A Comprehensive Analysis of the Patterns of Worldwide Mortality Evolution

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Abstract

A variety of literature deals with the question how the age distribution of deaths develops over time, and many different notions have been established for certain scenarios. In Börger et al. (2016), a classification framework has been developed that allows for a unique classification of mortality evolution patterns. In particular, the framework assigns a unique scenario to any possible mortality evolution. In contrast to many other classification approaches, this approach allows for so-called mixed scenarios, such as a combination of elements of compression and shifting mortality. Thus, it provides a more comprehensive picture of historical and potential future mortality evolution patterns.

In the present paper, we briefly summarize this classification framework and discuss issues in its practical application. Then we apply the framework to mortality data for different countries all over the world. This yields a complete picture of historical mortality evolution patterns in those countries and adds to existing analyses where only certain aspects of mortality evolution patterns have been considered (e.g., a test for one scenario like compression) for only one or a few countries. We then discuss similarities and differences in the historical mortality evolution patterns between different populations. We also apply the framework to different age ranges, since sometimes different scenarios can be observed for different age ranges, even within one population.

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1. Introduction

The age-specific structure of mortality changes probably all over the world. For instance, life expectancy has been increasing in most countries over the last decades. This, however, is only a particular symptom of the underlying change of the deaths curve (i.e., the age distribution at death). Thus, the question of how the structure of mortality changes in detail goes beyond the change in life expectancy: What are the main drivers of this evolution? Which age ranges are mostly affected by the change of the mortality structure? Moreover, potential dependencies and differences between the evolution of the structure of mortality for different populations often need to be analyzed in order to get an idea of supra-regional or even global trends in the deaths curve's evolution.

Many recent publications deal with the evolution of the mortality structure of single populations, including Nusselder and Mackenbach (1996) for the Netherlands, Cheung et al. (2009) for Switzerland, and Debón et al. (2011) for Spain. Of course, such an analysis usually is motivated by the objective of the research, but the results may be strongly affected by country-specific circumstances (e.g., the country's social structure or health care system). Moreover, supra-regional or even global trends in the deaths curve's evolution cannot be detected, and it is not possible to separate them from population-specific effects. That is why the analysis of mortality structures should be set in a context of coherent populations. In contrast, some authors analyze trends in mortality evolutions of different populations (e.g., Kannisto 2000; Edwards and Tuljapurkar 2005; Robine et al. 2008; Canudas-Romo 2008; and Thatcher et al. 2010). However, often such analyses are mere demonstrations of the framework presented in the particular articles, so the focus is less on the comparison of the evolution of mortality structures between different countries. Only a few authors focus on the comparison of the trends in mortality evolution, but they typically restrict their analyses to specific aspects of the mortality structure. For example, Ouellette and Bourbeau (2010) explore trends in the adult mortality structure of 10 different countries. They inspect the evolution of the two statistics, the modal age at death M and the standard deviation above the modal age at death, which they call SD(M+), , which were also suggested by, for example, Kannisto (2001). In a recent paper, Börger et al. (2016) observed that these two statistics are often not sufficient, particularly if the focus is on the change of the complete mortality structure, rather than on certain characteristics of its evolution. Also, Edwards (2011) has an exclusive focus on a certain characteristic of the deaths curve's evolution—in particular on the inequality of the age distribution of deaths. To this end, he analyzes the mortality data of 180 different populations. However, the inequality or dispersion of the distribution is, of course, just one aspect in which the structure of mortality can change. For instance, life expectancy might increase over time while the dispersion of the age distribution of deaths stays constant or even decreases. Finally, Viner et al. (2011) use data of 50 different populations, which are clustered in terms of amount of income. However, the focus of this research was specifically on "mortality trends in children and young people."

In summary, there is only little research on the differences in the development of the mortality structure between different populations. Authors covering that topic mainly focus on specific age ranges or particular characteristics of the mortality structure.

The present paper aims to fill this gap and provides a comparison of the past trends in the change of the (full) deaths curve of 34 different populations. To this end, we use a unique classification system for mortality evolution patterns that has been introduced by Börger et al. (2016). Based on the deaths curve, it can be applied to different age ranges and makes use of four statistics: The modal age at death M and the upper bound of the deaths curve's support UB measure changes in the position of a deaths curve over time, while the degree of inequality DoI and the number of deaths at the modal age at death d(M) display changes in the deaths curve's shape. The framework not only offers a classification of the trends in the deaths curve's evolution of one country, but also allows for a comparison of such trends between different populations (e.g., between countries) or subpopulations (e.g., between females and males, different age groups, or different socioeconomic or ethnic groups) at a glance.

For our analysis, we use sex-specific data and compute the age distribution at death for the starting ages 0 and 60. This provides insights into the trends in the mortality structure for the complete age range, as well as for the mortality structure of female and male retirees.

The remainder of this paper is organized as follows: In Section 2, we briefly describe the data and outline how we prepare our input data prior to the analysis. In Section 3, we discuss the classification framework we use. Section 4 covers some issues in the practical application of the described framework, such as the smoothing of time series of single statistics, mathematical methods for the detection of trends, and methods for analyzing the results. In Section 5, we present and discuss our results. Finally, Section 6 concludes.

2. Data

To the best of our knowledge, there is no single, publicly available database that includes mortality data for every population in the world, has a sufficiently long history, and has a data quality that allows for an immediate comparison of data from different populations. Thus, we have to find a trade-off between completeness, availability, quality, and standardization of our input data. We use data from the Human Mortality Database (HMD), since it provides mortality data that is highly standardized, of good quality, and easily available. Unfortunately, the HMD provides mortality data for only 38 countries. For some of these countries, the HMD offers data for subpopulations,[1] which leads to a total of 42 different populations. Besides these shortcomings in terms of completeness, we have decided to eliminate those calendar years in the history of each population's data where the HMD alerts the user to the lower quality of the (input) data. Moreover, for the discussion of the results in Section 5, we only focus on rather recent trends. Thus, we use only data after 1920—i.e., roughly one century back—for each population, even if the HMD offers data for the time prior to 1920.[2] Finally, our analysis requires input data for each population with a sufficiently long history, since we aim to detect long term trends in the mortality evolution, rather than short-term fluctuation of mortality evolution patterns. We have therefore decided to include only populations with a data history of at least 40 years. This leaves us with 34 populations, a large portion of which are European (see Section

[1] These subpopulations are East and West Germany for Germany, England and Wales, Scotland and Northern Ireland for the United Kingdom, and non-Maori and Maori populations for New Zealand.

[2] The choice of the year 1920 is reasonable, since especially in the 1910s we have several historic events that blur the long-term trends of the mortality evolution in many countries (for example, World War I and the Spanish flu).

4 for a list). For each population, we use sex-specific data, meaning we consider females and males separately.

We use deaths and exposure-to-risk data each with discrete calendar year and age subscript to calculate the discrete log force of mortality $\log(m_{x,t})$ for every age x between 0 and 109 and every calendar year t where we have data. Thereafter, we smooth and extrapolate these $\log(m_{x,t})$ in the age direction using P-splines. Given these smoothed log force of mortality curves for every calendar year, we calculate the deaths curves, which we scale such that they integrate to the value of 1 (i.e., we chose the radix 1 for the calculation of the corresponding survival curve). Hence, these curves can be interpreted as the density functions of the distribution of the age at death of a particular population. We calculate the curves for all populations, both sexes, and every calendar year. Moreover, we compute deaths curves for starting ages 0 and 60, which allows for a separate analysis of the trends of the deaths curve for the entire age range on the one hand, and the age range of retirement on the other hand. Note that the deaths curve with starting age 60 is the density function of the age distribution at death conditional on survival to age 60.

3. The Classification Framework

Recently, Börger et al. (2016) developed a unique classification framework for mortality evolution patterns, and they describe the advantages of this framework compared with previous approaches. For instance, the framework gives a clear definition for each scenario that might (or might not) prevail in the evolution of the deaths curve. Moreover, it allows for so-called mixed scenarios (e.g., compression and extension at the same time) and is able to measure any significant change in the deaths curve's position and shape over time. As we use this framework here, we give a short overview in this section.

The basic idea of this framework is to focus on the deaths curve and analyze its changes over time. To this end, the framework consists of four components:

The first component is the modal age at death, M. It is defined as the position of the deaths curve's peak. Whenever this peak moves to the left or to the right (i.e., M is decreasing or increasing), this is called *left-* or *right-shifting mortality.*

The second component is the upper bound of the deaths curve's support, UB, which is defined as the age where the survival curve reaches 0.[3] An increase or decrease of UB means that the support of the deaths curve extends or contracts. Consequently, UB indicates *extension* and *contraction*. These first two components, M and UB, together describe the evolution of the deaths curve's position over time.

The third component of the framework is the so-called degree of inequality DoI, which was developed in Börger et al. (2016) to measure *compression* and *decompression*. Following their definition, compression is a process in which the deaths curve becomes more unequal over time. A perfectly equal deaths curve over all ages in this respect would be a uniform distribution of the age at death on the age interval [0, UB]. Whenever the difference between this hypothetical and the realized deaths curve increases or decreases, this is called compression or decompression, respectively.

[3] We admit that in a theoretical, continuous setting, this age may not exist. However, we use data of finite populations and in a discrete setting. Thus, there is an age where the oldest member of a population dies.

Finally the fourth component is the number of deaths at the modal age at death d(M), which describes the relative importance of the age M compared with the remaining ages. Whenever d(M) increases or decreases, this is called *concentration* or *diffusion*, respectively. The latter two components, DoI and d(M), together describe the evolution of the deaths curve's shape over time.

Besides increasing and decreasing trends in these four statistics, the evolution of a statistic can, of course, be neutral, which means that no (significant) change has occurred over time in the concerned component. Thus, every possible evolution of a deaths curve between two points in time can be expressed as a four-dimensional vector, where each component denotes the trend of one statistic. For example, if we observe an increase in the first and third statistic while the second and fourth do not change significantly, this vector would be (right-shift, neutral, compression, neutral). Since we have four statistics and three attainable states of each component of this "scenario vector," we obtain 81 different theoretically possible scenarios. Note that some of these scenarios might not be relevant in practice. However, the complete set of the 81 scenarios guarantees that a scenario can be assigned to every possible mortality evolution. Also note that the framework only detects qualitative rather than quantitative changes. Thus, we do not answer the question of the pace of, say, right-shifting mortality, but answer the question if we observe it at all.

For the estimation of the statistics, we follow the methodology described by Börger et al. (2016). Note that, taking a theoretical perspective, both M and UB are independent from the starting age of the deaths curve (as long as the starting age is smaller than M), although in practice, the chosen estimators induce slight, nonsignificant differences between starting ages. That is why, in what follows, we only display and discuss M_0 and UB_0 (i.e., the time series for starting age 0) in order to reduce the number of charts and simplify the presentation of the results.

