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SOCIETY OF ACTUARIES

COURSE EA-2, SEGMENT A, STUDY NOTE

# ASSESSMENT AND SELECTION OF ACTUARIAL ASSUMPTIONS FOR MEASURING PENSION OBLIGATIONS 

by<br>Marilyn Oliver, FSA

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Marilyn Oliver, FSA
Sausalito, California
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## Chapter 1 - Introduction

This study note presents the factors to consider in setting demographic and economic assumptions used in measuring pension obligations. While its primary focus is on the general process of setting such actuarial assumptions, it also covers some of the specifics of assumption-setting relevant to the actuarial valuations used for pension funding in the United States.

Chapter 1 provides background on professional responsibilities in this area.
Chapter 2 describes the valuation process and the part played by actuarial assumptions.
Chapter 3 discusses two approaches for deriving assumptions: the "best estimate" approach and the "financial economics" approach.

Chapters 4-6 address demographic assumptions (assumptions used to project the future demographics of a group, such as probabilities of retirement and mortality rates).
Chapters 7-8 discuss economic assumptions (such as the discount rate, salary increase and inflation assumptions).

Chapter 9 concerns factors to keep in mind when setting assumptions - for instance the interrelationship of various assumptions and the purpose of the calculations being performed.
Chapter 10 discusses prescribed assumptions such as those contained in statutes and accounting standards (including those prescribed under the Pension Protection Act of 2006 and Financial Accounting Standard Statement Number. 158).
Chapter 11 provides background on how to monitor assumptions once they have been set.

The assumption-setting process is at the core of the actuary's work. It is one of the most challenging parts of the actuary's job because it relies heavily upon both professional judgment and technical knowledge. Not only is it important that the actuary be able to derive appropriate actuarial assumptions, but, in order for users to understand and use the actuary's work product effectively, it is important that the actuary be able to clearly explain and document the rationale behind each assumption.

Throughout this study note, new and existing terminology will be defined to establish common ground when applying and communicating practical concepts and approaches in the selection process for plan valuation assumptions.

## Actuarial Standards of Practice

Due to the degree of technical knowledge required to understand actuarial processes, users of actuarial work products must rely extensively on actuaries’ competence, abilities and integrity. This necessitates that actuaries maintain a high degree of professionalism both in the execution of and the communication of their work.

Actuarial Standards of Practice (ASOPs) "are intended to provide actuaries with a framework for performing professional assignments and offer guidance on relevant issues, recommended practices, documentation, and disclosure." Members of the American Academy of Actuaries, the American Society of Pension Professionals and Actuaries, the Casualty Actuarial Society, the Conference of Consulting Actuaries and the Society of Actuaries must abide by these standards when practicing in the United States. The standards also provide useful guidance on accepted practices for nonmember actuaries. The standards are issued by the Actuarial Standards Board, which is an entity established within the American Academy of Actuaries. It operates independently in establishing standards on behalf of the U.S. profession.

Current Actuarial Standards of Practice (ASOPs) particularly relevant to the assumption setting process include:

ASOP No. 4 Measuring Pension Obligations
ASOP No. 27 Selection of Economic Assumptions for Measuring Pension Obligations
ASOP No. 35 Selection of Demographic and Other Noneconomic Assumptions for Measuring Pension Obligations
ASOP No. 41 Actuarial Communications

These standards are subject to periodic update by the Actuarial Standards Board.

## Relationship of Study Note to Standards of Practice

This study note is intended for teaching purposes only. Material presented herein including statements using terminology such as "should" or its equivalent does not constitute a standard of practice.

## Chapter 2: The Reason for Assumptions

Actuaries prepare actuarial valuations for many purposes: to set contribution levels, to determine the funding status of pension plans, to support long-term budgeting, and to value potential plan design changes, to name a few. The users of these valuations are varied including plan sponsors, plan participants, bargaining groups, government agencies, accountants, investors, and other pension professionals.

Actuarial valuations of defined benefit retirement plans qualified under Section 401(a) of the Internal Revenue Code are generally prepared in conjunction with funding of retirement plan benefits before they become payable (prefunding). Ideally, prefunding enhances the security of participants’ retirement benefits by (1) maintaining a budgetable employer contribution level over time and thus increasing the likelihood that the plan will continue in operation and (2) building a trust fund balance sufficient to provide benefits should the employer fall upon hard times. ${ }^{1}$

## Overview of Valuation Process

The first step in a retirement plan actuarial valuation is to model the operation of a particular plan over time. This model must:
(1) project utilization and projected amounts of the benefits under the plan, as well as determine the present value of the resulting benefit streams as of the valuation date, and
(2) project other amounts (such as future payrolls and workforces) and determine the present value of these streams as necessary to calculate contribution rates and satisfy accounting requirements.

For most valuations (including those meant to satisfy statutory and accounting requirements) the valuation models a closed group comprised of active and inactive members, where inactive members include retirees, beneficiaries, and terminated members entitled to future vested benefits. (A closed group valuation reflects only those employees who are currently participating in the plan or who are currently employed by the plan sponsor as of the valuation date, and does not anticipate those who may be employed in the future.)

[^0]Generally benefits provided under a retirement plan will include:
(1) retirement benefits, which comprise the bulk of the cost of the plan, and
(2) ancillary ${ }^{2}$ benefits, such as disability and death benefits.

## Assumptions Necessary

In order to project utilization of the plan's retirement benefits, assumptions must be made regarding the timing of retirement and the probability of remaining in the covered employee group until that time. In order to project utilization of ancillary benefits and other benefits payable upon termination of employment, assumptions regarding the probabilities of the triggering events (e.g., disablement, death) must be made. In addition, benefits payable to surviving spouses and domestic partners upon death of the participant in active service require assumptions regarding marital / domestic partner status during employment and spouse / domestic partner ages, while those payable should the participant die after retirement may require assumptions regarding optional forms of payment taken at retirement, as well as marital / domestic partner status at retirement and spouse / domestic partner ages.

To estimate benefits that are based on pay or to project pay for other purposes, such as determining contribution amounts, assumptions regarding future salary increases must be made. If benefits are payable in annuity form, assumptions regarding post-retirement mortality rates must be made. To take into account a plan's provisions regarding calculation of lump sum payments (if any), it may be necessary to make special interest rate and mortality assumptions as well as assumptions of what proportion of participants take lump sums and under what circumstances.

Lastly, assumptions must be made regarding the interest rates to be used to discount the resulting payment streams to produce present values.

The major assumptions used in the valuation process are discussed in chapters that follow. They have been divided into two categories: demographic and economic. As their names suggest, demographic assumptions are used to project the demographic characteristics of the individuals covered by a pension plan while economic assumptions are related to the economy.

[^1]The list of assumptions discussed in this study note is not exhaustive. Additional assumptions may become necessary in practice. In other situations, however, it may not be necessary or appropriate to incorporate all of the assumptions discussed. The nature of the assumptions used will depend on factors such as the retirement plan's design, the group covered, and materiality to the purpose of the valuation, as well as legal constraints.

## Small Plans

Techniques and assumptions may differ based on the size of the retirement plan involved. Where appropriate, this study note will point out these differences and will, unless otherwise specified, use the following terminology:

Small Plans: Plans with 10 or more, but fewer than 200 actively employed participants.

Smallest Plans: Plans with fewer than 10 actively employed participants.

## Multi-Employer Plans

Information contained in this study note regarding the general derivation of actuarial assumptions is equally applicable to single-employer and multi-employer plans. Information regarding specific regulatory requirements generally concentrates on single-employer plans.

## Chapter 3 - Frameworks for Assumption Setting

Two common frameworks used in practice for assumption-setting are discussed below. The first, the Best Estimate Perspective, endeavors to set assumptions that best model the anticipated experience of the plan over the long term. The second framework, the Financial Economics Perspective, views the plan from a market-based perspective that generally leads to a different derivation of the interest rates used to discount liabilities (discount rates). Actuaries need not confine themselves to a particular perspective. Information from both perspectives may be useful to a plan sponsor. The actuary may also wish to use the different perspectives in different situations.

## Best Estimate Perspective

The goal of the best estimate perspective is to construct a set of assumptions that will project the most likely picture, in the actuary's judgment, of the plan's operation over the long term. Under the best estimate perspective the actuary bases assumptions on analyses of data and other available information including projected future trends. Where not set by statute, accounting requirements, or other regulation, best estimate discount rate assumptions will generally take into account anticipated earnings on the retirement plan's investments. The actuary's best estimate assumptions have historically been required when calculating minimum required contributions. However in more recent years individual assumptions have been increasingly legislated. Requirements under the Pension Protection Act of 2006 are illustrated below.

