TRANSACTIONS OF SOCIETY OF ACTUARIES 1950 REPORTS VOL. 2 NO. 4

## ANNUITY MORTALITY

WALTER G. BOWERMAN<br>SEE PAGE 76, NO. 3 of THIS VOLUME

## WILMER A. JENKINS:

I would like to compliment Mr. Bowerman on his very interesting paper, particularly his observations about the mortality differential between the sexes. The rather startling changes that have occurred over the last 30 years in the relationship between male and female mortality is perhaps most graphically illustrated by Mr. Bowerman's chart on page 89, which shows that at age 60 , for example, the $M / F$ ratio has increased from about $115 \%$ in 1922 to about $175 \%$ in 1947. The large changes that have occurred over age 55 or 60 are highly important in relation to annuity premiums and reserves, and doubtless have had their effect on the 4 -year age differential between the sexes originally assumed 20 years ago in the Combined Annuity Table and more recently extended to 5 or more years. The large changes below age 30 or 35 are significant in other connections, but are not important in relation to annuities. It should be observed that at the young ages the large percentage changes shown in the chart result from relatively small changes in the death rates themselves.

Mr. Bowerman develops a new 1950 annuity mortality table, which differs from the Annuity Table for 1949 in three respects. The first and most important respect is a reduction in the death rates at ages over 88. This reduction amounts to $3 \%$ at age $90,16 \%$ at age $95,30 \%$ at age 100 , with larger reductions over age 100 . This change extends the limiting age from 110 to 121. Mr. Bowerman's second departure from the 1949 Table is an extension of the table to age 0 , and the third consists of relatively minor adjustments, including one to improve the $\mathrm{M} / \mathrm{F}$ ratio at young ages, which do not affect annuity values materially.

As to the old-age adjustment, the paper isn't too clear but it is my impression that the 1950 Table is designed to represent current mortality levels with some conservatism. This would be a table requiring projection into the future in fixing annuity premiums and reserves. It would be informative if Mr. Bowerman would confirm or correct this impression in his reply to this discussion. But assuming that it is correct (and, in fact, even if it is not), I am not too impressed with the various reasons which Mr. Bowerman gives for the mortality rates he adopts at the very old ages, i.e., over age 95 or so.

The simple fact is that no one has satisfactory data as to annuitant mortality at these ages. We are all, therefore, forced to conjecture, analogy and judgment. Apparently Mr. Bowerman's table at the very old
ages is derived from three considerations. The first is the assumption of what is called an axiom-referred to at one point as a rule-that an annuity table should never show higher death rates than a comparable population table or a comparable table based on insured lives. In this connection Mr. Bowerman cites recent United States population life tables and the British A1924-29 life insurance table. As to the former, he did not comment on or make any adjustment for the well-known fact that population data tend to involve substantial overstatements of age at the very advanced ages. As to the A1924-29 table, he did not explain why he followed a foreign table when a United States table would seem to be a much better guide and one is readily available, i.e., the CSO underlying experience table.

Mr. Bowerman's second consideration seems to be that the limiting age of a mortality table should recognize that centenarians do sometimes live to the ages which he so interestingly demonstrated in his 1939 paper. However, the proved cases of centenarians outliving the limiting age of 110 in the 1949 table are so very rare that their statistical significance is, I think, very dubious. In recent years the oldest recorded age of survival among Civil War veterans was age 108. Unlike Mr. Bowerman, I do not see any great embarrassment when a very, very occasional annuitant outlives the mortality table.

As to the third consideration which apparently leads to his death rates at very advanced ages, Mr. Bowerman cites trends among other tables in the limiting age and trends in the value of the Makeham constant $\log c$. To me this consideration isn't very persuasive because trends of this kind change from time to time and frequently are very seriously distorted by the mechanics of graduation and other factors having no relationship to true mortality rates.

All in all, therefore, while I agree completely with Mr. Bowerman on the important principle that a conservative approach to annuity premiums and reserves is proper and necessary, I am not convinced by his arguments that his 1950 table is more proper as a conservative representation of current mortality at the very advanced ages than the Annuity Table for 1949. Moreover, I understand that Mr. Lew has written a discussion which presents data indicating that the 1949 table is sufficiently conservative.

In this connection, you will have noted that the Prudential 1950 Group Annuity Table presented at this meeting by Mr. Blagden, like Mr. Bowerman's table, is substantially more conservative at the very advanced ages than the Annuity Table for 1949. This, however, seems to be pure coincidence because, as Mr. Blagden states, in constructing the Prudential table he was not concerned greatly with the problem of individual
equity, i.e., the incidence of death rates at the various ages. Moreover, Mr. Blagden's table is designed to be used without projection so that, of necessity, it must embody somehow at some ages an element of conservatism not in the 1949 table which was designed to be used only with projection. Unlike Mr. Bowerman, Mr. Blagden does not contend that the Prudential table is representative of current mortality rates at all ages relatively.

In this connection also, you will have noted that Messrs. Fassel and Noback's Progressive Table shows mortality rates at ages 95 and over which, for many years, are generally comparable with those of the Annuity Table for 1949. The Progressive Table representations of current mortality rates are somewhat higher than the 1949 table for males and somewhat lower for females. That the Progressive Table death rates at ages 95 and over become progressively lower in the future, whereas the 1949 table rates do not, is attributable to the assumed projection scales for these tables and not to the levels of current mortality assumed in the tables themselves.

For the record, I would like to mention one other point. Mr. Bowerman implies that Mr. Lew and I forced the data with which we were working "into the Makeham mold," and that it would have been better not to Makehamize our table and to use the joint life scheme originated by Messrs. Elderton and King. The fact is that below age 50 or 60 the Annuity Table for 1949 was graduated by Makeham's formula in appearance but not in actuality, and below those ages we did utilize the Elder-ton-King scheme. Over age 50 or 60 the 1949 table was Makehamized in the usual way and, because Mr. Lew was largely responsible for this graduation, I can without immodesty say that in my opinion the resulting conformity of expected deaths to actual deaths is excellent. Mr. Lew and I agree completely with Mr. Bowerman's objection to Makeham graduations which force the data, and my point is that basic data for the 1949 table were, in fact, not forced.

It is pertinent to note that Mr. Bowerman agrees with Mr. Lew and me as to the situation in which actuaries find themselves in relation to the annuity mortality table they are now using. Like us, he is clearly unhappy with the 1937 Standard Annuity Table, and that, it seems to me, is the important problem now facing us all.

## W. RULON WILLIAMSON:

Mr. Bowerman's fascinating extension of the tabular "span of life" to age 120 or 121 shows the results of his wide reading and his roving imagination. I am discussing only one sentence of his paper: the next to the last one on page 76. "The population data, duly projected into the
future and modified for class selection, make a firmer basis than an extrapolation from annuity data alone."

If those population data are firm, we should certainly beware of the quicksands. I would like to tell a little about that "firm ground."

