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EXPERIENCE RATING

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IT HAS been said on innumerable occasions that the determination of a group experience rating formula is essentially a practical problem. The field of experience rating theory was apparently opened and closed for life actuaries with Mr. Keffer's paper, "An Experience Rating Formula," in *TASA* XXX. Recently Mr. Larson presented a paper, "A Method of Calculating Group Term Dividends" (*TSA* IV, 308), which describes a finished product in practical form with a minimum of reference to the underlying theory. Between these two papers lies an uncharted gulf of appearance and impression comparable in importance to the gap between a carefully defined q_x and a scale of gross premiums and dividends. The theory of life contingencies, while not directly producing gross premium scales, provides the actuary with tools for constructing and testing a consistent scale of gross premiums. Experience rating theory should similarly provide the means for constructing and testing a consistent experience rating plan.

The purpose of this paper is to present some of the theory underlying the experience rating process, together with the derivation and evaluation of several functions which are of use in constructing and testing practical surplus distribution formulas. The first portion of the paper is devoted to a brief discussion of the problems involved in experience rating and of the general methods of approach which may be adopted. The remainder of the paper deals with the mathematical formulas which may be used to evaluate several experience rating functions and the connection between these functions and practical distribution formulas. The appendix presents illustrative values of these functions for group life insurance.

I. GENERAL

A formula for distributing group surplus should return to a policyholder a portion of the aggregate divisible surplus which is consistent with the financial stability of the insurer and with the general objectives which underlie that formula. The return of such surplus is generally made in the form of a refund in cash, at the end of a policy year, of a portion of the premium paid during that policy year, or in the form of a discount, granted at the beginning of a policy year, to be applied to gross premiums falling due during that policy year. Whatever the form, refund or discount, the insurer is faced with the problem of establishing a consistent pattern for such returns, an experience rating plan, and of understanding the rationale which underlies that pattern.

As a starting point in the experience rating process, a group-writing company should have, for every group coverage, a set of standard or manual premium rates applicable to a given over-all classification of groups. As this standard premium scale will be independent of the experience or size of any individual case in the class of groups, it may be used as an experience base for an individual case in the sense that claims, expenses, and margins in any period may be expressed as a multiple of the total standard premiums for the insurance in force during the same period. Although the standard premium scales currently in use may not follow exactly the incidence of mortality by age, occupation, and other factors, the use of the standard premiums for an individual case as a statistical measure of exposed to risk can be justified on practical grounds. Thus a standard or manual loss ratio for a given period can be defined as total claims incurred during that period divided by total standard premiums for insurance in that period. Similarly, a standard or manual expense ratio can be defined as total expenses allocable to a given case in a particular policy year divided by the total standard premiums for the case in that policy year. Individual case statistics of this type can be determined on an accumulated-to-date basis as well as on a current basis with the policy year being a natural experience unit.

The standard loss ratio based on the combined experience of an entire class of groups in a one year period will usually be a stable enough quantity to exhibit long-term trends. For individual cases the standard loss ratios for the same period will vary about the over-all class loss ratio, because, first, some cases will be inherently poorer insurance risks than the over-all average while other cases will be inherently better and, second, the standard loss ratio for any one case will be subject to chance fluctuations because of the limited exposure in the experience period. Speaking mathematically, the standard loss ratio for a given case can be considered a random variable, the expected value of which may be termed the "true" loss ratio for that case. These "true" loss ratios will vary among cases due to basic differences in the underlying risks.

The probable loss ratio for an individual case may be defined as the best estimate the insurer can make as to the "true" loss ratio which that case would have experienced if the exposure had been large enough to make negligible the effect of any chance fluctuation. In practice the probable loss ratio is determined as a weighted average of the standard loss ratio to date for a given case and the standard loss ratio for the combined class of groups. The weight, or credibility, given the loss ratio for the individual case generally increases as the exposure size of the case increases. The individual case experience should, however, be restricted to relatively recent periods or be adjusted for any long-term trend in the over-all class experience if the probable loss ratio is to represent the current probable or expected level of losses. Mr. Keffer and others have thoroughly considered the evaluation of the probable loss ratio for an individual case.

The possible range in the chance fluctuation of the standard loss ratio for a given case about its probable loss ratio is perhaps best measured by means of the standard deviation in loss ratios. Mathematically, this will be the standard deviation of the random variable whose expected value is the "true" or probable loss ratio. For group life insurance, as is shown in the mathematical section, one approximation method for obtaining this standard deviation for a given experience period and a given case is to determine the value of the expected average amount per claim (or, more simply, per life) multiplied by the probable loss ratio and divided by the standard premium for that period and to extract the square root of this quantity. The standard deviation in loss ratios will increase with increase in the average amount per life and also with increase in the spread between the schedule maximum and the average amount per life because of the tendency of most variable amount schedules to concentrate the larger amounts at the older ages where the risk of loss is greatest. The standard deviation in loss ratios will decrease with increase in the standard premium and will also tend to decrease when small amounts are continued for pensioners because such a continuation program will decrease the expected average amount per claim. As a practical matter, differences between insurance schedules and their effect on loss ratio variance can be ignored if the schedules are limited by reasonable underwriting rules which are so graded by size of case as to keep the possible range in standard deviations roughly constant and relatively insignificant for each size class.

Some individual cases may be so large that any chance claim fluctuation would not unduly influence annual claim costs. Such a policyholder is, in effect, insured only against the occurrence of catastrophic losses and may demand some assurance that any excess of billed premiums over incurred claims and reasonable assessed expenses will be refunded at the end of each accounting period. From the insurer's point of view this implies carrying forward any deficit, resulting when incurred claims exceed premiums less assessed expenses, into the next accounting period with the only source of repayment being possible future experience gains. This general method of experience rating may be called the pure accounting method and is based on the postulate that the insurer should provide chiefly claim-paying and accounting services to very large policyholders.

The amount of any deficit which is to be carried forward under the pure accounting method must be restricted to such a reasonable level that the potential reduction in future refunds will not force an intelligent policyholder to take his insurance program elsewhere. If, however, total losses exceed this reasonable level and the excess losses are not to be carried forward as a deficit, the insurer must charge the general surplus or contingency funds with the amount of excess loss and should recover the amount of such loss, or fund it in advance, by means of some annual "insurance charge." This approach leads directly to the somewhat more general concept of modifying the pure accounting method by "insuring" losses in excess of a given insurance loss level and making a charge for that insurance. By means of this general concept, the rationale behind many types of distribution formulas in actual use can be examined.