## Minimum Funding Requirements under the Pension Protection Act of 2006 (PPA)

The best estimate perspective is required for assumptions other than those set in statute. For singleemployer plans, assumptions set by statute include the discount rate, mortality rates, and, in certain circumstances, such as for plans in at-risk status ${ }^{3}$, assumed retirement ages and optional forms of payment. With these exceptions, minimum funding requirement calculations are required to be based on non-prescribed actuarial assumptions:
"(A) each of which is reasonable (taking into account the experience of the plan and reasonable expectations), and
(B) which, in combination, offer the actuary's best estimate of anticipated experience under the plan."
"each of which is reasonable"
This requires the use of explicit assumptions - assumptions that can stand on their own.

This new requirement varies from prior minimum funding requirements for single-employer plans which allowed the use of implicit assumptions, - assumptions that are only reasonable when combined with another assumption and cannot stand on their own. Technically the prior law for single-employer plans did not require individual assumptions to be reasonable as long as the resulting contribution rate (based on reasonable aggregate assumptions) was unaffected ${ }^{4}$. For instance, in projecting the present value of future salaries a low assumed rate of salary increase could be used with a low assumed discount rate, if the understatement of future pay was exactly offset by the overstatement of the present value of the resulting salary stream due to the use of the reduced discount rate.

The prior law for multi-employer plans only required actuarial assumptions "which, in the aggregate, are reasonable."

Use of implicit assumptions is risky in that their impact on results may differ depending on the nature of the calculation and plan design - and that impact can also differ from year to year for a given plan as circumstances change. For this reason and because of concerns regarding transparency, they are generally not favored within the profession and they are not permissible under ASOP 27 and ASOP 35.

## "taking into account the experience of the plan"

The degree to which the experience of the plan is taken into account will depend on the size of the covered group and the materiality of the assumption (i.e. whether varying the assumption can have a significant impact on valuation results). For the smallest plans and for many small plans, past experience may not be considered indicative of future experience when the impact of possible random statistical fluctuations is taken into account. However, past experience may provide insights into an employer's policies that, after discussion with an employer, may be relevant to the assumption-setting process. For larger plans there may be sufficient data to consider the level of overall experience and, if credible (i.e. statistically significant) experience is available that is considered reliable in the judgment of the actuary, to analyze the data for trends. In either case, data should be analyzed with current conditions and future expectations in mind.

[^2]
## "reasonable expectations"

In meeting this requirement factors such as the experience of other similar groups, historic and projected economic trends and other factors anticipated to influence assumptions in the future should be considered. In addition, any expected changes in the individual employer's situation should be taken into account.
"in combination"
Each assumption is required to be reasonable. Also, the assumptions "in combination" must satisfy the best estimate criteria. Thus, each assumption must stand on its own, but does not operate in a vacuum and thus must be consistent with the other valuation assumptions. The "in combination" requirement implies that the assumptions chosen should be reflective of similar future environments. For example, if disability rates are increased materially due to a pattern of liberal determinations of disablement, post-disablement mortality rates should be reviewed to assure that they reflect the healthier condition of those receiving disability pensions. Under current law, this requirement only applies to non-prescribed assumptions (i.e. assumptions not prescribed by law).

## Financial Economics Perspective

Although the underlying economic theory has been in place for some time, the financial economics perspective is relatively new to most actuaries practicing in the pension area. Thus, in pension practice this is an area where applications are evolving and techniques are in flux. A brief discussion of the rationale for the discount rate basis used in determining actuarial liabilities under this perspective is given below. The financial economics perspective may be applied in many areas of retirement plan operation, including investments and plan design. It is generally not seen as applying to the derivation of noneconomic assumptions.

Financial economics uses a market-based approach. Under this approach the present value of future cash flows from pension plans are assigned values consistent with values that would be assigned in the capital market. If no default risk is present (i.e. the pension cash flows are guaranteed) this implies that a rate of return that does not incorporate a default risk (for instance, a long-term Treasury bond) would be used for discounting. If there is risk of default, a discount rate commensurate with the risk of default on the future pension cashflow would be used. Here the risk is defined and evaluated using the tenets of financial
economics. Though an in-depth discussion of these concepts is beyond the scope of this study note, generally, for plans covered by ERISA, these tenets lead to the use of a risk-free rate ${ }^{5}$.

Discount rates do not take into account or anticipate any reduction in plan costs due to future pension fund investment earnings in excess of the risk-free rate. This is consistent with another premise of financial economics that holds that market forces (the opportunity for arbitrage) are assumed to remove any risk premium that is not balanced by underlying risk. Thus, according to this theory, the prudent course is not to anticipate future earnings in excess of the risk-free rate, but to take into account any additional returns only as they occur.

Under this market-based approach, plan solvency calculations - for instance those performed to judge the funding progress of an ongoing plan - would generally be performed using a risk-free discount rate.

[^3]
## Chapter 4 - Demographic Assumptions - Post-retirement Mortality

Since pension benefits are often payable during the lifetime of a retiree (and often of a survivor), postretirement mortality rates play an important part in the valuation process.

Though mortality rates have been shown to vary by a variety of factors, in pension valuations, mortality rates are generally assumed to depend on:

- age
- gender and
- type of retirement (generally healthy or disabled).

Other factors that are considered in some cases include:

- whether the covered individual is a retiree or beneficiary, and
- . proxies for socio-economic status such as job type (white or blue collar) or salary/pension amount.

Factors influencing the use of these particular factors include significance and data available.

Mortality rates for disability retirements may vary based on the period since disablement as well as the factors outlined in the paragraph above. Also, they are influenced by the definition of disablement and its application in practice. For example, depending on the nature of the industry and the group and the definition of disablement, high rates of disablement may indicate more liberal qualification standards for disablement, thus leading to a relatively healthier disabled population.

## Standard Mortality Tables

For most plans mortality rates will be based on standard tables, such as those produced and published by the Society of Actuaries or governmental organizations, such as Social Security or Railroad Retirement, where appropriate.

Tables are generally named based on (1) the types and characteristics of data underlying the table and (2), because mortality rates generally change over time, the calendar year of experience that the mortality rates are assumed to represent. Detailed information regarding the source of data is generally provided in the report published with the table. In addition, a breakdown of table rates for subgroups may be provided.

To the extent possible, the table chosen should incorporate an experience base whose characteristics are similar to the group being valued. For instance, mortality tables based on individual annuities are generally not considered appropriate for retirement plan purposes because individual selection tends to result in lower rates of mortality. On the other hand, experience under group annuity contracts, where individual selection is reduced, is considered appropriate for setting mortality assumptions for retirement systems. Recently promulgated standards reflect the combined mortality experience of generally uninsured pension plan participants (e.g. RP2000).

Depending on the nature of the covered group, the plan's experience, and the actuary's judgment, mortality table rates for subgroups such as occupation (white collar versus blue collar) and income may be used. For larger groups having credible experience, that group's experience may be used for the underlying table, or as a basis for an adjustment to a standard table.

## Mortality Improvement

Current mortality tables that have been specifically designed for the retirement area generally contain no margin for future mortality improvement. However, projection scales for use in projecting future mortality improvement are provided with most Society of Actuaries mortality tables used in the retirement area. These scales usually vary by age and gender.

A section ${ }^{6}$ of Mortality Projection Scale AA compiled by the Society of Actuaries is shown below ${ }^{7}$.

| Age | Male | Female |
| :---: | :---: | :---: |
| 60 | .016 | .005 |
| 61 | .015 | .005 |
| 62 | .015 | .005 |
| 63 | .014 | .005 |

These scales are applied to reduce probabilities of death as follows:

Probability of death with n years of mortality improvement $=$ $($ mortality rate at age x$)(1 \text {-projection scale value at age } \mathrm{x})^{\mathrm{n}}$

[^4]
## Example of Application

The probability of death at age 60 in the SOA's RP-2000 table is .006747 for males and .005055 for females.

If we apply Scale AA to determine the probability of death for a male age 60 in 2010 (10 years after the rates in RP-2000 are assumed to apply) the calculation would be as follows:
$(.006747)(1-.016)^{10}=.005742$.

For a female the calculation would be:
$(.005055)(1-.005)^{10}=.004808$

## Generational Mortality Improvement

If it is assumed that the forces leading to mortality improvement will continue in the future, then mortality rates will vary by both age and the calendar year of attainment of age, since those attaining the age at a later date will be exposed to the forces leading to mortality improvement for a longer time period. Thus for example the probability of death at age 60 would be higher for an individual attaining age 60 in 2012 than for an individual attaining age 60 in 2016. Another way of looking at it is that different generations (those born in 1952 versus those born in 1956) will have different mortality rates at age 60 .