1. The age reporting for the census. Apparently between 1930 and 1940 such changes in reporting ages took place that in addition to the survival of those persons aged 55 and over in 1930 into the group 65 and over in 1940, there seem to have been some 600,000 persons net migrating across that 65 boundary line. That is, some $6 \frac{2}{3} \%$ of the $9,000,000$ claiming to be 65 and over were younger than that on the evidence of what had been reported 10 years earlier. I do know some now over 65 who have adopted a younger part to play. Many conversations with persons in many walks of life and with those directly in the Bureau of the Census show a pretty wide range of reasons for doubting the accuracy of the age tabulations making up the exposures-the denominator in the mortality rate fraction. This age reporting in the decennial census-taking-and particular interest attaches to what 1950 is going to show-shows that at least the following influences will affect the reports.
a) The determination to look younger.
b) The pride of fixing a higher reported age, to enhance the "new look."
c) The real employer preference for younger employees with more future.
d) The fancied job prejudice in favor of youth.
e) Among negroes, the pride in reaching extreme old age (I bave seen a tombstone for a negro aged 132).
$f$ ) The effect of Social Security benefits-both assistance and so-called insurance.
g) Gerontological philosophy and other theorizing as to the unimportance of chronological age.
h) Reports to census takers by relatives, neighbors or small children, not sure of the ages reported.
i) Passports, foreign travel, the too youthful picture and romance.
j) Many shifting whims as to the best age to report.
k) Carelessness.
l) Ignorance of the facts.
2. Incompleteness in census tabulations. There are always adjustments made in the Bureau of the Census, to correct for incomplete returns. At the low ages there has commonly been underregistration. There is not very much agreement among the different census experts as to how much the correction is-the fortunate mystery of truth! I found Dr. Greville noticeably diffident as to how the errors were to be located and corrected. Some people want to be lost. Some could get around, and under aliases be counted more than once!
3. Heaping. The happy days when mathematical formulae neatly corrected for heaping are gone. The sequence of the ages was supposed to be most orderly. Now we know better; and having seen nearly a doubling (as we think) from 1933 to 1947 in the births of a year, we can fancy that these big decisions among potential parents could have shown some erratic end products in such seemingly important times as the Civil War, the depressions of the '70s and the '90s. It is most probable that the smoothing of heaping makes as many errors as it corrects.
4. Denominator. We have vagaries in what people say about them-selves-in what others say about them-as to age, in the completeness of registration, in the mechanical smoothing by the compilers, in the meatax or more subtle approach to this correction. There are also possible mechanical errors in punching, tabulating, etc.-as I found out in some special jobs I got done. All told there is considerable doubt as to the correctness, the completeness and the extent of error generally in determining the exposure.
5. The Deaths. Many of the factors influencing the accuracy of the denominator also affect the numerator-the number of deaths. They can be overlooked, they can be handled somewhat differently than the supposedly linked exposure, and the motives for a particular answer, always made by another person than the deceased, perhaps more need analysiswhich I am not here going to give, merely to state.
6. The National Conference on Aging-August 13-15, 1950-I attended as a Population expert. One of the exhibits was a thin booklet labeled "some facts about OUR AGING POPULATION." On page 13 was a table from which $I$ am copying the headings and the last 5 lines.

## average Remaining Lifetime at Specified Ages

Death Registration States, 1901
and United States, 1948

| Age | Years Remannog |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1901 | 1948 |  |  |  |  |
|  |  | Total | White Males | White Fermales | NonWhite Males | NonWhite Females |
| 65 | 11.9 | 13.4 | 12.4 | 14.4 | 13.1 | 15.7 |
| 70. | 9.3 | 10.6 | 9.8 | 11.2 | 11.5 | 14.5 |
| 75 | 7.1 | 8.1 | 7.5 | 8.3 | 10.3 | 13.2 |
| 80. | 5.3 | 5.9 | 5.4 | 5.8 | 9.2 | 11.9 |
| 85. | 4.0 | 3.9 | 3.6 | 3.7 | 7.6 | 10.3 |

Dr. Greville's Life Table for Negro Females, 1939-41, leaving out Mongolians and miscellaneous "other races," showed 6.4 years for age 85. A gain in the somewhat noncomparable Negro Females and Non-White Females of $60 \%$ in 8 years, and a triple after-lifetime, when compared with the White Males, carries a suggestion of Lost Horizons. With avid readers of the Reader's Digest swallowing the yogurt of the Bulgars and the black-strap molasses of the Southern mammies, the sunset years should lengthen, and taking the sweet with the sour should "add life to the years."

## EDWARD A. LEW:

Mr. Bowerman has presented us with some interesting notes on a number of topics bearing on annuitant mortality. His extension of the Annuity Table for 1949 to age 0 and his modifications of that table at the younger ages appear to me to be reasonably well grounded, but I find myself in sharp disagreement with his approach to annuitant mortality at the older ages. Specifically, I would question whether Mr. Bowerman has made a case for lower death rates than those shown in the Annuity Table for 1949 at the advanced ages.

If I understand Mr. Bowerman correctly, his case rests partly on the proposition that even at the very high ages annuitant mortality rates should be lower than those shown in existing population or insured lives mortality tables; he regards this proposition as an axiom. He relies further on comparisons with death rates at ages above 95 as shown in the 1939-41 U.S. Life Tables and the British A1924-29 Table and on general considerations such as the desirability for conservatism at the older ages.

Since we do not have any reliable data as to American mortality at ages above 95 , it is impossible to test compliance with Mr. Bowerman's axiom or make any significant mortality comparisons at these ages. A review of the more important annuity tables compiled in the past indicates that there has been no general attempt to satisfy Mr. Bowerman's axiom. For instance, Dr. Hunter either did not make the test or did not regard Mr. Bowerman's proposition as an axiom at ages 96 and over, since the rates of mortality in the American Annuitants' Male Tables are higher at these ages than the white male death rates in the 1909-11 U.S. Original Registration States Tables. Mr. Robert Henderson also seems to have disregarded this axiom at ages 96 and over, in that the rates of mortality in the United States Male Annuitants' Tables exceed at these ages the white male death rates in the 1909-11 U.S. Original Registration States Tables. Messrs. Robert Henderson and J. D. Craig in constructing the Combined Annuity Table did not hesitate to adopt for males at ages 96 and over death rates which are higher than the white male rates in the

1909-11 U.S. Original Registration States Tables. The authors of the British Offices Annuitants Tables (1900-1920) and of the British Government Life Annuitants Tables (1900-1920) were similarly not deterred from adopting for males at ages 95 and over death rates which are higher than those shown in the British Life Table No. 8 (1910-12). There would seem, therefore, to be ample precedent for the Annuity Table for 1949 showing death rates which at ages 96 and over exceed those of the 1939-41 U.S. Life Tables.

The examples cited above suggest that actuaries have either not considered it worth while to make comparisons between annuitant mortality rates and population or insured lives death rates at the very old ages or have perhaps simply disregarded them, because of the general recognition that reliable death rates are not available at these ages and that the practical effect of variations in mortality at ages over 95 is usually negligible. This does not, of course, necessarily deny the good sense of Mr. Bowerman's comparison test where population or insured lives mortality rates are significant.

To the extent that Mr. Bowerman has based his argument for lower annuitant mortality rates at the older ages on the comparisons presented by him in Table F, I should say he has not proven his case that the death rates at the advanced ages in the Annuity Table for 1949 are higher than those recently experienced in the general population of the United States or among American insured lives. This is because Table F compares the mortality rates of the Annuity Table for 1949 with those in the 1939-41 U.S. Life Tables. The latter tables do not, however, purport to show actual death rates at the very old ages, as is made clear in a footnote in the official publication wherein it is stated that the mortality rates for white persons at ages above 92 were not based on actual statistics but were obtained by mathematical extrapolation from death rates at the younger ages. In the course of checking up on this matter, I wrote to Dr. T. N. E. Greville, the author of the 1939-41 U.S. Life Tables, who emphasized the point by saying: "As you know, the reported data at these ages are notoriously unreliable and I do not have at my disposal any additional data which would provide an adequate basis for making corrections in the reported figures."

Elsewhere in Table F, Mr. Bowerman compares the death rates of the Annuity Table for 1949 with those in the British Insured Lives A1924-29 Table. Although the mortality functions of the A1924-29 Table were for special purposes extended (apparently by mathematical extrapolation) through age 120, the basic experience from which this table was compiled included only nine deaths at ages 100 and over. This very small number of deaths gives an indication of the reliability of the mortality rates
shown in the A1924-29 Table at the advanced ages. It is questionable whether any conclusions can be drawn from a comparison between the Annuity Table for 1949 and the A1924-29 Table at ages over 95, if we take cognizance of the very limited significance of the A1924-29 Table death rates at ages over 95 and of the doubtful relevance of an experience among British insured lives to American annuitants.

