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ACTUARIAL NOTE: TERMINAL RESERVES FROM MEAN RESERVES AND NET PREMIUMS

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WILLIAM A. SPARE:

The United States Life is currently using Mr. Gustafson's formula (18) to calculate certain terminal reserves released as required for the Annual Statement in the "Analysis of Increase in Reserve During the Year."

Our company uses the seriatim method of valuation and our transaction cards show both the prior year's mean reserve and the face amount of insurance which, for level amount policies, is the amount in force at the end of the prior year as required by the formula. For limited-payment reducing term insurance policies (which have positive reserves), we cannot use the formula as our cards contain an equivalent level amount, which is not the actual amount in force at the end of the prior year. We are valuing these seriatim.

Instead of using attained age summations we employ an average attained age and, for level amount policies, sum the mean reserves and amounts of insurance for each valuation basis, using the average age constants which apply to that particular group. It will be noted that for any one attained age the constants vary only by the mortality table and interest rate employed.

The formula produces terminal reserves as of the anniversary in the current calendar year and hence it must be assumed that all transactions occur on the anniversary date. It must also be assumed that transactions are processed in the valuation file immediately after the effective date of the transaction. Before adopting the formula, tests were made which disclosed an error of less than 5% on surrenders and changes. On lapses and reinstatements the error was about 18% due to the time lag from the effective date of lapse or reinstatement to the date processed in the valuation file. Use of an average age (weighted only by number of policies) had very little effect on the results. It was decided that the formula could not be used for lapses or reinstatements and we are currently using a seriatim method for these transactions. However, reserves released by surrenders and changes are being calculated in bulk by the formula.

Transaction cards are divided into surrenders, outgoing changes and incoming changes and are then sorted by valuation basis. The cards pertaining to issues of the current year are removed because there is no prior year's mean reserve and the formula will not apply. All remaining cards are listed and totals of the prior year's mean reserves and face amounts are obtained for each valuation basis group. An average attained age is easily calculated for each of these groups by summing the office years of birth, dividing by the number of transactions and subtracting the result from the valuation year. To obtain the terminal reserves, formula (18) is applied to each group, using the constants pertaining to the valuation basis and average attained age for that group.

The formula must be modified for policies both issued and terminated in the current calendar year. On the average, such policies terminate or change at policy duration one-half. By using the current year's mean reserve and constants based on an average age at issue, we can obtain the first terminal reserve from formula (18). Taking one-half of the result gives an approximation to the terminal reserve at duration one-half.

(AUTHOR'S REVIEW OF DISCUSSION)

DALE R. GUSTAFSON:

I want to thank Mr. Spare for discussing a practical application of my formula (18).

It may be of interest to note that there are other relationships between terminal and mean reserves similar to formula (18). The following relation expresses the terminal reserve during a given calendar year in terms of the mean reserve at the end of that year, *i.e.*, in terms of the current year's mean reserve instead of the previous year's mean reserve; however, the net premium payable on the current anniversary is also required.

$$[{}_{t-1}V_{x:\overline{n}|}] = [{}_{t}MR_{x:\overline{n}|}] \cdot K_Y^{(6)} + [\text{Amount}] \cdot K_Y^{(5)} - [{}_{t-1}P^{N}_{x:\overline{n}|}],$$

where

$$K_Y^{(6)} = \frac{2 \cdot D_Y}{D_Y + D_{Y-1}} = K_Y^{(4)} \cdot \frac{1}{u_{Y-1}}.$$