The insurance loss level may be set so high that there is very little likelihood of a given case incurring total claims which exceed that level in any policy year through chance fluctuation alone. Such excess losses should then occur infrequently enough to be considered nonrecurring catastrophic losses. This implies that losses in excess of the catastrophe level may be excluded from an individual case's experience not only when applying the modified pure accounting method to that case but also when determining the probable loss ratio for that case. If this catastrophe level is defined, in terms of loss ratios for any experience period, as the sum of the probable loss ratio and some multiple of the standard deviation in loss ratios for that period, the probability of an individual case experiencing catastrophic losses in that period will be roughly constant for all exposure size classes. As an example, if this approach were taken and if loss ratios which exceed the probable loss ratio plus four standard deviations were considered to be catastrophic, a fifty-life case might well experience in some policy year a reasonable non-catastrophic loss, under group life insurance, of the probable losses plus 300% of the standard premiums for that policy year, whereas a 50,000 life case might experience a nonrecurring catastrophic loss which exceeds only the probable losses plus 10% of premiums. The probable or expected amount of losses in excess of the catastrophe level will usually be quite small, being in the neighborhood of 1% of standard premiums for losses exceeding the probable plus four standard deviations, and this amount may reasonably be included as a flat charge with the assessed expenses. If the probable excess loss were relatively large, it would be necessary, for conservatism, to base the amount of the insurance charge on probable funding in such a manner that the aggregate amount of excess losses for the over-all class of groups would be actually collected.

Under some of the distribution formulas in actual use, the refund payable at the end of the tth policy year is assumed to cover the entire t-year experience period and the previous refunds paid, if any, are assumed to be merely advance installments of the true t-year refund. The process involved is one of continual correction of the experience rating factors so that eventually the proper amount will have been returned to the policyholder. The general effect when such a t-year formula is applied is to increase the exposure size of the group by considering larger and larger experience periods, thus reducing the applicable loss ratio variance. This concept should not, of course, be applied when the possible fluctuation in claims for the next succeeding policy year is being considered.

One example of a t-year distribution formula in general use is mentioned in *Actuarial Studies No. 6* on page 117, and can be expressed as

$$\text{current refund } = aS' - \text{total previous refunds}$$

where a is an empirical reduction factor to reduce total paid refunds to total distributable surplus and S' is the total expected surplus based on the probable loss ratio at the end of the period. This formula may be thought of as the pure accounting method with a t-year accounting period modified by the insurance of all losses in that period which exceed the probable. The theoretically required aggregate assessment is "collected" by temporarily withholding all unexpected surplus which arises from experience better than the probable and the reduction factor corrects the assessment to the aggregate amount actually required. The insurance charge is thus collected only from cases having better than expected experience and the amount of actual surplus which should be withheld from an individual case at the end of any experience period can be obtained by multiplying the standard premiums to date by the excess of the probable loss ratio to date over the standard loss ratio to date. If the probable loss ratio remains constant over the experience period, this distribution formula is equivalent to granting a premium discount equal in decimal form to 1 less the probable loss ratio less the standard expense ratio. This type of formula may be called an expected surplus formula and its underlying objective is to see that each case is charged for its own expected claims.

A somewhat different type of t-year formula results if the underlying objective is to return to an individual policyholder as much of the actual surplus earned under his policy as is consistent with the financial stability of the insurer. This type of formula may be derived by modifying the pure accounting method so as to insure all losses in excess of billed premiums less assessed expenses, *i.e.*, the premium margin for claims. The required aggregate assessment may be collected by temporarily withholding a portion of actual surplus to date, in which event the distribution formula reduces to

$$\text{current refund} = JS - \text{total previous refunds}$$

where S is the actual surplus earned under the policy since issue and $(1 - J)$ is the most probable portion of the actual surplus which must be withheld in order to fund the probable excess loss. The factor J can be shown to depend essentially on the premium margin for claims and the standard deviation in loss ratio.

If the required aggregate assessment were to be funded by temporarily deducting some portion of standard premiums from actual surplus before paying any refund, the resulting distribution formula would reduce to

$$\text{current refund} = S - K \times (\text{standard premiums to date}) - \text{total previous refunds}$$

where K is the most probable portion of standard premiums which must be withheld in order to fund the probable excess loss. In practice an amount equal to K times standard premiums would be included with expenses. The factor K can be shown to depend on the premium margin for claims, the standard deviation in loss ratios, and the exposure size of the case. Methods for evaluating J and K will be discussed in the mathematical section and illustrative values for group life insurance are shown in the appendix.

When the experience period consists of the entire period since issue of a case, the distribution formula acquires a retrospective aspect and the insurer, in withholding a portion of the actual surplus earned by cases which have had good experience, usually has as a goal the accumulation of an amount sufficient to cover the actual deficits arising from those cases in the same exposure size class which have had poor experience. This implies that the insurance level for claims which applies to the entire t-year experience period in this type of formula must not exceed the premium margin for claims if an over-all deficit for the exposure size class is to be avoided. The insurance charge and the excess losses which are insured will be adjusted at the end of each year to include the entire experience to date and, as a result, losses which are insured in this manner at the end of a given experience period may be chargeable to the experience of the case in determining subsequent refunds. The actual surplus S in these formulas can, of course, be modified by the use of a catastrophe level which applies to each policy year as a separate unit, provided the amount of probable losses in excess of the catastrophe level is included each year as a flat charge with assessed expenses. It is also apparent that any of these t-year formulas may be modified so as to apply to the experience of any given policy year as an independent experience unit.

Discount actions following the actual surplus type of formula would reduce the premium margin for claims to such a point that the probable surplus arising from experience better than the margin will fund the probable losses in excess of the margin and thus would reduce the premium margin for claims to the probable loss ratio as under the expected surplus type. When discount action and refund action are combined, however, smaller discounts may be granted and refunds which are consistent with the increased premium margin for claims can be determined from the actual surplus type of formula. The actual surplus formulas therefore provide a consistent grading between the refund actions taken on very small cases, which are generally of the expected surplus type, and the refund actions taken on the very largest cases, which for competitive reasons may frequently be very similar to the pure accounting method.

The actual surplus type of distribution formula restricts the payment of refunds to cases which have effectively contributed positive amounts to the surplus currently divisible and thus tends to maximize the refunds paid to groups having good experience. On the other hand the expected surplus type of distribution formula may result in the payment of a refund to a case having an over-all actual deficit, a criticism which incidentally could also be directed at premium discounts in group insurance and post-mortem dividends in ordinary insurance. The rationale behind such a cash payment to an individual case which has already reduced the aggregate amount of divisible surplus through experience losses is at best obscure and seems inconsistent with the principle underlying the contribution plan of "returning to each policyholder that part of the divisible surplus which may be considered as having been contributed by him" (*Actuarial Studies No. 6*, page 24). This apparent inconsistency, which is to a certain extent eliminated if attention is focused on classes of groups, arises because the objective of the expected surplus formula is to provide equity between classes of groups and to charge each case within a class with its own expected claims. The objective of the expected surplus formula and the quoted objective of the contribution plan are thus not identical on the individual case level. Of course the particular type of distribution formula adopted by an insurer will depend on what the insurer is attempting to accomplish with that formula and quite probably the participating and nonparticipating approaches will differ.