4. Issues in Practical Application of the Framework

In this section, we cover issues that arise after the four statistics described in Section 3 have been calculated. Fig. 1 exemplarily shows the evolution of M for French males between 1920 and 2013. We do not smooth the input data by calendar years, so we observe considerable random fluctuations in this figure. These random fluctuations can blur the long-term trends in the time series, so we have to find methods to clearly decide when and where the statistic is increasing, neutral, or decreasing, respectively.

In this respect, we search for periods (with specified limits) on which the time series follow a linear trend. As a result, we get the direction of the respective trend (increasing, neutral, and decreasing) and the position of the trend changes (i.e., the limits of these periods). Trend changes can have two different qualities: either only the direction of the trend changes (we observe a continuous trend change within the time series) or both the direction of the trend and the level of the time series change at the same time (we observe a jump within the time series). Such jumps may be caused by historical events (for example, World War II or the fall of the Soviet Union) but also may be a result of changes in data-processing methods.

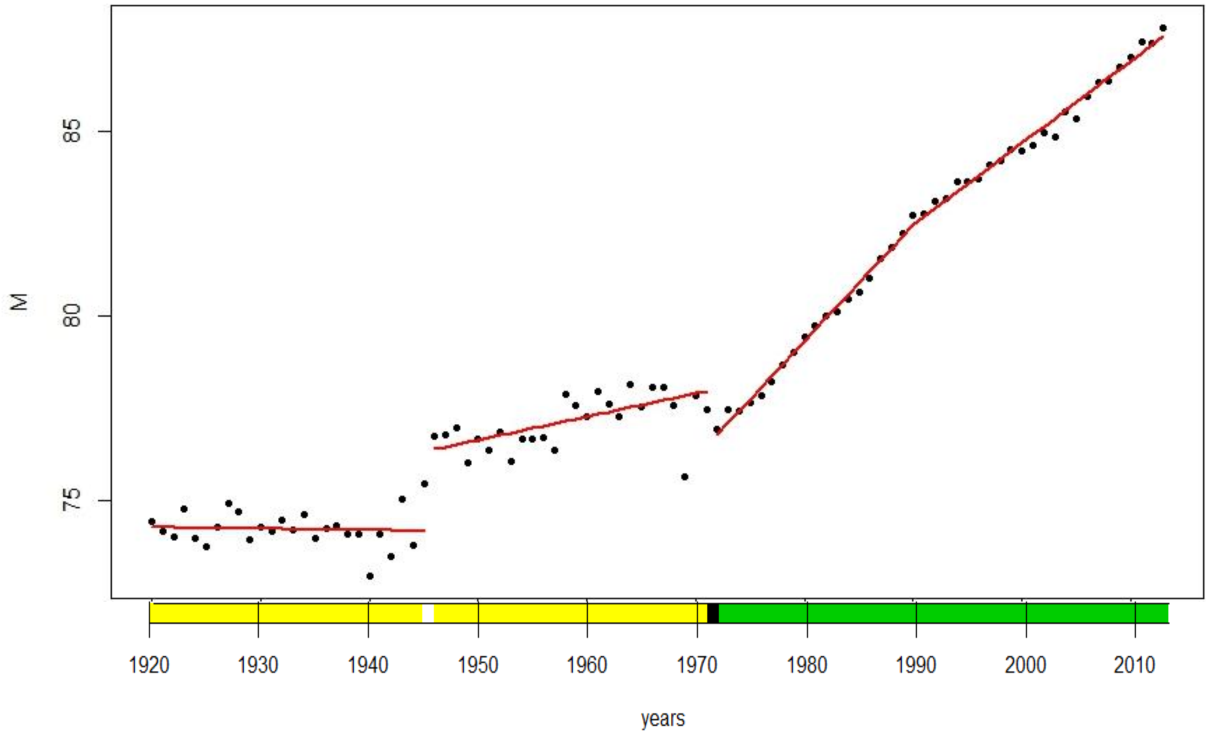

Fig. 1. Evolution of Modal Age at Death (M) for French Males, 1920–2013

Note: The red line shows the fitted polygonal function. On the time bar, colors identify trends: red = decreasing trend (not present in the example), yellow = neutral trend, green = increasing trend, white = upward jump, black = downward jump.

We take a three-step approach proposed by Börger et al. (2016) to find the trends and trend changes of the time series and refer to their paper for more details. Here we only give the basic idea: In the first step, we identify and eliminate outliers. For example, in Fig. 1, we can detect a considerable downward outlier in the year 1969, which has to be eliminated. In the second step, we identify time periods within which the time series follows constant linear trends. This particularly includes a determination of the optimal number of trend changes. As a result, we have determined a fit of the time series (see red line in Fig. 1). In the third step, for each period from the second step, we test whether a given trend is significantly different from 0. This gives us the desired direction of the trends (i.e., increasing, neutral, and decreasing). The results of this test for M for French males are illustrated with a colored time bar in Fig. 1.

To facilitate the analysis of the large number of results below, we will display time bar plots that allow for an easy and intuitive comparison of the trends of each statistic between different populations. To this end, we simultaneously display colored time bars (as in Fig. 1) for several populations in order to compare the trends and trend changes between the respective populations at a glance. Green intervals on such time bars indicate increasing trends, yellow and red intervals display neutral and decreasing trends, respectively. As mentioned above, we observe both continuous trend changes and jumps. In the time bar plots, the color changes only if the direction of the trend changes (e.g., from increasing to decreasing). If, however, the direction remains unchanged and only the slope changes, this cannot be detected in such time bar plots. For example in Fig. 1, the continuous trend change around 1990 is not shown in the time bar, as the direction of the trend does not change. In contrast, downward and upward jumps are always illustrated with a white or a black one-year trend, respectively, even if the direction of the trend does not change (see, for example, the upward jump in 1946 in Fig. 1).

For our analysis, we construct these plots separately by sex and starting age. Since neighboring countries frequently have similar trends in the mortality evolution, we sort the time bars by regional clusters:

- **Northwestern Europe**: Sweden, Norway, Iceland, Finland, Denmark, Scotland, England and Wales
- **Central Europe**: Netherlands, Belgium, Luxemburg, West Germany, Austria and Switzerland
- **Southwestern Europe**: France, Spain, Portugal and Italy
- **Eastern Europe**: East Germany, Poland, Czech Republic, Slovakia, Estonia, Latvia, Hungary, Bulgaria, Belarus, Ukraine and Russia
- **North America**: United States and Canada
- **Asia-Pacific area**: Japan, Australia and New Zealand (non-Maori and Maori)

Beyond displaying the trends in the four statistics over time for each population, we also analyze similarities and differences across populations. For this, we have developed a figure that we call relative similarity (RS). Since we have only three potential values for each trend process at each point in time, we use a rather simple approach: Let $T^{(\cdot)} = \{t_0^{(\cdot)}, \ldots, t_{n_{(\cdot)}}^{(\cdot)}\}$ be the time range of a considered time series, and let $(u_t)_{t \in T^u}$ and $(v_t)_{t \in T^v}$ be two trend processes. Then $u_{t_i^u} \in \{\text{decreasing, neutral, increasing}\}$ and $v_{t_j^v} \in \{\text{decreasing, neutral, increasing}\}$, respectively, with $i \in \{0, \ldots, n_u\}$ and $j \in \{0, \ldots, n_v\}$. Moreover, the number of common data points of these two trend processes is given by $N(u,v) = \max(t_0^u, t_0^v) - \min(t_{n_u}^u, t_{n_v}^v) + 1$. For these data points, we determine the relative similarity as

$$\mathrm{RS}(u,v) = \frac{1}{N(u,v)} \sum_{\tau \in T^u \cap T^v} \delta_\tau,$$

where

$$\delta_\tau = \begin{cases} -1 \text{ if } (u_\tau = \text{decreasing and } v_\tau = \text{increasing}) \text{ or } (u_\tau = \text{increasing and } v_\tau = \text{decreasing}), \\ 0 \text{ if } (u_\tau = \text{neutral and } v_\tau \neq \text{neutral}) \text{ or } (u_\tau \neq \text{neutral and } v_\tau = \text{neutral}), \text{ and} \\ 1 \text{ if } u_\tau = v_\tau. \end{cases}$$

Note that the value of RS is always in the interval $[-1, 1]$, where –1 means perfect dissimilarity and 1 means perfect similarity. This concept not only allows us to compare single statistics between different populations; we can also calculate the relative similarity of two populations—for example, for females with starting age 0 in total. This can be done by computing the average of the relative similarities for the four statistics.

5. Results

In this section, we present and analyze our results. We first analyze the trends (and trend changes) in the mortality evolution of 34 populations for males with starting age 0 (i.e., the complete age range) as a reference. We focus on the identification of supra-regional patterns, and thereby we identify most similarities or differences in the trends of the deaths curve's evolutions of the considered populations. In the second part of this section, we also analyze the trends in the mortality evolution of females and the trends in old-age mortality (i.e., with the starting age 60). For the sake of brevity, however, we then only highlight differences compared with the findings for males with starting age 0.

5.1. Reference Trends in the Mortality Evolution

In this subsection, we discuss the time bar plots for each statistic in detail before we summarize these findings in order to provide a complete picture of the recent mortality evolution of males with starting age 0. Since we have data for only few countries for the early years after 1920, searching for supra-regional patterns is difficult. The first year where we have data for each population is 1971. Hence, we mainly focus on rather recent trends when we discuss the results.

Fig. **2** displays the time bars for the modal age at death M. Prior to 1960, we find hardly any supra-regional patterns in the trends of M. The only observation worth noting in this period is an upward jump in the middle of the 1940s for many European populations, but also for Canada. This is probably caused by World War II. During the 1960s, however, we observe a relatively short period of left-shifting mortality (i.e., a decrease in M), particularly for most northern and central European populations. However, for the majority of the populations outside eastern Europe (and few other exceptions), we observe an inversion of this trend almost at the same time around 1970 and an increase in the modal age at death afterward until the end of the observation period.

