somewhat more in the projection potentialities than is justified, short of refinements of highly fascinating academic vicissitudes. When he suggests bringing in the economists and government specialists to further refine the "realism" of our projections, I question whether they will gain in consistency or meaningfulness. The first problem will be to find two of these gentlemen with the same theories. I suggest that the author give up that project and solve

it very simply—pick a high cost set of assumptions and a low cost set of assumptions; in short, the "range" concept will do a better job.

I, for one, am very glad that a paper on this subject has been presented and congratulate the author on his work. He even has been courageous enough to get into the subject of widows' and orphans' benefit projections which has been rather a "pseudo-science" in my experience.

ROBERT F. LINK:

I was personally greatly delighted to see the title of Mr. Niessen's paper, and I was even more pleased to read the paper itself. Pensions have been with us for a long time, but seem lately to be undergoing a rather sudden increase in importance which has not been matched in our literature by expositions of theory and technique adapted to the problems which we face today (which are not entirely the same as those faced by George King and his contemporaries). Those of us who are come lately to the pension field feel a real lack of specific information as to modern actuarial techniques in the field of valuation, projection and presentation. This paper breaks important ground in this field. I hope it and other recent papers such as Mr. Bronson's and Mr. Noback's herald a series which will bring our literature up to date. The techniques presented, and even more the general comments in the text, should provide valuable guides to correct thinking and procedure in the problem of projections.

Mr. Niessen's techniques are adapted to a presentation in which the objective is to relate valuation figures in a general way to a projection of benefit payments, showing whether a given scale of contributions is adequate or not, and what a change in the proposed scale of contributions would imply with respect to the solvency of a fund. It is possible to pose more exacting questions, and such questions do come up in practice. For example, suppose a valuation indicates a certain past service liability and normal cost, and suppose further that the assumptions of this valuation contain a certain margin of conservatism, and that the actual experience is expected to be more favorable than that assumed. When, and in what amounts, will gains emerge? If our conservative assumptions are followed in making payments, what would the actual payments into the fund turn out to be over the course of a number of years? In order to give an answer to this question which makes actuarial sense, it is necessary to make a projection of benefits which follows the assumptions as to actual experience quite slavishly. Otherwise, gains or losses will appear which have nothing at all to do with the margins of conservatism in the valuation. Or another example: it is desired to analyze the operation of a group

annuity dividend formula, showing the incidence of emergence of margins under certain assumptions as to actual experience, to determine whether the dividends emerge too quickly or have any other undesirable characteristics. Here there must be a thoroughly mathematical correspondence between payments and the reserves and benefits arising from them. No leeway at all can be allowed. Mr. Niessen's suggestion of redistributing the retirement benefits with respect to vested withdrawals would be unacceptable in these cases.

When projections require this sort of internal consistency, there is a tendency to pare them to the bone—to make them as simple as possible because they must satisfy more requirements than that of merely predicting some benefits. If a simple stationary population is made the tool for performing a projection, it is amenable to some rather nice mathematical manipulations which ease the work of computing four or five different kinds of benefits (payments at death or withdrawal, pension payments, employer withdrawal credits) and reserve or liability figures for many future years. Simplicity is an aid to seeing what you are doing and spotting sources of distortion in the final results. Even with a single average entry age, an average age for deaths, another for withdrawals and another for retirements, one can juggle things around so as to come fairly close to the financial characteristics of a particular group. The most likely difference is an excess of retirements in the early years in the projection.

This subject really requires not a brief paper and discussion, but a book. I am appending an outline of a projection problem which illustrates the mathematical advantages of a simple population in projection work, as a subchapter to such a book.

Illustrative Projection

A group is negotiating for a group annuity contract. Preliminary cost estimates have been made, but the group has been told that they may expect, in the absence of any significant change in the factors affecting cost (such as number and age distribution of lives and salary levels), that the actual cost will be lower than that indicated by the estimates. Employer withdrawal credits will certainly emerge as an offset, and dividends should also emerge. It is impossible to predict the amounts of these elements, but the group would like a projection which would illustrate, in principle, their probable incidence and size under certain assumptions as to future experience.

The proposed plan is a contributory unit plan with both past and

future service. The future service unit is a certain percentage of current salary for each year of service. The past service unit is computed in the same way as future service, but using a salary current at the register date of the contract and a somewhat lower percentage basis. Employee contributions are a constant multiple of the future service unit. The employer contributes an additional amount with respect to each employee each year, such amount being sufficient to purchase that part of the future service unit not purchased by the employee contribution. The employer also buys all past service annuities. The normal annuity form is a life annuity, and normal retirement age is 65.

If an employee dies, there is a return of all of his contributions, accumulated at 2% interest. There is no return to the employer in this case. If an employee withdraws, there is a return to the employee equal to the return in case of death. In this case there is also a return to the employer of employer contributions accumulated at 2% interest, this amount being reduced by 4% of the total return to both the employer and the employee. The return to the employer is contingent upon the furnishing of evidence of good health.