The concept of modifying the pure accounting method by the introduction of an insurance level and an insurance charge can be used to obtain many other types of distribution formulas. As an example, if an annual accounting period is used with the insurance loss level set at the premium margin for claims, either of the actual surplus formulas could be used to determine a preliminary refund. Part or all of this preliminary refund might then be withheld by the insurer to establish, for that individual case, a claim fluctuation fund which in effect would increase the premium margin for claims used in the actual surplus formula for the second policy year. Such a claim fluctuation fund is probably necessary if the total of the refunds paid to a case since issue to is be independent of the year by year order in which claims occur. As a second example, an annual accounting period can be used with the insurance loss level set, for each size class, at a level which the insurer feels reasonably certain will not force a lapse. This would seem to imply that the insurer can "trust" the policyholder to repay out of future experience gains any deficit not exceeding the insurance loss level less the premium margin for claims and, in consequence, to repay any insurance charge made within the same limit. The resulting distribution formula might be called a theoretical surplus formula and in effect assumes that a portion of general surplus is assigned to each case as an initial claim fluctuation fund. While the interests of conservatism suggest basing the probable funding of the insurance charge, and as a result its size, on a level lower than the insurance loss level, it would be undesirable to base the actual collection of the insurance charge on such a lower level, not only because of the resulting inconsistency in the treatment of the insurance charge as compared with the treatment of claims and expenses but also because of the peculiar effect of such a lower level on the marginal charge for actual claims.

II. MATHEMATICAL DEVELOPMENT

The following general notation has been used where the items refer to a particular policy year:

A' = expected average amount per claim
B = advance discount rate
C = expected claims by over-all class experience

D = actual claims
E = standard expense ratio
P = standard premiums
R = refund paid
T = insurance level for claims
U = premium margin for claims divided by standard premiums = $(1 - B - E)$
Z = credibility factor

The standard loss ratios, x, which are actually experienced in any period for which the probable loss ratio is Q will be assumed to follow a probability distribution having mean Q, standard deviation σ, frequency function $f(x)$, distribution function

$$F(x) = \int_{-\infty}^{x} f(y)\,dy\,,$$

and range 0 to M, where MP represents the maximum possible amount of claims under the given case during that period. For group life insurance the upper limit of the range does not affect the shape of $f(x)$ to any noticeable degree since M will usually exceed $Q + 50\sigma$. For most group casualty coverages the effect of M on $f(x)$ will also be negligible. Any "bunching" of loss ratios at 0, the lower limit of the range, must be limited to those cases which vary from the mean Q by Q/σ standard deviations. The positive skewness which the lower limit imposes on $f(x)$ can thus be expected to wear off whenever Q/σ, a measure of the exposure size of the case, exceeds 4 or 5, beyond which it has been assumed that $f(x)$ will be approximately normal. The general shapes of $f(x)$ and $F(x)$ will be determined, for practical purposes, by the values Q and σ and the type of group coverage involved.

For relatively small exposure sizes, where Q/σ is less than 1 say, the mode of $f(x)$ will be at zero and at the positive abscissas $f(x)$ will be small and relatively haphazard. The distribution function $F(x)$ on the other hand will be relatively stable, having a large "step" at zero and being almost linear thereafter. For purposes of evaluating functions dependent on the distribution of loss ratios about the probable, it will therefore be convenient, when small exposure sizes are considered, to express the functions in terms of $F(x)$ rather than in terms of $f(x)$.

For very large exposure sizes it will be convenient to express the functions in terms of the standardized normal frequency function $\phi(x)$ and the standardized normal distribution function $\Phi(x)$, inasmuch as tables of values for the latter functions will be readily available.

To obtain representative values of $f(x)$ and $F(x)$ for group life insurance either the Poisson distribution can be arbitrarily used or an actual distribution of one year standard loss ratios for cases insuring, say, 50 to 99 lives can be used on the assumption that such a distribution would, to a large extent, represent chance fluctuation while keeping the effect of variation in the true loss ratio to a minimum. If the latter approach is taken, the probabilities in the actual one year distribution may be easily projected to the end of the second, fourth, eighth, etc., years to obtain distributions with the same value of Q which will apply to larger exposure size cases and thus smaller values of σ. It is not necessary to graduate the probabilities $f(x)$ in the initial distribution, because the projection process tends to smooth out these probabilities for the longer exposure periods and for the short exposure periods $F(x)$ will be quite stable. An actual distribution of 2,000 cases was used to obtain values of $F(x)$ and, by approximate integration, a table of values of the function

$$\int_{-\infty}^{x} F(y)\, dy$$

both initially and for several projected durations. The values shown in the appendix were then derived from this table. Needless to say, the initial distribution will be dependent on company underwriting rules as to schedules and standard industrial classes as well as on the composition as to standard rate basis and the proportions in each subsize bracket. The values in the appendix are thus intended to be illustrative only. The method which was used, however, is a practical one and should enable the actuary to test the factors used in an actual surplus type of distribution formula.

The expected losses in excess of an amount TP can be expressed as

$$P\int_{T}^{+\infty} [x-T]\, f(x)\, dx = P \times L(T)\,,$$

where $L(T)$ is the expected excess loss as a multiple of the standard premium, x is a particular standard loss ratio and $f(x)$ is the probability that losses of exactly xP will occur. To evaluate $L(T)$ for small exposure sizes, this expression may be divided by P and, using the relations

$$\int_{-\infty}^{+\infty} x f(x)\, dx = Q \qquad \text{and} \qquad \int_{-\infty}^{x} f(y)\, dy = F(x)\,,$$

we obtain

$$L(T) = \int_{-\infty}^{T} F(x)\, dx - (T-Q) \qquad (1)$$

and, if a table of values of

$$\int_{-\infty}^{x} F(y)\,dy$$

is available, the values of $L(T)$ can be readily obtained. For large exposure size classes $f(x)$ will be normal (Q, σ) and using the relations

$$f(x) = \frac{1}{\sigma}\,\phi\left(\frac{x-Q}{\sigma}\right) \qquad \text{and} \qquad u\phi(u)\,du = -\,d\phi(u)$$

equation (1) becomes

$$L(T) = \sigma\left[\phi\left(\frac{T-Q}{\sigma}\right) - \left(\frac{T-Q}{\sigma}\right) + \left(\frac{T-Q}{\sigma}\right)\Phi\left(\frac{T-Q}{\sigma}\right)\right]. \quad (1)'$$

It will be convenient, in practice, to compute values for $g(x) = \phi(x) + x\Phi(x)$ and express equation (1)′ in the form

$$L(T) = \sigma g\left(\frac{T-Q}{\sigma}\right) - (T-Q). \qquad (1)''$$

If a proportion of actual surplus is to be withheld to exactly fund, in probability, the expected excess loss $L(T)$ and if $(1 - J)$ is the required proportion

$$(1-J)P\int_{-\infty}^{U} [U-x]\,f(x)\,dx = P \times L(T)$$

where $P[U - x]$ is the actual surplus earned if the case experiences a standard loss ratio of x. Using similar methods this expression reduces to

$$(1-J) = \frac{L(T)}{L(U)+(U-Q)} \text{ or } J = \frac{L(U)+(U-Q)-L(T)}{L(U)+(U-Q)}. \qquad (2)$$

The maximum refund allowable can be expressed as $J[UP - D]$. When T is set equal to U, equation (2) becomes

$$(1-J) = \frac{L(U)}{L(U)+(U-Q)} \quad \text{or} \quad J = \frac{(U-Q)}{L(U)+(U-Q)} \qquad (2)'$$

and the illustrative values for J have been determined from equation (2)′. When claims in excess of probable are insured and surplus arising only from experience better than probable is used to fund the excess loss, equation (2)′ may be modified by setting $U = Q$ from which it is apparent that J is zero and that all such surplus must be withheld in the expected surplus type of formula.