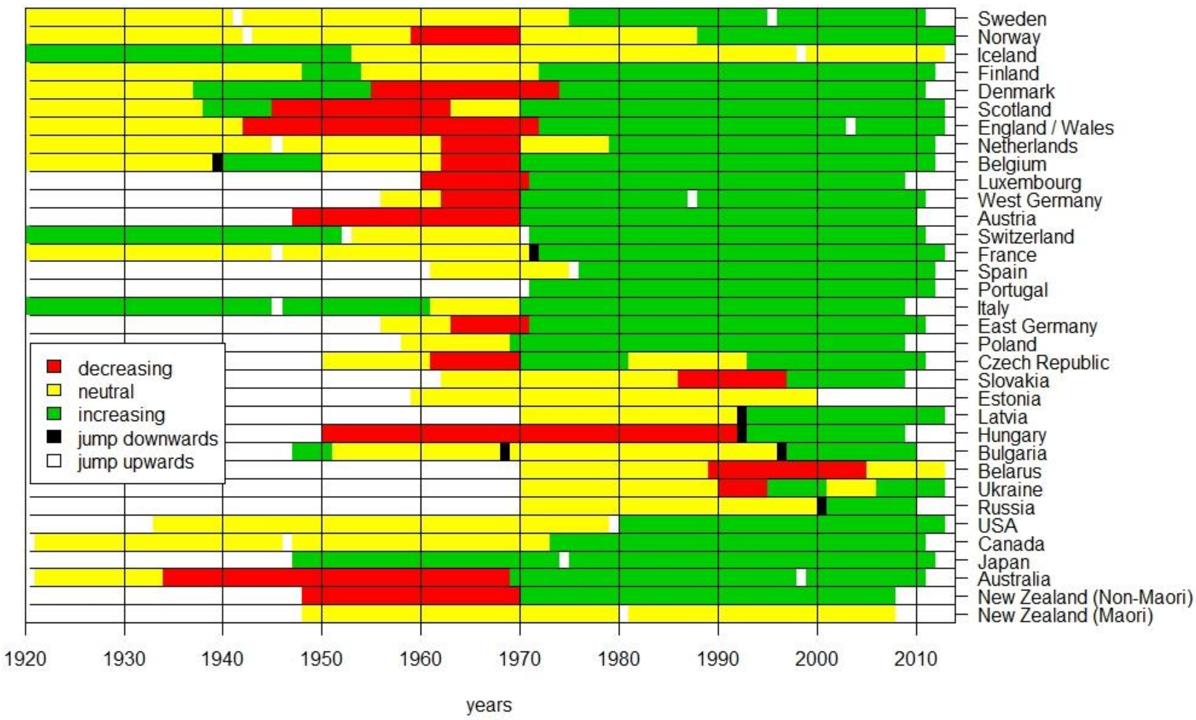

Fig. 2. Trends in the Evolution of Modal Age of Death (M): Males, Starting Age 0

In contrast, the modal age at death in most eastern European populations (except East Germany and Poland) does not follow these trends. In particular, after 1970, we observe only neutral trends or even left-shifting mortality at least until the early 1990s. It seems that the fall of the Soviet Union for these countries had a certain impact on the evolution of the deaths curve, because since then, we observe right shifts for many eastern European populations. However, for the easternmost European populations, this trend change is less clear than for the other eastern European populations, and thus we observe a certain degree of heterogeneity within the eastern European cluster after 1990.

Fig. 3 shows the time bars for the upper bound of the deaths curve's support UB. Also here, we can find apparent differences between eastern European populations and the other populations. Around 2000 at the latest, however, the direction of the trends of UB for all eastern European populations assimilates to the direction of the trends for the other populations. Moreover, we also find considerable differences within the eastern European cluster: On the one hand, we observe contraction for the easternmost populations until the late 1990s. On the other hand, we find only neutral periods (neither extension nor contraction) for the other eastern European populations before 1990. Thus, the long-term trends within this cluster are not homogeneous prior to 1990.

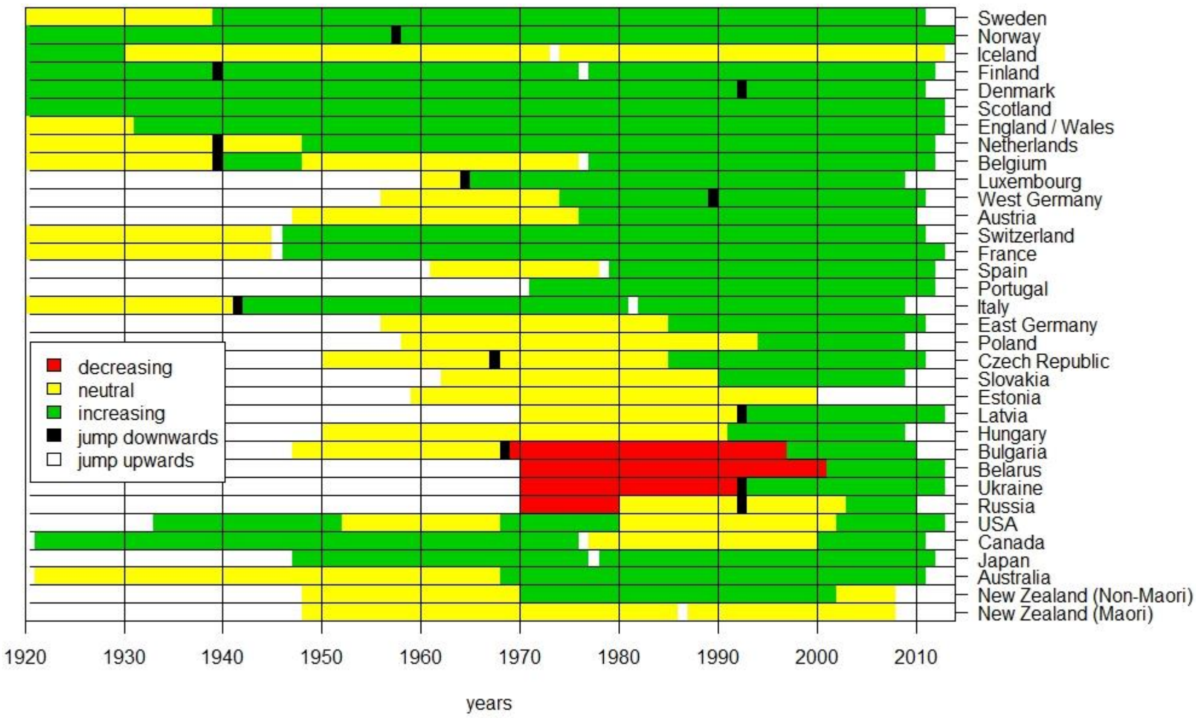

Fig. 3. Trends in the Evolution of the Upper Bound of the Deaths Curve's Support (UB): Males, Starting Age 0

Apart from the observations for the eastern European cluster, there are hardly any supra-regional trend changes for UB, especially during the first few decades. However, for example, almost all northern European populations (except Iceland, which also shows different patterns for M) experience extension over almost the entire observation period. This, however, could be misleading, since there are single downward jumps in these time series, which interrupt the overall increase in UB. Thus, if we would not admit for jumps in the time series (e.g., if we would fit a straight regression line to the time series during this period), the observed long-term trend might be not be significantly increasing any more. Also for most central European and Asia-Pacific populations, we observe long-term extension at least after the 1970s. In the United States and Canada, we have a two-decade period during the 1980s and 1990s where we observe a neutral trend. Thus, the North American cluster is homogeneous in this respect but shows special trends compared with the other populations.

Fig. 4 shows the trends in the evolution of the degree of inequality DoI_0 for males. Until the middle of the 1950s, we mostly observe compression with few exceptions, in particular around the time of World War II. For the subsequent about two decades, there are several in parts even opposing trends (e.g., in the northern European cluster). After that, however, we find compression for almost all populations except eastern Europe until the end of the observation period.

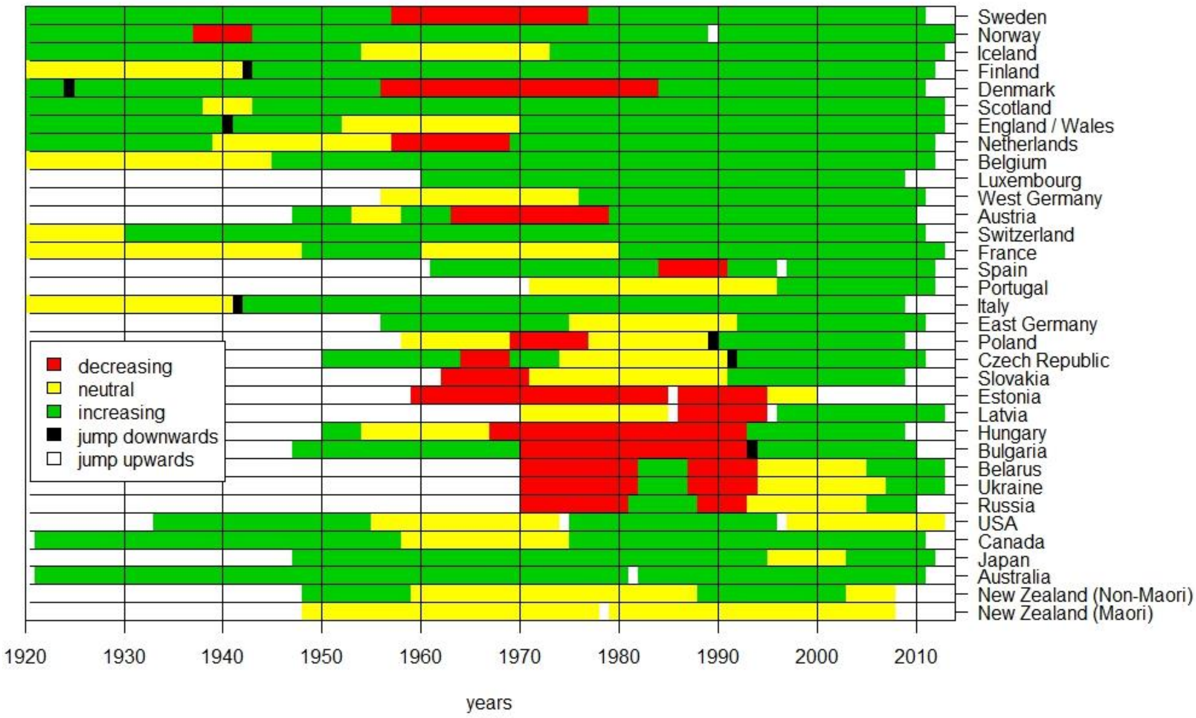

Fig. 4. Trends in the Evolution of Degree of Inequality (DoI_0): Males, Starting Age 0

Also with respect to the trends in DoI_0, the eastern European and the other populations have different trends, and we observe a similar heterogeneity in the direction of the trends, as for UB (decrease in the easternmost populations and neutral in the other populations until about 1990). For a few years during the 1980s, we observe an increasing trend for Belarus, Ukraine and Russia. Almost at the same time, we find an upward jump in the trends of Estonia and Latvia. Thus, for those populations, we have a short but significant period of compression (or an upward jump, respectively), even prior to the fall of the Soviet Union. For the easternmost European populations as well as outside Europe, we observe several neutral periods during the most recent decades. In particular, for the United States, we find almost two decades of neutral trend, which may compensate for the upward jump in the middle of the 1990s.

Finally, Fig. 5 displays the time bars for the numbers of deaths at the modal age at death $d(M)_0$ for males. For most populations, the trends in $d(M)_0$ and DoI_0 are very similar. However, there are some exceptions. For example, in the United States, DoI_0 did not change significantly during the 2000s, but we observe diffusion (decreasing $d(M)_0$) during that period. Moreover, for Austria, we observe decompression (decreasing DoI_0) during the 1960s and 1970s while there is concentration (increasing $d(M)_0$) at the same time. Also, in Austria and West Germany, we find a neutral period in the trend of $d(M)_0$ during parts of the 1980s and 1990s, which has no counterpart in the trends of DoI_0. In Japan, we observe diffusion during the 1990s and the first half of the 2000s, while we only observe a shorter neutral period for DoI_0. Such examples show that DoI and d(M) are not fully correlated and that these statistics indicate different phenomena.

