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MINIMUM STATUTORY NONFORFEITURE VALUES FOR RETIREMENT ANNUITY CONTRACTS

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GARDNER F. KNIGHT:

Our thanks are due the author for his excellent presentation of a subject which has hitherto received little formal attention.

We must not overlook, however, that the author, a strictly honest man, has reminded us in his conclusion that his remarks are predicated on, and limited to, a very definite type of deferred annuity contract. In particular, he obviously has in mind a contract which, per $100 of gross annual premium, has cash values based upon the accumulation of a net first year's premium in the amount of $63 and net level renewal premiums in the amount of $89. Such contracts fit in with his conclusions provided they run for a minimum term of ten years and are not issued above age 60. These conditions are generally met by the companies issuing the type of Retirement Annuity under discussion.

Where such contracts are used in connection with Pension Trust agreements it frequently becomes necessary to issue for shorter terms than ten years, and at the older ages. Under such circumstances it will be found that the predetermined scale of cash values will not meet the minimum cash value requirements of the New York law. In such cases the decision of the issuing company will probably be to use cash values based on the mortality table and with a level renewal premium, varying by issue age and duration, and a first year premium 80% of that amount where the deferred period is five or less years, or $(90 - 2n)\%$ thereof where the n-year deferred period is greater than five years. As a practical matter this reduction in first year premium may be limited to the ratio 63:89 which is normal for durations of ten years and over.

This is acceptable under the New York law. But if I may be pardoned for the heretical expression that the New York statutes are not all-controlling in the insurance industry, it must be pointed out that the cash surrender values brought out under the New York statute do not meet the minimum requirements of all states. In particular, domestic companies in Massachusetts are required under Section 144, Subsection 9, Chapter 175 G.L., to provide under every deferred annuity contract other than a single

premium contract that "in the event of the nonpayment of any premium after three full years' premiums have been paid" it shall "be converted into a paid-up annuity for such proportion of the original annuity as the number of completed years' premiums paid bears to the total number of premiums required under the contract." Where the usual paid-up cash value at retirement is obtained by accumulating at interest the cash value at date of default to the retirement date, it will be found that for the shorter durations from issue to retirement the minimum cash values according to the New York law may not meet the Massachusetts minimums. In particular, where the cash value at retirement is equal to the standard cash value on the \$63/\$89 scale of premiums, the third year Massachusetts minimum cash value will be higher than that provided by the New York scale. It will be necessary, therefore, to base the table of cash values in such cases on a renewal premium fixed at a level to provide such minimum third year cash values, setting the first year premium at the level required to bring out the cash value at the retirement age. For such contracts, if the third year value meets the minimum, subsequent values will also comply.

Let

$(\mathrm{PV})_{x+t}$ = present value of contract benefits according to the mortality table and interest assumption

$(\mathrm{MCV})_{x+t}$ = required minimum cash value (generally $t = 3$)

$(\mathrm{MV})_n$ = maturity cash value

P_x^a = desired adjusted stipulated (renewal) premium

P_x^1 = desired first year premium

n = years from issue to retirement date

Then

$$(\mathrm{MCV})_{x+t} = \frac{t}{n} \cdot v^{n-t} (\mathrm{MV})_n$$

(Note that for an assigned value of $(\mathrm{MV})_n$ $(\mathrm{MCV})_{x+t}$ does not vary with age at issue x)

$$\mathrm{P}_x^a = \frac{(\mathrm{PV})_{x+t} - (\mathrm{MCV})_{x+t}}{\ddot{a}_{x+t:\overline{n-t}|}}$$

$$\mathrm{P}_x^1 = (\mathrm{PV})_x - \mathrm{P}_x^a \cdot a_{x:\overline{n-1}|}.$$

This point, already known to those who are issuing such contracts, should be noted in the discussion of a paper otherwise so complete.

HARRY WALKER:

Mr. Boermeester's paper considers the calculation of minimum nonforfeiture values under Section 208-a of the New York Insurance Law in the case of a Retirement Annuity contract under which contractual values are based on the 1937 Standard Annuity Mortality Table. It may be of interest to the members of the Society if I outline the analysis made by the Equitable to prove compliance with Section 208-a in the case of our current issues of Retirement Annuity contracts under which contractual values are based on the ELAS Life Income Mortality Table, described in my paper submitted at the meeting of the Society last April.