If a proportion of standard premiums is to be withheld to exactly fund, in probability, the expected excess loss $L(T)$ and if K is the required proportion

$$KP\int_{-\infty}^{U-K} f(x)\,dx + P\int_{U-K}^{U} [U-x]\, f(x)\,dx = P\times L(T).$$

The first integral represents the probable amount collected from cases having good enough experience to pay the full charge KP and the second integral represents the probable amount collected from cases able to fund only a portion of KP. This expression reduces to

$$\int_{-\infty}^{U} F(x)\,dx - L(T) = \int_{-\infty}^{U-K} F(x)\,dx \qquad (3)$$

and K can be obtained from a table of values of

$$\int_{-\infty}^{x} F(y)\,dy$$

for small exposure sizes. For large exposure sizes where $f(x)$ is normal (Q, σ) this expression becomes

$$g\left(\frac{U-Q-K}{\sigma}\right) = g\left(\frac{U-Q}{\sigma}\right) - g\left(\frac{T-Q}{\sigma}\right) + \left(\frac{T-Q}{\sigma}\right) \qquad (3)'$$

where as before

$$g(x) = \phi(x) + x\Phi(x).$$

The maximum refund allowable will be given by $(U - K)P - D$. When $T = U$ equation (3) reduces to

$$(U-Q) = \int_{-\infty}^{U-K} F(x)\,dx$$

while equation (3)′ reduces to

$$g\left(\frac{U-Q-K}{\sigma}\right) = \frac{U-Q}{\sigma}.$$

If, in equation (3), U and T are set equal to Q, as in the expected surplus type formula, $K = Q$ and the entire amount of surplus derived from experience better than probable would be withheld as before.

In general, the K method collects the full insurance withholding before any refund is payable while the J method tends to "soak the best" by collecting larger insurance charges from cases with very good experience and lesser charges from cases having near premium margin experience. In rationale the J method lies midway between a flat charge regardless of actual surplus as represented by the K charge method and a variable charge levied only against cases with better than probable experience as represented by the expected surplus method. Both the J method and the K method will approach the expected surplus method as the premium margin for claims approaches the probable losses.

In applying the actual surplus formulas to individual cases it may be convenient to determine the standard deviation in loss ratios by approximate methods. Given a group of n lives with a flat scheduled amount A and assuming that the true probability of death q_r is known for the rth individual, the variance in claim amount σ_A^2 for the group is given by the expression

$$A^2 \sum_{i=1}^{n} q_i (1 - q_i) .$$

Letting q_0 equal the largest of the q_i and

$$q = \frac{1}{n} \sum_{i=1}^{n} q_i ,$$

it follows that

$$(1 - q) A^2 n q \geqslant \sigma_A^2 \geqslant (1 - q_0) A^2 n q .$$

Considering the usual size of q then, A^2nq can be taken as a conservative approximation to σ_A^2. The standard deviation in loss ratios σ can be obtained from the relationship

$$\sigma = \frac{\sigma_A}{P} .$$

The approximation

$$\sigma^2 \doteq \frac{A^2 n q}{P^2}$$

can be expressed in terms of readily available factors by assuming that

$$QP \doteq Anq$$

in which case

$$\sigma^2 \doteq \frac{AQ}{P} .$$

When a schedule with various amounts of insurance A_j ($j = 1, \ldots, k$) applies to the given group, the approximation for the variance in claim amount becomes

$$\sigma_A^2 = \sum_{j=1}^{k} \sigma_{A_j}^2 \doteq \sum_{j=1}^{k} A_j^2 n_j q_j ,$$

where n_j is the number of lives insured for amount A_j and q_j is the average probability of death for the n_j lives. By means of the rough approximation

$$\frac{\sum_{j=1}^{k} A_j^2 n_j q_j}{\sum_{j=1}^{k} A_j n_j q} \doteq \frac{\sum_{j=1}^{k} A_j n_j q_j}{\sum_{j=1}^{k} n_j q_j}$$

and the assumption that

$$QP \doteq \sum_{j=1}^{k} A_j n_j q_j ,$$

the variance in claim amount can be roughly approximated by

$$\sigma_A^2 \doteq A'QP ,$$

where the expected average amount per claim

$$A' = \frac{\sum_{j=1}^{k} A_j n_j q_j}{\sum_{j=1}^{k} n_j q_j} .$$

From this it follows that

$$\sigma^2 \doteq \frac{A'Q}{P} .$$

This approximation will, in general, have a tendency to understate the value of σ^2. In practice the expected average amount per claim A' may not be readily available and further approximation may be necessary, *e.g.*, A' can be expressed as θA where A is the average amount per life for the given case. For individual cases with reasonable age-amount distributions θ will generally range from 1.0 to 1.25 if there is no schedule decrease at the higher ages. When reduced amounts are continued for pensioners θ might be as small as .65 or so. Since individual cases are usually permitted to choose suitable schedules within rather broad limits, θ might be determined as a companywide average suitably loaded so as to offset any lack of conservatism in the basic approximation for σ. As a very rough guide θ may be arbitrarily set at 1, resulting in the approximate relationship

$$\sigma^2 \doteq \frac{AQ}{P} .$$

In developing the t-year distribution formulas we can define

$$Q\sum_{i=1}^{t} P_i = \sum_{i=1}^{t} C_i + Z_t \sum_{i=1}^{t} (D_i - C_i)\,; \; T = U;$$

$$U\sum_{i=1}^{t} P_i = \sum_{i=1}^{t} P_i (1 - B_i - E_i)\,; \quad \text{and} \quad \sigma^2 = \frac{A'Q}{\sum_{i=1}^{t} P_i}$$

where the subscripts refer to a particular policy year to which the given item applies. If J is determined from these values of Q, U, T, and σ, the distribution formula which results can be expressed as

$$J\left[U\sum_{i=1}^{t} P_i - \sum_{i=1}^{t} D_i\right] - \sum_{i=1}^{t-1} R_i = R_t$$

or

$$J \times (\text{actual surplus earned in first } t \text{ policy years}) - \sum_{i=1}^{t-1} R_i = R_t\,.$$