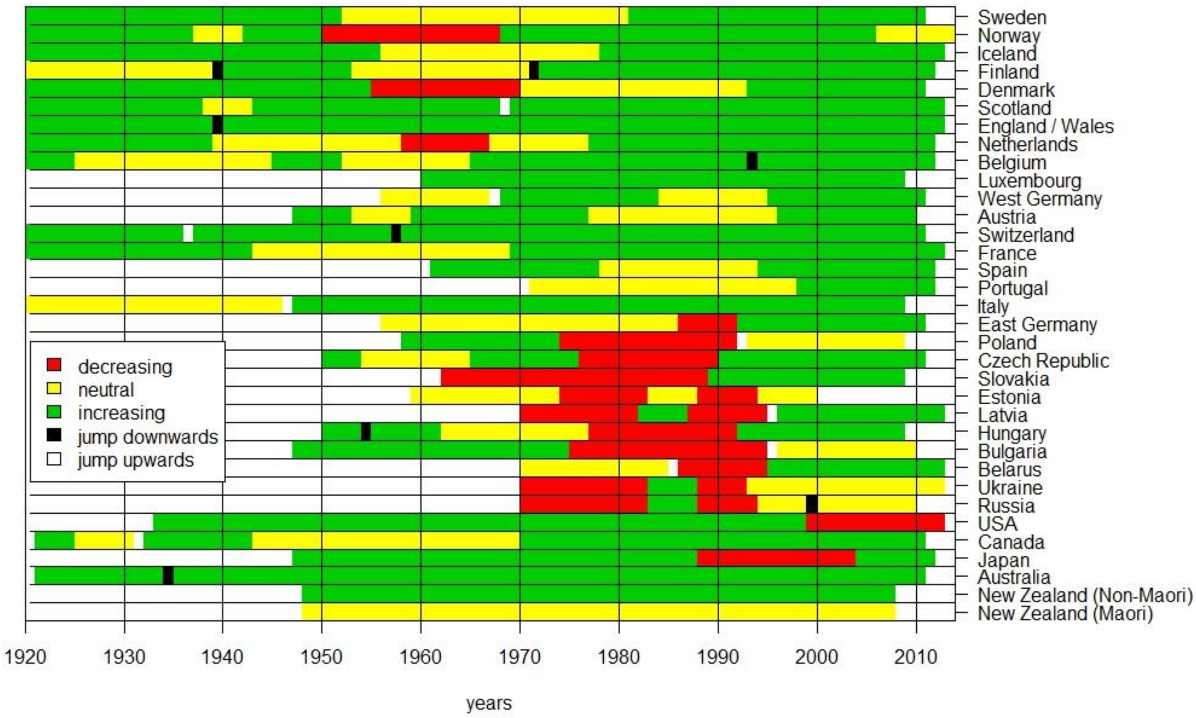

Fig. 5. Trends in the Evolution of the Number of Deaths at Modal Age of Death ($d(M)_0$): Males, Starting Age 0

Fig. 6 shows the relative similarity (RS) between each pair of the 34 populations (averaged over all four statistics) and provides a summary of the similarities and differences of the trends in the deaths curve's changes. Where there is a higher RS between two populations, the corresponding cell in this figure is brighter. Hence, in this figure, areas with white/yellow shades point to clusters with higher relative similarity, whereas areas with more reddish shades indicate larger differences. At first glance, we can detect bright areas in every corner of the figure. This means we have high relative similarities among the northwestern, central and southwestern European clusters, as well as the North American and Asia-Pacific clusters. Within these areas, we find some horizontal and vertical patterns. These patterns identify potential outliers regarding trends in the evolution of the deaths curve in their respective neighborhood. We observe such vertical and horizontal patterns, for example, for Iceland and the Maori population of New Zealand.

As expected, the relative similarity between the eastern European populations and the other populations is comparatively low, which again points to the difference in the trends between the eastern European cluster and the other populations. Though within this cluster, the relative similarity appears to be comparatively high, we also detect outliers here.

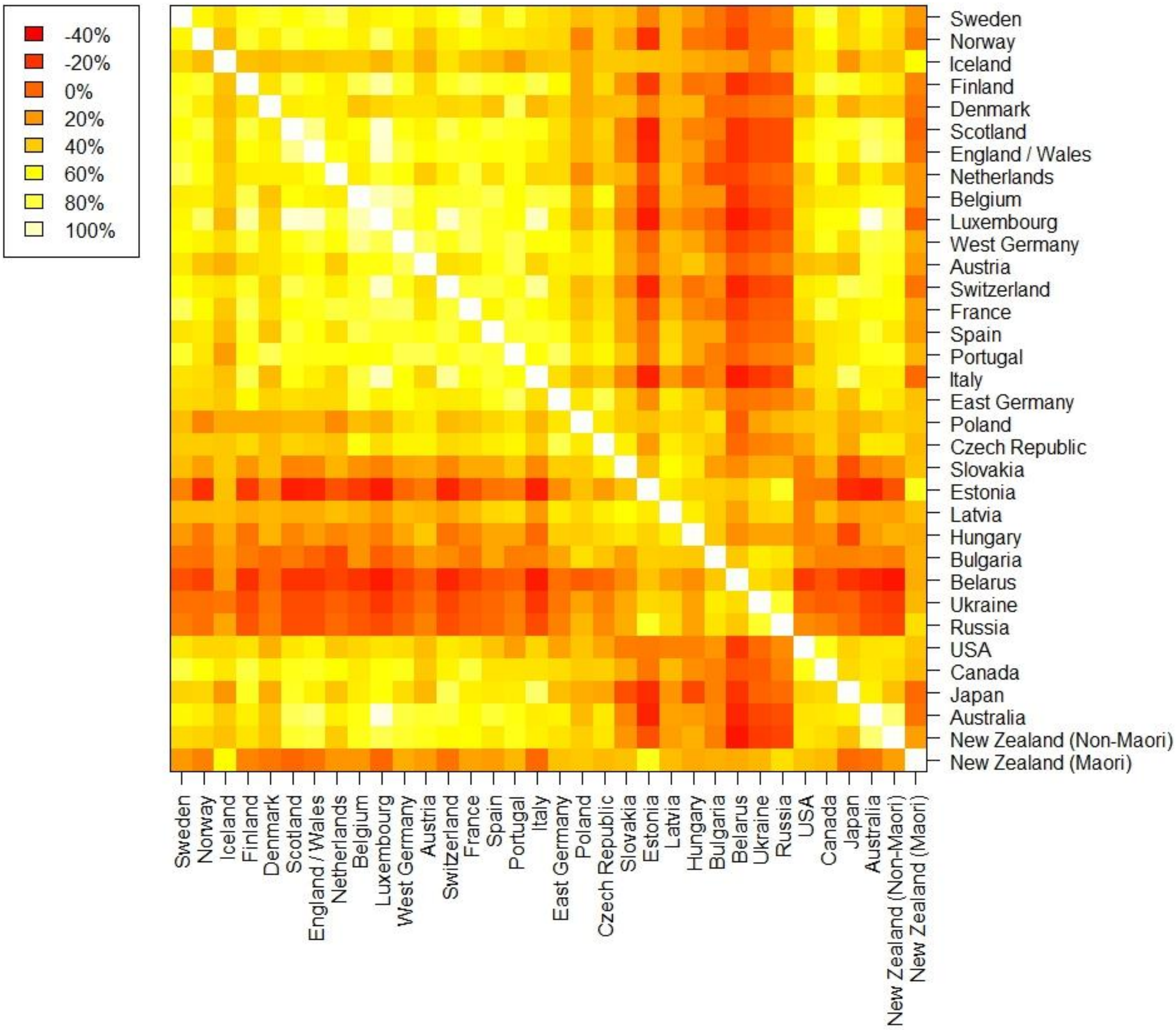

Fig. 6. Average Relative Similarities for all Populations: Males, Starting Age 0

For the most recent decades, we can summarize three major observations. First, toward the end of the observation period for most populations, we observe an increase for the statistics measuring the position of the deaths curve (i.e., M and UB). This means that during that time period, the deaths curves shifted to the right, and at the same time, the deaths curves' support extended. Some eastern European populations adopted these trends in the 1990s, which leads to an almost global scenario of right-shifting mortality and extension at the same time. For the majority of the other populations, this trend started even in the 1970s and thus is rather long-term. The general trend toward increasing statistics can also be observed for the statistics measuring changes in the shape of the deaths curve. However, here we observe more exceptions, both quantitatively and qualitatively, in particular in the eastern European cluster and outside Europe. In summary, the global reference scenario—derived from the trends we observed for males and apart from the exceptions mentioned above—for the most recent decades is (right shift, extension, compression, concentration).

Second, around 1970 (plus or minus about a decade), we observe a considerable accumulation of trend changes. For many populations, we observe a long-term increasing trend in many statistics after that. Especially for M, we observe a trend change around 1970 for many populations at the same time.

Finally, the trends in each of the four statistics for most eastern European populations experience a change shortly after 1990. Prior to this trend change, the trends of the eastern European and the other populations differ in general. Compared with the other populations, the eastern European cluster appears to be rather homogeneous, although the trends in the easternmost European populations and the other eastern European populations often are different.

5.2. Comparisons of the Trends in the Mortality Evolution

In this subsection, we analyze the mortality evolution of females with starting age 0 and the evolution of old-age mortality (i.e., starting age 60) for both sexes, and we compare the results with the results from Subsection 5.1. These analyses address two questions in particular: Are there significant differences in the trends of mortality evolution between females and males? And can we detect significant differences between the trends in the mortality evolution for the complete age range versus old-age mortality?

5.2.1. Females vs. Males

Fig. 7 displays the time bars for the modal age at death M for females. At first glance, the patterns in the trends and trend changes for females are quite different from those of males. We observe much more long-term right-shifting mortality, there is no left-shifting mortality during the 1960s, and we cannot find the almost global trend change around 1970 that we observed for males.