To reflect this difference, projection scales can be applied for the number of years between the calendar year of the valuation and the calendar year the individual turns a given age. This is called the generational method for projecting mortality improvement.

For example, if a valuation is being performed as of $1 / 1 / 2014$ using a mortality table with mortality rates representative of mortality in 2014, a present value factor at age x would use the following mortality rates, where the superscript represents the calendar year that the individual attains a given age.

```
q}\mp@subsup{q}{}{2014}\mp@subsup{}{x}{}
```



```
and q}\mp@subsup{q}{}{2016}\mp@subsup{}{x+2}{}=\mp@subsup{q}{}{2014}\mp@subsup{}{x+2}{(1-\mp@subsup{\mathrm{ scale }}{x+2}{*}\mp@subsup{)}{}{2},\ldots...
```

A numerical example for a male age 61 in payment status as of $1 / 1 / 2014$, using a hypothetical mortality table assumed to be applicable in 2014 and Scale AA is shown below.

| Age | $\mathrm{q}^{2014}{ }_{\mathrm{x}}$ | Year retiree <br> reaches age | Years <br> from 2014 | Scale AA <br> factor at age | Mortality rate in year <br> retiree reaches age |
| :---: | :---: | :---: | :---: | :---: | :--- |
| 61 | .0070 | 2014 | 0 | .015 | $.0070=$ <br> $.0070^{*}(1-.015)^{\wedge} 0$ |
| 62 | .0080 | 2015 | 1 | .015 | $.0079=$ <br> $.0080 *(1-.015)^{\wedge 1}$ |
| 63 | .0090 | 2016 | 2 | .014 | $.00875=$ <br> $.0090^{*}(1-.014)^{\wedge 2}$ |

In contrast, a numerical example for a male age 40 in 2014 with payments starting at age 61 on 1/1/2035 is shown below.

| Age | $\mathrm{q}^{2014}{ }_{\mathrm{x}}$ | Year retiree <br> reaches age | Years <br> from 2014 | Scale AA <br> factor at age | Mortality rate in year <br> retiree reaches age |
| :---: | :---: | :---: | :---: | :---: | :--- |
| 61 | .0070 | 2035 | 21 | .015 | $.0051=$ <br> $.0070^{*}(1-.015)^{\wedge} 21$ |
| 62 | .0080 | 2036 | 22 | .015 | $.0057=$ <br> $.0080^{*}(1-.015)^{\wedge 22}$ |
| 63 | .0090 | 2037 | 23 | .014 | $.0065=$ <br> $.0090^{*}(1-.014)^{\wedge 23}$ |

As illustrated above, application of a mortality projection scale on a generational basis will result in recognizing mortality improvement for a longer period when calculating annuity factors for younger plan participants (employees versus retirees) resulting in increased present values.

## Other Methods of Recognizing Mortality Improvement

Simpler methods of projection can also be used. A method often used is to apply an adjustment to the age used to look up rates in a mortality table. If a setback (younger age) is used, lower mortality rates will result. Conversely if a setforward (older age) is used, mortality rates will be increased. Other methods of projecting mortality rates include projecting all mortality table rates forward for a fixed number of years or applying a fixed margin (such as $5 \%$ or $10 \%$ ) to table mortality rates.

All of these simpler methods assume the same degree of mortality improvement regardless of the year in which the participant attains a given age. Thus, the probability of death one year from the valuation date is assumed to decline by the same proportion as the probability of death 20 years from the valuation date.

Theoretically this treatment will initially result in understatement of mortality rates (actuarial gains) followed by overstatement (actuarial losses) in later years. However, from a contribution rate perspective, results may not be unreasonable depending on the frequency at which the table and projection are updated. Depending on the circumstances the complexity involved in using the generational approach may not be justified.

## Application:

## Set-back / set-forward:

This is the simplest method, since the setback or forward can just be applied to the age used in calculating annuity factors (for instance, an annuity factor at age 60 can be used instead of an annuity factor at age 61 if a 1-year setback is used). A rule of thumb is that for each year of setback, mortality rates are reduced by about $10 \%$. As would be expected, this is not accurate at all ages or for all tables. However, it generally seems to provide a rough basis for comparison purposes.

## Fixed Years Projection

For example, assume a $1 \%$ per year mortality improvement scale, if 10 years of mortality improvement are to be projected, all rates in the valuation mortality table would be multiplied by (1-.01)^10. This new table would then be used for several valuations until the actuary decided it was appropriate to either update the table to a more current one or project the mortality rates further. A study Impact of Mortality Projection Scales on Defined Benefit Pension Plan Valuations available on the Society of Actuaries Pension Section website discusses the ramifications of this projection method in terms of patterns of gains and losses and funded status and compares results to those obtained using the generational method.

This technique is also used to update an existing mortality table to the valuation date. For instance, when performing a $1 / 1 / 2010$ valuation using generational mortality techniques and the RP-2000 Healthy Annuitant Table, the RP-2000 Table must first be updated to the year 2010. This is accomplished by rolling all table mortality rates forward for 10 years using a projection scale that in the actuary's best judgment is appropriate.

## Fixed Margin

Under this approach all mortality rates are multiplied by (1-margin). For instance if the margin were 10\%, all mortality rates would be multiplied by $90 \%$.

## Projection of Annuitant versus Non-Annuitant Mortality

Under all three of these approaches, in order to mimic the impact of generational mortality improvement, it is not unusual to use different levels of adjustment for annuitants (retirees and beneficiaries in payment status) and non-annuitants (including active employees and terminated participants entitled to deferred benefits), since non-annuitants would be expected to receive payments further in the future and thus to be exposed to the benefits of future mortality improvement for a longer period. For example, for valuations in 2008,

- Set-back method: The post-retirement mortality table for retirees in payment status might be the RP-2000 Table projected to 2008 with a 1-year setback while the post-retirement mortality table for employees and terminated vested would be the RP-2000 table projected to 2008 with a 3 -year setback
- Fixed Years Projection method: The post-retirement mortality table for retirees in payment status might be the RP-2000 Table projected to 2008 and then further projected an additional 7 years (to 2015), while the post-retirement mortality table for employees and terminated vested would be the RP-2000 table projected to 2008 and then projected an additional 15 years ( to 2023).
- Fixed Margin: The post-retirement mortality table for retirees in payment status might be the RP-2000 Table projected to 2008 with a margin for future mortality improvement, while the post-retirement mortality table for employees and terminated vested would be the RP-2000 table projected to 2008 with a larger margin for future mortality improvement.

The number of years projected and the set-backs and margins used will depend on the average number of years until payments are expected to be received. This can be approximated by the duration ${ }^{8}$ of the liabilities. As well as differing between annuitants and non-annuitants, duration of normal cost will generally differ from that of actuarial accrued liabilities, and for the same group these durations can differ depending on the actuarial cost method used.

Actual set-backs and margins, as well as the projection scale used, will depend on the Actuary's conclusions regarding prospects of future mortality improvement, including to what extent differences by gender and age should be forecast.

[^5]The actuary is expected to make an explicit assumption about the projection of future mortality rates. If mortality tables are used without any projection then the actuary has assumed no future mortality improvement after the date of the table.

## Sources of Mortality Tables and Studies

## Society of Actuaries (SOA) Studies

Reports are available on the SOA web site for a variety of mortality studies. In the retirement area, the most commonly used tables are the RP-2000, UP-94, 1994 Group Annuity, and 1983 Group Annuity mortality tables.

## RP-2000 (Retired Pensioner) Mortality Table

The RP-2000 (Retired Pensioner) Mortality Tables are based on experience under private-sector retirement plans. There are three main tables:

1. An Employee Table with mortality rates through age 70,
2. A Healthy Annuitant Table applicable to the post-retirement period with rates starting at age 50, and
3. A Combined Healthy Participant Table that is formed by blending the mortality rates of the Employee and Healthy Annuitant Tables.
Mortality rates in the Combined Table are lower than those in the Healthy Annuitant Table because of the inclusion of active employees in the base (who experience shows are generally healthier than healthy retirees of the same age).

Tables are provided for subgroups primarily composed of white collar or blue collar workers. A table of disability mortality rates is also provided. However the definitions of disablement in the plans used in the construction of the table are not uniform. Since mortality rates can vary considerably based on the definition of disablement (higher mortality rates for stricter definitions of disablement and lower mortality rates for more liberal definitions of disablement), the RP-2000 disability mortality table rates are not recommended for use with plans with particularly strict or liberal definitions of disablement.