It would seem to me that more satisfactory criteria for judging the conservatism of the mortality rates shown in the Annuity Table for 1949 at the advanced ages are to be found in the actual experience data relating to recent American death rates at the older ages. Even though such data do not yield any significant death rates at ages beyond 95 , they do give us some indication of the gradient of mortality rates in this age range and hence permit very guarded conclusions about mortality at such very old ages. In order of their relevance to the Annuity Table for 1949, I would list these actual experience data as follows:

Individual Immediate Nonrefund Annuities
Intercompany Experience between 1946 and 1948 Anniversaries by Number of Contracts-Contract Years 2 and Over

| Attained Age Group | Exposure |  |  | Actual Deaths |  |  | Ratio of actual to Expected Deates by Annuity Table for 1949 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Male | Female | Total | Male | Female | Total | Male | Female | Total |
| 80-89 | 5,774 | 17,761 | 23,535 | 690 | 1,711 | 2,401 | 101\% | $110 \%$ | 107\% |
| 90 and over. | 400 | 1,118 | 1,518 | 121 | 217 | 338 | 122 | 91 | 100 |
| Total. | 6,174 | 18,879 | 25,053 | 811 | 1,928 | 2,739 | 104\% | 108\% | 106\% |

Individual Immediate Refund Annuities
Intercompany Experience between 1946 and 1948 Anniversaries by Number of Contracts

| Atcained <br> Age Group | Exposure |  |  | Actual Deaths |  |  | Ratio of Actial to Expected Deaths by Annoity Table ror 1949 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Male | Female | Total | Male | Female | Total | Male | Female | Total |
| $80-89$. | 9,898 | 27,687 | 37,585 | 1,189 | 2,732 | 3,921 | 104\% | $112 \%$ | 109\% |
| 90 and over. | 611 | 1,950 | 2,561 | 175 | 413 | 588 | 116 | 98 |  |
| Total. | 10,509 | 29,637 | 40,146 | 1,364 | 3,145 | 4,509 | 105\% | $110 \%$ | 109\% |

## Group Annuities Matured Lives*

Intercompany Experience during 1946 and 1947 by Number of Lives

| Attadned <br> Age Group | Exposure |  |  | Actual Deates |  |  | Ratio of Actual to <br> Expected Deaths by <br> Annuty Table <br> por 1949 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Male | Female | Total | Male | Fermale | Total | Male | Female | Total |
| 81-85 | 1,898.14 | 140.16 | 2,038.30 | 264 | 22 | 286 | 124\% | 206\% | 128\% |
| 86 and over. | 472.42 | 45.50 | 517.92 | 96 | 9 | 105 | 110 | 135 | 112 |
| Total. | 2,370.56 | 185.66 | 2,556.22 | 360 | 31 | 391 | 120\% | 179\% | 123\% |

* Retired on or after normal retirement date.

> STANDARD OrDinary Insurance in Force
> 15 Years or Longer
> Intercompany Experience from 1946 to 1948 Anniversaries
> by Amounts of Insurance (In Thousand Dollar Units)

| Attained Age Group | Exposure | Actual Deaths | Ratio of Actual to Expected Deaths by Male Annuity Table for 1949 |
| :---: | :---: | :---: | :---: |
| 80-84 | 260,036 | 30,388 | 114\% |
| 85-89. | 56,368 | 9,880 | 119 |
| 90-94. | 8,376 | 2,186 | 115 |
| 95 and over | 377 | 140 | 117 |
| Total. | 325,157 | 42,594 | 115\% |

In so far as the above figures go, I believe they do indicate that the mortality rates at ages 80 and over in the Annuity Table for 1949 are reasonably conservative.

WILLIAM H. KELTON:
Mr. Bowerman is to be congratulated on the usual thoroughness with which he has proceeded with the extension of the Jenkins and Lew mortality table at both extremes to present us with a modern annuity mortality table covering the entire range of life.

His mortality rates at the infantile and juvenile ages are derived principally from Ordinary business of Industrial companies. The death rate
for age 0 is derived from experience of the first policy year on policies issued up to age six months and appears to represent that for the year following approximate age two months. The Travelers first year death rate for policies issued up to six months exposed from the anniversaries in 1944 to the anniversaries in 1949 has been about 2.4 per 1,000 , as compared with Mr. Bowerman's 4.0 for males and 3.2 for females. Our death rate is based on only 11 deaths, however, and can be given little credence as yet. It is also possible that our average age at issue may be more than two months.

Regarding the extension of the table to age 120, I agree with Mr. Bowerman that some such conservative assumption is advisable for purposes of nonparticipating premium computations where there will be no later opportunity to correct a current error in judgment. It is not necessary to be so conservative in valuing old business since valuation bases may easily be changed as experience develops. However, it is to be noted from Table G that annuity values by Mr. Bowerman's 1950 table exceed those of the Jenkins-Lew 1949 Table appreciably only for ages above 80. Hence, the additional reserves built up by using 120 as the limiting age may be sufficiently deferred and so small in volume as to cause little present concern.

A most interesting feature of Mr. Bowerman's table is his comparison of male and female death rates. The relative excesses of the former over the latter have grown surprisingly in recent years and Mr. Bowerman is the first to so forcibly bring this feature of annuity experiences to our attention. I have computed our own ratios of male to female death rates on annuities and find that they result in a curve very similar to that developed by Mr. Bowerman's tables, the principal difference in the Travelers' experience being ratios about twenty points higher between ages 50 and 70 . Our ratio of male to female mortality on annuitants exposed between the anniversaries in 1936 and 1949 was $247 \%$ for ages $50-59$ and $208 \%$ for ages $60-69$. For ages above 69 our ratios are similar to those from Mr. Bowerman's 1950 table.

The fluctuations by age in ratios of male to female death rates in Mr . Bowerman's Table B raise considerable doubt with regard to the use of a setback in age to measure female mortality. Messrs. Fassel and Noback, however, have indicated in their paper presented at this meeting that the setback device may still produce values which are sufficiently accurate for practical purposes, particularly in view of the fact that it does not appear from past experience that we can estimate future annuity mortality rates with any great degree of accuracy.

## B. FRANKLIN BLAIR:

Mr. Bowerman's very interesting paper covers a number of points bearing on the subject of the mortality of annuitants. However, I will confine my remarks to just one of these points-the mortality differential by sex.

The most striking features of the charts of the $M / F$ ratios for U.S. whites for recent years shown by Mr. Bowerman on page 89 are the two peaks at about ages 20 and 50 . The peaks are even higher in the graph on page 92 showing Metropolitan Life Industrial experience for the first nine months of 1949.

The accompanying Table 1 and chart indicate that these two peaks (and the trough between them) result mainly from variations by age in the effect of the sex differentials in the death rates for three important groups of causes of death. Deaths from violence and accidents are mainly responsible for the peak at about age 20 . The other peak, near age 50, results largely from sex differentials in death rates for circulatory diseases and to a lesser extent from differentials in death rates from violence and accidents. The trough between the two peaks is somewhat accentuated by a group of causes of death peculiar to women in the age groups from about 15 to 45-diseases of pregnancy, childbirth and the puerperium.

When the differential effects of these three causes of death are removed, the chart of the $\mathbf{M} / \mathbf{F}$ ratios loses its camel-like look and becomes somewhat concave. As shown in column (8) of my table, the ratio of the residual death rates is somewhat over $100 \%$ during childhoood and adolescence, falls to just about $100 \%$ for ages from about 20 to 40 and then rises to a plateau at about $130 \%$ for ages 55 to 74 with possibly a slight tendency to a downward turn beginning around age 65 .

My method of analysis, while not highly scientific, is a simple device for throwing light on the causes of death which are most responsible for the unusual shape of the graph of the $\mathrm{M} / \mathrm{F}$ ratio. As a result, it might be helpful in any attempt to forecast the future trends of this ratio.

