TRANSACTIONS OF SOCIETY OF ACTUARIES
1950 REPORTS VOL. 2 NO. 4

ACTUARIAL NOTE: THE VALUATION OF SELF-INSURED RETIREMENT PLANS

JOSEPH C. NOBACK

SEE PAGE 49, NO. 2 OF THIS VOLUME

GEOFFREY O'H. HARRISON:

This excellent paper seems certain to be added to the recommended Course of Reading for Part 8.

Much of the material is not new, but Mr. Noback has prepared a valuable summary of the subject. Some of the original references were listed by Mr. Bronson in *TSA* I, 242.

This discussion deals with the valuation of salaried benefits after a number of years of service (pp. 61 ff.). Past service here means since the plan inception date.

The author gives an example where benefits are based on average salary and points out that past service and future service benefits must be valued separately. For past service, we value the benefits arising from actual past salaries. For future service, we value the benefits arising from current salary, then the benefits arising from next year's expected salary, and so on. Unless we were using select tables, and of course these are not always available, the assumption that current salary had been attained by salary increments in proportion to increments in the salary scale would increase rather than decrease the work involved, even though it would enable us to value together benefits arising from past and future expected salaries; for if we were to make this assumption our commutation columns starting at each attained age would be different for each age at entry to the plan. Actual past salaries should not be ignored since all of them affect the benefits.

Where benefits are based on final average salary, it is usually assumed that current salary has been attained by salary increments in proportion to increments in the salary scale. Benefits arising from past and future expected salaries are usually valued together. We value the benefit if the service table decrement takes place this year, then the benefit if the decrement takes place next year, and so on.

However, where the final average is the average of a large number of years, it is desirable to compare the approximate results obtained in a few cases on the usual assumption with the results obtained using actual past salaries, and to make an adjustment if it is indicated. The use of

actual past salaries is awkward, since in our calculations we begin after a while to drop one year's actual past salary as we pick up one year's expected future salary.

The following problem, taken from the 1947 Part 8 examination, calls for the use of actual past salaries although benefits are based on final average salary.

A pension fund grants a pension on retirement at any age over 60 and under 66 of the following percentages of total salary excluding salary earned more than 30 years prior to retirement:

Retirement Age Last Birthday	Percentage of Salary
60	1.0
61	1.1
62	1.2
63	1.3
64	1.4
65	1.5

Upon withdrawal prior to age 60 and after completion of 5 years of membership in the plan, a deferred annuity is granted whereby an income is paid commencing at age 65 equal to 1% of total salary excluding salary earned more than 30 years prior to withdrawal.

Given a service table and salary scale, derive formulas for:

(1) the annual contribution as a level percentage of salary at any entry age;

(2) the present value of future benefits in respect of both past and future service for an employee now age 50 who entered service at age 25, assuming that records of all past salaries are available.

(1) Using Mr. Noback's notation, if the contribution rate is c, the present value of all future contributions is

$$c \cdot (\mathrm{AS})_{[x]} \cdot \frac{{}^{S}\bar{\mathrm{N}}_{[x]}}{{}^{S}\mathrm{D}_{[x]}}.$$

The value of the pension benefit is

$$.001\ (\mathrm{AS})_{[x]} \left\{ \frac{10\ {}^{Z}\mathrm{M}^{ra}_{[x]+60-x} + {}^{Z}\mathrm{R}^{ra}_{[x]+61-x}}{{}^{S}\mathrm{D}_{[x]}} \right\}$$

where

$$Z_{[x]+r} = \begin{cases} S_{[x]} + \ldots + S_{[x]+r-1} + \frac{1}{2} S_{[x]+r} & \text{for } r < 30 \\ \frac{1}{2} S_{[x]+r-30} + S_{[x]+r-29} + \ldots + S_{[x]+r-1} + \frac{1}{2} S_{[x]+r} & \text{for } r \geqq 30 \end{cases}$$

The value of the withdrawal benefit is

$$.01\ (\mathrm{AS})_{[x]} \cdot \frac{{}^{Z}\mathrm{M}^{wa}_{[x]+5}}{{}^{S}\mathrm{D}_{[x]}}$$

where

$$\mathrm{C}^{wa}_{[x]+m} = \mathrm{C}^{w}_{[x]+m} \times {}_{65-x-m-1/2}|\,\bar{a}_{[x]+m+1/2}.$$

(2) Let $S'_{[25]+t}$ be the employee's actual salary during the year of life $[25]+t$ to $[25]+t+1$, so that $(AS)_{[25]+25} = S'_{[25]+25}$, and let

$$Z_{[25]+r} = \begin{cases} S'_{[25]} + \ldots + S'_{[25]+25} + \dfrac{S'_{[25]+25}}{S_{[25]+25}} \\ \qquad \times (S_{[25]+26} + \ldots + S_{[25]+r-1} + \tfrac{1}{2} S_{[25]+r}) \quad \text{for } r < 30 \\ \tfrac{1}{2} S'_{[25]+r-30} + S'_{[25]+r-29} + \ldots + S'_{[25]+25} + \dfrac{S'_{[25]+25}}{S_{[25]+25}} \\ \qquad \times (S_{[25]+26} + \ldots + S_{[25]+r-1} + \tfrac{1}{2} S_{[25]+r}) \quad \text{for } r \geqq 30 \end{cases}$$