In order to project the cost to the employer, and the reductions due to dividends and withdrawal credits, it is necessary to project the following items: annuity payments, employee death returns, employee withdrawal returns, employer withdrawal credits, expenses, gross purchase payments by employer and employee, and the actuarial reserve under the contract, for each year involved in the projection. The report to the employer will then include an exhibit showing gross purchase payments and the offsetting items of dividends and withdrawal credits. Dividends are paid out of the margins arising from the realized investment rate, actual mortality experience, excess of loading over actual expenses, excess of reserve released over withdrawal returns or credits. In calculation, dividends are some function of the asset share under the contract, the reserve under the contract, and contingency reserves. It is clear that, in order to present a sensible picture of dividends, the projection must be internally consistent as to all actuarial assumptions. It should also be reasonably consistent with the general appearance of the group.

A very crude approach to this problem which may be satisfactory for this purpose is to construct a stationary active life population which reproduces approximately the average entry age, the average premium per dollar of deferred annuity, the number of deaths expected and the number of withdrawals, as well as their average duration. This can be done surprisingly well with a stationary population which involves only five vari-

ables (other than the total number of lives, which equals the number in the actual group): an entry age, an age at withdrawal, an age at death, a number of annual withdrawals, and a number of annual deaths. Since it will usually not be true that the group actually is a stationary population, and since the difference is mainly a reflection of past growth, this arrangement will probably overstate the number retiring in early years as compared to the number actually expected to retire, but this is not a serious defect.

For retired lives, one merely constructs a continuation of the stationary population, based on the desired mortality assumption, and rounded to integral numbers of lives.

Now an average salary is determined which reproduces the total expected employee cost under the plan. The employer cost is then automatically reproduced (as a result of constructing the stationary population so as to reproduce the average premium per dollar of deferred annuity). Parenthetically, it might be remarked that the use of a flat salary assumption understates retirement benefits in early years, which is an offsetting error to one mentioned above. This assumption has the advantage of producing a future service unit and a past service unit which are constant dollar amounts per life.

Finally, a past service formula is determined which is consistent with that under the contract and reproduces the past service liability. A modification of the maximum number of years or minimum age for which past service will be credited is in order here, since, in the absence of such a modification, the past service liability might be inflated, due to recent growth of the group. This step sounds messy, but it only takes about fifteen minutes if properly arranged.

A few miscellaneous assumptions should be noted. The contract is effective on a January 1. Future service premiums are assumed to be paid each January 1. All past service annuities are purchased in full on January 1 of the first year. Retirements occur on January 1, and the first retirements under the plan occur as of January 1 of the *second* year. Employee birthdays are all January 1. Entry into the plan occurs on January 1. Deaths and withdrawals occur on July 1.

We are now ready to proceed with the projection. For each quantity to be projected, an initial formula (IF) is written down which expresses the requirements of the problem. This initial formula is then transformed into a working formula (WF) which is mathematically equivalent but expresses the method of calculation. The working formula is made the basis for a calculation sheet.

Initial and working formulas are presented for all items required in the projection except expenses, which are a rather individual proposition.

t = number of full years contract in effect as of next anniversary.
x = attained age.
y = entry age.
z = retirement age.
w = age on January 1 of year of withdrawal.
u = age on January 1 of year of death.
p = age below which no past service is credited.
m = maximum number of years of credited past service.
l_x^a = number of active lives at age x.
l_x^r = number of retired lives at age x.
W_w = number of withdrawals in a year.
d_u = number of deaths in a year.
V_x^{db} = reserve at age x for a death benefit of 1 plus interest at 2% from age x to age at death; benefit payable only at death before age z.
V_x^{da} = reserve at age x for a monthly deferred annuity of 1 per year, first payment on attainment of age z.
V_x^r = reserve at age x with respect to a monthly annuity of 1 per year currently payable to a retired life.
${}_rP_x$ = gross premium payable by the employer which, together with an employee premium of r, will purchase a deferred monthly annuity of 1 per year, first payment at attainment of age z.
P_x = gross premium payable by the employer which will purchase a deferred monthly annuity of 1 per year, first payment at attainment of age z.
A = future service annual income credited for one year of service.
B = past service annual income credited for one year of service.
r = ratio of employee contribution to future service unit.

Annual Future Service Cost

Employee:

$$ {}^{fs}C^{ee} = r \cdot A \cdot \sum_{x=y}^{z-1} l_x^a. \qquad \text{(IF and WF)} $$

Employer:

$$ {}^{fs}C^{er} = A \cdot \sum_{x=y}^{z-1} {}_rP_x \cdot l_x^a. \qquad \text{(IF and WF)} $$

Past Service Single Sum Cost at Issue

$$ {}^{ps}C^{er} = B \cdot \sum_{x=p+1}^{z-1} [(x-p) \dot{-} m] \cdot P_x \cdot l_x^s . \quad \text{(IF and WF)} $$

Employee Withdrawal and Death Benefits

Withdrawal:

$$ WB_t = W_w \cdot A \cdot r \cdot s_{\overline{t \dot{-} (w-y+1)|}}^{.02} \cdot (1.02)^{1/2} . \quad \text{(IF and WF)} $$

Death:

$$ DB_t = d_u \cdot A \cdot r \cdot s_{\overline{t \dot{-} (u-y+1)|}}^{.02} \cdot (1.02)^{1/2} . \quad \text{(IF and WF)} $$

Employer Withdrawal Credit

$$ \begin{aligned} WC_t = .96 W_w \Big\{ & A \cdot \sum_{k=1}^{t \dot{-} w-y+1} {}_rP_{w+1-k} \cdot (1.02)^{k-1/2} \\ & + [0 \dot{-} (w-p-t+1) \dot{-} m] \cdot B \cdot P_{w+1-t} \cdot (1.02)^{t-1/2} \Big\} \\ & - .04 WB_t . \quad \text{(IF and WF)} \end{aligned} $$