The form of contract we issue involves level gross annual premiums, optional retirement ages, and death benefits equal to the sum of the gross premiums paid or the cash value, whichever is greater, and provides for a 10 years certain life income at the optional retirement ages with the income based on the actuarial equivalent of the cash value, using interest at $2\frac{1}{4}\%$ for the annuity certain portion of the settlement and the ELAS Life Income Mortality Table with interest at $2\frac{1}{2}\%$ for the deferred annuity portion. The interest basis during the deferred period prior to the commencement of income is $2\frac{1}{2}\%$. The cash values at all optional retirement ages are equal to the accumulation at $2\frac{1}{2}\%$ interest of a net premium which is expressed as a percentage of the gross premium, the percentage varying with issue age. The cash value and death benefit of the paid-up annuity available as a nonforfeiture option is equal to the cash value at the date of default accumulated at $2\frac{1}{2}\%$ interest.

In order to comply with Section 208-a, the following three conditions must be satisfied:

1. The present value as of the date of default of the paid-up annuity commencing at the latest optional retirement date (but not beyond the anniversary nearest age 70) shall be not less than the excess of (a) over (b), where:
   (a) = the present value of future guaranteed benefits that would have been provided if there had been no default, and
   (b) = the present value of "adjusted stipulated payments" (as defined in the Section).
2. In the event of default on an optional retirement date, the present value of the paid-up deferred annuity commencing at the latest optional retirement date (but not beyond the anniversary nearest age 70) shall be not less than the present value of the annuity commencing at the date of default.
3. If a cash value is allowed on default, the cash value must be at least equal to the present value of the paid-up nonforfeiture benefit then available.

In calculating the present values and the "adjusted stipulated payments," we have used the 1937 Standard Annuity Table and the rates of

interest specified in the contract for the calculation of nonforfeiture benefits—*i.e.*, $2\frac{1}{2}\%$ interest prior to the commencement of income, and $2\frac{1}{2}\%$ after the commencement of income except that $2\frac{1}{4}\%$ interest is used in determining the value at maturity of the annuity certain payments.

Demonstration that condition 1 is satisfied:

The "adjusted stipulated payments," π^a, are defined by formula as follows:

$\pi^a \ddot{a}_{x:\overline{r-x}|}$ = present value at issue of all future benefits $+.20\ \pi^a + .02\ \pi^a (r - 5 - x)$, where r is the latest optional retirement age, not exceeding 70, and x is the issue age.

Condition 1 may be expressed as follows:

Value of paid-up benefits at default $\geq$ Minimum Value defined by Section 208-a.

Value of paid-up benefits (*i.e.*, benefits prior to maturity plus deferred income benefits) is

$$\sum_{t=1}^{70-y} [L(1+i)^t]\ v^t \frac{d_{y+t-1}}{l_y} + K'\, v^{70-y} \cdot {}_{70-y}p_y \cdot {}^{10c}\ddot{a}_{70}^{(12)},$$

whence the condition may be expressed

$$L(1 - {}_{70-y}p_y) + K'\, v^{70-y} \cdot {}_{70-y}p_y \cdot {}^{10c}\ddot{a}_{70}^{(12)} \geq M,$$

where:

y = attained age at default,

L = cash value available at age y on default at age y,

K' = Paid-up deferred life income, ten years certain (commencing at age 70), available on default at age y,

M = minimum value defined by Section 208-a,

${}^{10c}\ddot{a}_x^{(12)}$ = value at age x of \$1.00 a year of life income, ten years certain, payable monthly beginning at age x, based on the 1937 Standard Annuity Table and the interest rates specified in the contract.

But

$$K' = \frac{L}{v^{70-y} \cdot {}^{10c}\ddot{a}'^{(12)}_{70}},$$

where the prime denotes use of the ELAS Life Income Mortality Table instead of the SA Table. Therefore the condition becomes

$$L\left[1 - {}_{70-y}p_y\left(1 - \frac{{}^{10c}\ddot{a}_{70}^{(12)}}{{}^{10c}\ddot{a}'^{(12)}_{70}}\right)\right] \geq M.$$

If the expression in brackets is denoted by R, then condition 1 is satisfied if

$$R \geq M \div L .$$

We computed values of R and $M \div L$ for representative ages and durations covering the ranges of issue ages and duration under the contract. At all points R is greater than the Minimum Value divided by the cash value. Thus the contract complies with condition 1 at all ages and durations.

Demonstration that condition 2 is satisfied:

Condition 2 may be expressed as follows:
Value of paid-up benefits on default at an optional retirement age $\geq$ value of life income ten years certain, payable monthly beginning immediately, then available.

We showed above that the left-hand side was equal to $L \times R$. The right-hand side may be expressed as

$$K \cdot {}^{10c}\ddot{a}_y^{(12)} ,$$

where

K = the amount of life income, ten years certain, payable monthly beginning immediately, available at optional retirement age y.