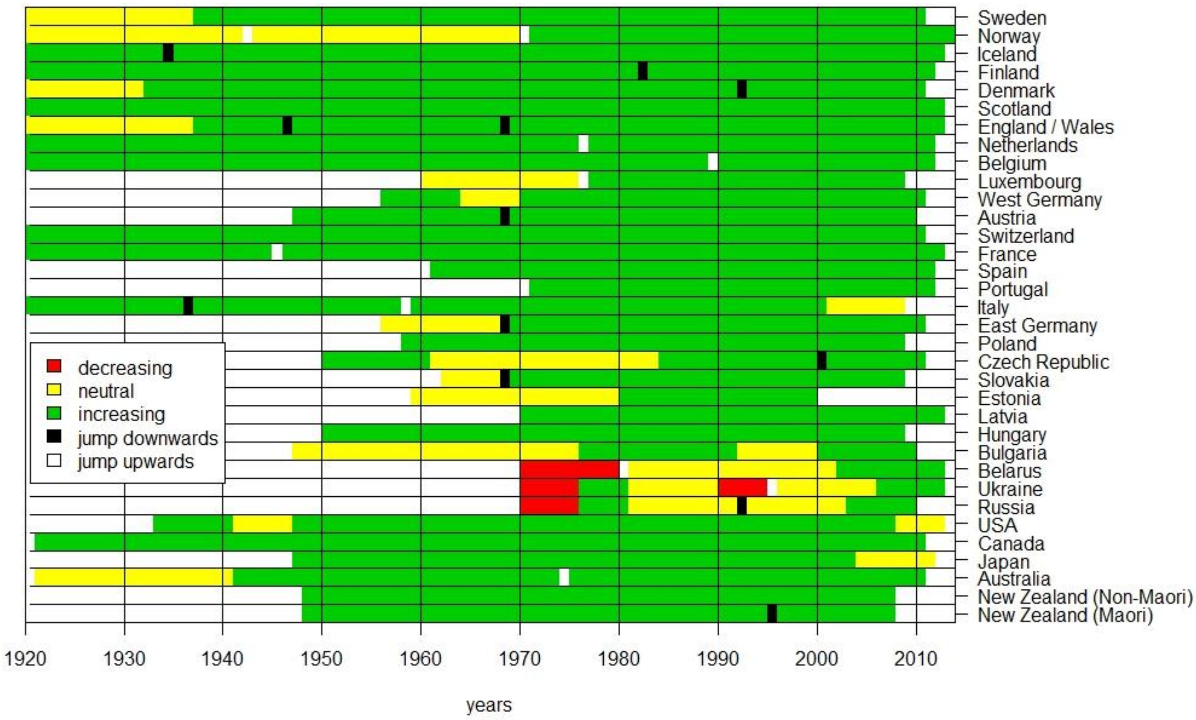

Fig. 7. Trends in the Evolution of Modal Age at Death (M): Females, Starting Age 0

However, though we do not observe a decrease in M during the 1960s for females, we can observe downward jumps for some populations at the end of the 1960s (e.g., England and Wales and Austria). This effect might be connected to the left shift for males at that time. Further, we can find a difference in the trends of some eastern European populations and the other populations, although this is only true for the easternmost populations. Moreover, the right shift during the most recent decades for most populations seems to be a unisex phenomenon. Thus, there are some similarities between the trends of females and males, but in general, the sex-related differences in the trends in M are considerable.

The trends in UB for females are shown in Fig. 8. The differences between the trends of UB for females and males are immaterial for some populations (e.g., the southwestern European cluster). The absence of sex-specific differences in the trend of UB for these populations might point to a decreasing importance of the individual's sex for the mortality structure with increasing age. The difference between the trends for eastern European populations and most of the other populations is also apparent for females. Also, we observe a very long-term increase for females, which starts even earlier for some female populations than for the corresponding male populations (e.g., in Belgium, Austria and the non-Maori population of New Zealand). Some populations seem to follow this long-term extension, but these trends are interrupted by downward jumps (e.g., in Norway, Luxembourg and West Germany).

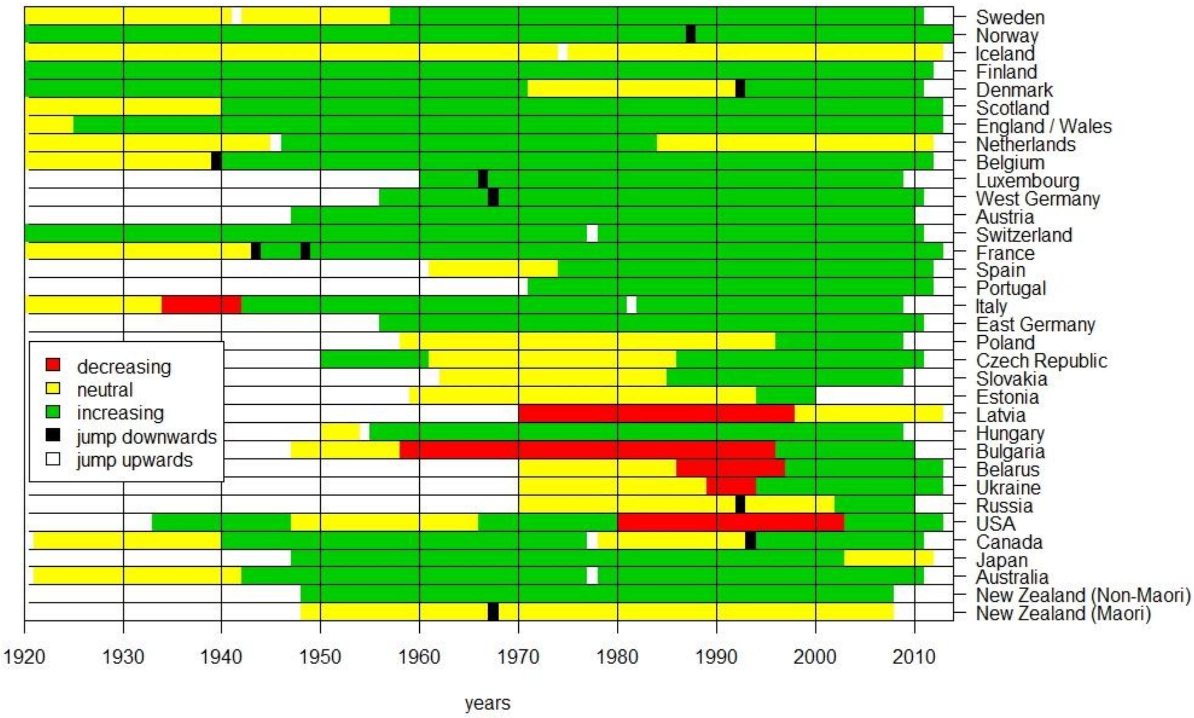

Fig. 8. Trends in the Evolution of the Upper Bound of the Deaths Curve's Support (UB): Females, Starting Age 0

Fig. 9 shows the time bars for the degrees of inequality (DoI_0) for females. The differences between females and males here have the same quality as the difference between sexes for UB. For some populations (e.g., Denmark), the sex-specific differences appear to be immaterial; for others (e.g., Austria), we observe a very long-term increase (i.e., compression) for females, where we observed periods of decrease (i.e. decompression) for males. Also, the sex-specific trends in DoI_0 are completely different in certain single populations (e.g., the Maori population of New Zealand). All in all, we can state two things: First, the supra-regional patterns we identified in the trends of DoI_0 for males seem to be unisex phenomena. Second, however, when looking at single countries, we sometimes find considerable differences between sexes.

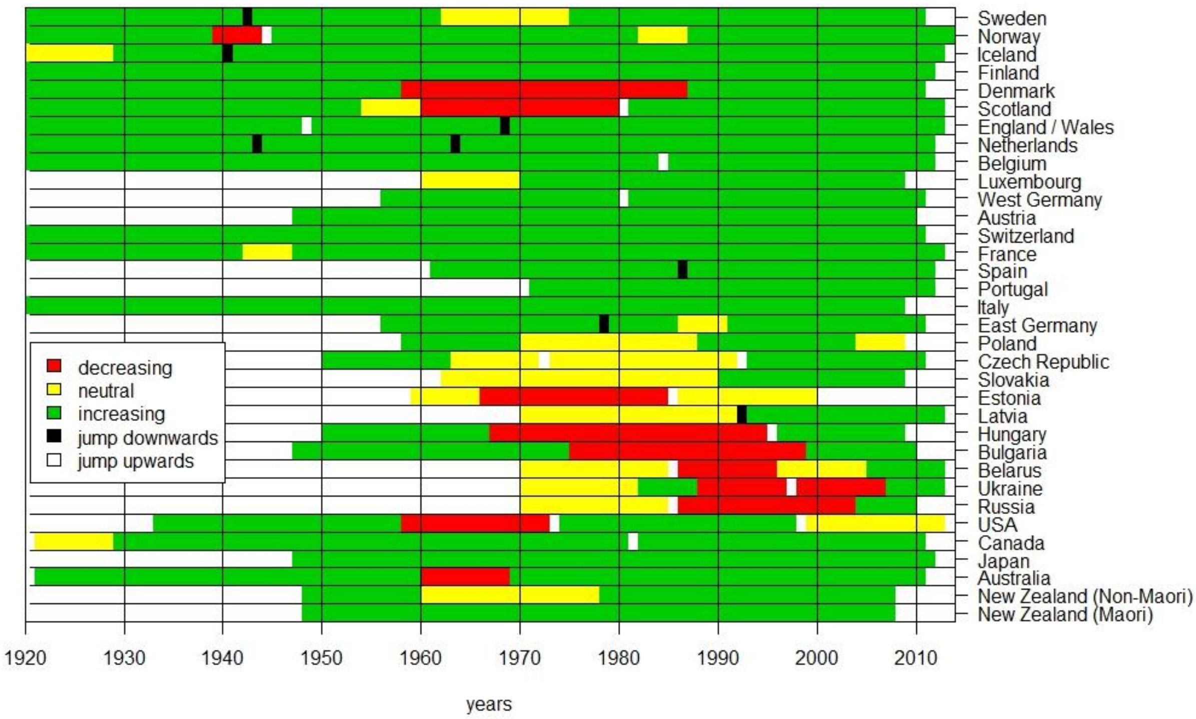

Fig. 9. Trends in the Evolution of Degrees of Inequality (DoI_0): Females, Starting Age 0

The time bars for the trend in the numbers of deaths at the modal age at death $d(M)_0$ for females are displayed in Fig. **10**. As for the other statistics, we also observe more long-term increases in $d(M)_0$ (i.e., concentration) than for males. In particular, for most central, southwestern and eastern European populations, these differences between sexes become apparent. For the eastern European cluster in general, we observe less diffusion for females than for males. In contrast, for few northern European populations (e.g., Denmark), we observe diffusion between about 1960 and 1990, which we cannot find for males.