The value of the pension benefit is

$$.001 \left\{ \frac{10\, {}^{z}\mathrm{M}^{ra}_{[25]+35} + {}^{z}\mathrm{R}^{ra}_{[25]+36}}{\mathrm{D}_{[25]+25}} \right\}.$$

The value of the withdrawal benefit is

$$.01 \, \frac{{}^{z}\mathrm{M}^{wa}_{[25]+25}}{\mathrm{D}_{[25]+25}}$$

where

$$\mathrm{C}^{wa}_{[25]+m} = \mathrm{C}^{w}_{[25]+m} \times {}_{40-m-1/2}\,|\,\bar{a}_{[25]+m+1/2}.$$

PHILIP D. SLATER:

Mr. Noback has furnished the Society a lucid and concise note developing formulae for cost estimates and the valuation of so-called self-insured plans. This paper will be of value to students and actuaries in this country as a reference, and will avoid the frequent need to refer to the old classical English papers which are not as accessible.

It is perhaps unfortunate that the title to this note may give the impression that the methods developed apply only to self-insured plans. Valuations of the type outlined in the paper are frequently desirable and necessary in connection with insured plans of the deposit administration type or as supplementary studies in connection with insured plans of other types where comparisons with other bases of funding are needed for the purpose of studying the incidence of costs.

My initial remark concerns the last section of the note on the effect of changes in salary scale. There appears to be some confusion in Section B between the salary scale function and the actual annual rate of salary which might be troublesome to students, and which the author may care to clarify in his remarks on the discussions to his paper.

I have no comment to make in this discussion on the mathematical development of the various formulae in his note other than to point out that,

although the formulae flow quite neatly from basic principles, the work of computing the various functions can be quite tedious and expensive particularly when a fair amount of experimenting with assumptions is found to be necessary. It is hoped that with current developments in electronic computing devices it may soon be possible to cope with these problems economically and quickly.

In applying these techniques to pension cost calculations or valuations, appropriate assumptions must be made of the various basic items such as mortality before and after retirement, rates of retirement, disability and withdrawal, and probable future rates of interest and salary increases. Some of these items such as interest and mortality are relatively stable within each group and beyond much direct control of the group. Others such as rates of withdrawal and salary changes are sometimes subject to rather violent fluctuations even though the group can exercise some degree of control by changes in personnel policies. We therefore encounter some of the same types of problems that occur in connection with disability or sickness insurance where there is dependence of basic factors on economic conditions which are beyond the control of the actuary or the group. It is here, therefore, that the actuary is faced with one of the most difficult parts of his task in making appropriate assumptions. And since he is one that must live with the plan, it is extremely advantageous that the choice be well made.

Experience has shown that the selection of appropriate salary scales and withdrawal rates at the inception of the plan, even if it is decided to draw heavily upon the past experience of the group, is usually made difficult by the lack of appropriate records, and by the fact that it takes a long time for a group to develop its own experience. Because of the difficulty in determining these factors, and restrictions which may be made by the Internal Revenue Bureau on the choice of appropriate salary scales where its approval is required, there has been a tendency in recent times to set up cost estimates and valuations assuming no withdrawals in the future and indefinite continuation of the current salary.

One thing that can be said for such a procedure is that it does simplify the calculations considerably. More important than this, however, is the fact that it takes the emphasis away from quibbling about assumptions which can never be more than good educated guesses and permits a more thorough study of the more important function of developing methods of funding which are more flexible and fit the needs of the particular group.

The statement is frequently made that the ultimate cost of a plan will depend entirely on the benefits which are eventually paid out. This is an accurate statement only if we consider as "benefits" the interest earnings

of the fund and its expenses including federal income taxes. The financial impact of withdrawals and salary changes on the ability of the group to meet its commitments at various times during the existence of the plan is of great importance. The difference in the incidence of total ultimate costs of a plan under various funding methods in a pension fund can be more striking than such differences in the costs of permanent ordinary insurance over term insurance. This analogy goes one step further. Just as the loan value of an ordinary policy may serve as a contingency reserve to help keep a policy in force when premiums cannot be met, a pension plan where the funding has been kept well abreast of earned benefits has a better chance of surviving a period of difficult times.

Now that Mr. Noback's note has conveniently set forth and made accessible mathematical tools for formalizing basic assumptions and studying their interrelationships, it is hoped that his note, together with other recent papers on pension plans which have appeared in the *Transactions*, will tend to stimulate more research on the impact of modern funding methods on the cost pattern of pension plans and means of safeguarding the continuity of plans under varying conditions.

CECIL J. NESBITT:

This paper gives a well organized presentation of the mathematical functions required in valuing a self-insured retirement plan. The presentation is on a select basis which raises the question of the value of such a basis for pension plan calculations.