If K is similarly determined the resulting distribution formula can be expressed as

$$(U - K) \sum_{i=1}^{t} P_i - \sum_{i=1}^{t} D_i - \sum_{i=1}^{t-1} R_i = R_t$$

or

$$(\text{Actual surplus earned in first } t \text{ policy years}) - K\sum_{i=1}^{t} P_i - \sum_{i=1}^{t-1} R_i = R_t\,.$$

Using the same notation, the distribution formula on page 112 of *Actuarial Studies No. 6* can be expressed as

$$a\left[\sum_{i=1}^{t} P_i (1 - B_i - E_i) - \sum_{i=1}^{t} C_i - Z_t \sum_{i=1}^{t} (D_i - C_i)\right] - \sum_{i=1}^{t-1} R_i = R$$

or

$$a\,(U - Q) \sum_{i=1}^{t} P_i - \sum_{i=1}^{t-1} R_i = R_t$$

or as

$$a \times (\text{expected surplus based on } Q \text{ at end of period}) - \sum_{i=1}^{t-1} R_i = R_t\,.$$

An empirical reduction factor similar to a could also be applied to S in the actual surplus formulas and for participating insurance such an approach seems reasonable. This reduction factor would presumably be the ratio of divisible surplus to the current increment in general surplus and contingency funds. The nonparticipating approach would be to adopt conservative values for J or K and accept some fluctuation in the annual increments to company surplus and contingency funds. Whatever approach is taken, the factors in the distribution formula should be obtained with at least the same consideration for policyholder equity that is used in distributing the premium charges for the coverage.

APPENDIX

TABLE 1

VALUES OF $L(T)$, THE EXPECTED LOSS IN EXCESS OF TP AS A MULTIPLE OF STANDARD PREMIUMS, WHEN THE PROBABLE LOSS RATIO $Q = .50$

Insurance Level less Probable Loss Ratio	50–99 Life Exposures ($\sigma = .77$)	400–799 Life Exposures ($\sigma = .32$)	1,600–3,199 Life Exposures ($\sigma = .17$)	Normal Curve ($\sigma = .10$)	Normal Curve ($\sigma = .05$)
1.0 σ......	.13	.04	.02	.01	.004
1.5 σ......	.09	.02	.01	.003	.002
2.0 σ......	.06	.01	.006	.001	....
3.0 σ......	.04	.006	.002	....	....
4.0 σ......	.03	.004	.001	....	....

TABLE 2

VALUES OF $J(U)$, THE PROPORTION OF ACTUAL SURPLUS WHICH MAY BE RETURNED TO THE POLICYHOLDER IF LOSSES IN EXCESS OF THE PREMIUM MARGIN U ARE TO BE INSURED, WHEN THE PROBABLE LOSS RATIO $Q = .50$

Premium Margin* less Probable Loss Ratio	50–99 Life Exposures ($\sigma = .77$)	400–799 Life Exposures ($\sigma = .32$)	1,600–3,199 Life Exposures ($\sigma = .17$)	Normal Curve ($\sigma = .10$)	Normal Curve ($\sigma = .05$)
0.0........	.00	.00	.00	.00	.00
.1 σ......	.23	.23	.23	.22	.22
.2 σ......	.40	.40	.40	.39	.39
.4 σ......	.61	.63	.64	.64	.64
.6 σ......	.73	.76	.77	.78	.78
.8 σ......	.80	.84	.86	.87	.87
1.0 σ......	.85	.89	.91	.92	.92
1.5 σ......	.93	.95	.97	.98	.98
2.0 σ......	.96	.98	.98	1.00	1.00
3.0 σ......	.98	.99	1.00	1.00	1.00

* Where the premium margin for claims, UP, exceeds one year's standard premiums, a claim fluctuation fund is implicitly assumed.

TABLE 3

VALUES OF $K(U)$, THE PROPORTION OF STANDARD PREMIUMS TO BE WITHHELD FROM ACTUAL SURPLUS TO INSURE LOSSES IN EXCESS OF THE PREMIUM MARGIN U, WHEN THE PROBABLE LOSS RATIO $Q = .50$

Premium Margin* less Probable Loss Ratio	50–99 Life Exposures ($\sigma = .77$)	400–799 Life Exposures ($\sigma = .32$)	1,600–3,199 Life Exposures ($\sigma = .17$)	Normal Curve ($\sigma = .10$)	Normal Curve ($\sigma = .05$)
0.0........	.50	.50	.50	.50	.50
.1 σ......	.44	.23	.14	.10	.05
.2 σ......	.39	.17	.10	.07	.04
.4 σ......	.29	.11	.06	.04	.02
.6 σ......	.24	.08	.04	.03	.01
.8 σ......	.19	.06	.03	.02	.008
1.0 σ......	.15	.04	.02	.01	.005
1.5 σ......	.10	.02	.01	.003	.002
2.0 σ......	.07	.01	.006	.001	....
3.0 σ......	.04	.007	.002	....	....

* Where the premium margin for claims, UP, exceeds one year's standard premiums, a claim fluctuation fund is implicitly assumed.

DISCUSSION OF PRECEDING PAPER

ARTHUR G. WEAVER:

Mr. Jackson is to be congratulated upon his contribution to experience rating theory. In particular, the mathematical development provides a valuable tool for constructing and testing the risk spread charge in experience rating formulas.

The author has referred to a recent paper, "A Method of Calculating Group Term Dividends," by Robert E. Larson (*TSA* IV, 308). It is appropriate that he should do so, since in a sense Mr. Jackson has picked up where Mr. Larson left off. For instance, Mr. Larson suggested that claims in excess of 150% of the standard premium be pooled and assessed as a risk spread charge against all policyholders rather than be charged against the specific policyholder with poor experience. This intriguing approach has certain practical objections which largely disappear when the 150% figure is graded in such a way that the probability of exceeding the graded limit is approximately constant for all sizes of policies. For example, if all claims in excess of the probable loss ratio plus 1.5σ be pooled, Mr. Jackson's Table 2 would indicate the following pattern for Group Life with a probable loss ratio of 50%. Of course, in addition to the charge for risk

Life Exposures	Pooling Level (Percentage of Premium)	Risk Spread Charge (Percentage of Premium)
50– 99	165%	10%
400– 799	98	2
1,600–3,199	75	1

spread, a charge for contingency reserves and contribution to surplus should also be made.

Such a device is practical for Group Life with a loss ratio of 50%; for coverages with a much higher probable loss ratio, the risk spread charge can become a prohibitively high proportion of the premium. In the only too common Group A & H situation where the loss ratio approaches the premium margin for claims, we must decide whether the poor claim experience is accidental or indicative of a trend. In the first case, contingency reserves and surplus can properly be used to avoid too drastic a cut in dividends. Otherwise the only satisfactory procedure is to increase premium rates.

In his mathematical development, the author requires, for each group coverage, a set of standard or manual premium rates applicable to a given over-all classification of groups. He then relates claims, expenses and margins for each individual case as a multiple of the appropriate standard or manual premiums. In the John Hancock it would be additional work to prepare such a standard premium for each individual case, particularly for Group Accident and Health coverages. Consequently I have been interested to know if the earned premium, which is readily available, can be substituted for standard premium in Mr. Jackson's formulas.