TABLE 1
United States Vital Statistics for Whites--1947

| Ages | Death Rates PER 1,000 |  | Percentage of Total Deatis Resulting prom |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Diseases of Pregnancy, Childbirth and the Puerperium | Violence and Accidents |  | Diseases of the Circulatory System |  |
|  | Male $(1 A)$ | Female (1B) | Female <br> (2) | $\begin{gathered} \text { Male } \\ \text { (3A) } \end{gathered}$ | Female (3B) | Male <br> (4A) | Female <br> (4B) |
| 0. | 33.94 | 26.02 |  | 2.9 | 3.1 | 0.2 | 0.2 |
| 1-4. | 1.61 | 1.32 |  | 30.1 | 24.3 | 1.4 | 1.6 |
| 5-9 | 0.79 | 0.54 |  | 45.2 | 31.8 | 3.0 | 4.5 |
| 10-14. | 0.77 | 0.47 | 0.3 | 52.0 | 26.6 | 6.3 | 10.5 |
| 15-19. | 1.43 | 0.79 | 7.3 | 61.1 | 32.0 | 5.7 | 8.5 |
| 20-24. | 1.92 | 1.02 | 13.2 | 64.7 | 21.9 | 4.9 | 8.5 |
| 25-29. | 1.96 | 1.24 | 12.5 | 53.2 | 16.9 | 8.1 | 10.1 |
| 30-34. | 2.36 | 1.62 | 9.2 | 42.6 | 13.9 | 13.2 | 12.5 |
| 35-39 | 3.41 | 2.30 | 5.8 | 31.5 | 11.8 | 21.2 | 14.8 |
| 40-44. | 5.31 | 3.29 | 1.9 | 22.0 | 8.7 | 29.6 | 17.3 |
| 45-49. | 8.26 | 4.94 | 0.2 | 15.3 | 6.5 | 36.1 | 21.5 |
| 50-54. | 13.17 | 7.42 | * | 10.5 | 5.0 | 40.8 | 24.8 |
| 55-59. | 19.39 | 10.96 | * | 8.3 | 3.9 | 42.7 | 28.9 |
| 60-64. | 29.22 | 17.48 |  | 6.6 | 3.2 | 43.6 | 33.9 |
| 65-69. | 43.66 | 28.92 |  | 5.4 | 3.2 | 44.4 | 37.8 |
| 70-74. | 63.61 | 46.57 |  | 4.5 | 3.7 | 45.3 | 42.0 |
|  | Ratio of Male to Female Deati Rateg |  |  |  |  | Residual Deate Rates Excluding Deaths from Tefse |  |
|  | All Causes of Death | Excluding Deaths from |  |  |  |  |  |
|  |  | Pregnancy, Childbitth and the Puerperium (6) | Pregnancy, etc., Violence and Accidents (7) | Preceding Causes and Circulatory Diseases (8) |  |  |  |
|  |  |  |  |  |  | Males (9A) | Females (9B) |
| 0. | 130\% | 130\% | 131\% | 131\% |  | 32.88 | 25.18 |
| 1-4. | 122 | 122 | 113 | 112 |  | 10 | 0.98 |
| 5-9. | 146 | 146 | 116 | 121 |  | 41 | 0.34 |
| 10-14. | 164 | 164 | 109 | 110 |  | 32 | 0.29 |
| 15-19. | 181 | 195 | 117 | 115 |  | 47 | 0.41 |
| 20-24. | 188 | 217 | 103 | 102 |  | . 58 | 0.57 |
| 25-29. | 158 | 180 | 105 | 101 |  | 76 | 0.75 |
| 30-34.. | 146 | 161 | 108 | 100 |  | . 04 | 1.04 |
| 35-39. | 148 | 157 | 123 | 103 |  | 61 | 1.56 |
| 40-44. | 161 | 164 | 141 | 108 |  | 57 | 2.37 |
| 45-49. | 167 | 168 | 152 | 114 |  | . 02 | 3.54 |
| 50-54. | 177 | 177 | 167 | 123 |  | 41 | 5.21 |
| 55-59.. | 177 | 177 | 169 | 129 |  | . 51 | 7.37 |
| 60-64.. | 167 | 167 | 161 | 132 |  | 56 | 10.99 |
| 65-69.. | 151 | 151 | 148 | 129 |  | . 91 | 17.05 |
| 70-74.. | 137 | 137 | 135 | 126 |  | . 88 | 25.28 |

[^0]PERCENTAGE EXCESS OF MALE DEATH RATE OVER FEMALE
U.S. WHITES-1947

_ All causes of death.
————Excluding deaths from diseases of pregnancy, childbirth and the puerperiun.
———- Excluding deaths from violent and accidental causes and from diseases of pregnancy, childbirth and the puerperium.
.......... $=$ Residual death rates excluding deaths from preceding causes and also deaths from circulatory diseases.

## (AUTHOR'S REVIEW OF DISCUSSION)

WALTER G. BOWERMAN:
Benjamin Franklin said years ago that, after you have done something, if you cannot figure out about six good reasons why you did it, it is too bad. A similar comment can be made about anybody who writes a $100-$ page paper; if he cannot find somebody else to make a few improvements here and there, it would be very remarkable. A year ago, in this room, I expressed my tribute and appreciation to the work that Mr. Jenkins and Mr. Lew had done and in my paper I expressed my personal reluctance to have found an improvement here and there in their massive achievement. What I did was in accordance with the modern scientific method of drawing a little circle and spending one's time within that circle. I did practically nothing about the projection proposition. As I believe I said at the beginning of my paper, I was dealing with current mortality only and looking only at the 1949 table without projection.

I also did practically nothing at that time about the proposition of joint lives. I did look into the value of $\log c$ and made a brief reference to it. It would be very interesting to me to know how the Fassel-Noback progressive tables show up on the joint lives. Apparently they did not present any numerical data on that phase.

Since last June, I have done further work about the joint lives. At that time, I said I would think for what I call the 1950 table the $\log c$ of .043 and .049 would probably be all right. Since then, we have made a number of fairly extensive calculations in the New York Life and we find, instead of .043 and .049 , we get definitely better results using .040 for the male lives and .046 for the females. We tried very hard to get something which we thought was satisfactory using . 043 for each sex. Then we also tried .046 for each sex. We did not find those deviations were small enough to satisfy us. I think Fassel and Noback, if they work it out, will find their deviations on the joint lives are markedly higher than those of Jenkins and Lew. I deem it to be too large to be appropriate for practical use. Of course, that is a matter of business judgment.

The tables which have now been prepared on the 1950 basis correspond to Tables 32 to 36 of the Jenkins-Lew paper (TSA I, 455-459). Where an interest rate is involved it is at $2 \frac{1}{2} \%$.

For the 36 examples in Table III the percentage error averaged twothirds as much as in the 1949 table. The maximum percentage error was 1.6 as against 3.0 in the Jenkins-Lew Table 34. Furthermore positive and

TABLE I
Joint Life annuities for Two Lives at Equal ages ON THE 1950 TABLE AT $2 \frac{1}{2} \%$ INTEREST