## UP-94 (Uninsured Pensioner) and 1994 Group Annuity Mortality (GAM) Tables

The UP-94 (Uninsured Pensioner) and 1994 Group Annuity Mortality (GAM) Tables are based on the same underlying experience (a combination of insured group annuity experience at ages 65 and over, of Civil Service data for those 55 and under, and blended experience from the two sources between
ages 55 and 65). In developing the 1994 Group Annuity Mortality Static ${ }^{9}$ Table, a margin of $7 \%$ was included (mortality rates were decreased by 7\%) to allow for variations in experience due to factors such as size of the insurer's book of annuity business and demographic factors such as industry. This $7 \%$ margin was considered necessary for group annuity reserving purposes because mortality tables are changed infrequently due to regulatory requirements. Use of this margin was not considered necessary in the pension area where the actuary often has the ability to adopt and change mortality assumptions as circumstances dictate and gains and losses due to random fluctuations are automatically incorporated in the funding process. Thus no margin was incorporated in the UP-94 Table.

## 1983 Group Annuity Mortality Table

The 1983 Group Annuity Mortality Table was derived from the experience base used for the 1971 Group Annuity Mortality Table projected to 1983. Final smoothed mortality rates were then reduced by a $10 \%$ margin. Published with the table was Projection Scale H, which was derived for use in projecting future mortality improvement after 1983.

The table had been in use in Current Liability calculations, but based on a reevaluation by the IRS and Treasury, it was considered to no longer be appropriate for reflection of current mortality patterns effective with plan years starting in 2007. When compared to the RP-2000 table projected to 2007, patterns of mortality rates varied significantly by both age and gender. More details may be found in Internal Revenue Bulletin 2005-51, beginning on page 1186 (REG-124988-05).

## Conference of Consulting Actuaries

The 1985 Disability Study conducted by William H. Blake and published in the Proceedings of the Conference of Consulting Actuaries Volume XXXVI contains mortality rates for disability retirees.

## Social Security Administration (SSA) Studies

Mortality rates for disabled annuitants may be found in "Social Security Disability Insurance Program Worker Experience", Study Number 118, June 2005 on the Office of the Actuary web site http://www.ssa.gov/OACT/NOTES/s2000s.html. Disabled life mortality tables compiled by Social Security are based on the Social Security definitions of disablement.

[^6]
## Railroad Retirement

Mortality Rates of Railroad Disability Annuitants may be found in the Railroad Retirement Actuarial Valuation Report on the Railroad Retirement Board web site at http://www.rrb.gov/bfo/bdgt_fin.asp. Two tables are shown - one for retirees meeting Social Security disability requirements (labeled "with Disability Freeze") and one for those not meeting Social Security disability requirements. Social Security requires total and permanent disability for eligibility for disability benefits. Railroad Retirement requires permanent disability for work in the employee's regular railroad occupation. As would be expected, mortality rates for retirees meeting the more stringent Social Security disability requirements are higher than mortality rates for retirees not meeting those requirements.

Department of Treasury
Revenue Ruling 96-7 includes disability mortality tables for disabled lives in general and disabled lives eligible for Social Security disability Benefits. See Chapter 10, Assumptions Prescribed by Statute / Regulations for more information.

## Industry Tables

Various unions and industry groups have prepared mortality studies for specific industries.

## Sources of Projection Scales

Society of Actuaries Studies
A scale for projecting future mortality improvement, Scale AA, is included in the reports containing the RP-2000 and UP-94 Mortality Tables. Scale AA is based on age and gender and assumes a continuation of current trends in mortality improvement (including lower rates of mortality improvement for females than for males). It is based on an average of Civil Service and Social Security experience between 1977 and 1993 with a minimum rate of $.5 \%$ for ages under 85 . Assuming a 7\% discount rate, use of the UP-94 Table with Scale AA mortality projection on a full generational basis increases deferred annuities at age 62 for 25 year old plan participants by $15 \%$ and 5\% for males and females, respectively. For 65 year old plan participants, immediate annuities are increased by $2.5 \%$ and $1.2 \%$ for males and females respectively.

## Social Security Administration

Low, Medium, High Projections for Long-term annual decreases in mortality rates may be found in the Annual Trustees Report.

## Use of a Flat Rate, Adjustment of Existing Projection Scales

Alternatively, the actuary may wish to use a flat rate, for example $1 \%$ per year, or a percentage of an existing projection scale based on his or her judgment concerning levels of future mortality improvement. Sources used in forming this judgment may include historic experience and other available data and studies, as well as opinions of demographers, scientists, and other experts. When reviewing this data, the underlying population should be considered, since factors influencing mortality improvement may vary by factors such as socio-economic group.

Use of a single flat rate to adjust male and female mortality tables implies that rates of mortality improvement will be constant for all ages and for both genders.

## Comparison of RP-2000 and UP-94 Tables

The ratio of annuity factors (i.e. deferred to age 65 for ages under 65, and immediate for ages 65 and over) at 7\% using the RP-2000 Combined Healthy Table to those using the UP-1994 Table and the UP-94 table projected to 2000 using Scale AA are illustrated below.

|  | RP-2000 / UP-1994 |  | RP 2000 / <br> UP-1994 Projected to 2000 |  |
| :---: | :---: | :---: | :---: | :---: |
| Age | Males | Females | Males | Females |
| 30 | 1.05 | .98 | 1.02 | 0.97 |
| 40 | 1.04 | .98 | 1.01 | 0.97 |
| 50 | 1.04 | .98 | 1.01 | 0.97 |
| 60 | 1.03 | .98 | 1.01 | 0.97 |
| 65 | 1.02 | .98 | 1.00 | 0.98 |
| 70 | 1.01 | .98 | 0.99 | 0.97 |
| 80 | .98 | .98 | 0.96 | 0.97 |
| 90 | .94 | 1.01 | 0.93 | 1.00 |

Since the ratio of annuity values is generally higher for males and lower for females at most retirement ages, the results for a given group will depend on its male / female composition.

## Special Considerations

In the case of plans where annuities are purchased at retirement, anticipated annuity purchase prices at the time of retirement should be taken into account in setting the post-retirement mortality rate assumption,
except where not permitted by law, as in calculating minimum contribution levels. Generally, these assumptions are more conservative (that is, they tend to produce a higher cost) than assumptions for benefits paid from the plan trust, because the insurance company does not usually have the opportunity to adjust for adverse experience (such as improvements in mortality) after the annuities are purchased. In addition, annuity prices generally include provision for commissions, premium taxes, contract administration expenses and some margin for risk and profit for the insurance company.

## Chapter 5 - Demographic Assumptions (At or Before Retirement) and Expenses

## Pre-retirement Decrements

The most important demographic assumptions before retirement are:
Retirement rates
Termination rates
Disability rates, and
Death rates.

## Retirement Rates

Retirement rates are usually based on age. Generally service will also be considered if early retirement subsidies differ significantly by service. (For instance, if the plan provides for actuarially reduced early retirement benefits at age 55 with 10 years of service and unreduced early retirement benefits at age 55 with 30 years of service, different retirement rates would be set for those with 30 or more years of service and those with 10 but less than 30 years of service.) Furthermore, the actuary may anticipate that retirement rates for a less valuable benefit may decrease as the participant approaches eligibility for a more valuable benefit. For instance, in the above example, a participant age 55 with 29 years of service may be less likely to retire than a 55 -year-old with 10 years of service, because the 29-year employee is likely to work one more year to qualify for the unreduced 30-year benefit. Eligibility criteria for other post-retirement benefits, such as significant postretirement medical benefits should also be considered. Retirement rates may or may not be dependent on gender, depending on whether gender differences are seen as material.

Depending on the size of the group, retirement rates may be based on the plan's experience, on experience of similar groups, on published tables or on available tables developed at the consulting firm level, as appropriate. In any case, it is important to consider the plan's benefit provisions and those of other post-retirement benefit programs, such as employer-provided health care, Social Security, and Medicare benefits. Factors that may influence observed experience may include economic environment, the financial health of the employer, other plans of the employer (e.g. a 401(k) plan) and health (depending on the availability / eligibility provisions of disability retirement benefits).

For small plans a single assumed retirement age may be appropriate - provided that, in the actuary's judgment, the results adequately reflect any subsidies in the plan's early retirement provisions and that where material, for instance for the smallest plans, the retirement age adequately reflects the expected retirement dates of the principals and owners, as long as this is the actuary's best estimate.