To test the propriety of this substitution, I have prepared tables of $f(x)$, $F(x)$ and $\int_{-\infty}^{x} F(y)dy$ based on several thousand Group Life and Group A & H policies representing a cross section of John Hancock 1952 experience. The values of $L(T)$ which result for Group Life are close to those given by Mr. Jackson and we conclude that earned premiums can be used without serious error. This is understandable when we realize that all renewal underwriting action is directed toward reflecting the true underlying mortality or morbidity of the individual case in the premium rates charged. For this reason earned premiums for all cases in aggregate should follow the incidence of mortality or morbidity by age, occupation and other factors at least as well as standard premiums.

VALUES OF $L(T)$, THE EXPECTED LOSS IN EXCESS OF TP AS A MULTIPLE OF STANDARD PREMIUMS (JACKSON) AND EARNED PREMIUMS (JOHN HANCOCK), WHEN THE PROBABLE LOSS RATIO Q = .50 (GROUP LIFE)

POOLING LEVEL	50–99 LIFE EXPOSURES		400–799 LIFE EXPOSURES	
	Jackson $\sigma=.77$	John Hancock $\sigma=.80$	Jackson $\sigma=.32$	John Hancock $\sigma=.42$
$Q+1.0\,\sigma$	.13	.11	.04	.05
$Q+2.0\,\sigma$	.06	.05	.01	.01
$Q+3.0\,\sigma$	.04	.02	.006	.003

Our tests also permitted us to check Mr. Jackson's approximate formula for evaluating the standard deviation in loss ratios. The comparison is shown below.

Life Exposures	Standard Deviation in John Hancock Loss Ratio			
	Life		Weekly Indemnity	
	Formula	Actual	Formula	Actual
Under 25.	1.44	1.58	.45	.65
25– 49.	.95	.90	.30	.56
50– 99.	.49	.80	.22	.42
100–999.	.40	.53	.13	.34

The actual standard deviations for Group Life are nearly double the corresponding deviations for Weekly Indemnity policies, at least for groups involving under 1,000 lives. As might be expected because of the lower average claim payment, the standard deviations for Hospital Expense policies are slightly lower than for Weekly Indemnity policies. However, the loss ratio Q for Hospital Expense policies is considerably higher.

Mr. Jackson's approximate formula for standard deviations in the loss ratio attempts to recognize the greater deviation resulting from a graded schedule of benefits compared with a flat schedule. However, it may seriously understate the true value of σ where a number of older executives are insured for several times the average.

Mr. Jackson suggests that insurance schedules be limited by reasonable underwriting rules in order to keep the possible range in standard deviations roughly constant and relatively insignificant for each size class. While this limitation is desirable if a single scale of risk spread charges is to be used, it is unduly restrictive and produces schedules which are competitively unrealistic. It may be preferable to permit other schedules provided an additional risk spread charge is made.

The frequency function $f(x)$ is significantly skew for medium and small life exposures. This is particularly noticeable for Group Life policies and less so for Group Accident and Health policies. While our statistics for larger cases are limited they suggest that this skewness in distribution is reduced but never completely eliminated. For this reason the standardized normal frequency and distribution functions should be used only for determining expected losses $L(T)$ under larger policies where adequate experimental data are not available.

The following table shows the extent of this skewness in the Group Life distribution of John Hancock loss ratios for selected life exposure classifications. Clearly such a distribution of loss ratios reflects underwriting

Loss Ratio	Life Exposures				
	Under 25 Lives	25–49 Lives	50–99 Lives	100–499 Lives	500–999 Lives
Exactly 0%	87%	83%	67%	37%	4%
0– 50%	0	0	3	22	48
50–100%	0	1	11	26	43
100–150%	0	4	9	9	4
150–200%	1	3	3	4	1
200% and over	12	9	7	2	0
Total	100%	100%	100%	100%	100%

rules and practices of a single company, and would not necessarily be representative of the distribution to be expected in another company.

WILLIAM I. STRUBLE:

Mr. Jackson has described concisely the rationale of two basic rating theories by starting with the pure accounting method of experience rating. The first theory leads to an expected surplus type of distribution formula which, as he points out, occasionally has the somewhat startling effect of producing a refund on a case which itself has produced no divisible surplus. The second basic theory deals with actual case surplus and the formulae are presented in two alternate forms. The first is referred to as the J method, which uses a percentage of the surplus of surplus-producing risks to offset the negative actual margin on losing risks. The second alternate of the actual surplus type formula is referred to as the K method, which uses a percentage of the standard premium of all cases to offset the negative actual margin on losing risks. As Mr. Jackson points out, the J method tends to "soak the best," whereas the K method results in an insurance charge, which is equal for all cases in the class and which is independent of the actual experience of the individual case.

To the above basic theories can be added one which "soaks the best" to an even greater extent than the J method. This theory—not used by our company, by the way—could be referred to as a "maximum retro" theory in that each case would receive as a refund its own actual surplus subject to a maximum refund as determined by class. The cases within a class of cases are in effect divided into three groups. Those cases with actual claim and expense charges in excess of premium receive no refund.

Those cases with claim and expense charges less than premium but in excess of a certain minimum charge receive as a refund their own individual total actual surplus. The third group consists of cases with claim and expense charges less than the minimum charge and such cases receive as a refund the difference between premium and this charge, that is, the "maximum retro" for the class of cases. This method results in the average case being self-rated and in charging entirely against the very good cases the negative margins of the poor cases. The maximum refunds on the best cases are, therefore, less than those under Mr. Jackson's J method. When the method is applied to accumulated experience, a small case soon reaches a self-rated basis. Large cases because of their size do not exhibit the wide chance fluctuations in loss ratios noted on small cases and, therefore, under any theory can be self-rated, provided the level of advance discount is low enough or alternately the advance premium is high enough so that such cases may be considered as renewed on a participating basis. A charge for catastrophic claims beyond some selected level expressed as a multiple of the standard deviation of the expected loss ratio for the class of risks can be made against all risks in a manner similar to that under Mr. Jackson's K method. Claims beyond this point on a given case would be excluded if the theory were applied to accumulated experience so as to reduce the negative margin carried forward and thereby lessen the chance of the policyholder's switching carriers.

FRED H. HOLSTEN:

Mr. Jackson has presented for the first time in the *Transactions* a mathematical approach toward evaluating the risk, or insurance, element under various types of Group refund formulas. He applies his techniques to several of the traditional types of formulas and analyzes their similarities and differences. He further provides tables of functions derived from frequency distributions of standard Group Life claim ratios, as obtained from actual cases and the Normal law, from which the insurance charge for various types of refund formulas may be calculated. For the most part he refrains from discussing methods for the determination of such vital components as the probable claim ratio (Q) and the premium margin for claims (U) and centers his attention on the objective mathematics of the problem.