Reserve at End of Year for Deferred Annuities

Future Service:

$$ {}^{fs}R_t^{da} = A \cdot \sum_{x=y+1}^{z} [(x-y) \dot{-} t] \cdot V_x^{da} \cdot l_x^s . \quad \text{(IF)} $$

$$ {}^{fs}R_t^{da} = A \cdot \sum_{k=y+1}^{y+t} \sum_{x=z}^{k} V_x^{da} \cdot l_x^s \qquad \text{for } t \dot{-} z-y $$

and is constant for years $z-y$ and later. (WF)

Past Service:

$$ {}^{ps}R_t^{da} = B \cdot \sum_{x=p+t+1}^{z} [0 \dot{-} (x-p-t) \dot{-} m] \cdot V_x^{da} \cdot l_x^s . \quad \text{(IF)} $$

$$ \begin{aligned} {}^{ps}R_t^{da} &= B \cdot \Big\{ \sum_{k=z}^{p+t+1} \sum_{x=z}^{k} V_x^{da} \cdot l_x^s - \sum_{k=z}^{p+t+m+1} \sum_{x=z}^{k} V_x^{da} \cdot l_x^s \Big\} \quad \text{for } t \leqslant z-p-m-1 \\ &= B \cdot \sum_{k=z}^{p+t+1} \sum_{x=z}^{k} V_x^{da} \cdot l_x^s \qquad \text{for } z-p-m-1 < t \leqslant z-p-1 \\ &= 0 \qquad \text{for } t > z-p-1 . \quad \text{(WF)} \end{aligned} $$

Reserve at End of Year for Employee Death Benefit

$$R_t^{db} = r \cdot A \cdot \sum_{x=y+1}^{z} V_x^{db} \cdot l_x^s \cdot \ddot{s}_{\overline{(x-y) \dot{-} t}|}^{.02}. \qquad \text{(IF)}$$

$$R_t^{db} = r \cdot A \cdot \sum_{k=y+1}^{y+t} \left\{ (1.02)^{k-y} \cdot \sum_{x=z}^{k} V_x^{db} \cdot l_x^s \right\} \qquad \text{for } t \dot{>} z-y$$

and is constant for years $z-y$ and later. (WF)

Income for Retired Lives

Annual income to one life retiring in year t:

$$I_t^{(1)} = [(t-1) \dot{>} (z-y)] \cdot A + [0 \dot{>} (z-p-t+1) \dot{>} m] \cdot B$$

except that $I_1^{(1)} = 0$.

Total future service income paid during year t:

$$I_t^{fs} = A \cdot \sum_{k=2}^{t} [(k-1) \dot{>} (z-y)] \cdot l_{z+t-k+1/2}^r. \qquad \text{(IF)}$$

$$I_t^{fs} = 0 \qquad \text{for } t = 1$$

$$= A \cdot \sum_{k=1}^{t-1} \sum_{h=1}^{k} l_{z+h-1/2}^r \qquad \text{for } 1 < t \leqslant z-y+1$$

$$= A \cdot \left\{ \sum_{k=1}^{t-1} \sum_{h=1}^{k} l_{z+h-1/2} - \sum_{k=1}^{t-(z-y)-1} \sum_{h=1}^{k} l_{z+h-1/2} \right\}$$

$$\text{for } t > z-y+1. \qquad \text{(WF)}$$

Past service income paid during year t:

$$I_t^{ps} = B \cdot \sum_{k=2}^{t} [0 \dot{>} (z-p-k+1) \dot{>} m] \, l_{z+t-k+1/2}^r. \qquad \text{(IF)}$$

$$I_t^{ps} = 0 \qquad \text{for } t = 1$$

$$= B \cdot m \cdot \sum_{k=1}^{t-1} l_{z+k-1/2}^r \qquad \text{for } 1 < t \leqslant z-p-m+1$$

$$= B \cdot \left\{ m \cdot \sum_{k=1}^{t-1} l_{z+k-1/2}^r - \sum_{k=1}^{t-(z-p-m)-1} \sum_{h=1}^{k} l_{z+h-1/2}^r \right\}$$

$$\text{for } z-p-m+1 < t \leqslant z-p+1$$

$$= B \cdot \left\{ m \cdot \sum_{k=1}^{t-1} l_{z+k-1/2}^r - \sum_{k=1}^{t-(z-p-m)-1} \sum_{h=1}^{k} l_{z+h-1/2}^r + \sum_{k=1}^{t-(z-p)-1} \sum_{h=1}^{k} l_{z+h-1/2}^r \right\} \qquad \text{for } t > z-p+1. \qquad \text{(WF)}$$

Reserve at End of Year t for Retired Lives

The method for reserve follows that shown above for income, except that $l^r_{x+1/2}$ is replaced by $l^r_{x+1} \cdot V^r_{x+1}$.