Then condition 2 may be expressed as

$$L \cdot R \geq K \cdot {}^{10c}\ddot{a}_y^{(12)} ,$$

or

$$R \geq \frac{K}{L} \cdot {}^{10c}\ddot{a}_y^{(12)} .$$

Since

$$K = L \div {}^{10c}\ddot{a}'^{(12)}_y ,$$

this inequality becomes

$$1 - {}_{70-y}p_y \left(1 - \frac{{}^{10c}\ddot{a}^{(12)}_{70}}{{}^{10c}\ddot{a}'^{(12)}_{70}} \right) \geq \frac{{}^{10c}\ddot{a}^{(12)}_y}{{}^{10c}\ddot{a}'^{(12)}_y} .$$

This condition can be transformed to

$${}_{70-y}p_y \left(1 - \frac{{}^{10c}\ddot{a}^{(12)}_{70}}{{}^{10c}\ddot{a}'^{(12)}_{70}} \right) \leq 1 - \frac{{}^{10c}\ddot{a}^{(12)}_y}{{}^{10c}\ddot{a}'^{(12)}_y} ,$$

which, since ${}_{70-y}p_y < 1$, holds if

$$1 - \frac{{}^{10c}\ddot{a}_{70}^{(12)}}{{}^{10c}\ddot{a}'^{(12)}_{70}} \leq 1 - \frac{{}^{10c}\ddot{a}_{y}^{(12)}}{{}^{10c}\ddot{a}'^{(12)}_{y}}$$

or

$$\frac{{}^{10c}\ddot{a}'^{(12)}_{70}}{{}^{10c}\ddot{a}^{(12)}_{70}} \leq \frac{{}^{10c}\ddot{a}'^{(12)}_{y}}{{}^{10c}\ddot{a}^{(12)}_{y}}.$$

This inequality holds for ages of y from 55 to 69 inclusive (the range of optional retirement ages), as may be seen from Table 1.

TABLE 1

Optional Retirement Age y	Male	Female
	${}^{10c}\ddot{a}'^{(12)}_{y} \div {}^{10c}\ddot{a}^{(12)}_{y}$	${}^{10c}\ddot{a}'^{(12)}_{y} \div {}^{10c}\ddot{a}^{(12)}_{y}$
55	1.1119	1.1053
56	1.1123	1.1044
57	1.1123	1.1031
58	1.1119	1.1014
59	1.1110	1.0992
60	1.1096	1.0966
61	1.1076	1.0933
62	1.1050	1.0896
63	1.1016	1.0853
64	1.0975	1.0804
65	1.0928	1.0750
66	1.0874	1.0690
67	1.0813	1.0624
68	1.0747	1.0553
69	1.0676	1.0477
Retirement Age 70	1.0600	1.0397

${}^{10c}\ddot{a}'^{(12)}_{y}$ = Value of monthly immediate ten years certain life annuity, based on ELAS Life Income Table and the interest rates specified in the contract.

${}^{10c}\ddot{a}^{(12)}_{y}$ = Value of monthly immediate ten years certain life annuity, based on SA Table and the interest rates specified in the contract.

Demonstration that condition 3 is satisfied:

Condition 3 may be expressed as follows:

Cash Value at default $\geq$ Present Value of benefits at default. The left-hand side of this inequality is L, and the right-hand side is equal to $L \times R$, as shown above.

Then condition 3 is satisfied if $L \geq L \cdot R$, *i.e.*, if

$$R \leq 1 .$$

Since

$\frac{{}^{10c}\ddot{a}_{70}^{(12)}}{{}^{10c}\ddot{a}'^{(12)}_{70}} < 1$ as shown in Table 1, and ${}_{70-v}p_v < 1$, R is always less than 1.

Conclusion: One purpose that may be served by this discussion is to point out the complexities of Section 208-a of the New York law, particularly as it affects a company that has decided to adopt a more modern table than the 1937 Standard Annuity Table as its actuarial basis for deferred annuity contracts. I suggest that serious consideration be given to urging an amendment of this statute for the sake of simplification and to avoid the necessity of complying with a set of conditions which in the present law have been defined in terms of the 1937 Standard Annuity Table.

ROBERT W. WALKER:

Mr. Boermeester is to be complimented on drawing this facet of the nonforfeiture law of New York to the attention of the members of the Society. His note attracted my attention for I had been interested in 1947 in examining into the problem in an effort to assure that our policy values were adequate. It does not suffice to assume that they will be; we must actually test. It should be noted, of course, that the law relates only to the stipulated premium contract, single premium contracts being specifically excluded.