| Age | Two Males $a_{x x}$ | Two Females $a_{x x}$ | Male and Female $a_{x x}$ | Age | Two Males $a_{x x}$ | Two Females $a_{x x}$ | Male and Female $a_{x x}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 30.965 | 32.285 | 31.572 | 45 | 16.761 | 19.657 | 18.064 |
| 1 | 30.997 | 32.306 | 31.598 | 46. | 16.305 | 19.230 | 17.621 |
| 2 | 30.872 | 32.204 | 31.483 | 47. | 15.851 | 18.797 | 17.176 |
| 3 | 30.700 | 32.055 | 31.322 | 48. | 15.400 | 18.360 | 16.730 |
| 4 | 30.513 | 31.891 | 31.144 | 49 | 14.951 | 17.917 | 16.284 |
| 5 | 30.313 | 31.716 | 30.955 | 50. | 14.507 | 17.470 | 15.837 |
| 6. | 30.104 | 31.531 | 30.757 | 51. | 14.067 | 17.019 | 15.391 |
| 7 | 29.884 | 31.338 | 30.549 | 52. | 13.630 | 16.562 | 14.945 |
| 8 | 29.655 | 31.138 | 30.333 | 53. | 13.199 | 16.100 | 14.500 |
| 9 | 29.416 | 30.930 | 30.108 | 54. | 12.771 | 15.634 | 14.054 |
| 10. | 29.171 | 30.716 | 29.877 | 55. | 12.347 | 15.164 | 13.609 |
| 11. | 28.918 | 30.496 | 29.639 | 56. | 11.928 | 14.691 | 13.165 |
| 12. | 28.659 | 30.270 | 29.395 | 57. | 11.512 | 14.214 | 12.722 |
| 13. | 28.395 | 30.040 | 29.146 | 58. | 11.099 | 13.735 | 12.279 |
| 14. | 28.127 | 29.804 | 28.892 | 59. | 10.690 | 13.254 | 11.838 |
| 15. | 27.855 | 29.564 | 28.635 | 60. | 10.283 | 12.773 | 11.397 |
| 16. | 27.579 | 29.320 | 28.373 | 61. | 9.878 | 12.291 | 10.958 |
| 17. | 27.300 | 29.071 | 28.107 | 62. | 9.475 | 11.809 | 10.520 |
| 18. | 27.016 | 28.817 | 27.837 | 63. | 9.076 | 11.329 | 10.084 |
| 19. | 26.727 | 28.558 | 27.560 | 64. | 8.679 | 10.851 | 9.651 |
| 20. | 26.432 | 28.293 | 27.279 | 65. | 8.288 | 10.375 | 9.222 |
| 21. | 26.131 | 28.022 | 26.991 | 66. | 7.901 | 9.903 | 8.797 |
| 22. | 25.822 | 27.745 | 26.696 | 67. | 7.519 | 9.436 | 8.378 |
| 23. | 25.507 | 27.463 | 26.396 | 68. | 7.144 | 8.974 | 7.964 |
| 24. | 25.185 | 27.175 | 26.088 | 69. | 6.776 | 8.519 | 7.557 |
| 25. | 24.855 | 26.881 | 25.774 | 70. | 6.415 | 8.071 | 7.158 |
| 26. | 24.519 | 26.580 | 25.454 | 71. | 6.063 | 7.631 | 6.766 |
| 27. | 24.174 | 26.274 | 25.126 | 72. | 5.718 | 7.199 | 6.384 |
| 28. | 23.822 | 25.961 | 24.791 | 73. | 5.384 | 6.778 | 6.010 |
| 29. | 23.462 | 25.642 | 24.449 | 74 | 5.058 | 6.367 | 5.647 |
| 30. | 23.095 | 25.316 | 24.101 | 75. | 4.743 | 5.967 | 5.295 |
| 31. | 22.720 | 24.984 | 23.745 | 76. | 4.438 | 5.579 | 4.953 |
| 32. | 22.337 | 24.646 | 23.382 | 77. | 4.144 | 5.204 | 4.623 |
| 33. | 21.948 | 24.301 | 23.012 | 78. | 3.861 | 4.842 | 4.305 |
| 34. | 21.551 | 23.949 | 22.635 |  | 3.590 | 4.494 | 3.999 |
| 35. | 21.148 | 23.591 | 22.252 | 80. | 3.329 | 4.159 | 3.706 |
| 36. | 20.737 | 23.227 | 21.862 | 81. | 3.081 | 3.839 | 3.426 |
| 37. | 20.319 | 22.856 | 21.464 | 82. | 2.843 | 3.534 | 3.158 |
| 38. | 19.894 | 22.478 | 21.060 | 83. | 2.618 | 3.244 | 2.904 |
| 39. | 19.462 | 22.094 | 20.649 | 84. | 2.405 | 2.971 | 2.664 |
| 40. | 19.023 | 21.703 | 20.232 | 85. | 2.206 | 2.715 | 2.440 |
| 41. | 18.578 | 21.306 | 19.808 | 86. | 2.024 | 2.480 | 2.234 |
| 42. | 18.128 | 20.903 | 19.378 | 87. | 1.858 | 2.265 | 2.046 |
| 43. | 17.673 | 20.493 | 18.944 | 88. | 1. 709 | 2.070 | 1.876 |
| 44. | 17.217 | 20.078 | 18.505 | 89. | 1.575 | 1.895 | 1.724 |

TABLE I-Continued

| Age | $\begin{gathered} \text { Two } \\ \text { Males } \\ c_{x x} \end{gathered}$ | Two Females $a_{x x}$ | Male and Female $a_{x x}$ | Age | $\begin{gathered} \text { Two } \\ \text { Males } \\ a_{x x} \end{gathered}$ | Two Females $a_{x x}$ | Male and Female $a_{x x}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 90. | 1.457 | 1.737 | 1.588 | 105. | 538 | . 541 | 540 |
| 91. | 1.351 | 1.596 | 1.467 | 106. | 488 | 489 | 488 |
| 92. | 1. 258 | 1.469 | 1.358 | 107 | 435 | 435 | 435 |
| 93. | 1.173 | 1.354 | 1.259 | 108. | 380 | . 379 | 379 |
| 94. | 1.097 | 1.250 | 1.171 | 109 | . 324 | . 323 | . 324 |
| 95. | 1.028 | 1.156 | 1.090 | 110. | 270 | . 269 | 270 |
| 96. | . 965 | 1.070 | 1.016 | 111. | 219 | . 218 | 219 |
| 97. | . 907 | . 992 | . 948 | 112 | 172 | . 172 | . 172 |
| 98. | . 854 | . 920 | . 886 | 113. | . 131 | . 131 | . 131 |
| 99. | . 804 | . 856 | . 830 | 114 | . 096 | . 096 | . 096 |
| 100. | . 758 | . 797 | . 777 | 115. | . 066 | . 067 | . 067 |
| 101. | . 714 | . 742 | . 728 | 116. | . 043 | . 043 | . 043 |
| 102. | . 672 | . 691 | 681 | 117. | . 025 | . 025 | 025 |
| 103. | . 629 | . 641 | . 635 | 118. | . 012 | . 012 | . 012 |
| 104. | . 585 | . 592 | . 588 | 119. | . 004 | .004 0 | . 004 |

negative errors tend to balance each other more than in the earlier table. Thus the results may be deemed satisfactory for practical purposes.

Tests were made using $\log c=.043$ for each sex and separately using $\log c=.046$ for each sex, but the deviations from the exact calculations of $a_{x x}$ were not deemed sufficiently small. Accordingly the same procedure was followed as in Tables 35 and 36 of the Jenkins-Lew paper.

The percentage errors in Table V are on the average about one-eighth smaller than those of the Jenkins-Lew Table 36; this is on the basis of their values without regard to sign. When plus and minus values are dealt with, the net total of errors in Table V below is $56 \%$ of that in the earlier paper. Thus the present proposal may be deemed satisfactory.

In my paper the single-life annuity values were shown for every fifth age only. The complete tables are now appended. Any member of the Society who wishes may upon request have the $\mathrm{N}_{x}$ and $\mathrm{D}_{x}$ columns or the corresponding joint life columns for two lives.

As a further supplement to the paper I will append a chart showing for the year 1947 the relative excess of male mortality over female for the three countries: New Zealand, Australia and the United States (Whites). These figures are in harmony with those in the paper and with the comments thereon.

Even as we meet here today, the Saturday Evening Post (November 11, 1950) is being distributed with an article telling of a marvelous treatment

TABLE II
Additions to Produce Equal ages for Two Males or Two Females for Use in Computing Joint Life Annuities on the 1950 Table
$a_{x: x+h}=a_{x+t ; x+t}$

| Difference of Age in Years $h$ | Addition to Younger Age in Years |  | Dtrference of Age in Years $h$ | Addition to Younger Age in Years |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Two Males } \\ t \\ (\log c= \\ .040) \end{gathered}$ | $\begin{gathered} \text { Two Females } \\ t \\ (\log c= \\ .046) \end{gathered}$ |  | $\begin{gathered} \text { Two Males } \\ \vdots \\ (\log c= \\ .040) \end{gathered}$ | $\left\{\begin{array}{c} \text { Two Females } \\ t \\ (\log c= \\ .046) \end{array}\right.$ |
| 1. | 512 | . 513 | 31 | 24.082 | 24.800 |
| 2. | 1.046 | 1.053 | 32 | 25.030 | 25.764 |
| 3 | 1.603 | 1.619 | 33. | 25.982 | 26.733 |
| 4 | 2.183 | 2.211 | 34. | 26.938 | 27. 705 |
| 5 | 2. 785 | 2.828 | 35 | 27.898 | 28.679 |
| 6 | 3.409 | 3.470 | 36. | 28.861 | 29.656 |
| 7 | 4.055 | 4.136 | 37 | 29.828 | 30.636 |
| 8 | 4.721 | 4.824 | 38. | 30.797 | 31.617 |
| 9. | 5.407 | 5.536 | 39 | 31.769 | 32, 601 |
| 10. | 6.113 | 6.269 | 40 | 32.744 | 33.585 |
| 11. | 6.837 | 7.021 | 41. | 33.720 | 34. 572 |
| 12 | 7.580 | 7.792 | 42. | 34.699 | 35.559 |
| 13. | 8.340 | 8.581 | 43. | 35.679 | 36.549 |
| 14. | 9.116 | 9.389 | 44. | 36.661 | 37.538 |
| 15. | 9.907 | 10.212 | 45. | 37.645 | 38.529 |
| 16 | 10.714 | 11.049 | 46 | 38.630 | 39.522 |
| 17. | 11.534 | 11.901 | 47 | 39.616 | 40.514 |
| 18 | 12.368 | 12.765 | 48 | 40.604 | 41.508 |
| 19. | 13.214 | 13.641 | 49 | 41.593 | 42.501 |
| 20. | 14.072 | 14.528 | 50. | 42.582 | 43.497 |
| 21. | 14.940 | 15.425 | 51. | 43.573 | 44.492 |
| 22. | 15.819 | 16.333 | 52. | 44.564 | 45.488 |
| 23. | 16.707 | 17.247 | 53. | 45.556 | 46.485 |
| 24. | 17.604 | 18.169 | 54. | 46.542 | 47.481 |
| 25. | 18.509 | 19.100 | 55. | 47.543 | 48.478 |
| 26. | 19.422 | 20.036 | 56. | 48.537 | 49.476 |
| 27. | 20.342 | 20.978 | 57. | 49.531 | 50.473 |
| 28. | 21.268 | 21.927 | 58. | 50.526 | 51.471 |
| 29. | 21.474 | 22.881 | 59 | 51.522 | 52,469 |
| 30. | 23.139 | 23.838 | 60. | 52.517 | 53.467 |