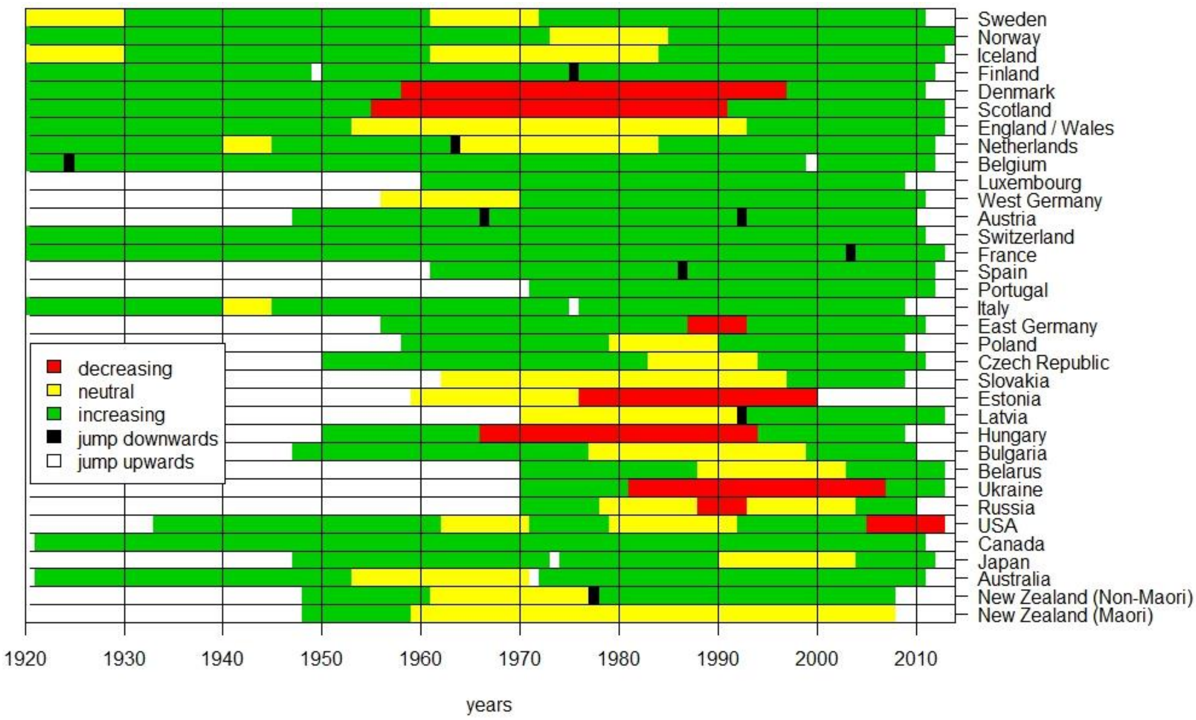

Fig. 10. Trends in the Evolution of the Numbers of Deaths at the Modal Age at Death ($d(M)_0$): Females, Starting Age 0

Across statistics, we find increasing trends for females during the most recent years. Thus, then the trends of the deaths curve's evolution coincide for both sexes for most populations. However, there are apparent sex-specific differences in the trends of the deaths curve's evolution over the entire observation period. This is supported by Fig. 11, which shows the relative similarity (RS) between sexes for the starting age 0.[4] For the majority of the populations (27 out of 34), the relative similarity is smaller than 70 percent, and there are only four populations (Luxembourg, Switzerland, Italy and Japan) where the RS exceeds 80 percent. Not least, this finding illustrates the importance of a sex-specific consideration of mortality evolutions, as females and males have experienced different trends in the mortality evolution during the last 100 years. Thus, for example, for the usage of unisex models, it must be carefully checked whether the model is applicable and reasonable, depending on the question at hand.

[4] We first calculate the relative similarity between sexes (instead of populations) per statistic and population. After that, we determine the arithmetical mean between relative similarities for the four statistics per population. The formulae we use here are analogous to those described in Section 4.

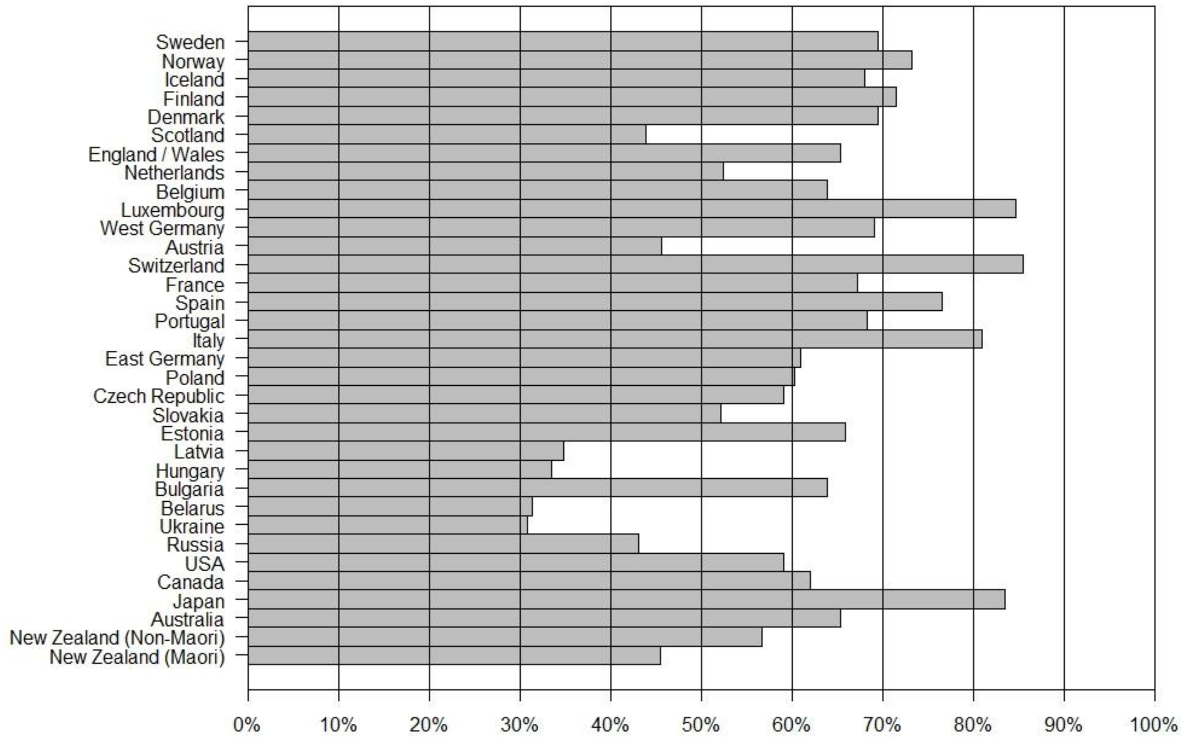

Fig. 11. Relative Similarity (RS) Between Females and Males: Average of All Statistics, Starting Age 0

Moreover, we can see from Fig. 11 that the sex-related differences of the trends in the deaths curve's evolution are relatively high (i.e., low RS) for most of the eastern European populations. For this cluster, we can find only three populations whose RS exceeds 60 percent: East Germany, Estonia and Bulgaria. In comparison, the RS between sexes for central and southwestern European populations, for example, is rather high. Thus, we can identify regions where the sex-related differences in the deaths curve's evolution are usually larger than in other regions. Moreover, also in neighboring countries (e.g. Austria and Switzerland), the RS between sexes can differ significantly. Note, however, that a low RS for sexes does not necessarily imply a great difference in the level of mortality between both sexes.

To conclude the analysis of the sex-related similarities and differences of the trends in the deaths curve's evolutions, we can state three major findings:

1. Regarding left- or right-shifting mortality, the differences between females and males appear to be significant. However, during the most recent years, the direction of the trends for both females and males tends to converge toward right-shifting mortality.
2. Regarding compression or decompression, we cannot find any clear difference between females and males. Despite some sex-specific differences for some populations, the generally observed trends seem to be unisex phenomena.
3. Regarding extension or contraction and concentration or diffusion, respectively, there are few sex-related differences. However, these differences cannot be regarded as immaterial.

5.2.2. *Starting Age 0 vs. 60*

Before we compare results for different age ranges, we recall that, as mentioned in Section 3, the modal age at death and the upper bound of the deaths curve's support are independent of the choice of the starting age. Thus, periods of left- or right-shifting mortality as well as extension or contraction coincide for both age ranges. Therefore, in this subsection, we only consider the trends in DoI_{60} and $d(M)_{60}$ and compare them with DoI_0 and $d(M)_0$, respectively.

Fig. 12 displays the time bars for the degrees of inequality DoI_{60} for males. At the first glance, we can see that there are many more and much longer periods of neutral trends or even decreases in DoI_{60} than in DoI_0. Indeed, for DoI_0, we observed an increasing trend for many European populations after World War II, which seemed to be interrupted by relatively short periods of decompression or jumps. Though we observe a long-term compression for the retirees in a few populations (e.g., Sweden, the Netherlands, Belgium, Japan and Australia), the majority of the European populations do not experience an increase in DoI_{60} before the 1980s or even later.

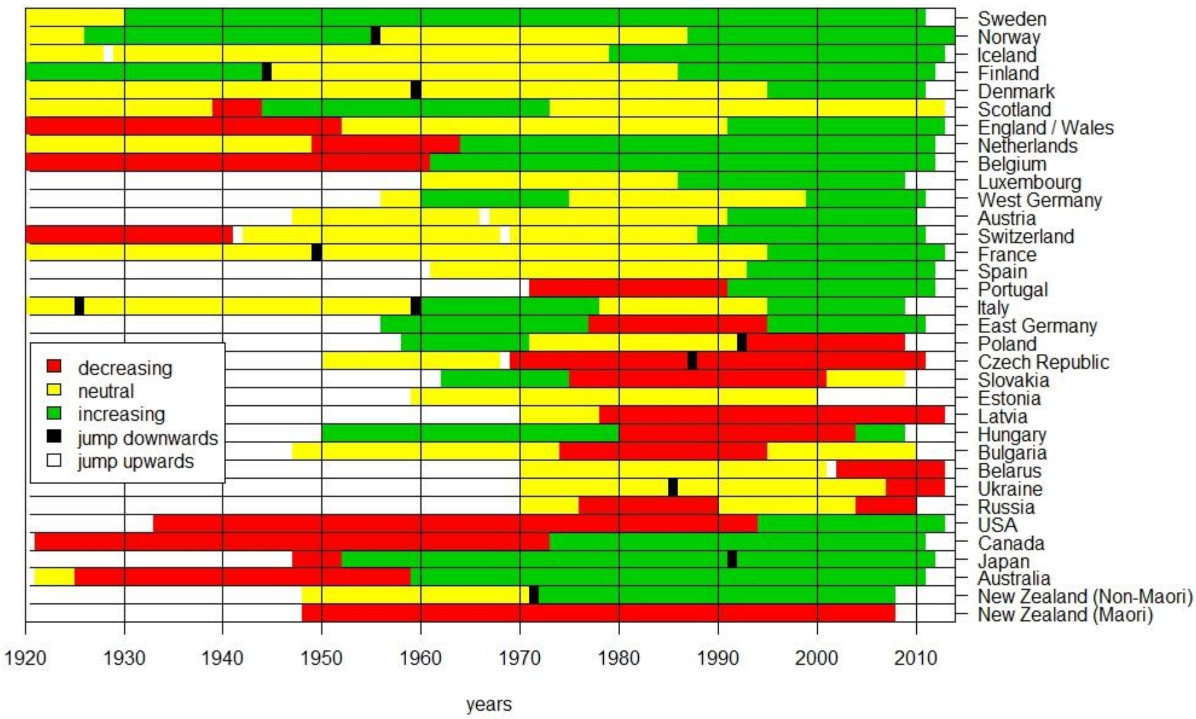

Fig. 12. Trends in the Evolution of the Degree of Inequality (DoI_{60}): Males, Starting Age 60

For most of the eastern European populations, we do not even observe any period of compression, but rather long-term decompression for the starting age 60. Thus, we have no apparent trend change there during the 1990s. For the other European populations (except Italy), however, we do not observe any decompression after the early 1960s, but rather neutral trends. This observation again underlines the differences in the trends of the mortality evolution between the eastern European populations and the other (European) populations even for the starting age 60.