Assumptions for estimating the cost of an early retirement incentive program present a special challenge. Rates of acceptance will differ by age and often by service. Gender may also be a consideration. The design of the program, utilization of similar programs, historic retirement experience, special circumstances accompanying the program, and the demography of the covered group are all factors that may influence utilization. In addition, other sources of retirement income including the availability of other employment should be considered. Because of the uncertainty in this process, illustrating results for a range of assumptions may be appropriate. In addition, the actuary should consider reducing the shortterm rates of retirement after an early retirement incentive program, since those employees who declined to retire under the incentive program are less likely to less likely to retire -- without an incentive -- shortly after that program is closed.

Similarly, it is very difficult to set an assumption for shutdown and other infrequently-occurring events. The actuary should take into account the business conditions of the employer, the past history and other factors, as well as the materiality of any additional benefits payable as a result of the shutdown. The assumptions should then be monitored in future years and adjusted as necessary with changing business conditions.

## Termination

Termination rates are generally defined to include termination from service for reasons other than retirement, death, and disablement. They are affected by the plan's provisions as well as industry and occupation. Rates may vary by age, service, or a combination of age and service. Gender may also be considered. Service is an important predictor of termination. Properly adjusting termination rates to reflect higher rates of termination for shorter service employees and lower rates of termination for longer service employees can have a significant impact on contribution levels.

Depending on the size of the group, termination rates may be based on the plan's experience, on experience of similar groups, or on available tables as appropriate. Where termination rates are based on existing tables, adjustments may be made to reflect the group's experience to the extent it is considered credible or other factors such as industry.

For the smallest plans, the actuary may decide not to use termination rates if they are judged to be immaterial. Owners, who often represent the bulk of liabilities, would generally not be assumed to terminate employment prior to retirement. Rates of termination for other employees are often difficult to predict because the plans have too few participants to provide credible experience.

## Disability Rates

Rates of disablement are used when the plan contains provisions for special benefits upon disability. If this is not the case, they are generally incorporated in the termination assumption. If sufficient credible experience is available, rates may be based on an experience study. For smaller plans, publicly available tables may be used - such as those compiled by the Society of Actuaries, Social Security, or Railroad Retirement. If standard tables are used, it is important that to the extent possible the table be based on the definition of disablement and administrative policies used to determine eligibility for the plan's disability benefit. In addition, occupation and industry should be considered. For many plans the definition will be based on eligibility for Social Security disability benefits. In this case disability rates based on Social Security data may be used as a guide. However, it should be kept in mind that these rates, ignoring other factors, may tend to overstate a plan's disability rates because workers who have been out of the work force for a number of years may apply for Social Security Disability Benefits ${ }^{10}$.

## Recovery from Disablement

Depending on the plan's provisions regarding recovery from disablement, the plan's definition of disablement, and the magnitude of disability benefits, recovery may be a material assumption. Recovery assumptions can either be based on the plan's experience or on data from an outside source with similar demographic and disability provisions. Recovery rates used for this purpose should be adjusted to recognize the impact of any vested or accrued benefits that might be payable at a later date to the recovered individual. Another approach to recognizing recovery rates is to reduce the rate of disablement assumption.

## Pre-retirement Mortality Rates

Standard mortality tables are often used for pre-retirement mortality rates. For a given age and gender, pre-retirement mortality rates are usually lower than post-retirement mortality rates. Where this difference is considered material, different mortality tables are used for the pre-retirement and post-retirement periods. Unless large pre-retirement mortality benefits are provided, pre-retirement mortality assumptions are generally not incorporated in the valuation of the "smallest plans" because the impact of such an assumption is generally considered immaterial.

[^7]
## Use of Standard Tables

## Termination of Employment and Retirement Studies

Termination and retirement rates are highly specific to individual plans. Thus standard tables generally are not appropriate for direct use; however they may be useful in quantifying factors that may influence rates or exploring possible rate patterns.

The report of the SOA Pension Plan Termination and Retirement Study, finalized in 2003 and based on 1994-2000 data provided by 32 contributors for 112 plans, is available on the SOA Pension Section web site http://www.soa.org/research/pension/research-2003-soa-pension-plan-turnoverstudy.aspx. Findings included:

- Comparing age-based, service-based, and age/service-based termination rate tables, the study notes that tables based on age and service are the best predictors of termination rates, followed by service alone.
- The pattern of termination rates differed significantly from those under the Crocker Sarason Straight tables, a set of age-based termination rate ( T ) tables reflecting different levels of termination that were published in 1955 and have been used over the years in the industry. Thus, the continued use of these tables was called into question. The aggregate termination rates from the SOA study on an age basis are compared to the T table rates in the chart that follows. Note how the rates in the SOA study decline abruptly from age 22 to age 35, while the Sarason T Table rates are based on concave curves with rates declining much more gradually and thus potentially overstating termination rates for employees in their 30's and 40's.


## Termination



The report includes rates of termination of employment by age, service, and age and service for the following groups:

- total experience
- small plans (defined in the study as plans contributing experience of under 1,000 lives per year to the study)
- salaried workers
- hourly union workers
- hourly nonunion workers.

Results from the service-based tables are illustrated below. These results are shown to illustrate the considerable variation that may be found between rates of termination for different groups. They should not be used for valuations of an individual plan without corroboration based on the plan's historic experience and anticipated future plan circumstances.

Termination Rates for:

| Completed Years <br> of Service | All | Small <br> Plans | Salaried $^{11}$ | Hourly <br> Union $^{12}$ | Hourly <br> Nonunion |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $17.1 \%$ | $19.8 \%$ | $12.7 \%$ | $2.9 \%$ | $33.3 \%$ |
| 1 | $16.0 \%$ | $18.1 \%$ | $11.9 \%$ | $2.8 \%$ | $29.6 \%$ |
| 2 | $15.6 \%$ | $16.5 \%$ | $11.0 \%$ | $2.6 \%$ | $26.2 \%$ |
| 3 | $12.0 \%$ | $15.0 \%$ | $10.2 \%$ | $2.5 \%$ | $23.0 \%$ |
| 4 | $9.9 \%$ | $13.6 \%$ | $9.5 \%$ | $2.4 \%$ | $20.1 \%$ |
| 5 | $8.3 \%$ | $12.3 \%$ | $8.8 \%$ | $2.3 \%$ | $17.5 \%$ |
| 10 | $4.9 \%$ | $7.4 \%$ | $6.0 \%$ | $1.8 \%$ | $8.2 \%$ |
| 15 | $3.7 \%$ | $4.4 \%$ | $4.0 \%$ | $1.4 \%$ | $3.7 \%$ |
| 20 | $2.8 \%$ | $2.5 \%$ | $2.7 \%$ | $1.1 \%$ | $1.6 \%$ |

Separate rates of retirement are also included in the SOA study, but they are based on the combined experience of plans with a wide variety of different early retirement benefit eligibility and benefit provisions, and so are not recommended for general use.

Another set of termination tables, based on substantially less data, was produced by Roger Vaughn and printed in the 1992 Pension Forum which is available on the SOA web site at (http://www.soa.org/professional-interests/pension/research-thinking-ahead/pen-experiencestudies.aspx). This study includes a select and ultimate termination rate table ${ }^{14}$ and compares termination rates for a small sample of plans from three industries - banking, healthcare, and manufacturing. The study found that ultimate termination rates, when graphed by age, followed a pattern similar to the pattern observed in the SOA termination and retirement study.

## Disability Studies

Disability incidence rates may be found in "Social Security Disability Insurance Program Worker Experience", Study Number 118, June 2005, on the Social Security Office of the Actuary web site. Railroad Retirement disability retirement rates may be found in the Railroad Retirement Actuarial

[^8]Valuation on the Railroad Retirement Board website. (These rates are shown separately based on qualification for Social Security disability benefits, [referred to as with disability freeze]).Various unions and industry groups have prepared disability studies for specific industries.

The 1985 Disability Study conducted by William H. Blake and published in the Proceedings of the Conference of Consulting Actuaries Volume XXXVI ${ }^{15}$ contains disablement rates based on the Social Security definition of disablement. Experience for four different occupational subgroups is also provided.

## Other Studies

The reader should check the sources listed for new tables and updates to existing tables.

## Other Assumptions

## Optional Form Choices

If calculations of lump sum amounts and/or benefit reductions for optional payment forms are based on assumptions that differ from valuation assumptions, it may be necessary to take into account the cost impact of election of these payment forms in the valuation. Thus, it will be necessary to estimate the percentages of participants electing these payment forms. Even if the variation between assumptions for optional forms and those in the valuation is not material for valuation purposes, optional payment form election assumptions may be necessary for cash-flow modeling. This is particularly true when lump sums are offered. (For example, consider the difference in projected cash flow between a lump sum option and a $100 \%$ joint and survivor option.).