TABLE III
Test of Method for Obtaining Joint Life Annuities for Two Males or Two Females on 1950 Table at $2 \frac{1}{3} \%$ Interest

| Younger Life x | Older Lipe $\boldsymbol{y}$ | Two Males |  |  |  | Two Females |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Exact Value $a_{x y}$ | $\begin{gathered} \text { Approxi- } \\ \text { mate } \\ \text { Value } \\ a_{\text {wow }}(\log \\ c=.040) \end{gathered}$ | Error |  | Exact Value <br> $a_{x y}$ | Approxi- <br> mate <br> Value <br> $a_{\text {nev }}$ (log <br> $c=.046$ ) | Error |  |
|  |  |  |  | $\begin{gathered} a_{x y}- \\ a_{\text {towo }} \end{gathered}$ | $\%$ of <br> $a_{x y}$ |  |  | $\begin{gathered} a_{x y}- \\ a_{x w} \end{gathered}$ | $\%$ of <br> $a_{x y}$ |
| 25. | 35 | 22.667 | 22.677 | $-.010$ | 0.0\% | 24.896 | 24.893 | . 003 | 0.0\% |
|  | 45 | 19.419 | 19.430 | $-.011$ | 0.1 | 21.890 | 21.888 | . 002 | 0.0 |
|  | 55 | 15.530 | 15.338 | +. 192 | 1.2 | 17.970 | 17.989 | -. 019 | 0.1 |
|  | 65 | 11.381 | 11.205 | . 176 | 1.6 | 13.370 | 13.454 | -. 084 | 0.6 |
|  | 75 | 7.302 | 7.301 | . 001 | 0.0 | 8.649 | 8.748 | -. 099 | 1.1 |
|  | 85 | 3.997 | 3.998 | $-.001$ | 0.0 | 4.670 | 4.679 | $-.009$ | 0.2 |
| 35. | 45 | 18.537 | 18.527 | . 010 | 0.1 | 21.194 | 21.198 | -. 004 | 0.0 |
|  | 55 | 15.099 | 14.919 | . 180 | 1.2 | 17.652 | 17.681 | $-.029$ | 0.2 |
|  | 65 | 11.209 | 11.042 | . 167 | 1.5 | 13.240 | 13.332 | -. 092 | 0.7 |
|  | 75 | 7.248 | 7.240 | . 008 | 0.1 | 8.605 | 8.708 | $-.103$ | 1.2 |
|  | 85 | 3.982 | 3.979 | . 003 | 0.1 | 4.657 | 4.669 | $-.012$ | 0.3 |
| 45. | 55 | 14.085 | 14.018 | . 067 | 0.5 | 16.877 | 16.896 | $-.019$ | 0.1 |
|  | 65 | 10.726 | 10.661 | . 065 | 0.6 | 12.924 | 13.000 | $-.076$ | 0.6 |
|  | 75 | 7.064 | 7.093 | $-.029$ | 0.4 | 8.495 | 8.593 | -. 098 | 1.2 |
|  | 85 | 3.926 | 3.933 | $-.007$ | 0.2 | 4.626 | 4.638 | $-.012$ | 0.3 |
| 55. | 65 | 9.815 | 9.832 | $-.017$ | 0.2 | 12.133 | 12.161 | $-.028$ | 0.2 |
|  | 75 | 6.686 | 6.750 | $-.064$ | 1.0 | 8.217 | 8.282 | -. 065 | 0.8 |
|  | 85 | 3.797 | 3.823 | $-.026$ | 0.7 | 4.547 | 4.550 | $-.003$ | 0.1 |

TABLE IV
Table for Obtaining Equal Ages for One Male and One Female in Computing Joint Life Annuities on the 1950 Table

$$
a_{x y}=a_{\text {trese }}^{*}
$$

| Age in Years $x, y, w$ | Male Life 1,000b - $\mathrm{c}^{+5}$ <br> (1) | Female <br> $1,000 B$ <br> - $C^{+5}$ <br> (2) | $\begin{gathered} \text { Male and } \\ \text { Female } \\ 1,000\left(b c^{+0+5}\right. \\ \left.+B C^{+5}\right) \end{gathered}$ <br> (3) |
| :---: | :---: | :---: | :---: |
| 0. | . 04913 | . 01274 | . 06187 |
| 1. | . 05387 | . 01416 | . 06803 |
| 2. | . 05907 | . 01574 | . 07481 |
| 3. | . 06477 | . 01750 | . 08227 |
| 4. | . 07102 | . 01946 | . 09048 |
| 5. | . 07787 | . 02163 | . 09950 |
| 6. | . 08538 | . 02405 | . 10943 |
| 7 | . 09362 | . 02673 | . 12035 |
| 8. | . 10265 | . 02972 | . 13237 |
|  | . 11255 | . 03304 | . 14559 |
| 10. | . 12341 | . 03673 | . 16014 |
| 11. | 13532 | . 04084 | 17616 |
| 12. | . 14838 | . 04540 | . 19378 |
| 13. | . 16269 | . 05047 | . 21316 |
| 14. | . 17839 | . 05611 | . 23450 |
| 15. | . 19560 | . 06238 | . 25798 |
| 16. | . 21447 | . 06935 | . 28382 |
| 17. | . 23516 | . 07710 | . 31226 |
| 18. | . 25785 | . 08572 | . 34357 |
| 19. | . 28272 | . 09529 | . 37801 |
| 20. | . 31000 | . 10594 | . 41594 |
| 21. | . 33991 | . 11778 | . 45769 |
| 22. | . 37270 | . 13094 | . 50364 |
|  | . 40866 | . 14557 | . 55423 |
| 24. | . 44809 | . 16183 | . 60992 |
| 25. | . 49132 | . 17991 | . 67123 |
| 26. | . 53872 | .20001 | . 73873 |
| 27. | . 59069 | . 22236 | . 81305 |
| 28. | . 64768 | . 24721 | . 89489 |
| 29. | . 71017 | . 27483 | . 98500 |
| 30. | . 77868 | . 30554 | 1.08422 |
| 31. | . 85381 | . 33966 | 1.19347 |
| 32. | . 93619 | .37763 | 1.31382 |
| 33. | 1.02651 | . 41982 | 1.44633 |
| 34. | 1. 12554 | . 46673 | 1. 59227 |
| 35. | 1.23413 | . 51887 | 1.75300 |