Once the working formulas are derived, the rest of the work is easy, or at least straightforward. Double summation signs have a tiresome look, but the work involved in evaluating them is quite simple. The two work sheets for retired lives are by far the worst. Headings for the income sheet are shown as an illustration (column (6) is future service, and column (11) is past service):

	(1)	(2)	(3)	(4)	(5)	(6)
t	$l^r_{x+t-1/2}$	$\sum_1^t (1)$	$\sum_1^t (2)$	$(3)_{t-(z-y)}$	$(3)-(4)$	$A \cdot (5)_{t-1}$

(7)	(8)	(9)	(10)	(11)	(12)
$m \cdot (2)$	$(3)_{t-(z-p-m)}$	$(3)_{t-(z-p)}$	$(7)-(8)+(9)$	$B \cdot (10)_{t-1}$	$(6)+(11)$

Note that nine out of twelve columns involve only copying, adding or subtracting.

Comments on the Projection

First, it might be desirable to recapitulate, in general terms, what the method is. In essence, two major steps are involved. One recasts the problem into an equivalent one which contains certain mathematical properties (such as that of the stationary population). Then one utilizes these mathematical properties to transform rather extensive calculations into simpler ones. The principal device is that of obtaining summation formulas in place of formulas involving an array of multiplications. The illustration is rather specialized, being a typical group annuity plan, but I believe that the approach is general and can be adapted to other types of plans and funding methods. Once the first step has been completed (that of recasting the problem in mathematical terms), one has forged the tool and can use it for a variety of purposes.

The formulas given above do not define the method, but merely illustrate it. They should be derived afresh to meet the conditions of each new problem. This is not as difficult as it might seem. My own method is to draw diagrams (the actuarial equivalent of counting on one's fingers) rather than to use mathematics. In some cases, the working formula need not be written down at all; reasoning carries one directly from the initial formula to the work sheet.

Although it is not explicitly shown in the illustration for all cases, each working formula ultimately reaches a constant value. The value of t at

which this constant value is reached is easily determined (for example, the reserve for active lives becomes constant when $t = z-y$). This constant value can be determined independently from the initial formula, and furnishes a convenient extra check on the correctness of the working formula and resulting arithmetic.

The constant value represents the ultimate situation under the contract; if the projection is to run for a relatively short period, it may be desirable to use initial formulas to show what the ultimate situation would be.

If the assumption of a single age for deaths and another for withdrawals is regarded as too crude, it is possible to use decrement rates in deriving a more realistic population in which these decrements occur at a number of ages. The formulas for death and withdrawal benefits become somewhat more complicated but still follow the same basic principles.

The treatment of past service in the illustration is rather unrealistic (normally, it would be paid over a period of years, and this makes considerable difference in the dividends). However, past service tends to be tricky, whatever the method of the projection. I believe something better could be worked out.

The reserves involved in this projection depart from standard valuation procedure in that no reserve is held with respect to the excess of surrender value over true net premium reserve on employee premiums. If this were important in a particular projection, it could be handled by an approximation; or it might be handled very nicely by another working formula.

The treatment of vested withdrawals under this method is somewhat weak; in essence, it is assumed that there are no withdrawals beyond the vesting age. This may tend to overstate the proportion of higher age employees accruing benefits under the plan. Mr. Niessen's method is much superior in this respect.

The mathematical method is not well suited to answering the employer's question, "What amounts are likely to be paid out in the next twenty years to retired lives?" In this case, the projection should be based on the actual age and salary distribution of the group in question, without any modification. Mr. Niessen's method does this very nicely.

One must conclude that the preparation of projections is a highly individual business, depending on the person doing the projection, the type of problem, the amount of clerical help available and other factors.

W. RULON WILLIAMSON:

Mr. Niessen's most interesting paper comes at a time when almost everyone is taking a shot at pension discussion. I am talking with people

with and without actuarial training a good deal, and recently I made a couple of cut-outs to illustrate some of this projection business.

One of them was essentially a stationary population table, and the other was the q_x curve. Obviously the stationary population table curves down to zero at 100 or 110 or 120, and the q_x curve starts high at zero, plummets down, hits low at age 10, goes pretty well across the page at a gentle slope and then rides high at the right-hand edge. In using the curve I start at the low point, and as the curve approaches the end it takes a lot of room.

The population table shows the confines of the business for an Ordinary Life contract. The exposure drops down over the years, the important ones in scope coming before attained age 70. In most exposures there is a reducing amount at risk per capita, and the financial situation is relatively well in hand in the life insurance company using the contract.

In the Government-sponsored annuity programs, the exponential type of curve shows the growing financial importance of the remote years. The first fifty years do not much matter when your thoughts are on infinity.

The knowledge of the present is most significant in the Ordinary Life business; it has little to do with the government concern with way out yonder.

A narrow contingency margin is valid for the first; there is no evidence how wide a contingency margin would be needed in underwriting the second.

In private business the solvency of the insurer is of the essence; in public business the insurer is the Government—the whole body of taxpayers.

In Ordinary Life it is assumed that the policyholder dislikes increasing costs and would rather pay more now and less later. In the public plans, it is common knowledge that no one wants to pay anything, but never as much as would be wise.

In Ordinary Life the policyholder expects to pay all his share of the costs. In any contributory plan the idea of paying one's share is "a laugh." So far, the OASI old age beneficiaries have perhaps paid 2% of their prospective benefits, and a large category of the new start people can expect the chance to get a 200 to 1 return. In Federal Civil Service in 1948 some 100,000 beneficiaries were reported to have paid 11% of the cost of their pension.