He does, however, devote some attention to the derivation and analysis of an approximate formula for obtaining the highly important standard deviation of the Group Life standard claim ratio. The approximate formula given for this standard deviation is the square root of the quantity AQ/P, which is probably the most practical form for computation. However, care must be exercised in the interpretation of this formula so as to allow for possible interdependence of the factors A and P. For example, a common situation leading to an increase in the average amount of insurance per life (A) would occur when the schedule of insurance is being increased on some or all of the lives, and in this case the increase in A would be accompanied by a commensurate increase in the standard premium (P). Thus, if the schedule increase were to consist simply of a proportionate increase on all lives, the standard premium would also increase in exactly this proportion. In these circumstances, it would be misleading to say that "the standard deviation in loss ratio will increase with increase in the average amount per life" without adding the qualification that the standard premium is assumed to remain fixed. Probably what Mr. Jackson had in mind is the special situation where an increase in the average amount per life is brought about by an increase only on some of the lives and in such fashion that the greater degree of dispersion in amounts of insurance offsets the increase in standard premium.

The author indicates that the above formula is intended to include approximate allowance for the greater variance brought about by typical departures in scheduled amounts from a flat benefit, and the tendency for the mortality rate to be higher in most cases on those lives having the higher amounts, where the average degree of dispersion in amounts is controlled by underwriting rules graded by size of group. For analysis purposes, however, it might often be more convenient to start with another form of the formula applicable to a group with a flat schedule, Q divided by the square root of nq (which uses factors that are reasonably independent of one another) and then, if other than a flat schedule is involved, to modify the conclusion in the light of possible accompanying changes in the degree of dispersion in amounts of claim. With this formula, for example, one could readily conclude that the standard deviation of the standard claim ratio will tend to decrease when insurance is continued for pensioners, because both n and q would increase while Q (based on intercompany mortality experience including waiver of premium disability claims) would remain substantially constant in the average case; and that, when dealing with a flat schedule, this reduction would be greatest if full amounts were continued, since this would have a stronger effect on increasing the average q and, furthermore, would keep the dispersion in amounts of claim at zero. Where other than a flat schedule is involved, a a greater reduction could well be effected if there were a moderate reduction in insurance on pensioners (assuming they would otherwise have had more than the average amount of insurance) because this would tend to reduce the dispersion in amounts of claim.

Mr. Jackson indicates that a company using an actual surplus type of refund formula can, by decreasing its relative discount action with increasing size of group, gradually increase the difference between the premium margin for claims and the probable standard claim ratio, and thereby effect a grading between the equivalent of an expected surplus type of refund for very small groups to something approaching the pure accounting method for very large groups. It is possible to effect this sort of grading independently of the discount action—in fact, even if no discounts are made at all—by using as a refund formula for a given policy year

$$UP - kP - z\,(D - kP) - KP,$$

where z would be graded by formula from zero on very small groups to unity for the larger groups, and k is the expected standard claim ratio based on experience to the beginning of the policy year adjusted for a year's trend in the over-all class experience.

Assuming that the average deficiency in k (if extreme losses are excluded from its determination) is included in U, as well as provision for possible future contingencies, the factor k would provide for the insurance of the excess of $z\,(D - kP)$ over $(TP - kP)$ instead of the excess of $(D - QP)$ over $(TP - QP)$ and could be obtained from implicit formulas worked out by the methods presented in the paper.

Setting the insurance loss level (T) in what Mr. Jackson calls a theoretical surplus formula involves a number of considerations, some of them intangible or at least difficult to express in quantitative terms. On the one hand, one has to consider the policyholder with good experience (that is, where D does not exceed U) or the prospect anticipating good experience, both of whom expect a maximum of emphasis on the pure accounting approach. This would lead to setting a very high insurance loss level in order to produce a very small insurance charge. On the other hand, the higher the insurance loss level the more the insurer is counting on the average policyholder in a particular class not canceling because of excess losses being carried over into the determination of future refunds. From a strict dollars-and-cents viewpoint the problem is one of achieving an optimum balance. For one side of the balance the insurer must attempt to ascertain what a policyholder's or prospect's ideas of a reasonable insurance charge (something that they often look upon as "profit" to the insurer) would be, and then estimate the consequences of possible dissatisfaction and loss of existing and new business resulting from deviations from these ideas. The other side of the balance involves the insurer's judgment of the evaluation that an average policyholder in a particular deficit position would place on such matters as the acquisition expense charges reincurred with another carrier, the inconvenience and expense to him of disturbing an existing program, the search for another carrier, the re-enrollment of employees and the establishment of new records, and on his satisfaction with the quality of service and other contacts with the present carrier. It also depends upon the chance of this average policyholder being aware of the general effect of a carry-over of a particular size of deficit on his future refunds, and on the confidence the present carrier has in its conservation measures and personnel. A large part of the question of fixing the "proper" insurance loss level therefore is a matter of subjective decisions, but we are indebted to Mr. Jackson for providing the tools wherewith the objective aspects of the problem may be substantially resolved.

HERBERT J. STARK:

Mr. Jackson's paper offers a fertile field for comment, both from theoretical and practical aspects, to the actuary responsible for Group surplus distribution.

In a limited sense, "experience rating," as I understand the term, relates to the determination of the probable claim rate, which taken in relation to the standard premiums as defined by Mr. Jackson is the probable loss ratio Q. This probable loss ratio, as Mr. Jackson points out, has been thoroughly considered by Mr. Keffer and others. The theory implies that the distribution of the standard loss ratios of a series of groups would be "flatter" than that expected as the result of chance fluctuations alone, since the "true" loss ratios for the groups would vary over an appreciable range.

Thus it is puzzling to me that the actual distribution of several thousand cases is used by Mr. Jackson to obtain values of a distribution function, unless of course the cases were so selected as to have within narrow limits a uniform value of Q. It seems to me that only on this basis could Mr. Jackson justify his assumption "that such a distribution would to a large extent represent chance fluctuation while keeping the effect of variation in true loss ratio to a minimum."

Mr. Jackson's paper indicates a variety of methods theoretically possible for distributing excess losses among profitable groups. It seems to us that, as implied by Mr. Jackson, a formula of the expected surplus type which can return a dividend to a group which has not actually contributed to surplus is theoretically undesirable. Also it seems that this type of formula is undesirable in practice because it necessarily requires somewhat greater retentions from groups which are uniformly profitable to provide for the amounts distributed to groups which have shown an actual loss.

For similar reasons we use a formula of the K type rather than the J

type, since the K type formula provides for a more uniform year-to-year retention on each group and for dividends which more closely follow that group's current experience. Finally, the same line of reasoning has led us to a t-year formula, since the return to a consistently profitable group will evidently be larger when a formerly unprofitable group is required to make up its prior loss before becoming eligible for dividend distribution.

Mr. Jackson does not discuss at length the accumulation of a contingency reserve. He suggests the use either of an empirical reduction formula applied to the surplus earnings of a group in addition to the distribution formula proper, or the use of conservative values for J or K in the distribution formula. We have felt that specific provision should be made for the accumulation of contingency reserves and that that contribution should be proportionate to the standard deviation of the expected claims under each group for the current year.