[^1]TABLE IV-Continued

| Age in Years $x, y, w$ | $\begin{gathered} \text { Male Life } \\ 1,000 b \\ \cdot \sigma^{*+b} \end{gathered}$ <br> (1) | Female <br> $1,000 B$ <br> - $C^{y+5}$ <br> (2) | Male and Female $1,000\left(6 c^{0+5}\right.$ $\left.+B C^{w+\sigma}\right)$ <br> (3) |
| :---: | :---: | :---: | :---: |
| 36. | 1.35320 | 57685 | 1.93005 |
| 37. | 1.48375 | . 64130 | 2.12505 |
| 38 | 1.62690 | . 71295 | 2.33985 |
| 39 | 1.78386 | . 79261 | 2.57647 |
| 40. | 1.95597 | . 88117 | 2.83714 |
| 41. | 2.14468 | . 97963 | 3.12431 |
| 42. | 2.35159 | 1.08908 | 3.44067 |
| 43 | 2.57847 | 1.21077 | 3.78924 |
| 44. | 2.82723 | 1.34605 | 4.17328 |
| 45. | 3.10000 | 1.49645 | 4.59645 |
| 46 | 3.39908 | 1.66365 | 5.06273 |
| 47 | 3.72702 | 1.84953 | 5.57655 |
| 48 | 4.08659 | 2.05618 | 6.14277 |
| 49. | 4.48086 | 2.28592 | 6.76678 |
| 50. | 4.91317 | 2.54133 | 7.45450 |
| 51 | 5.38718 | 2.82528 | 8.21246 |
| 52. | 5.90693 | 3.14095 | 9.04788 |
| 53. | 6.47682 | 3.49190 | 9.96872 |
| 54. | 7.10169 | 3.88205 | 10.98374 |
| 55. | 7.78685 | 4.31580 | 12.10265 |
| 56. | 8.53811 | 4.79801 | 13.31612 |
| 57. | 9.36185 | 5.33410 | 14.69595 |
| 58. | 10.26506 | 5.93009 | 16.19515 |
| 59. | 11.25542 | 6.59267 | 17.84809 |
| 60. | 12.34132 | 7.32928 | 19.67060 |
| 61. | 13.53199 | 8.14819 | 21.68018 |
| 62. | 14.83753 | 9.05860 | 23.89613 |
| 63. | 16.26903 | 10.07074 | 26.33977 |
| 64. | 17.83864 | 11.19596 | 29.03460 |
| 65. | 19.55968 | 12.44690 | 32.00658 |
| 66. | 21.44676 | 13.83762 | 35.28438 |
| 67. | 23.51591 | 15.38372 | 38.89963 |
| 68. | 25.78468 | 17.10257 | 42.88725 |
| 69. | 28.27234 | 19.01347 | 47.28581 |
| 70. | 31.00000 | 21.13787 | 52.13787 |
| 71. | 33.99082 | 23.49964 | 57.49046 |
| 72. | 37.27020 | 26.12530 | 63.39550 |
| 73. | 40.86596 | 29.04433 | 69.91029 |
| 74. | 44.80863 | 32.28950 | 77.09813 |
| 75. | 49.13169 | 35.89726 | 85.02895 |
| 76. | 53.87183 | 39.90812 | 93.77995 |
| 77. | 59.06928 | 44.36713 | 103.43641 |
| 78. | 64.76818 | 49.32434 | 114.09252 |
| 79. | 71.01690 | 54.83544 | 125.85234 |
| 80. | 77.86846 | 60.96229 | 138.83075 |

TABLE IV-Continued

| Age in Years $x, y, w$ | Male Life <br> 1,000b - $c^{x+5}$ <br> (1) | $\begin{aligned} & \text { Female } \\ & 1,000 B \\ & \cdot C^{y+b} \end{aligned}$ <br> (2) | Male and Female 1,000 ( $b c^{*+5}$ $\left.+B C^{\infty+6}\right)$ <br> (3) |
| :---: | :---: | :---: | :---: |
| 81 | 85.38109 | 67.77372 | 153.15481 |
| 82 | 93.61850 | 75.34619 | 168.96469 |
| 83 | 102.65065 | 83.76475 | 186.41540 |
| 84 | 112.55420 | 93.12393 | 205.67813 |
| 85 | 123.41322 | 103.52883 | 226.94205 |
| 86. | 135.31991 | 115.09628 | 250.41619 |
| 87 | 148.37533 | 127.95619 | 276.33152 |
| 88. | 162.69031 | 142.25296 | 304.94327 |
| 89 | 178.38638 | 158.14712 | 336.53350 |
| 90. | 195.59678 | 175.81718 | 371.41396 |
| 91. | 214.46760 | 195.46153 | 409.92913 |
| 92 | 235.15905 | 217.30079 | 452.45984 |
| 93. | 257.84677 | 241.58018 | 499.42695 |
| 94 | 282.72336 | 268.57235 | 551.29571 |
| 95. | 310.00000 | 298.58040 | 608.58040 |
| 96. | 339.90825 | 331.94131 | 671.84956 |
| 97 | 372.70198 | 369.02968 | 741.73166 |
| 98. | 408.65959 | 410.26201 | 818.92160 |
| 99. | 448.08634 | 456.10129 | 904.18763 |
| 100. | 491.31690 | 507.06227 | 998.37917 |

TABLE V
Test of Method for Obtaining Joint Life Annuities for One Male and One Female on 1950 Table at $2 \frac{1}{2} \%$ InTEREST

| Male <br> Ace <br> $\boldsymbol{x}$ | $\begin{gathered} \text { Fe- } \\ \text { MALE } \\ \text { Age } \\ y \end{gathered}$ | Exact Valde $a_{x y}$ | Approxi- <br> mate <br> Value <br> $a_{\text {now }}$ | Error |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{gathered} a_{x y}- \\ a_{u v w} \end{gathered}$ | $\begin{gathered} \% \text { of } \\ a_{x y} \end{gathered}$ |
| 25 | 35 | 24.144 | 24.361 | $-.217$ | -0.9\% |
|  | 45 | 21.479 | 21.744 | $-.265$ | $-1.2$ |
|  | 55 | 17.790 | 17.864 | $-.074$ | $-0.4$ |
|  | 65 | 13.370 | 13.303 | $+.067$ | +0.5 |
|  | 75 | 8.626 | 8.669 | $-.043$ | -0.5 |
|  | 85 | 4.663 | 4.605 | $+.058$ | +1.2 |
| 35. | 25 | 23.243 | 23.102 | . 141 | 0.6 |
|  | 45 | 20.300 | 20.403 | $-.103$ | -0.5 |
|  | 55 | 17.185 | 17.199 | $-.014$ | -0.1 |
|  | 65 | 13.057 | 13.048 | $+.009$ | $+0.1$ |
|  | 75 | 8.549 | 8.583 | $-.034$ | $-0.4$ |
|  | 85 | 4.644 | 4.584 | $+.060$ | +1.3 |
| 45. | 25 | 19.712 | 19.596 | . 116 | 0.6 |
|  | 35 | 19.212 | 19.156 | . 056 | 0.3 |
|  | 55 | 15.834 | 15.862 | -. 028 | -0.2 |
|  | 65 | 12.398 | 12.470 | $-.072$ | -0.6 |
|  | 75 | 8.297 | 8.368 | $-.071$ | -0.9 |
|  | 85 | 4.570 | 4.528 | $+.042$ | +0.9 |
| 55. | 25 | 15.654 | 15.536 | . 118 | 0.8 |
|  | 35 | 15.430 | 15.341 | . 089 | 0.6 |
|  | 45 | 14.879 | 14.831 | . 048 | 0.3 |
|  | 65 | 11.184 | 11.274 | $-.090$ | -0.8 |
|  | 75 | 7.791 | 7.890 | $-.099$ | $-1.3$ |
|  | 85 | 4.404 | 4.388 | $+.016$ | +0.4 |
| 65. | 25 | 11.427 | 11.382 | . 045 | 0.4 |
|  | 35 | 11.336 | 11.308 | . 028 | 0.2 |
|  | 45 | 11.111 | 11.095 | . 016 | 0.1 |
|  | 55 | 10.544 | 10.524 | . 020 | 0.2 |
|  | 75 | 6.879 | 6.915 | $-.036$ | -0.5 |
|  | 85 | 4.100 | 4.071 | +.029 | +0.7 |
| 75. | 25 | 7.319 | 7.390 | $-.071$ | -1.0 |
|  | 35 | 7.287 | 7.363 | $-.076$ | $-1.0$ |
|  | 45 | 7.208 | 7.282 | $-.074$ | $-1.0$ |
|  | 55 | 7.007 | 7.062 | $-.055$ | -0.8 |
|  | 65 | 6.475 | 6.502 | $-.027$ | -0.4 |
|  | 85 | 3.467 | 3.436 | $+.031$ | +0.9 |
| 85. | 25 | 4.002 | 4.058 | $-.056$ | -1.4 |
|  | 35 | 3.992 | 4.049 | $-.057$ | -1.4 |
|  | 45 | 3.968 | 4.023 | $-.055$ | -1.4 |
|  | 55 | 3.908 | 3.957 | $-.049$ | $-1.3$ |
|  | 65 | 3.740 | 3.773 | $-.033$ | $-0.9$ |
|  | 75 | 3.305 | 3.322 | $-.017$ | -0.5 |