With this slender participation in meeting the costs, it is interesting to me that the workers—a very Junior Partner indeed—should care much for the single "right projection." It is obvious that there *is* no single right projection.

An employer watches his budget, an insurance company wants to stay

solvent, a state insurance department tries to protect the policyholder for his own good, of course. The welfare state lacks these checks and balances. The Bureau of the Budget acts episodically here. The citizens rarely get much concerned—and when they do there is a pretty well-oiled hearing machinery so that their comments are viewed through the wrong end of the telescope, and the right testimony through the right end. Some witnesses are heard after decisions have already been reached. Two widely different projections are hard to rationalize. The record looks better with one. I believe only one projection inadvisable under these conditions.

HERBERT J. STARK:

Mr. Niessen has prepared an interesting and helpful analysis of the part which projections may play in the efforts of actuarial consultants to educate employers and employee representatives in the principles of financing of their pension plans. There is no doubt that projections can be extremely useful in this education. Projections clearly depict the steady long-term increase in pension outlays, and they can show strikingly the effect of advance funding in reducing the contributions necessary after the plan has become mature below the outlays which then would be required on a pay-as-you-go basis. Mr. Niessen ingeniously points out the ways in which much additional clarification can be secured by the use of projections and adjustments in them to take into account variations in the funding practices and changes in valuation assumptions which may be under consideration.

However, near the close of his paper, Mr. Niessen finds it necessary to point out the danger that "the projection will be looked upon as a forecast" and that "the actuary will be called to task if actual experience does not conform with the projection." I should like to discuss some of the reasons why, in my opinion, it is difficult to prepare a projection which is also a successful forecast. I wish also to indicate a method of producing a forecast which may be found helpful in certain cases.

The usual service table treats each rate of decrement as a function of the attained age of the employee, and may be considered as having been prepared on an aggregate basis with respect to the duration of employment of the employees. The formulae in Mr. Niessen's paper imply the use of a service table of this type. Actually, such studies as we have made indicate clearly that length of service plays a major part in determining some of the rates of decrement involved. In particular, our studies indicate that the rates of withdrawal from service (otherwise than by retirement or death) are to a far greater extent a function of the length of service of the employee than a function of his age as such.

If all employees entered the service of the employer at the same age, their attained ages would of course exactly correspond with their lengths of service. If it can be assumed that ages of entry into service are concentrated in a narrow age grouping, it is likewise satisfactory to use a service table which does not reflect variations in length of service at each attained age. I like to speculate that our actuaries tend to use a service table of this type largely because pension theory happened to be developed in Great Britain during the quiet and stable period preceding the first World War, when the range of ages at entry into service was a narrow one in many cases.

In our current United States and Canadian practice, however, the number of employees at relatively advanced ages but with short service is substantial. This occurs in part because of relatively high job mobility on this continent. In greater degree, I believe, it occurs because of the rapid expansion in recent years of the work forces of many of the employers with whom we deal, and because much of this expansion has involved the erection of brand-new plants in new areas, with a work force recruited from those available, and including, at date of recruiting, a substantial number of workers at relatively advanced ages.

The effect of these facts on a projection tends to be twofold. First, if the rates of withdrawal used are derived from actual experience subdivided by age alone, the crude rates of withdrawal at the older ages are relatively high, since many of the employees at those older ages had short service and thus high withdrawal rates. This leads the actuary into the temptation to give an undue weight to these crude figures, and to use a service table showing appreciable rates of withdrawal at the older ages. My impression is that most consultants resist this temptation and conservatively use a service table showing minimum or zero rates of withdrawal at the older ages, whether or not the experience of the actual employer is taken into account with respect to the withdrawal rates used at younger ages.

For valuation purposes this conservative approach seems sound, and if carried out should tend to avoid the eventual development of substantial deficits as a result of overestimated withdrawals; but from a projection (or forecast) viewpoint this conservatism tends to lead the actuary astray since in the early years of actual experience it is likely that there will be a substantially greater number of withdrawals at the older ages (among short-service employees) than he had assumed and an apparent corresponding overconservatism in the projection. In later years, if conditions are fairly stable, the older employees will more largely be survivors of those entering service at a young age, and the apparent overconservatism should disappear. Hence a projection may be consistent with a valuation

which will ultimately prove to be satisfactory, but in the early years the projection may seem overconservative. This may raise doubts as to the necessity of advance funding to the extent required by the valuation and recommended by the actuary.

Similar considerations apply to rates of retirement, which may be materially lower among short-service employees whose pensions are least adequate, than among long-service employees whose pensions are relatively greater. If a minimum service requirement for retirement is included in the plan, it seems particularly urgent that the effect of this in postponing retirement of short-service employees be taken into account. Prior to application of the valuation factors to the actual distribution of the existing employees, there should be excluded for special treatment those employees whose length of service would not qualify them for retirement at the usual ages.

Still another factor making it difficult to utilize projections as forecasts is the treatment of future new entrants into the employer's service. In actual practice, of course, these new entrants will emerge as pensioners to a negligible extent in the 20 to 25 years following the preparation of a particular projection. If, after the projection is made, new entrants are hired in significant numbers at relatively high ages the age distribution of the work force may become materially different from that implicit in the projection, with consequent difficulty in reconciling the experience with the projection.