Election percentages are generally based on the plan's experience [and expectations of future experience]. Provisions of the plan should be considered, including whether early retirement subsidies and subsidized joint and survivor options are included in the value of the optional form of payment. Consideration of the impact of lump sum and optional benefit elections is specifically required in the new PPA provisions for single-employer plans.

[^9]
## Marital Percentage / Spouse Age

Since data typically used in retirement plan actuarial valuations do not include marital status and spouse demographic information for active employees it is necessary to make assumptions in these areas when valuing pre-retirement survivor benefits and post-retirement subsidized joint and survivor benefits. Because the cost of the benefits involved is generally small, these are often not based on plan sponsor-specific studies. However, if benefits are significant and data is available, use of an assumption based on plan-specific data should be considered.

Marital status assumptions will also be necessary for valuing pre-retirement survivor benefits Again, because the cost of the benefits involved is generally small, these are often not based on plan sponsorspecific studies.

If significant post-retirement death benefits are provided in the smallest plans, actual data on marital and spouse age information should be obtained and used.

## Expenses

The other common assumption that does not come under the heading of economic or demographic is the assumption for expenses paid out of the plan trust.

## Investment Expenses

Investment expenses are often taken into account by a reduction in assumed rates of investment return.

## Administrative Expenses

Administrative expenses are often reflected as an addition to normal cost or a percentage of normal cost based on budgeted amounts. Alternatively, administrative expenses may be prefunded by either:

- an explicit adjustment to the investment rate of return, or
- (though not commonly used at present) by projecting future administrative expenses taking into account anticipated future fixed and per participant costs

Some practices involving reducing the assumed rate of investment return for administrative expenses may distort results due to differences in nature and timing of these expenses relative to returns.

Caveat: When the discount rate is set by law or other outside entity, it cannot be adjusted to reflect expense, and other means must be used to take expenses into account, to the extent permitted.

# Chapter 6 - Demographic Assumptions - Choosing 

## General Considerations

## Plan Provisions

The plan documents will contain information that impacts the types of assumptions necessary and their structure, as well as their values. Thus, they should be carefully reviewed before proceeding and any material impact on the assumption-setting process should be taken into account. Examples of items that can influence the assumption setting process include:

- benefit formulas and benefit eligibility requirements
- status of plan (ongoing, frozen)
- definition of disablement
- forms of payment of retirement benefits
- basis for optional forms of payment (including lump sums).


## Other Information

General information such as the planned retirement date of the owners in the smallest plans; and provisions of collective bargaining agreements relative to force reductions and future salary increases should be reflected in assumptions as appropriate.

## Characteristics of the Group

The characteristics of the group may impact assumption choice in a variety of ways. For instance, job type (white-collar vs. blue-collar) may influence the choice of demographic assumptions, including rates of termination, retirement, disability and mortality.

## Consideration of Past Experience, and Current and Anticipated Future Conditions

Assumptions used should, as nearly as possible, be representative of anticipated future experience under the retirement plan. Thus the actuary should consider not only past experience but also current experience and anticipated trends in future experience.

## Margins for Conservatism

Whether a margin for conservatism should be included depends on the nature and purpose of the project and the type of assumption. For instance, different considerations apply when calculating minimum
required contributions and maximum deductible contributions as opposed to calculating a worst-case scenario for a potential plan change.

For minimum and maximum funding for single-employer plans, calculations must be based on actuarial assumptions:
"(A) each of which is reasonable (taking into account the experience of the plan and reasonable expectations), and
(B) which, in combination, offer the actuary's best estimate of anticipated experience under the plan Thus, the use of margins for conservatism is somewhat limited. However, to the extent that two values for an assumption are equally likely in an actuary's view, a more conservative assumption might be used in consideration of one of the purposes of the minimum required calculation - which is to provide security for plan members' benefits.

It should be kept in mind that the impact of deviations in experience will automatically be taken into account in contribution rate determinations through the gain / loss amortization process. Relatively short periods for amortization of gains and losses (for example for minimum funding purposes under IRC section 430) reduce the need for margins for conservatism except where the margin is being used to recognize an anticipated change in future experience - for example mortality improvement.

In some situations, it may be appropriate to include a margin for statistical fluctuation based on the size of the plan. For further information on methodology, the report for the GAM 1994 Table and the GAR 1994 Table (which is available on the SOA web site at http://www.soa.org/library/research/transactions-of-society-of-actuaries/1990-95/1995/january/tsa95v4722.pdf) illustrates the methodology used to derive the margin $\left(5 \%{ }^{16}\right)$ that was incorporated in the table to produce annuity reserve values adequate for random variation of two standard deviations for a 3,000-life block of business. (Such a margin may not be appropriate for calculating minimum and maximum funding contributions unless it reflects the actuary's best estimate of future mortality experience.)

[^10]
## Materiality

In deciding on the process to be used in setting an assumption, materiality to the purpose at hand should be considered. The refinement or inclusion of an assumption anticipated to have a small impact on liabilities may not be necessary.

## Using Past Experience

Credibility
In reviewing a plan's experience an actuary must judge whether the experience is significant enough to factor into the assumption setting process, and if so, to what degree. Though sophisticated statistical techniques may be applied, in the retirement area this process of determining credibility generally involves a synthesis of the actuary's judgment, the sources available for setting the assumption, and standard tests of statistical significance (such as the use of confidence intervals).

In general, the evaluation of the credibility of past data considers the nature and frequency of the decrement involved, the size of the group and the consistency of the experience between years, where for a given decrement larger groups with more consistent experience generally produce higher credibility.

## Analyzing the Data

Ideally these assumptions should be based on experience data that reflects current conditions. The data should be reviewed considering any occurrences during the experience period that might significantly impact the incidence and distribution of decrements (such as changes in plan provisions or administrative procedures, early retirement incentive programs and workforce reductions). Depending on the actuary's judgment, this may lead to the exclusion of data for certain periods (such as the period prior to a plan change). Data surrounding a change should also be closely reviewed for anomalies. For instance, for early retirement incentive programs, a decrease in the incidence of retirements would be expected between the announcement and implementation of the program and also after the program's completion (because participants who would normally be expected to retire during that period may retire earlier than anticipated in order to utilize the program).

Where significant, assumptions should be adjusted for factors that will affect experience in the future but did not impact participants fully during the experience period such as:

- changes in plan provisions with eligibility requirements that participants could not have satisfied during the experience period
- known future changes in relevant Federal Programs such as Social Security and Medicare, including changes in the Social Security age for unreduced benefits
- anticipated changes in the employer's workforce structure, lines of business, prospects, etc.
- changes in related benefits provided by the employer, such as postretirement medical coverage


## Trending

If there are sufficient credible data, experience should generally be reviewed by year as well as in the aggregate, and by demographic subgroups to the extent possible. Data from prior experience studies and experience studies from other, similar groups may also be included in the process, particularly when setting assumptions for decrements like disablement or death where the number of occurrences is relatively low.

An example of the use of trends in setting termination rate assumptions follows.

Example:
A company's termination rates are reviewed every 3 years. It has a workforce of approximately 7,000 employees during the study periods. The 2009-11 study is the first study conducted for the plan sponsor by the actuary's firm. The chart summarizes the ratios of actual terminations to expected terminations (A/E Ratios) and the adjustments made by the actuary. (An A/E ratio of less than 1 indicates that fewer terminations are being experienced than expected and that retirement liabilities may be understated.)

|  | 2009-2011Study | 2012-2014 Study |
| :---: | :---: | :---: |
| A/E Ratios: |  |  |
| Year 1 of study | 96\% | 100\% |
| Year 2 of study | 94\% | 103\% |
| Year 3 of study | 92\% | 99\% |
| Total | 94\% | 101\% |
| Other <br> Information | - Recession period. <br> - Data confirms current assumption structure and relationship of rates <br> - Current assumptions are relatively low compared to those of similar industries in the same geographical area | - Data confirms structure and relationship of rates |
| Adjustment to assumption | Recognize half of deviation of actual from expected | None |
| Rationale: | Part of decline in terminations may be temporary due recession and ensuing unavailability of other jobs. May also be due to random fluctuations. | No material deviation from assumption. |
| A/E Ratio after Assumption Update | 97\% | 101\%, no adjustments |

## Using Published Tables

Published tables are often used as a basis for setting certain demographic assumptions. In particular, published tables are used when:
(1) the number of observations for the particular group may be unavailable or too small to be statistically valid or
(2) where in the actuary's judgment, the group's experience is more likely to "mimic" that of a wider population than to vary from the wider population (e.g. for mortality rates).