TABLE VI
Annuity Values on 1950 Table at $2 \frac{1}{2} \%$ Interest

| Age <br> $x$ | $\begin{gathered} \text { Male } \\ a_{x} \end{gathered}$ | Female $a_{x}$ | $\underset{x}{\text { Age }}$ | $\begin{gathered} \text { Male } \\ a_{x} \end{gathered}$ | $\begin{gathered} \text { Female } \\ a_{x} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0. | 32.699 | 33.679 | 45 | 20.119 | 22.450 |
| 1 | 32.652 | 33.632 | 46. | 19.697 | 22.057 |
| 2. | 32.521 | 33.520 | 47. | 19.273 | 21.659 |
| 3. | 32.364 | 33.382 | 48 | 18.848 | 21.253 |
| 4. | 32.197 | 33.234 | 49. | 18.421 | 20.842 |
| 5. | 32.021 | 33.079 | 50. | 17.993 | 20.424 |
| 6. | 31.839 | 32.918 | 51. | 17.564 | 19.999 |
| 7 | 31.650 | 32.751 | 52. | 17.136 | 19.569 |
| 8 | 31.454 | 32.578 | 53. | 16.706 | 19.131 |
| 9 | 31.251 | 32.400 | 54. | 16.277 | 18.687 |
| 10. | 31.042 | 32.216 | 55. | 15.847 | 18.237 |
| 11. | 30.828 | 32.028 | 56. | 15.416 | 17.782 |
| 12. | 30.608 | 31.835 | 57. | 14.985 | 17.321 |
| 13. | 30.384 | 31.638 | 58. | 14.554 | 16.854 |
| 14. | 30.155 | 31.436 | 59. | 14.121 | 16.383 |
| 15. | 29.922 | 31.229 | 60. | 13.688 | 15.908 |
| 16 | 29.685 | 31.019 | 61. | 13.253 | 15.429 |
| 17. | 29.444 | 30.804 | 62. | 12.818 | 14.947 |
| 18. | 29.198 | 30.584 | 63. | 12.382 | 14.462 |
| 19. | 28.947 | 30.360 | 64. | 11.946 | 13.975 |
| 20. | 28.691 | 30.130 | 65. | 11.511 | 13.486 |
| 21. | 28.429 | 29.895 | 66. | 11.077 | 12.997 |
| 22. | 28.161 | 29.654 | 67. | 10.646 | 12.508 |
| 23. | 27.886 | 29.408 | 68 | 10.217 | 12.019 |
| 24. | 27.605 | 29.157 | 69. | 9.791 | 11.532 |
| 25. | 27.318 | 28.900 | 70. | 9.370 | 11.048 |
| 26. | 27.023 | 28.638 | 71. | 8.954 | 10.567 |
| 27. | 26.723 | 28.369 | 72. | 8.543 | 10.089 |
| 28. | 26.415 | 28.095 | 73. | 8.138 | 9.617 |
| 29. | 26.100 | 27.814 | 74. | 7.740 | 9.151 |
| 30. | 25.778 | 27.528 | 75. | 7.350 | 8.691 |
| 31. | 25.449 | 27.235 | 76. | 6.968 | 8.240 |
| 32. | 25.113 | 26.936 | 77. | 6.595 | 7.797 |
| 33. | 24.770 | 26.631 | 78. | 6.231 | 7.364 |
| 34. | 24.420 | 26.319 |  | 5.878 | 6.941 |
| 35. | 24.063 | 26.001 | 80. | 5.536 | 6.530 |
| 36. | 23.699 | 25.676 |  | 5.205 | 6.131 |
| 37. | 23.328 | 25.344 | 82. | 4.885 | 5.745 |
| 38. | 22.949 | 25.006 | 83. | 4.579 | 5.374 |
| 39. | 22.564 | 24.661 | 84. | 4.287 | 5.019 |
| 40. | 22.171 | 24.310 | 85. | 4.012 | 4.683 |
| 41. | 21.772 | 23.951 | 86. | 3.755 | 4.367 |
| 42. | 21.366 | 23.586 | 87. | 3.518 | 4.073 |
| 43. | 20.954 | 23.214 | 88. | 3.300 | 3.801 |
| 44. | 20.539 | 22.835 | 89. | 3.102 | 3.551 |

TABLE VI-Continued

| ${ }_{x}^{\text {Age }}$ | $\underset{a_{x}}{\text { Male }}$ | Female <br> $a_{x}$ | $\underset{\sim}{\text { Age }}$ | $\underset{a_{\mathbf{z}}}{\text { Male }}$ | Female $a_{x}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 90. | 2.922 | 3.321 | 105 | 1.315 | 1.319 |
| 91 | 2.759 | 3.110 | 106 | 1.214 | 1.215 |
| 92 | 2.610 | 2.917 | 107. | 1.107 | 1.107 |
| 93 | 2.474 | 2.738 | 108 | . 997 | . 996 |
| 94. | 2.349 | 2.573 | 109. | . 886 | . 885 |
| 95. | 2.233 | 2.421 | 110. | . 776 | . 775 |
| 96. | 2.125 | 2.280 | 111. | . 669 | . 669 |
| 97. | 2.024 | 2.149 | 112. | 568 | . 568 |
| 98. | 1.929 | 2.028 | 113. | . 474 | . 474 |
| 99. | 1.840 | 1.916 | 114 | . 387 | . 387 |
| 100. | 1.754 | 1.810 | 115. | . 309 | . 309 |
| 101. | 1.670 | 1.710 | 116 | 237 | 238 |
| 102 | 1.586 | 1.613 | 117 | . 173 | 174 |
| 103. | 1.500 | 1.517 | 118. | . 116 | 116 |
| 104. | 1.410 | 1.420 | 119. | . 063 | .063 |
|  |  |  | 120 | 0 | 0 |

for apoplectic stroke. It gives a new lease on life to people who have had paralysis from this cause. The Metropolitan Life's Statistical Bulletin (September, 1950) shows that death rates from pneumonia ("the old man's friend") have decreased by $80 \%$ in the last 20 years, due largely to the use of the sulfa drugs, penicillin, aureomycin, etc. Every week some news of great promise and practical import is presented dealing with beneficial treatment of arthritis, rheumatic heart, diabetes, and even cancer. Yet to most of us age 90 seems a long way off, and what happens after true age 100 is shrouded in dark mystery. There are some who believe that the almost continuous wars upon which we are now embarked will lead to a revival of Spartan spirit and mores, including optional euthanasia for all over 90 , say. That seems harsh today, but is very moderate compared to the dictum of Anthony Trollope that "it might be a good thing if all were peacefully chloroformed at age sixty."* The thirty year interval between sixty and ninety is a measure of how far longevity has advanced during the first half of the twentieth century.

[^2]PERCENTAGE EXCESS OF MALE DEATH RATE OVER FEMALE-1947




[^0]:    *Less than 0.05\%.

[^1]:    * To compute $a_{x y}$ on a male life age $x$ and female life $y$, add the respective data from columas (1) and (2) and use the sum to enter col. (3) and determine the age $w$ therefrom. In this table $1,000 b=031$; $1,000 B=.0075 ; \log c=.040$ (male); $\log C=.046$ (female). The extent of the errors from use of this approximate method appears in the next table.

[^2]:    * It was a jesting reference to this from The Fixed Period and Sir William Osler's statement that "men are useless above sixty years of age and should stop work then" (1905) which caused Osler (1849-1919) to be headlined as the advocate of chloroform after sixty and the enemy of old age.