May I offer a very simple suggestion with respect to the problem of new entrants? It is that the projection be carried forward with respect solely to the existing group of employees; on this basis it will, of course, be found that the pension roll will reach a maximum some twenty-five or thirty years after the date of the forecast and thereafter will decline to zero. The projection may then be completed by assuming that the approximate effect of new entrants (disregarding expansion of work force) would be merely to offset that decline and to maintain the pension roll indefinitely at the maximum level projected for the present employees. In actual practice there would, of course, be some contribution to the pension roll by new entrants prior to the date the maximum pension roll among employees active at the date of the projection was reached, but this may, I think, be disregarded. This suggestion would facilitate reconciliation of the projection with the valuation, since the valuation on the entry age normal cost basis assumes that the liability for pensions of new entrants is exactly offset by the level contributions made with respect to them. Thus reconciliation with the valuation may be made, taking into account only the

projection carried to completion with respect to the original group of employees.

For pension plans newly undertaken, it would be considerable additional labor to use a modification of Mr. Niessen's approach which would take variations dependent on length of service into account. However, where a significant volume of prior experience under the pension plan in question is available, it would seem possible, without an undue amount of work, to prepare a projection which is also a forecast. I had occasion to make such a forecast a good many years ago and it might be helpful to indicate briefly the methods used.

The work was carried out in groupings, each comprising the employees at five consecutive ages and five consecutive lengths of service in years. These were derived from a census of the employees at a fixed date. There were available also records of the actual employees affected by each type of decrement for a period of several years prior to the date of the census. These decrements were added back into the census groupings thus giving the base for obtaining a rate for each type of decrement for each such grouping used.

It should be noted that the employees whose ages at the date of the census centered at, say, age 42 and whose lengths of service centered at 22 years were, a year prior to that, centered at points one year less in each respect, and two years prior, at points two years less in each respect. Thus the work was carried out not only with respect to the group centered at, say, age 42 and service 22 years, but with respect to groups centered at age 41, service 21 years, and at age 40, service 20 years, etc.

The projection was prepared by carrying forward the census groupings, just as the rates of decrement had been prepared by carrying them backward. Graduated factors for each type of decrement were applied to the number in each of the census groups to yield totals for the anticipated withdrawals, deaths, retirements, etc., of the first year. Those expected to survive in service were carried to the start of the second year at a central age and length of service each one year greater and decrement factors corresponding thereto were applied so as to give the decrements of the second year. This process was continued for the full period the projection was to cover. The retirements of each year were, of course, added into the pension roll at the appropriate ages and the pension roll carried forward by use of a mortality table based on the actual prior experience.

While new entrants were taken into account based on the distribution of those hired in the years preceding the census, they had of course a

negligible effect on retirements during most of the period for which the forecast was carried.

While a substantial number of rates of decrement must be derived and used, this method is less laborious than might at first appear, since the number of combinations of age and length of service is limited and since some types of decrement, such as retirement, do not appear until relatively high ages are reached. It will be noted that no tables representative of general experience were used and that the sole assumptions involved were that decrements (and to a minor extent new entrants) would follow the pattern theretofore prevailing. This may prove a great advantage in dealing with an employer who tends to be suspicious of general mortality tables, etc., either selected by the actuary or derived from the employer's own experience through processes which he cannot readily understand.

Finally, it may be worth while to remark that it was possible to compare the pension outlays of the pension plan in question with those forecast by the projection, for about ten years after the date of the work. The comparison was quite satisfactory.

GEOFFREY N. CALVERT:

I find projections are the most useful tool that the actuary has for inducing a company to fund a pension fund instead of letting it go on a pay-as-you-go basis. Projections can be used very effectively in bringing out the tremendous cost in a pay-as-you-go system in later years as compared with a funded plan.

In comparing the outlay on a funded plan with that on a pay-as-you-go plan, it is advisable to insert in the projections at every tenth year a statement of the lump-sum liability for pensioners then in existence or in prospect who would continue to draw the pension if the whole plan were terminated at that time. What I want to bring out is the situation where, under a pay-as-you-go plan, the employer is fully responsible for paying each pension, and the claim rests on the employer, since there is no fund, as compared with a situation where there is a funded plan and the claim is on the fund. If both plans are terminated, the employer having the funded plan has no further payments to make and is relieved of further costs at once. Under the pay-as-you-go plan the employer has to continue to pay the pensions to all existing and prospective pensioners.

Projections, when used in that connection, should therefore be accompanied by statements showing at periodic points of time the liability which has accrued but not yet been met—in other words, the liability in

respect to pensions which still have to be paid beyond each successive point of time. Just a straight projection, as such, does not quite do the whole job.

There is a peculiar kind of projection around. It is being prepared by the UMW. I have run into it in consulting work in several places. The UMW headquarters are preparing a projection that runs for 20 or 25 years and stops at that point. The pension cost is really rising about then, but they stop the whole thing at that point and find the total amount of money which would be required over the period from the beginning of the plan to the end of the 20-year or 25-year period, and then they spread that uniformly over the period and convert that into cents per hour, and demonstrate, as they think, in a very conclusive manner, that only 6 cents an hour put into a pool is quite sufficient to keep the thing running for at least the whole period covered—and nobody can see beyond that point. If the projection had been run for 50 years, *i.e.*, to the point where the outlay stabilizes, and allowance made for the period beyond that, then maybe, instead of 6 cents an hour, it would be found that 11 or 12 cents an hour would be necessary to provide the same benefits.