Published tables are also used as a vehicle for smoothing data or to produce a table when sufficient information is not available that varies by demographic variables (such as age). For example, standard tables may be used for pre- or post-retirement mortality rates where there may be insufficient data available to construct a custom or plan-specific table or where the construction of a separate table based on employer data is considered an unnecessary refinement. Considerations in choosing the appropriate table are set out below.

## Purpose of Table

The purpose of the published table may impact the level of conservatism reflected in the derivation of the final rates. For example, mortality tables used for group annuity reserving generally include margins to provide for items such as random fluctuations in experience and differences in characteristics of business between companies. On the other hand, mortality tables designed for use in retirement plan valuations generally do not include these margins because variations in experience are automatically recognized in future contribution requirements by the amortization of gains and losses.

## Source of Data

It is important to read the background statements regarding the derivation of the experience to ensure that the underlying data set is, as nearly as possible, consistent with the group under consideration, and that any caveats to the results are considered in the assumption-setting process.

Particular care should be taken in applying any retirement rate or termination rate table based on experience other than that of the employer under consideration. This is because plan sponsor-specific factors such as personnel policies, industry, job type, and plan provisions can significantly affect both termination and retirement rates. Also, in some termination studies, rates for retirement, termination, disablement, and death are combined.

## Period Covered

The experience base underlying the table should be reviewed. Ideally, relevant conditions should not be materially different between the experience period used in deriving the table and the period over which the table is anticipated to be used. Thus, for instance, when using published mortality tables, the actuary should consider adjusting the tables for mortality improvement between the year of the table and the valuation date.

## Comparisons to Actual Experience

If credible data are available, expected decrements can be calculated using the published table's rates and compared to the actual number of decrements ${ }^{17}$. The published table's rates may then be adjusted to replicate a desired ratio of actual to expected decrements. This desired ratio may be less than, greater than, or equal to 1.0 depending on the actuary's judgment as to factors such as the credibility of the data and the continuation of observed trends.

Adjustments may be made to the rates in the published table for specific age and/or service groups or in the aggregate.

Often adjustments are made in the aggregate, by applying setbacks or setforwards to the published table. However, it should be kept in mind that use of these types of adjustments assumes that the underlying curve generally mimics the pattern of rates in the table. This may not always be the case as illustrated earlier regarding the Crocker Sarason Straight termination tables.

## Developing Tables Based on Actual Experience

If appropriate published tables are unavailable, it may be necessary to develop a table based on the plan's experience. Steps in this process are briefly outlined below.

## Collect, review, and analyze appropriate data

Sufficient historical data must be obtained to produce credible results, but data must also be representative of the current environment.

A thorough review of the data is essential. In areas such as mortality and disability, claims in process or unreported claims should be considered and handled in a consistent manner from study to study so that no

[^11]occurrences are missed or double-counted. Year-to-year data should be separately compiled and analyzed for trends and any unusual results should be checked with the plan sponsor for explanation.

The easiest way to collect experience data is to collect and reconcile the necessary data each year as part of the valuation process. If data are gathered separately at the time of the experience study, it may be difficult to reconcile any apparent inconsistencies or errors in the data.

As discussed previously, it is important to adjust for any significant changes in conditions during the period. This may lead to placing less credibility on certain data or eliminating certain periods of data.

## Consider the Appropriate Table Form

Table forms vary by type of decrement. Common forms were discussed earlier and are summarized below:

| Type of decrement | Rates generally vary by: |
| :--- | :--- |
| Pre-retirement Mortality | Age, gender |
| Disablement | Age, generally gender, job type or industry |
| Termination | Age and service, service only, age only, status |
|  | (salaried, hourly nonunion, hourly union), gender <br> (infrequently) |
| Retirement | Age, service (as impacts subsidized early |
|  | retirement eligibility), gender (infrequently) |
| Post-retirement Mortality | Age, gender |
| Post-disablement Mortality | Age, gender, period since disablement |

In certain cases, rates may be varied by two variables during an initial period and then by only one variable thereafter. The initial period is referred to as the select period. Rates during the period which follows are referred to as ultimate rates. The two areas where this is most often applied are termination rates and disability mortality rates.

The best practical predictor of termination rates is generally thought to be select and ultimate tables that vary by age and service during a select period, generally lasting 3 -to 5 (and sometimes 10) years in practice, and by age only thereafter.

Disability mortality rate tables are often in select and ultimate form, with rates during the select period varying by age and period since disablement, and rates during the ultimate period varying only by age. For most private sector pension valuations, tables are based on age alone since disability benefits are generally not a large component of overall costs.

## Smoothing

Unadjusted rates of retirement, death or other decrement are based on the ratio of number of decrements to number of lives exposed to the risk of decrement which are calculated from the experience data. These q's must generally be smoothed to produce an orderly progression of rates. Many methods are available for this process including:

- graphic methods,
- finite difference methods, that smooth by maintaining constant differences between successive values, and
- graduation methods, that allow for varying degrees of smoothness versus fit.

For most purposes, graphic methods are sufficient. For retirement rates, smoothing may be minimized since rates will not generally follow a smooth pattern - reaching peaks at eligibility for subsidized early retirement benefits, Social Security early retirement benefits, the plan's normal retirement age, Medicare eligibility age and Social Security normal retirement age.

## Extrapolation

Where data are insufficient near end points, probabilities may be extrapolated based on a continuation of the data trend or with reference to an existing table.

## Information Available from Prior Actuary

When undertaking new cases, it is often helpful to discuss existing assumptions and their rationale with the prior actuary, though this does not absolve the actuary from making his or her own determination of which assumptions are appropriate. Other items that can be discussed with the prior actuary include:

- Information regarding past plan changes and other factors that may have influenced historic experience, and
- Results of prior analysis of experience data.

Where considered necessary, efforts may be made to obtain prior valuation data sets for use in future experience studies.

## Chapter 7 - Economic Assumptions -

The economic assumptions most often applicable in pension plan valuations are:

- Discount rate
- Salary increase rate
- Social Security Taxable Wage Base increase rate
- Cost-of-living increase rate
- For cash balance plans, interest-crediting rate.

Discount Rate: The discount rate is the assumed rate of interest used to convert payment streams to present values as of the valuation date.

Salary increase rate (or Salary Scale): This is the pay increase assumption used to project the future pay levels of each current active plan participant.

Social Security Taxable Wage Base increase rate: This is the assumption regarding increases in the Social Security national average wage index that is used to project future Social Security Taxable Wage bases (as well as bend points and individual's average earnings under the Social Security formula). This assumption is necessary to calculate projected benefits when a pension plan's benefit formula takes into account Social Security benefits.

Cost-of-living increase rate: This assumption can be necessary for a variety of purposes:

1. Internal Revenue Code limitations on benefits and compensation are increased based on increases in the Consumer Price Index. Though these increases may not be recognized in valuations for minimum contribution purposes, they are relevant for other purposes, such as accounting, forecasting and cash flow analysis and calculating maximum deductible contributions.
2. For employees retiring after age 62, an adjustment for CPI increases is incorporated in the Social Security formula. This may impact benefit amounts under pension plans that integrate with Social Security using the offset method.
3. Retirement plans may include automatic cost-of-living adjustments, or may have a pattern of granting special (ad hoc) cost of living increases to retirees’ benefits. Future ad hoc increases are generally not taken into account except in forecasting or, depending on the circumstances, for FAS 87/158 accounting purposes.
4. For negotiated plans with retirement benefits based on fixed dollar amounts rather than pay, it may be necessary to project future increases in the fixed dollar amount for planning or for FAS87/158 purposes.

## Interest Crediting Rate for Cash Balance Plans

The interest crediting rate for cash balance plans, if based on an index or similar "floating" assumptions, should reflect the actuary's long-term expectations relative to that rate. Thus the interest crediting rate as of the valuation date should not be used unless it reflects the actuary's best estimate of the applicable long-term interest crediting rate. The derivation of this long-term interest crediting rate must be consistent with the derivation of the discount rate and, also, with the derivation of the other economic assumptions.

## Consistency of Assumptions

It is necessary to make sure that assumptions are consistent with a given economic scenario - for instance, if the actuary assumes relatively high post-retirement cost-of-living increases (i.e., high inflation), then he/she may also need to assume higher increases in compensation.

## Building Block Approach

A tool that allows a set of economic assumptions to be derived in a consistent manner and thereby theoretically assures consistent recognition of common underlying components is called the Building Block Approach. Under this approach, each assumption is divided into underlying components. An assumption for each component is then developed based on a synthesis of historic data, outlooks for the future, and other factors considered relevant by the actuary. The component assumptions are then aggregated to produce the final economic assumptions.