For the type of contract described by Mr. Boermeester, a contract issued by Northwestern and other companies, Section 208-a 7 defines the adjusted stipulated premium to be used in computing the nonforfeiture benefits as:

$$P' = \frac{K\, v^{r-x}}{\ddot{a}_{\overline{r-x}|} - .2 - .02\,(r-x-5)}$$

where

K = maturity value

r = retirement age

x = issue age.

The net level valuation premium for the benefits is, of course:

$$P = \frac{K\, v^{r-x}}{\ddot{a}_{\overline{r-x}|}}.$$

Since a prospective valuation procedure is set forth, it follows that if there be no surrender deduction the policy values must all be adequate. If, however, the policy values be other than net level premium reserves a question arises. In the case in point the approach taken in setting up the criterion for establishing adequate values was to establish the maximum value of the surrender deduction exacted from what would otherwise be the net level premium reserve, as follows:

New York minimum cash value at duration n

$$= K\, v^{r-x-n} - P'\ddot{a}_{\overline{r-x-n}|}$$

$$\text{Company cash value} = K\, v^{r-x-n} - P\ddot{a}_{\overline{r-x-n}|} - D_n$$

$$\text{where } D_n = \text{surrender deduction.}$$

$$\text{Difference} = P\ddot{a}_{\overline{r-x-n}|} - P'\ddot{a}_{\overline{r-x-n}|} + D_n\,.$$

It follows that the maximum permitted value of D_n is such that this expression equals zero, *i.e.*, the value of the expression may not be greater than zero. Equating the expression to zero thus sets up the criterion for the maximum values of D, the surrender charge, for the full range of values of the variables at hand.

This approach may not have the algebraic finesse of the general thesis of the paper but is a very practical one.

JAMES E. HOSKINS:

Mr. Boermeester discusses the situation where annuity options are equivalent to cash values on the basis of the 1937 Standard Annuity Table with a uniform setback.

Mr. Harry Walker's discussion, presented at this meeting, shows that immediate annuity options may be made equivalent to cash values on the basis of some other table than the 1937 Standard if certain conditions are met, one of them being that the ratio of the yield on the actual table to that on the 1937 Standard Table at ages prior to 70 is not greater than the corresponding ratio at 70 (or the latest age at which an immediate annuity

option is available if that occurs before 70). This criterion had previously been given in substance by Mr. Waugh in *TSA* IV, 376.

A table is theoretically possible which does not meet this criterion when the standard of measurement is the 1937 Standard Table without setback but does meet Mr. Walker's criteria if the standard of measurement is the 1937 Standard Table with a setback of not more than three years. Under these conditions the table may be used to compute options, but the amount of setback by which minimum values are defined must be stated in the contract.

Obviously, the criteria will be met if the setback used is that which at age 70 reproduces the automatic option and if, at any prior age, the setback on the 1937 Standard Table which produces the equivalent of the true age on the table actually used is not less than the setback at maturity.

(AUTHOR'S REVIEW OF DISCUSSION)

JOHN M. BOERMEESTER:

Mr. Knight need not be accused of heresy, since he merely made the observation that Massachusetts, too, has a statute governing minimum values for annuities. His demonstration of the determination of minimum values for the Commonwealth of Massachusetts serves to emphasize the point that a statute with relatively simple provisions might possibly be quite acceptable for annuity contracts.

Mr. Harry Walker has done a notable service by presenting a discussion for a practical filing problem under which technical difficulties were encountered with Section 208-a of the New York Law. These difficulties, he points out, arise because conditions in the law have been defined in terms of the 1937 Standard Annuity Table. In particular, Mr. Walker points out the peculiar problem presented by condition 2 which in effect stipulates that if a company wishes to provide an early retirement income, then the present value at such early retirement age of the paid-up nonforfeiture benefit must never be less than the corresponding present value of the early retirement option benefits.

Mr. Walker did not show the development of minimum statutory values as required when the ELAS Table is used to establish life income values. I assume that in computing any particular minimum value he first established the present value of contractual life income benefits payable after retirement on the basis of the unmodified 1937 Standard Annuity Table in lieu of the presumably larger corresponding present value of a pure endowment due at retirement of an amount equal to the contract cash value.

Mr. Walker suggests that serious consideration be given to amending the statute. I certainly agree that a reappraisal should be made at this time, particularly in view of the publication of several new annuity tables.

Mr. Robert W. Walker presents a method for determining margins for a contract under which values are to be based on the 1937 Standard Annuity Table. The method he suggests is a practical one as long as the influence of life contingencies prior to retirement age is insignificant.

Mr. Hoskins' discussion is again so stated in his usual precise style that I find it impossible to add anything further.

I wish to thank these four gentlemen for their comments which have helped me to become aware of a number of problems that heretofore had escaped my notice.