I wanted to mention that in case anyone runs into it. I checked into the method of preparing the UMW figures in their own head office, and I was surprised.

Mr. Bronson was entirely correct in his statement about investment yields of trust funds. The lowest of them are way down below the general average of insurance company yields. The highest have been much higher. I know of a large trust fund which has a trust agreement which prohibits the trustee from investing in any securities except government bonds maturing in ten years or less. That fund is yielding very much below 2 per cent. Another pension trust I have seen had all its investment in six different assets, one of which was a machine tool company. The trustee bought all the shares of the machine tool company for a million dollars. It made a million dollars profit in the first year. The trustee wrote the asset down to $1.00 and made a million dollars on that in the second year. Then the employer sold off one of the subsidiaries and extinguished most of the liability for past service. After the employer had made three installments on the past service cost, it was fully liquidated, and the trust fund was still making a million dollars a year from the asset valued at $1.00.

Maybe somebody could tell me what interest rate the actuary should use in evaluating the liability of that fund.

(AUTHOR'S REVIEW OF DISCUSSION)

A. M. NIESSEN:*

When I cautiously invited a discussion of my paper, I did not expect a response as gratifying and stimulating as I actually received. Apparently, the subject of projections is now of considerable interest to actuaries, as evidenced by the wide variety of interests of the members of the Society who participated in the discussion. We find there Messrs. Stark and R. F. Link, representatives of two giant life insurance companies; Mr. Myers, the chief actuary of the colossal OASI system; and finally, Messrs. Bronson, Calvert, and Williamson, all of whom are well known consulting actuaries. In my opinion, the discussion as a whole was very stimulating, and I wish to express my sincere thanks to all the gentlemen who participated in it.

Mr. Stark gave us a penetrating analysis of the factors which make a forecast-projection difficult. One of the troubles pointed out by Mr. Stark is the changing age distribution of employees and particularly the admission of older new entrants during recent years. For an employee population with such age and service characteristics, Mr. Stark correctly points out that the use of an aggregate service table is not appropriate. My paper gives the impression that we rely on an aggregate service table. However, the truth of the matter is that we at the Railroad Retirement Board use select service tables throughout. In the projection which we recently made, the typical age distributions for withdrawals, retirements, and deaths in a year were obtained from expected separations computed on the basis of select service tables. In my paper I left out as much procedural detail as possible and concentrated on the general aspects of the problem of projections.

Mr. Stark's suggestion of preparing a short range projection without considering new entrants is very interesting indeed. I would even go a little further and leave out also present employees with very short periods of service. In a system which has already acquired a relatively heavy benefit load, the leaving out of new entrants and of short service present employees would not make any appreciable difference in the first 10 or even 15 years, as pointed out by Mr. Stark. There is no doubt that this simplified procedure would result in great savings in computational work, especially since new entrants usually offer rather troublesome problems. I would like, at this point, to mention that the method suggested in my

* The opinions here expressed are those of the author and do not necessarily represent the official views of the Railroad Retirement Board.

paper does not involve separate dealings with new entrants and present employees. The special "index numbers" which I use are supposed to take account of all new retirements or deaths, regardless of whether they come from present employees or new entrants.

Mr. R. F. Link shares my feeling that projections are a very useful tool in the actuarial workshop. I fully subscribe to his opinion that there is still a great deal to say about pensions and projections, and I hope with him that perhaps some authority in the field will undertake the labor of writing a technical book on the subject, something in the nature of an actuarial study. What interested me a great deal in Mr. Link's discussion are the techniques which he describes as applicable for insured plans in general and group annuities in particular. I recognize that the insured plans may require different projection techniques than large government plans. Mr. Link gives us the details of a projection technique for a group annuity plan. This type of information, as well as the remarks made by Mr. Stark, are very valuable contributions to the theory and practice of pension plans.

Mr. Myers has in recent years had more occasions than anyone I know to use the projection method in estimating cost for the most important retirement and survivor insurance system we have in America today. For that system, the OASI, Mr. Myers has found the projection method most satisfactory. Judging from his writings, he believes that for other retirement systems, whether private or public, the projection method has advantages which cannot be overstressed. I share Mr. Myers' high regard for projections and join him in hoping that actuaries will become more projection-conscious than they are now.

While on the subject of projections, I wish to point out that the methods advocated in my paper would probably not be suitable for a national all-inclusive scheme like the OASI. The special techniques developed by the former and present actuaries of the Social Security Administration (Messrs. Williamson, Bronson, Myers, Shudde, and others) are described in a special House of Representatives document entitled *Issues in Social Security* (H. Res. 204, 79th Congress, First session, U.S. Government Printing Office, 1946, pp. 184–256). It is my belief that no student of the actuarial aspects of social security can afford not to read these pages of the document otherwise known as the Calhoun Report. Anyone who would undertake the labor of writing a book on the subject of projections, as suggested by Mr. Link, would undoubtedly draw heavily from this excellent material as well as other material developed since Mr. Myers has taken over the duties of the chief actuary of the Social Security Administration.