Of the various basic factors—interest, mortality, salary scale, retirement, withdrawal—the last may vary with duration as well as age and, under special circumstances, so also may the salary scale, the retirement factor, and mortality after retirement. In the case of smaller funds, however, there may not be an adequate experience basis for deriving select rates and, in that case, it may be better to use conservative aggregate rates and adjust for gains and losses in periodic valuations. Large funds may justify the use of a select basis. Notable examples in this regard are the Railroad Retirement System and the United States Civil Service Retirement and Disability Fund.

The British actuaries, G. Heywood and W. F. Marples, in their paper "Pension Funds: Some Practical Points" (*TFA* XIV, 105) doubt the value of a select basis. In discussing the withdrawal factor, they state (p. 123):

> Withdrawal is as much affected by duration of employment as age at short durations and there is no doubt that select rates should be used in theory. This would, in our view, involve undue labour in relation to any practical value achieved, as it would assume an accuracy in the results which is quite unattainable in the combination of estimates of future events.

It appears to me that the additional labor of using a select basis may be reduced by using age-service groupings of the valuation data. It is possible that a select basis valuation applied to such age-service groups may give a better estimate of the plan costs and liabilities than a more detailed valuation on an aggregate basis. I would be interested to know in how much detail Mr. Noback would apply a select basis valuation.

There is a minor question about the formulas on pp. 59–60 for the valuation of a disability annuity subject to a minimum. It would seem unusual to have a disability benefit available during the first year after entry but the $K_{[x]+m}$ definition indicates that such a benefit is considered here. The formulas on p. 60, however, provide only for disability in the second and subsequent years. Provision for disability in the first year could be accomplished by using a special definition for ${}^{s'K}\mathrm{M}^{ha}_{[x]}$, namely

$$ {}^{s'K}\mathrm{M}^{ha}_{[x]} = S_{[x]} \sum_{n=0}^{\infty} K_{[x]+n}\, \mathrm{C}^{ha}_{[x]+n}. $$

On p. 67 the Normal Cost accrual rate $c^{Er}_{[x]}$, determined on the basis of a retirement annuity of 2% of average salary for each year of service in plan, is applied towards the funding of 1% past service benefits and 2% future service benefits. This inconsistency between the basis for the accrual rate and the basis for benefits might, for some age-service subgroups in the valuation, lead to negative "Supplementary Costs" in regard to those groups. The negative amounts would probably be relatively unimportant in this case.

There is a somewhat related problem in regard to the Aggregate Cost Method where the negative amounts may be more significant. A single accrual rate for all employees may be in excess of the contribution rate required for younger employees, and when one takes credit for such excess contributions it is equivalent to taking credit for negative reserves. One suggestion for eliminating or reducing such negative reserves would be to grade the accrual rate in some simple manner by attained age.

(AUTHOR'S REVIEW OF DISCUSSION)

JOSEPH C. NOBACK:

This paper was written to assist the student in studying the mathematics of those Retirement Plans in which:

(1) benefits are provided in the event of death, disability, and withdrawal as well as in the event of retirement, and
(2) the formulae used to determine the benefits and the contributions are functions of salary.

In order to achieve this objective, the subject was developed from a theoretical point of view, assuming that all of the basic factors were available.

Mr. Slater and Dr. Nesbitt have noted that in actual practice it is often not possible to forecast withdrawal rates and salary scales satisfactorily and that, consequently, the use of a select basis for valuation is subject to some practical limitations. Mr. Slater has indicated that one solution is to assume the continuation of the current salary and the absence of any future withdrawals.

The author is grateful to each of these men for bringing into focus the practical problems involved in the valuation of pension plans and he would like to take this opportunity to thank them and Mr. Harrison for their contributions to the discussion.

Mr. Slater raises a question with regard to the four paragraphs on page 69. These paragraphs attempt to touch upon the consequences of a change in "salary scale." Mr. Slater suggests that it would be clearer to discuss the change with relation to actual salaries. On review it would seem that these last paragraphs of the paper are rather cryptic. The following paragraphs are now offered to help.

If the change takes the form of a fixed percentage increase in actual salaries and if it may be assumed that the salary scale is not affected, then the value of $c_{[x]}^{Er}$ is, in general, not changed. This conclusion applies to both average salary plans and final salary plans. Such a development, however, would tend to create a deficit in the fund because additional benefits will accrue to present contributors for which an adequate charge is not made. In a final salary plan, the deficit would tend to be greater than in an average salary plan. Furthermore, the deficit would tend to be greater where the salary scale is flatter.

If the change takes the form of a flat addition to actual salaries and if it may be assumed that the salary scale itself is thereby flattened, then the value of $c_{[x]}^{Er}$ tends to be reduced. This applies to both average salary plans and final salary plans. The effect of such a change upon the fund cannot readily be determined by general reasoning because the additional benefits which will accrue to present contributors may or may not be offset by the fact that contributions in the past have been at a higher rate than are now to be required for new entrants.