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Measuring the Rate of Retirement in an Interval Beginning with an Eligibility Change Point

by William H. Blake, Jr.

The exposure theory that I studied some 30 years ago was similar to that described in Chapter 6 of Dick London's *Survival Models and Their Estimation*, the SOA's current textbook on the subject—the ratio of the observed number of deaths to the scheduled exposure in an age interval is a estimate of the rate of mortality and the ratio to the exact exposure is an estimate of the central death rate, which then can be converted to an estimate of the mortality rate. In most instances, the estimate of q_x assumes that terminations are distributed over the interval of age. The purpose of this article is to call attention to the heaping of retirements that can occur following a point at which eligibility requirements change and to illustrate that the traditional estimators generally overstate the rate of retirement in the unit interval beginning at such a point.

Heaping at Initial Eligibility

The example I have chosen is taken from the retirement experience of a particular plan for a recent calendar year. The plan provides that an employee must have attained age 55 and completed 10 years of vesting service in order to retire prior to the normal retirement age of 65. Retirements occur on the first day of a calendar month. For the year in question, 80 employees elected to retire at age 55 from among 187 who were eligible at some time during the year. Table 1 on page 14 shows the distribution of retirements by month of eligibility. Note that 32% occurred in the first month and 52% occurred by the third month of eligibility.

Because retirements occur at 12 discrete points during the year, it is easy to calculate the rate based on the number exiting each month relative to the number then eligible. In our example, 115 participants reached their first month of eligibility during the year and of these 26 retired, so the first eligibility month retirement rate for the observation year is 0.226. The complement of the retirement rate is the survival rate and the product of these for the 12 months is the complement of the retirement rate at age 55 for those eligible during the year, 0.637 in this case. London calls this the product limit estimator and describes its properties in Section 7.6 of his book. In particular, the estimator is unbiased and does not depend on the distribution of retirements over the interval.

For the scheduled exposure method, we increase the number entering each month of age by the number that would have entered except for the fact that they retired in an earlier month. The number entering the first month remains unchanged. For the second month, 25 of the 26 retirements that occurred in the first month remain under age 56 and so are added to the 100 employees active in the second month of eligibility sometime during the observation year. Continuing thus, we find a total of 1,323 employee-months of scheduled exposure at age 55 during the observation year. This yields an estimated rate of retirement of 0.726, noticeably higher than the 0.637 that is our unbiased estimate.

For the exact exposure method, we use the number entering each month of age without adjustment. Relating the 80 retirements to the 890 employee-months of exposure at age 55 yields 1.079 as an estimate of the central retirement rate. The estimated rate of retirement is 0.701 if the central rate is converted assuming a linear distribution for l_x over the age interval and 0.660 if the central rate is converted assuming an exponential distribution. Both of these are higher than 0.637, but the exponential estimate is closer. Starting with the linear estimate but making an ad hoc adjustment for the average month of retirement yields an estimate of 0.619.

Anniversary to Anniversary Measures

Using the calendar year as the observation period means that the experience at an age, 55 in our example, consists of the latter part of a year of age for one cohort and the first part of a year of age for a second cohort. The scheduled exposure estimate, at least, should be better if we measure experience with respect to a single cohort between the 55th birthday and the 56th birthday for the respective members, because the exposure calculated on this basis is less sensitive to the distribution of retirements over the age interval.

Table 2 on page 15 shows the retirement experience for the 120 employees covered by the plan under discussion who turned age 55 in the calendar year of observation until they reached age 56 in the following year. Forty-seven retired, including 25 in the first month of eligibility. The product limit estimate for the retirement rate is 0.394. The scheduled exposure estimate is 0.400. All of the lives that enter the year of age are exposed for a full year except for one death in the fourth month and four employees who had less than 10 years of service at age 55. The exact exposure estimate of m_x yields several different estimates of q_x, depending on what is assumed for the distribution of l_x over the age interval.

Heaping at Other Points

The plan under discussion provides a flat dollar benefit for each year of service up to a maximum of 30 years. Benefits commencing prior to age 62 are reduced for early retirement. Therefore, we might expect to find a concentration of retirements immediately following either age 62 or 30 years of service.