Mr. Williamson gives us, in his usual witty and incisive manner, a series of pertinent remarks covering the fields of private and public insurance. I am pleased to note that Mr. Williamson appreciates the difficulties which the actuarially uninitiated layman encounters while trying to make some practical sense of two projections or cost estimates that are miles apart. Mr. Williamson, as probably would Messrs. Bronson and Myers, would nevertheless insist on presenting two projections, at least when they pertain to a public retirement plan. If I may disagree with these gentlemen, who are recognized authorities in the pension field, I would still prefer to present only a single projection on the grounds that two projections remain only a matter of record, while actual decisions are based on some intermediate or average series of cost figures. Curiously enough, such intermediate projections have often been prepared by the actuary himself. I therefore fail to see why a single projection will not be sufficient, provided it is accompanied by a statement that the actual story may lie quite far on either side of the figures presented. However, this point, although a very important one, is not one of the basic considerations of my paper. What I wanted to convey is the importance of the projection method in general, and I am pleased to note that on this point all discussants, with the possible exception of Mr. Bronson, seem to agree with me.

Mr. Bronson's critical comments were most welcome, especially since I hoped for some friendly criticism which would point out the weak spots of my paper. I am glad that Mr. Bronson himself noted the difference between his "laymen" on the management side and my "laymen" on the employee side. I venture to say, though, that in times less prosperous than we are having now, perhaps even the management "laymen" will need a little convincing that not everything is rosy with a pension fund just because it has a lot of money and because its current income exceeds current disbursements.

Concerning the matter of "range" estimates versus a single one, it is my impression that Mr. Bronson would impose such a requirement only on public plans like social security, railroad retirement, civil service, etc. For private plans, I assume Mr. Bronson would still favor single cost figures, a view which he expressed in his most interesting discussion of Mr. Myers' paper, "Some Considerations in Pension Fund Valuation" (see *TASA* XLVI, 397-401). Although I recognize the basic differences between public and private plans with respect to accounting procedures, income tax laws and the like, I still cannot see why the government actuary should not be guided by the same or similar practical considerations as the consulting actuary. In my opinion, the government actuary could sufficiently

protect his reputation for professional integrity by pointing out that the actual costs may be much higher than what is shown in his report. The actuary should also try to make some rough estimate of the range in which costs may lie and present this range to the interested parties. I will go even further and say that the private actuary should also do the same.

Mr. Bronson takes me to task for giving recognition to the effect of a different interest rate while neglecting to mention what would happen to the projection if we were to assume substantially different mortality rates. My general approach was to prepare one basic projection and then utilize it to answer questions which do not involve a great deal of additional work. The effect of interest rates, if considered by itself, can be established with very little work, while to introduce the effect of different mortality rates would practically require a whole new projection. The inconsistencies which Mr. Bronson found in my paper also stem from this same general attitude of mine. What I had in mind was to project the "typical groups" with great care and on a basis which is sufficiently realistic in the sense of not being unconservative or overconservative. Once the projection of the typical groups is accomplished, the remaining portion of the computational work is not too heavy. Thus, if the method suggested in my paper is followed, it is possible to establish a relatively durable core of worksheets upon which modified projections can be built. This durability, which I assign to the major part of the work connected with the projection, is not inconsistent with a statement which I make in another place that a projection cannot be expected to agree closely with actual experience beyond the first few years.

Concerning the question of consulting with economists and other specialists on the formulation of economic assumptions, the disagreement between Mr. Bronson and myself is more apparent than real. Mr. Bronson, who favors a range in projections, has obviously no need for such consultations, since the actuary himself, without any outside guidance, can decide upon high and low cost assumptions, especially if the cost figures so produced are rather far apart. I, on the other hand, favor a single projection, so that I would like to have the benefit of someone else's thinking as to future economic conditions. I did not mean to imply that the actuary should take the economist's word for it. However, a knowledge of other people's opinions, especially if those people are considered qualified in the field, should be helpful.

I was rather intrigued by Mr. Bronson's final comment regarding the projection of widows' and orphans' benefits. It may require a lot of courage to make such a projection, as Mr. Bronson points out. However, if we deal with a system which provides benefits of this kind, and if we are to

prepare a projection for that system as a whole, we have no alternative but to acquire the courage to make this kind of projection. Personally, I cannot see why a projection of widows' and orphans' benefits is theoretically so much more difficult than a projection of retirement benefits. It surely involves more work and other types of information, but the basic techniques, in my opinion, remain the same. I imagine that Mr. Bronson himself, in preparing *Actuarial Study No. 19* at the Social Security Board, was guided by the same motives of necessity, when he undertook to project widows' and orphans' benefits up to the year 2000.

Mr. Calvert shares my opinion that the management "layman" also needs a projection to induce him to fund a pension system instead of letting it go on a pay-as-you-go basis. Mr. Calvert's discussion brought out two very important points. One is that a projection will be greatly enhanced in value if it would show at stated time intervals the liability with respect to pensioners then on the rolls. Such a figure is very nice to have, since it can be compared with the reserves which will be available at the same point of time. As a matter of fact, I made a similar suggestion in my paper, and I am pleased to note that Mr. Calvert attaches considerable importance to this point. The other important comment made by Mr. Calvert is that a short range projection may give a completely erroneous picture of the financial condition of the fund. Mr. Calvert mentions a specific short term projection which is now being circulated. In my opinion, when an actuary is asked to prepare such a short term projection, he should do so, but the projection must be accompanied by a statement containing an actuarial appraisal of the fund as a whole and what is likely to happen to the fund at a point of time not reached by the projection itself.