For some non-European populations, we found some neutral periods in the most recent decades for the starting age 0. This also is not true for the starting age 60. Moreover, in the early decades of the observation period, we find compression for the starting age 0 and decompression for the starting age 60 for these populations. Thus, we can even observe opposing trends at that time for different starting ages.

Fig. **13** shows the trends in the numbers of deaths in the modal age at death $d(M)_{60}$ for males. For d(M), the differences between the starting ages are even bigger than for DoI. Before the late 1960s, concentration occurred only for single populations and for rather short time periods. Instead, we globally observe neutral trends or even diffusion for most populations. Starting in the late 1960s, we find a short trend of concentration for single populations across the world (e.g., England and Wales, Italy, East Germany, Bulgaria and the United States). However, during the 1980s, we have a rather comprehensive trend of diffusion. Only in the last few decades do we observe a comprehensive trend of concentration for almost all non-eastern European populations. In contrast, for most eastern European populations, we observe a continued diffusion until the end of the observation period.

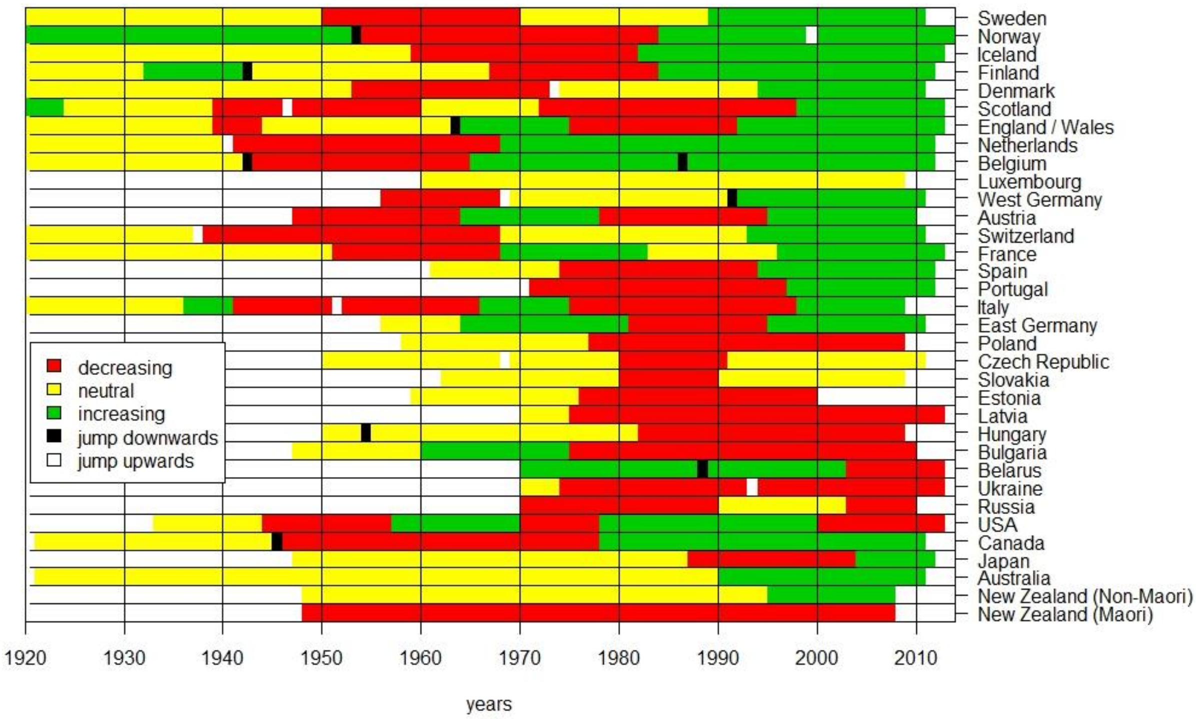

Fig. 13. Trends in the Evolution of the Numbers of Deaths at the Modal Age at Death $d(M)_{60}$: Males, Starting Age 60

For females, the differences between DoI_0 and DoI_{60} and between $d(M)_0$ and $d(M)_{60}$, respectively, are much smaller than for males (see Fig. 16 and Fig. 17 in Appendix 0 for starting age 60). Since we cannot obtain any new insights here, we do not carry out the comparisons in detail. However, in what follows, we briefly discuss the similarity (or dissimilarity) of the trends in the deaths curve's evolution between the starting ages for both sexes.

Fig. 14 and Fig. 15 show the relative similarities between the starting ages 0 and 60 for males and females, respectively, as an average over all four statistics over the entire observation period. We observe that the relative similarities between the different starting ages are much smaller for males than for females in general. For females, the trends in the deaths curve's evolution appear to be dominated by changes in the mortality at older ages, thus leading to similar trends for both starting ages. For males, in contrast, changes in the deaths curve below age 60 seem to have a more relevant impact, as they significantly change the trends observed only for ages above 60. This once again illustrates the differences in the trends of the mortality evolution between females and males.

In conclusion, we can state that there are considerable differences in the trends of DoI and d(M) between the starting ages 0 and 60. These differences are more significant for males than for females.

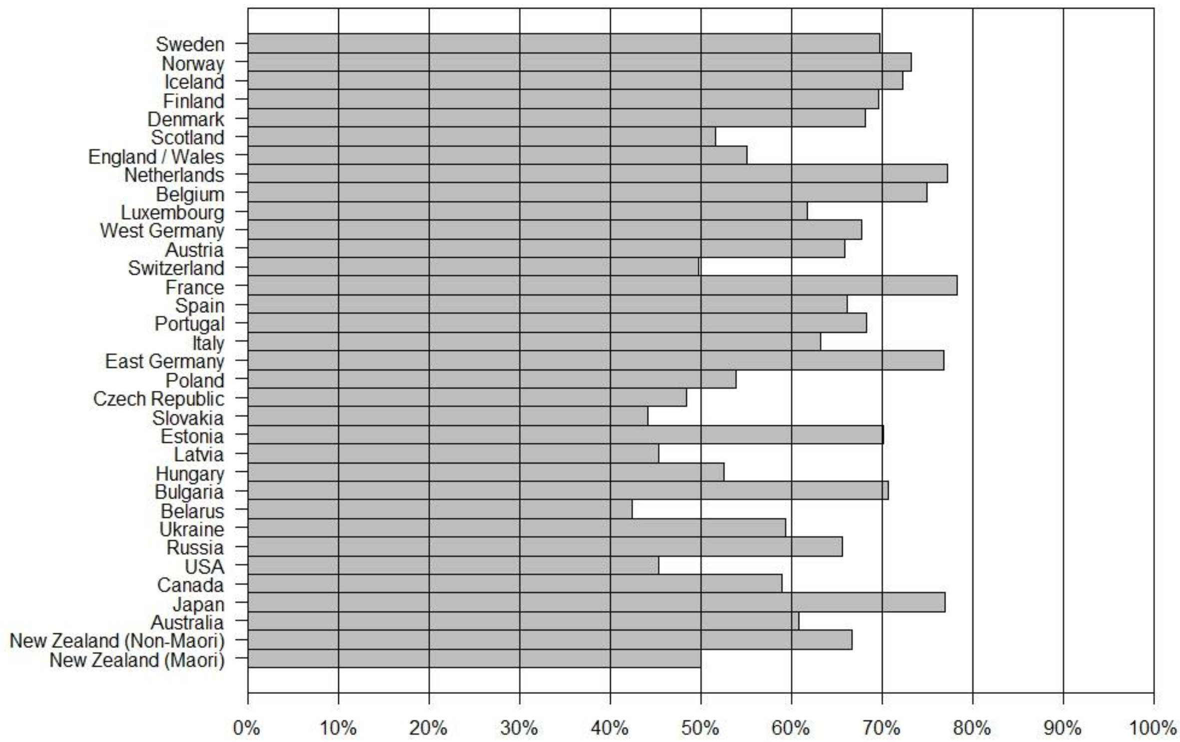

Fig. 14. Relative Similarities Between Starting Ages 0 and 60: Average of All Statistics, Males

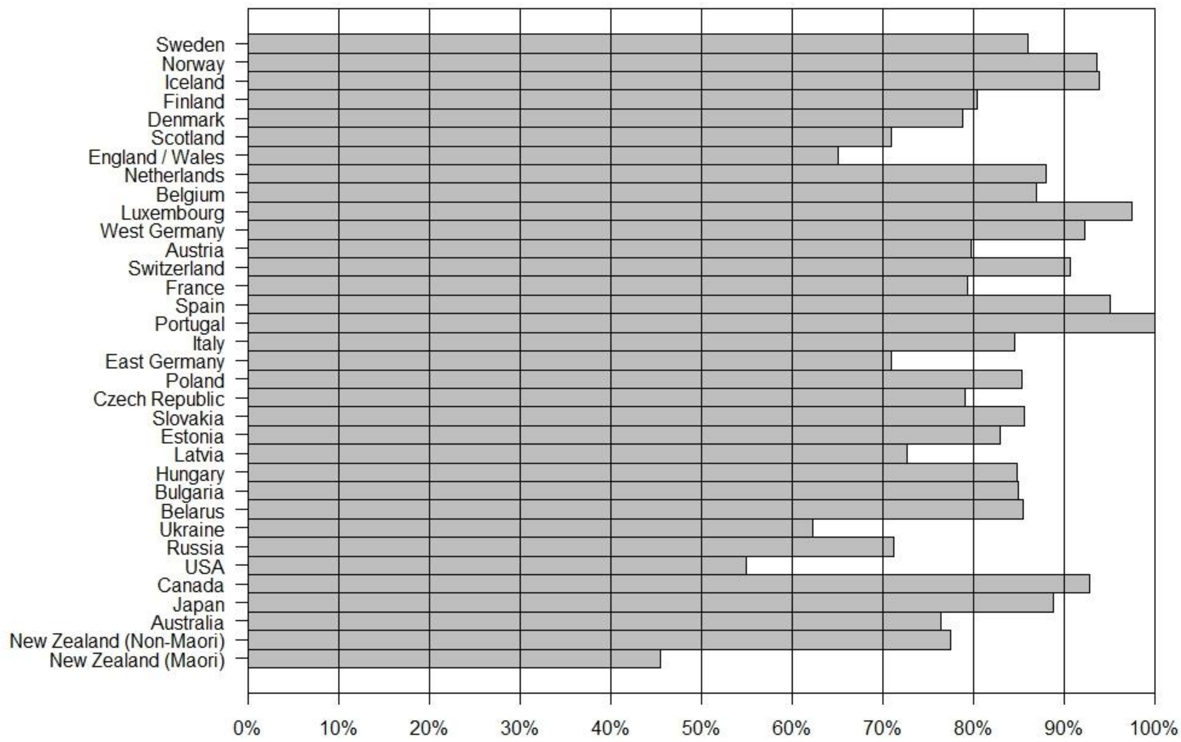

Fig. 15. Relative Similarities Between Starting Ages 0 and 60: Average of All Statistics, Females

6. Conclusion

In this article, we discuss and apply a classification framework for mortality evolution patterns that was recently introduced by Börger et al. (2016). This framework consists of four components, which together uniquely define a scenario of mortality evolution. The modal age at death M and the upper bound of the deaths curve's support UB indicate changes in the position of the deaths curve, where M measures right- or left-shifting mortality and UB measures extension or contraction, respectively. The degree of inequality (DoI) and the number of deaths in the modal age at death d(M) indicate changes in the shape of the deaths curve, of which DoI measures compression or decompression and d(M) measures concentration or diffusion, respectively.