To test for heaping, I tabulated the number of retirements from active employment over a four-year period by completed years and months of age and by completed years and months of credited service as of the benefit commencement date. The service distribution had no noticeable heaping of retirements by month at 30 years or any other point.

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TABLE 1
Estimated Probability of Retiring at Age 55
Based on Experience for One Plan during One Calendar Year

	Month of Eligibility Measured from the Later of Age 55 and 10 Years of Service												
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Total**
Distribution of Retirements													
No. of Retirements	26	7	9	10	6	5	5	2	3	2	1	4	80
Cumulative Percentage	0.325	0.413	0.525	0.650	0.725	0.788	0.850	0.875	0.913	0.938	0.950	1.000	
Avg. Month of Retirement													4.050
Product Limit Estimate													
No. Entering Month	115	100	96	83	80	70	68	62	58	57	54	47	890
No. Retiring in Month	26	7	9	10	6	5	5	2	3	2	1	4	80
Proportion Retiring in Month	0.226	0.070	0.094	0.120	0.075	0.071	0.074	0.032	0.052	0.035	0.019	0.085	
Cumulative Survival Probabillity	0.774	0.720	0.652	0.574	0.531	0.493	0.457	0.442	0.419	0.404	0.397	0.363	
Probability of Retirement	1 – 0.363 = 0.637												
Schedule Exposure Estimate													
No. Scheduled to Enter Month	115	125	127	120	123	117	115	109	102	98	90	82	1,323
No. Retiring in Month	26	7	9	10	6	5	5	2	3	2	1	4	80
Probability of Retirement	12 × 80/1,323 = 0.726												
Exact Exposure Estimates													
No. Entering Month	115	100	96	83	80	70	68	62	58	57	54	47	890
No. Retiring in Month	26	7	9	10	6	5	5	2	3	2	1	4	80
Central Rate of Retirement	12 × 80/890 = 1.079												
Probability of Retirement Assuming:													
Linear Distribution for l_x	1.079/(1 + 1.079/2) = 0.701												
Exponential Distribution for l_x	1 – exp (–1.079) = 0.660												
Adjusting for Avg. Month of Retirement	1.079/(1 + 1.079 × (1 – 4.050/13)) = 0.619												

TABLE 2
Estimated Probability of Retiring at Age 55
Based on Experience for One Plan Between Birthdays in Succeeding Calendar Years

	Month of Eligibility Measured from the Later of Age 55 and 10 Years of Service												
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Total**
Distribution of Retirements													
No. of Retirements	25	7	7	3	0	1	0	0	2	2	0	0	47
Cumulative Percentage	0.532	0.681	0.830	0.894	0.894	0.915	0.915	0.915	0.957	1.000	1.000	1.000	
Avg. Month of Retirement													2.468
Product Limit Estimate													
No. Entering Month	120	95	88	79	76	76	74	74	73	71	68	68	962
No. Retiring in Month	25	7	7	3	0	1	0	0	2	2	0	0	47
Proportion Retiring in Month	0.208	0.074	0.080	0.038	0.000	0.013	0.000	0.000	0.027	0.028	0.000	0.000	
Cumulative Survival Probabillity	0.792	0.73	0.675	0.649	0.649	0.641	0.641	0.641	0.623	0.606	0.606	0.606	
Probability of Retirement	1 – .606 = 0.394												
Schedule Exposure Estimate													
No. Scheduled to Enter Month	120	120	120	118	118	118	117	117	116	116	115	115	1,410
No. Retiring in Month	25	7	7	3	0	1	0	0	2	2	0	0	47
Probability of Retirement	12 × 47/1,410 = 0.400												
Exact Exposure Estimates													
No. Entering Month	120	95	88	79	76	76	74	74	73	71	68	68	962
No. Retiring in Month	25	7	7	3	0	1	0	0	2	2	0	0	47
Central Rate of Retirement	12 × 47/962 = 0.586												
Probability of Retirement Assuming:													
Linear Distribution for l_x	0.586/(1 + 0.586/2) = 0.453												
Exponential Distribution for l_x	1 – exp (0.586) = 0.444												
Adjusting for Avg. Month of Retirement	0.586/(1 + 0.586 × (1 – 2.468/13)) = 0.397												

