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ACTUARIAL NOTE: VALUATION OF POLICY DEPOSITS AT AN INTEREST RATE LOWER THAN THE GUARANTEED RATE

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FOR many years most insurance contracts have permitted policy proceeds to be placed with the company at a guaranteed rate of interest. The company makes interest payments monthly, quarterly, semiannually or annually in accordance with the terms of the supplementary contract established.

In many cases the interest rate guaranteed under older contracts exceeds the interest rate which a company can earn at the present time. Under these circumstances a company may adopt one of the following three procedures:

1. Interest losses may be taken each year.
2. Existing amounts on deposit may be strengthened to a lower interest rate. New deposits as they occur would be strengthened to the same basis.
3. Existing reserves may be adequately strengthened and additional strengthening for anticipated future losses under new deposits may also be established.

For practical reasons most companies confine themselves to either the first or second procedure described above. One method of strengthening existing deposits is described below. Rates of withdrawal are determined by duration for funds on deposit. Let $(wq)_n$ equal the probability that \$1 on deposit at the start of the nth contract year terminates during that year for any reason. Further define f_n as the required reserve per dollar on deposit at the start of the nth contract year, let i equal the strengthened interest rate and i' the guaranteed rate basis. Assuming that withdrawals occur on the anniversary, we have for an annual case

$$f_n(1 + i) = [1 - (wq)_n]f_{n+1} + i' + (wq)_n . \qquad (1)$$

Strengthening factors can be determined from these rates of withdrawal by means of equation (1) working backwards from

$$f_{\omega-1} = v(1 + i') \quad \text{since} \quad (wq)_{\omega-1} = 1 .$$

Equation (1) may also be expressed as

$$f_n(1 + i) + (wq)_n(f_{n+1} - 1) = f_{n+1} + i' . \qquad (2)$$

Equation (2) has exactly the same form as the annuity equations in Mr. Shepherd's analysis of the Gain and Loss Exhibit *RAIA* XXXI, 440–49, if we define tabular reserves released as $(wq)_n(f_{n+1} - 1)$ and tabular payments as i'. Mean reserves are of the form

$$\frac{f_{n-1} + f_n + i'}{2}.$$

Following the pattern of Mr. Shepherd's analysis, we can develop the following equation over a calendar year period:

$$\Sigma q V^w + \Sigma I = \Sigma_2 M + \Sigma V^w + \Sigma B - \Sigma_1 M - \Sigma \pi f_1$$

in which

$\Sigma q V^w$ = Tabular reserves released by termination (as defined above)
ΣI = Tabular interest
$\Sigma_2 M$ = Mean reserve at end of year on strengthened basis
ΣV^w = Terminal reserves released by terminations less the amount on deposit surrendered
ΣB = Actual interest payments plus deposits surrendered
$\Sigma_1 M$ = Mean reserves at beginning of year on strengthened basis
$\Sigma \pi f_1$ = New considerations strengthened to the valuation basis

Tabular interest is computed by the formula

$$\Sigma I = \frac{i}{2}\left[\Sigma_1 M + \Sigma_2 M - (\Sigma q V^w + \Sigma I - \Sigma V^w)\right]$$

which assumes that new considerations are strengthened as they occur and that interest on the reserve released by withdrawals in excess of the withdrawal value is needed for the remainder of the calendar year.

Tabular less actual reserves released are then determined as the balancing item. By comparing tabular less actual reserves released over a period of years a company can determine whether the reserve strengthening has been adequate.

Adjustments for payments on other than an annual basis can be made in the manner indicated by Mr. Shepherd for annuities. In any case, no great theoretical accuracy is attainable in view of the fact that rates of withdrawal for amounts on deposit will vary not only by duration but also with a company's contract provisions, the guaranteed interest rate, economic conditions and a host of other factors.