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COURSE EA-1 STUDY NOTE

MEASUREMENT OF INVESTMENT RETURN

by

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The Education and Examination Committee provides study notes to persons preparing for the examinations of the Society of Actuaries. They are intended to acquaint candidates with some of the theoretical and practical considerations involved in the various subjects. While varying opinions are presented where appropriate, limits on the length of the material and other considerations sometimes prevent the inclusion of all possible opinions. These study notes do not, however, represent any official opinion, interpretations or endorsement of the Society of Actuaries or its Education and Examination Committee. The Society is grateful to the authors for their contributions in preparing the study notes.

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MEASUREMENT OF INVESTMENT RETURN

History of Measurement Techniques

The measurement of investment performance is an evolving process. Measurement techniques are used to see if the investor is achieving his goals. Investment goals are changing and becoming more targeted, and the measurement of investment performance is changing along with the goals.

In the 1950s the investment of pension funds was largely debt oriented with a buy and hold policy. As a result, the book or cost value of assets was generally used to measure investment return. Market values were often looked on with suspicion because market value fluctuations were felt to frequently differ from the long-term underlying value of the security.

During the late 1950s and early 1960s equities gained favor and (at least through 1966) the market value of many pension funds rose dramatically above book value. Buy and hold strategies gave way to active asset management. The accountants recommended recognition of a part of unrealized appreciation of a portfolio. Measurement of investment return swung from book values to market values as the only true current measure of a security's worth. Capital market studies indicated that, while equities were riskier, in the long run they always seemed to out perform other assets and a pension fund could afford to weather the short-term fluctuations.

During the period starting in the mid-1960s and continuing through 1981, inflation averaged over 5 percent annually and the Dow-Jones industrial average remained flat. Inflation peaked at over 10 percent annually in the years 1979 through 1981. Pension fund sponsors found that the downside risk of equity investment could last for very long periods of time (10 to 15 years for example). Measurement of investment risk became the new measurement technique -- in an effort to avoid repeating the losses of the stock market declines of 1973 and 1974.

Two other changes in the 1970s deserve comment. The overall growth of institutional funds (led by pension and profit sharing plans) changed the relative mix of the United States capital markets from individual ownership toward institutional ownership. Second, the growth of computer technology allowed vast amounts of data to be stored, retrieved and manipulated efficiently. Comparison of institutional performance against competition became a major business in its own right.

For the 1980s, a period of disinflation relative to the 1970s, investment returns were uniformly high compared to the underlying inflation and salary increase patterns that influence pension plan liabilities. The 10 percent inflation of the late 1970s, early 1980s, has abated to rates in the 3 percent to 5 percent range.

Computerization and the large investment data bases now allow plan sponsors to be more precise and more targeted in their measurement techniques. Sponsors focus on the "risk adjusted return," that is, a comparison of an asset's volatility compared to its return. A sponsor may measure the fund's rate of return less a "safe" return available from government treasury bills. The extra return is the premium for taking risk. During the last decade that extra return has been high, largely the result of the lowering of

inflation. In addition, sponsors focus on a "style adjusted return." In other words, managers with different investment styles (for example, growth oriented, value oriented or a rotational manager) will be compared to other managers with similar styles in order to evaluate the performance of a specific pool of assets.

Asset Base

The measurement of investment rates of return can be based on any one of several asset valuation bases, for example: book (or cost) value, market value or even on the basis of the actuarial value of assets used to determine plan contributions.

The market value of assets is generally used for most purposes. The actuarial value of assets (with a dollar-weighted rate of return) is the appropriate base to quantify the effect of a given year's investment experience on the pension contribution for the year. Insurance companies generally state rates of return on funds held in their general account on essentially an amortized cost basis.

The amortized cost of a debt instrument (a bond, for example) is equal to its cost plus a portion of the premium or discount compared to its maturity value. The premium or discount is usually accrued over the period from purchase to the date of maturity or to the date that the debt may be called.

Cumulative Rates of Return

After each time period's rate of return has been computed, the rates can be combined to measure rate of return over longer periods on a time-weighted basis (see formulas on pages 6 and 7). A dollar-weighted return can be computed by considering the actual level of assets and cash flow of the fund during the total period. Both the time-

weighted rate and the dollar-weighted rate measure the rate of investment growth of a fund over a given time period. The rates differ in the weighting that is given to the level of assets and the cash flow that occurs during the period. The time-weighted rate of return is calculated in a manner to eliminate the effect of different asset values and cash flow. It is considered to be a more appropriate measure of return for the purpose of comparing the performance of investment managers. The dollar-weighted rate of return, on the other hand, gives full weighting to the actual value of assets and cash flow of the fund. This rate provides the actual rate of return earned by the fund.

Comparative Data

Comparative data on investment performance usually includes measurement for the fund as a whole and, in addition, measurement by asset type (equity, equity plus cash equivalents, debt, real estate, etc.). Time-weighted rates of return are almost always used for comparison to other funds or standard indices (most commonly Standard and Poor's 500 for equities and Shearson Lehman or Salomon Brothers bond index for bonds). Comparative data on asset mix (currently and over various time periods) and risk exposure is common.

Cumulative rates of return are usually compared to other rates in order to assess whether or not an investor's expectations are being met. Comparison may be made to absolute measures such as:

-- a fixed rate set as an objective for the fund, or
-- the actuarial assumption.

Relative rates of return are often used to evaluate performance. For example, a fund may compare its performance to:

-- the rate of inflation plus some targeted "real" rate of return,
-- the return on the S&P 500 stocks,
-- the first quartile rate of return in an investment survey,
-- all "value" oriented equity managers (for a value oriented manager), or
-- international asset funds (for an international asset pool).