Measuring the Rate of Retirement
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The distribution at age 55 was similar to what we have already seen in Table 1. Above age 55, there was some concentration of retirements in the early months of age. At age 60 and above, the observations were too scattered to show any pattern. Combining all ages over 55, 12% of retirements occurred in the first month of age, 34% occurred in the first three months of age, and 62% occurred in the first six months of age. Of course, the number of retirements in the earlier months of age would be greater than the number in the later months if the force of retirement is constant over the interval.

I also tabulated retirements for a plan that allows early retirement after 30 years of credited service regardless of age. During the same four-year period, 48% of the retirements at 30 years occurred in the first month of eligibility, 66% in the first three months and 78% in the first six months.

Summary

An employee must satisfy certain requirements set forth in the plan in order to retire or to qualify for enhanced benefits. This can lead to a concentration of retirements at or immediately after age and/or service combinations at which eligibility requirements change. The heaping within the interval of age or service may invalidate the assumptions underlying some of the commonly used exposure formulas. Constructing rates based on months measured from each eligibility change point provides an unbiased estimate of the retirement rate. Using scheduled exposure appears to work better if the observation period extends from anniversary to anniversary than if it is defined in terms of calendar years. Using exact exposure requires an assumption for the distribution of retirements over the interval that is reasonably related to the experience.

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Randolph's Bonanza Bigger than Expected

by M.D. Drysdale

Editor's Note: *The following article originally appeared in* The Herald of Randolph *(Vermont) on August 21, 1997 and is reprinted here with permission.*

The Vermont State Retirement Board, meeting today, is expected to vote to reimburse the town of Randolph $431,145 for years of overpayments into the state retirement system.

The repayment is even more than Randolph officials hoped in May, when the Retirement Board agreed in principle that Randolph was owed the money.

At that time, estimates were that Randolph would receive $232,000 to $400,000. The passage of another fiscal year and some other findings brought the amount owed even higher, according to Town Manager Gwen Hallsmith.

"They topped our highest estimate," she declared. It was Hallsmith who discovered the systematic overpayments. Hiring an actuary on behalf of the town, she was able to convince skeptical state officials that Randolph was owed substantial payment.

The payments will come in the form of credits of $44,000 a year for 20 years. That totals $880,000, a figure which includes interest for the subsequent years.

In addition, Randolph will see a huge difference in the rate it pays in the future for being part of the state retirement plan. Last year, Hallsmith said, Randolph had to pay a whopping 14.5% of payroll into the retirement plan. Next year the town will pay only 8.2%.

With the first of the $44,000 credits, retirement payments will be only about $10,000 to $15,000, compared to the $111,728 that was paid last year, she estimated.

Bethel, Too

In Bethel, Town Manager Del Cloud said an actuary has just completed a study of that town's retirement payments over the years.

Bethel has been charged even a higher rate—15.34%—than Randolph, and the state has acknowledged that it too should get some money back.

Bethel's total retirement payments were about $30,000 last year. That annual rate should be cut almost in half if Bethel is allowed to use the state's rate of 8.2%.

Now that he's got the numbers, he is ready to "broach the subject" with the Retirement Board, Cloud said. "It shouldn't take too long. Randolph has established the methodology."

30-Year History

The state has been requiring Randolph and Bethel to pay a separate rate for retirement benefits ever since the two towns joined the retirement system in 1968. Only three towns are part of the state system.

Research by Hallsmith, however, indicated that since 1975 the state had performed no separate actuarial studies that would justify the towns paying a higher rate.

The state was at first reluctant to admit a mistake had been made, but after Randolph hired both an attorney and an actuary, the treasurer's office began to see the light.

Employee to Benefit

In a related matter Tuesday night, selectmen voted health benefits to the former town employee whose plight brought the entire retirement snafu to light.

Larry Haraden took early retirement last year from the town crew because of a health problem, relying on assurances from the state retirement policy that he would receive health insurance that would take care of some serious health problems, Hallsmith explained.

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