We calculate these statistics for 34 populations separately for males and females and for the starting ages 0 and 60. Thus, we obtain 544 time series, which we have to rework. For this purpose, we discuss several methods. In addition, we introduce the relative similarity (RS), which enables us to efficiently compare each pair of trend processes with three attainable states.

In the discussion of the results, we first focus on the trends in the deaths curve's evolution of males with starting age 0. Here we obtained three major findings. First, during the most recent years, all four statistics increase for almost all populations, which generally is a long-term effect for all populations outside eastern Europe. Second, at least until the 1990s, the eastern European populations experience different trends than the other populations. And third, during the 1960s, we observe a comprehensive decreasing trend, especially for M, and an inversion of this trend around 1970 for many populations at more or less the same time. This allows for two conclusions: there are supra-regional patterns in the trends of the change of the deaths curve, and there is no single global pattern for these trends. Both findings must be taken into account in future research on this topic.

By comparison of these findings to the trends in the deaths curve's evolution of females, we obtain different results. Regarding left- or right-shifting mortality (i.e., the trends in M), the differences between males and females are considerable. In contrast, the differences between females and males regarding compression or decompression (i.e., the trends in DoI) appear to be immaterial. Finally, for extension or contraction (i.e., the trends in UB) and concentration or diffusion (i.e., the trends in d(M)), respectively, we can find certain differences between sexes but also some similarities. However, the increase in all four statistics during the most recent decades seems to be an almost-global unisex phenomenon. Analyzing the RS between females and males per population, we indeed find large differences in the trends of the deaths curve's evolution between sexes. Moreover, we find regions where these sex-related differences are larger or smaller than in other regions. For instance, in eastern Europe, the RS between sexes in general is smaller (meaning larger differences) than for other populations.

The trends in DoI and in d(M), respectively, for the age range beyond age 60 in general show fewer long-term increasing trends than for the complete age range. Consequently, we observe multiple periods where these statistics show opposing trends for the two starting ages for males. In contrast, for females, the differences of the trends in these two statistics between the starting ages are rather small. We therefore analyze the RS between the starting ages for both sexes and find that these similarities indeed are smaller for males than for females. This repeatedly illustrates the need for sex-specific analyses of mortality structures and sex-specific mortality modeling.

The insights of the present analysis especially can be helpful for the application of mortality models. There, the first step, of course, is to find a suitable model. The second step, however, is the calibration of the model to historical data. Conventional mortality models do not incorporate trend changes. Therefore, such models should be calibrated to periods of time series where the change of the deaths curve follows a constant trend. With this analysis, we can exactly refer to such periods. Moreover, for the application of more sophisticated models (e.g. multi-population mortality models), this analysis provides findings about coherent populations. For example, is it reasonable to apply a multi-population mortality model to data from Belgium, the Netherlands and Luxembourg in order to obtain more reliable mortality rates for the comparatively small population of Luxembourg? And if that appears to be reasonable, is this still true if we considered sex-related mortality and/or old-age mortality?

Moreover, the methods introduced here can be very useful for governments, life insurers and pension funds whenever trends in the mortality evolution need to be analyzed in the context of surrounding, dependent or superior populations. For instance, these methods enable life insurers to test the trends in the mortality structure of their portfolios for consistency with the country's general public.

Not least, these findings hopefully will initiate further research in more explanatory disciplines, such as epidemiology, medicine, biology, sociology and demography. To our knowledge, for example, the sex-related differences in the mortality structure (and consequently also in the trends of the mortality evolution) are still an open field of research. Moreover, from our point of view, the differences in the trends of the deaths curve's evolution between the starting ages, especially for males, are worth further research.

Acknowledgments

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Appendix A. Trends in the Mortality Evolution of Females With Starting Age 60

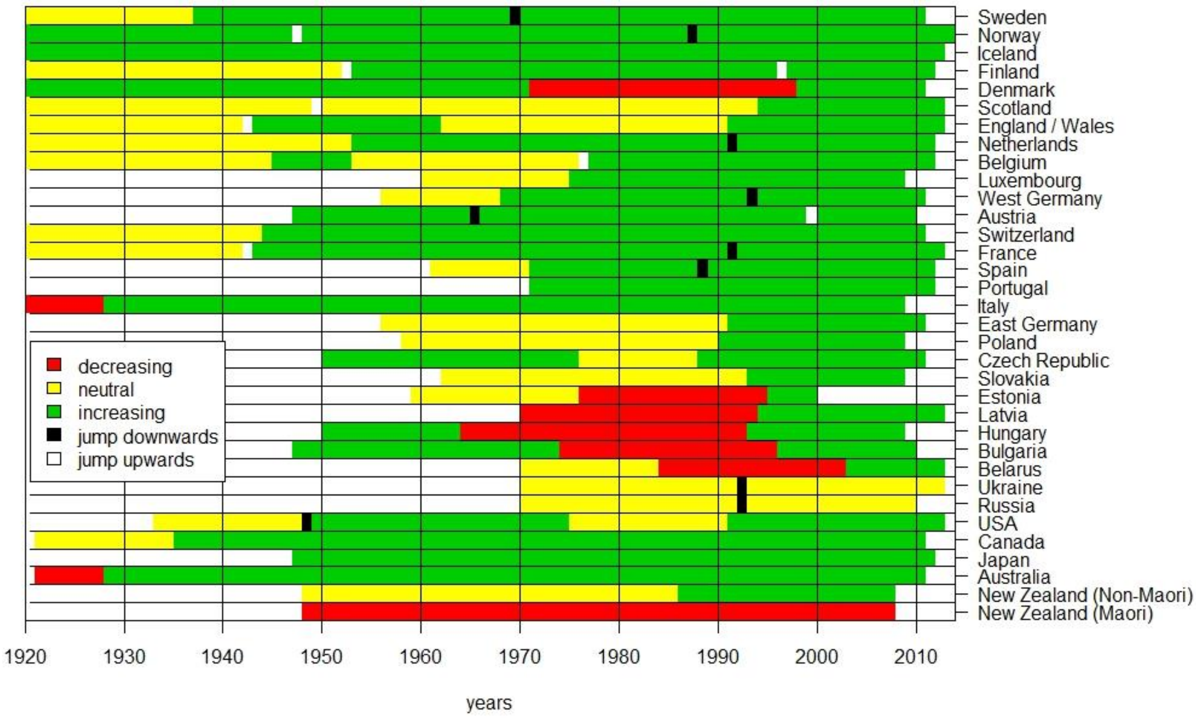

Fig. 16. Trends in the Evolution of Degrees of Inequality (DoI_{60}): Females, Starting Age 60

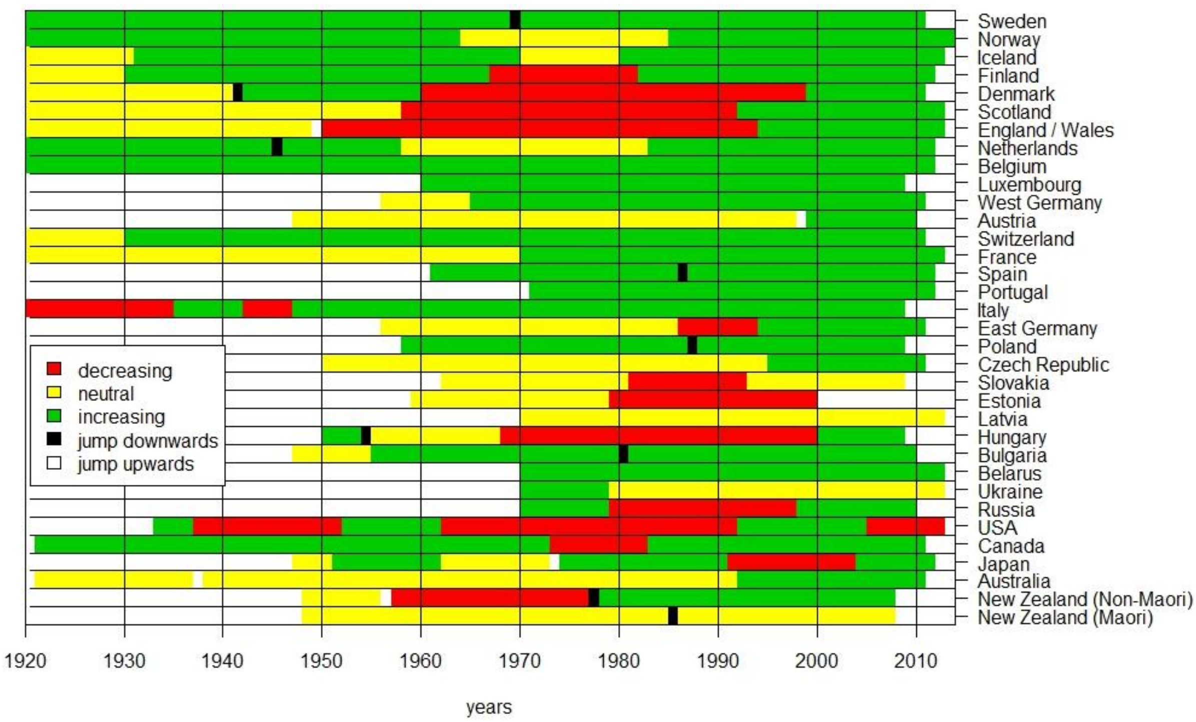

Fig. 17. Trends in the Evolution of the Numbers of Deaths at the Modal Age at Death $d(M)_{60}$: Females, Starting Age 60

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