As implied in *Actuarial Studies No. 6*, and as borne out approximately in Mr. Jackson's Appendix tables, the K required under a K type t-year formula is, to a first approximation, proportionate to the standard deviation of the expected claims for the entire policy duration. Thus, in the Metropolitan's formula, retention for contingency reserves and to offset losses involves two terms related respectively to the number of life years currently insured under the policy and to the number of life years covered since its issue. Both factors are read from a basic table with values inversely proportionate to the square root of the number of life years.

Under the Metropolitan's practice a simplified formula of a somewhat different type is applied to smaller groups. In this type of formula a direct charge for losses on unprofitable groups is made against profitable groups. This charge is varied roughly for the limited variations in size of the groups to which it applies and is kept in line with actual experience by tabulating the aggregate losses each year. In addition an annual charge for accumulation of contingency reserves is assessed against each profitable group.

It may be noted that Mr. Jackson's suggestion of a separate roughly constant charge for catastrophe losses is comparable to the purchase of excess loss reinsurance by the issuing company. The purchase of such insurance is a practice which we understand to be followed by some insurers.

One further comment may be desirable. This relates to Mr. Jackson's use of standard premiums as a basis for the distribution formula. Actually, equity seems to suggest substitution for these of hypothetical premiums based on the "probable loss ratios" (*i.e.*, Q premiums) of the particular groups. This is a much more important distinction in dealing with Hospital and Surgical coverages where even more of the variations in claims between individual groups seems to be intrinsic, and less of it dependent upon chance, than is true for Group Life insurance or the older forms of Group Accident and Health insurance. However, if an effort is made to keep the actual premiums charged particular groups in line with their accumulative experience, with due allowance for the relatively smaller margins required for the larger size groups, the actual premiums being charged can be taken as an approximation to the "true experience" or Q premiums. Thus the Metropolitan's formula does not include the step which Mr. Jackson suggests as a starting point in the experience rating process, that is, the establishment of a set of standard, or manual, premium rates applicable to the various classifications of groups. Our retentions for losses and contingency reserves are computed in relation to the actual premiums currently being charged.

(AUTHOR'S REVIEW OF DISCUSSION)

PAUL H. JACKSON:

Mr. Weaver presents expected loss values for various insurance levels derived from an analysis of several thousand Group Life and Group A & H policies and based on earned premiums rather than standard premiums. It is indeed surprising that the results of independent and widely differing studies should agree so closely. The actual standard deviations in loss ratio which he presents support the belief that the approximation formula for σ generally understates the true value. For Group Life insurance, the Aetna study indicated that a loading of 30% to 40% would be reasonable and the John Hancock results appear to bear this out. Apparently a loading of 100% or so would be required for Weekly Indemnity. While pure theory might suggest natural limitations for insurance schedules, Mr. Weaver quite properly points out that competitive considerations may lead to greater maximums for which a larger insurance charge must be made, and such an approach seems reasonable at least for the larger cases where the required insurance charge can be held to reasonable levels by retaining a sufficiently large premium margin for claims. Such greater maximums would have a tendency to prolong the skewness of $f(x)$ for the larger size classes.

Mr. Struble's "maximum retro" formula is an actual surplus type with the insurance loss level set at the premium margin for claims and with insurance charges collected by withholding any actual surplus earned by reason of experience being better than C where $P\int_{-\infty}^{C}(C - x)f(x)dx = P \times L(U)$. By modifying formula (2) in the paper, $C + L(C) = Q + L(U)$ and the "maximum retro" refund would be given by $(U - C)P$.

Mr. Holsten was apparently misled by the rather brief analysis of the factors which affect the standard deviation. The analysis assumed two group cases having common values for two of the three factors A, Q and P and that assumption should have been pointed out in the paper. Mr. Holsten has presented a very interesting formula which grades the expected surplus type formula into the actual surplus type without the use of discount action. For very small groups where $Z = 0$ it can be shown that $k - K = Q$ if the resulting formula is to be an expected surplus type and thus KP must cover the extreme losses excluded from the determination of k. If discount actions are not desirable perhaps the simplest approach would be through the use of automatic refunds whereby the premium margin for claims, after deducting the automatic refund, can be set at the probable loss ratio for small cases, grading by formula up to $Q + 3\sigma$ for large cases. Mr. Holsten outlines the many objective considerations involved in setting the insurance loss level in a theoretical surplus formula. A further consideration is the size of the accompanying claim fluctuation fund which is required if the particular class of groups is not to operate at a continual deficit.

Mr. Stark questions the assumption that each of the 2,000 cases studied had a probable loss ratio of .50. To begin with, after only one year's experience, the individual case Q's might range from .45 to .60 and the frequency function $f(x)$ could be corrected for any overstatement in the deviation about the actual case Q which results when the over-all average Q is used. However, the approximation formula for σ and the net values for K or J must be loaded for conservatism as well as to take into account the greater schedule maximums for larger cases and, under the t-year formulas, the deficits which result when the first $t - 1$ installments exceed the t-year refund. Thus, while theoretically proper, it would hardly be realistic to reduce the initial dispersion. I am certain that Mr. Stark would agree that very little credibility should be given one year's experience on a 50–99 life case and that the resulting actual case Q's would be very close to .50. Apparently he disagrees with my assumption that zero is a good approximation to that credibility.

Mr. Stark feels that the specific contribution of a case for the accumulation of contingency reserves should be proportionate to σ. The standard deviation is derived either from theoretical considerations as to the possible occurrence of n independent deaths or from several years' actual experience under a block of group cases. The contingency reserves, as set up in a company's annual statement, must cover, among other things, the possibility of n dependent deaths as well as disasters of the Texas City type. There are many theoretical reasons why a contingency charge should be proportionate to standard premiums, earned premiums, volume, lives or geographical concentration of the group. On the other hand, there is no theoretical reason why a contingency charge should not be proportionate to σ. From a practical point of view, such a charge places the heaviest load on the smaller cases which are likely to complain the least and the lightest load on the larger cases where competition is fiercest.

Mr. Stark's company is probably not alone in its use of one formula for large cases and a "simplified formula of a somewhat different type" for small cases. Such an approach creates the problem of where to draw the arbitrary line. The paper introduced the actual surplus formula in an effort to produce an experience rating formula that could equitably apply to all cases, large and small alike. The paper also presented simple methods for the relatively accurate evaluation of the factors involved and, by comparison, the assumption that K is, to a first approximation, proportionate to σ seems crude and unrealistic.

It was most interesting to learn that Mr. Stark's company uses a K-type t-year actual surplus formula. Rating formulas of this type are highly desirable because they provide for the greatest percentage return of actual surplus to cases having better than expected experience, thus giving the insurer competitive means to effectively strengthen his hold on the better business.

I want to thank Messrs. Weaver, Struble, Holsten and Stark for their constructive and carefully prepared discussions which add considerably to the material available on the subject of experience rating.