Obviously, other types of comparisons are possible. For comparative data see the Society of Actuaries' publication Economic Statistics for Pension Actuaries, published annually.

A key issue in investment return comparisons is the time frame over which the comparison is made. It is typical that the highest performers on a relative basis in one time period may be very low in a different time period.

Measurement of Risk

Risk measures generally relate to a security or portfolio's probable asset movement in relation to a market index (such as the Standard & Poor's 500 for equities). These risk measures (Beta, R^2, etc.) are statistical in nature and are beyond the intended scope of this paper. Other measures include the fluctuations in rates of return from an average.

Transaction Analyses

Some funds have developed detailed objectives, strategy and policy statements. These statements may include statements of policy regarding the actual administration of the investment process. An analysis of transactions (cash flow, sales and purchases) can indicate if policy is being followed, how rapidly funds are invested, if trades are made at market prices, if party-in-interest investments are being made, etc. This type of analysis is particularly relevant for the largest funds where administrative efficiency can mean significant dollars of investment return.

Formulas

(a) Basic Formula

Let the following symbols be defined:

M_1 - beginning market value
M_2 - ending market value
C - net external cash flow (contributions less benefit payments)*
I - investment income including dividends, interest, rents, realized and unrealized appreciation or depreciation
R - rate of return for the time period

Then

(1) $M_2 = M_1 + C + I$

and, assuming that net contributions are received uniformly throughout the time period:

(2) $M_2 = M_1 + C + (R \times M_1) + (R \times 1/2\ C)$.

Rearranging formula (2)

(3) $R = \frac{M_2 - M_1 - C}{M_1 + 1/2C}$

or, using formula (1) in the numerator of formula (3) we obtain

(4) $R = \frac{I}{M_1 + 1/2C}$

If the rate is desired in the form 1 + R, formula (3) can be revised to be

(5) $1 + R = \frac{M_2 - 1/2C}{M_1 + 1/2C}$

The Bank Administration Institute recommends these formulas be used to compute rate of return, preferably on a monthly basis. The annual rate is then the product of 12 monthly rates. In practice, quarterly rates are usually computed and then converted to annual rates.

* If the timing of contributions and benefit payments differ significantly, then separate treatment of contributions and benefits is appropriate. The Bank Administration Institute's recommendation for monthly computation is intended to minimize significant timing differences. For an approximation of annual rates where timing differences exist, treat contributions and benefit payments separately and weight them by t/12 where "t" is the average number of months from the date of contribution (and separately, the average number of months from the date of benefit payment) to the end of the year.

(b) Time-Weighted Return: The time-weighted rate of return is the geometric mean of the rates of return for "n" time periods with all periods given equal weight.

(c) Dollar-Weighted Return: The dollar-weighted rate of return is computed by taking into account in each period the assets held for that period.

An example shows the use of these different rates. Assume two fund managers (A) and (B) each have $1,000,000 on January 1 of year one. All benefits and contributions are paid or received on average as of June 30 of each year. Net additions for each year for each manager are:

	Net Additions	
Year	Manager A	Manager B
1	$ 500,000	$ 0
2	500,000	0
3	1,000,000	0

Asset values as of the end of each year are as follows:

	End of Year Asset Values	
Year	Manager A	Manager B
1	$ 1,625,000	$ 1,100,000
2	2,218,750	1,155,000
3	2,946,875	1,039,500

Rates of return for each manager may be computed from formula (5). For example, for manager A in year 1 the rate of return is 10%.

$$1+R = \frac{\$1{,}625{,}000 - \$250{,}000}{\$1{,}000{,}000 + \$250{,}000}$$

$$= \frac{\$1{,}375{,}000}{\$1{,}250{,}000}$$

$$= 1.1$$

Annual rates of return for each manager for each year are as follows:

	Annual Rates of Return	
Year	Manager A	Manager B
1	+ 10%	+ 10%
2	+ 5	+ 5
3	− 10	− 10

These results have been calculated using a one-year time period. The managers' results may be different if each year's rate is computed as the product of two six-month rates.

The time-weighted rates of return are the same for each manager. The rate for the three-year period is:

$$\text{3-year time-weighted rate of return} = (1.10 \times 1.05 \times .9)^{1/3}$$

$$= (1.0395)^{1/3}$$

$$= 1.013$$

The managers' time-weighted return is 1.3% annually.

The dollar-weighted rates of return differ for A and B. Formulas for computing dollar-weighted rates of return take into account the actual cash flow of the funds. Therefore:

Manager A:

$$MV_3 = MV_0 (1+i)^3 + C_1(1+i)^{2\ 1/2} + C_2(1+i)^{1\ 1/2} + C_3(1+i)^{1/2}$$

$$\$2,946,875 = \$1,000,000(1+i)^3 + \$500,000(1+i)^{2\ 1/2}$$

$$+ \$500,000(1+i)^{1\ 1/2} + \$1,000,000(1+i)^{1/2}$$

$$i = -1.0\%$$

Solving for i by trial and error we find the annual 3-year rate of return is −1.0% for Manager A.

Manager B:

$$MV_3 = MV_0(1+i)^3$$

$$\$1,039,500 = \$1,000,000(1+i)^3$$

$$i = 1.3\%$$

Manager B's dollar-weighted rate of return is +1.3% per year. This is the same as the time-weighted return for Manager B because there was no external cash flow to the fund.

While both managers had the same time-weighted rates of return, Manager B had a better dollar-weighted rate for the three-year period.