Basic components of the five major economic assumptions using this approach are shown below.


Note that since prospects for future inflation influence most aspects of the economy, an underlying inflation component is common to all of these assumptions.

## Example ${ }^{19}$ :

Assume the following component assumptions:
Inflation: 3.0\%
Real Rate of Return for Discount Rate: 4.75\%
Productivity Wage Increase: 1.0\%
Merit / Longevity Increase: 0.5\%
Real Rate of Return for Cash Balance Plan Interest Crediting Rate: 2.5\%
Then the derived assumptions are:
Discount Rate Assumption $=3.0 \%+4.75 \%=7.75 \%$
Salary Increase Assumption $=3.0 \%+1.0 \%+.5 \%=4.5 \%$
Social Security Wage Base Assumption $=3.0 \%+1.0 \%=4.0 \%$
Cost of Living Assumption $=3.0 \%$, prior to application of any caps or other post-retirement cost-of-living increase provisions.

Cash Balance Interest Crediting Rate Assumption $=3.0 \%+2.5 \%=5.5 \%$

The individual components are described in the paragraphs that follow.

[^12]
## Inflation

Inflation may be thought of as the general increase in prices throughout the economy. It is generally measured in terms of increases in such series as the Consumer Price Index (CPI) or the GDP Implicit Price Deflator. The Consumer Price Index measures changes in expenditures for a market basket of goods and services that is redetermined from time to time to reflect current consumption patterns. The GDP Implicit Price Deflator measures changes in prices of Gross Domestic Product (GDP). The major difference between the two indices is that the CPI reflects purchases of foreign, as well as domestic, goods and services while the GDP Implicit Price Deflator reflects only domestic goods and services.

It is important that the perspective used to set the inflation assumption be consistent with the perspective used to derive the other assumptions -- in particular the discount rate. For instance, when setting bestestimate assumptions during periods when current rates of inflation are believed to differ from those anticipated over the long-term, the inflation rate assumption should take into account expected long-term inflation rates and not be based solely on the current inflation rate. A technique that is sometimes used if the difference between short-term and long-term expected inflation is significant and is expected to significantly impact results over the transition period, is to grade the assumed inflation rates by plan year from current levels to the long-term assumption.

On the other hand, when setting the inflation assumption in a case where the discount rate is based on current market rates (as is often the case in the financial economics framework), the current market's inflation expectations should be used.

Historic values of the Consumer Price Index may be found on the Bureau of Labor Statistics web site www.bls.gov. Historic values of the GDP Implicit Price Deflator may be found on the Bureau of Economic Analysis web site at http://www.bea.gov/national/.

## Real Rate of Return:

Real rates of return are investment returns net of inflation. Real rates of return may be calculated directly by the formula:
$\left[\left(1+\right.\right.$ rate of return $\left._{t}\right) /\left(1+\right.$ inflation rate $\left.\left._{t}\right)\right]-1$, where $t$ refers to year.
When setting best estimate assumptions, the real rate of return assumption used in deriving the discount rate is the weighted average (generally based on a real or hypothetical target asset allocation) of the anticipated long-term rates of return net of inflation for each asset class included in a real or hypothetical
investment portfolio. More details on the derivation of this assumption will be provided in the next chapter.

For cash balance plans, the interest crediting real rate of return assumption would reflect anticipated real rates of return for the related index. Again, it is important that the perspective used to set this assumption be consistent with the perspective used to derive the other assumptions -- in particular the discount rate.

## Productivity Wage Increases

Productivity wage increases are increases in wages due to increased productivity within a particular industry, type of job, or the economy as a whole. They reflect the impact of factors such as technological advances, capital infusions, and additional training and education.

Historic values for the economy as a whole may be measured as increases in national real wages using increases in the National Average Wage Index net of changes in the CPI or GDP Implicit Price Deflator ${ }^{20}$. The National Average Wage Index is available on the web site of the Office of the Chief Actuary within the Social Security Administration web site at http://www.ssa.gov/OACT/COLA/AWI.html\#Series.

In analyzing these results keep in mind that changes in the productivity component of wage increases are sensitive to:

- changes in the allocation of additional revenue due to productivity increases between total compensation, return on capital, and shareholders, and
- for the portion of additional revenue due to productivity increases that is allocated to total compensation, changes in the allocation between wages and fringe benefits (such as medical insurance premiums).

Historic increases in productivity may also be reviewed. There are various measures of productivity; however, the one most relevant to wages is the ratio of output to hours worked (called labor productivity) - where output is based on real final output (as measured by real GDP). Increases in labor productivity may be due to factors other than increased productivity of the workforce. For instance, they may be due to advances in technology and capital investment. An assumption that wages increase at the same rate as increases in labor productivity is consistent with an assumption that wages will remain a constant percentage of GDP.

[^13]The Bureau of Labor Statistics provides historic labor productivity measures on their web site http://www.bls.gov/lpc/. Increases in productivity for major sectors are available on the site including breakdowns by business, non-farm business, and manufacturing. Increases are also available by industry (though for fewer years). The Bureau also provides data regarding wage increases by industry, geographical area, and job type.

Assumptions regarding the productivity component of wages may differ for a particular industry, job class or employer. However, it should be kept in mind that depending on the situation, differences from national or industry-wide averages may be relatively short term in nature because of broader job market forces. (For instance, competition for labor may force the employer to increase salaries in order to avoid losing its workforce to other employers.)

## Merit/Longevity Increases

This salary increase component represents expected pay increases due to performance and promotion, as well as step and longevity (length of service) increases. These increases are generally higher earlier in a person's career and vary by service and/or age. They also generally vary by type of job, industry and (since they are based on compensation policies) between companies.

## Setting the Merit/Longevity Assumption

Material regarding the employer's current compensation policies, as well as historic demographic information may be used in deriving this assumption.

If the employer is of sufficient size, a study of historic salary increases categorized by age and/or service may be conducted. For this purpose, merit/longevity increases for each individual are measured as the difference between the individual's annual salary increase rate and the general (cost-of-living plus productivity) salary increase rate for the employee group as a whole. Increases in average pay for the group as a whole are often used as a surrogate for general pay increases. This practice is based on the assumption that the employee group being studied approximates a stationary population. ${ }^{21}$

Changes in compensation structure and policies during the study period, as well as factors influencing levels of bonuses and overtime pay should be considered when reviewing historic information and

[^14]appropriate adjustments made to final assumptions or periods considered. Caution: As noted above, yearly increases in average payroll are approximations of general pay increases. Thus the rate of increase in a going concern's payroll budget for the coming year is not indicative of an individual's pay increase for the coming year, which includes general plus merit and longevity increases.

## Example (Merit and Longevity Increase Derivation):

Normal Company generally has around 5,000 full-time and no part-time employees. Its retirement plan is ongoing and covers new hires as well as continuing employees. All pay is in the form of salary. No bonuses or other forms of compensation are paid. The actuary decides to study of merit/longevity pay increases over a 4 -year period 2005 - 2008. Data from the $1 / 1 / 2005$ through $1 / 1 / 2009$ valuations is used.

The salary study steps are:
(1) Calculate total salary increases (inflation + productivity + merit \& longevity) for each employee for each year in which the employee was actively employed at both the beginning and end of the year.
(2) Summarize results by year for categories such as service, age, age and service, and gender. Select the most explanatory categorization taking into account materiality. In this case, groupings based on service were decided upon.
(3) Estimate general (inflation + productivity) salary increases for each year using average salaries at each valuation date. Compare results for reasonability to salary policies, bargaining agreements, and other available information.
(4) Calculate merit / longevity increases as:
[(1+step (2) result) / (1+step (3) result)] - 1.
Compare for reasonability with salary policy and bargaining agreement information.

Results are shown below.

| Steps (1) and (2) Results: Total Salary Increases (merit/longevity+inflation+productivity) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Service | 2005 | 2006 | 2007 | 2008 | 2006-2008 ${ }^{22}$ |
| 0-4 | 7.5\% | 7.6\% | 8.7\% | 7.1\% | 7.8\% |
| 5-9 | 4.5\% | 4.5\% | 4.5\% | 5.1\% | 4.7\% |
| 10+ | 9.2\% | 4.0\% | 4.75\% | 3.75\% | 4.2\% |
| Step (3) Results: General Salary Increases (inflation+productivity) |  |  |  |  |  |
| Initial | 5.0\% | 4.5\% | 4.0\% | 3.5\% | 4.0\% |
| Adjusted | 5.0\% | $3.5 \%{ }^{23}$ | 4.0\% | 3.5\% | 3.7\% |
| Step (4) : Merit/longevity Increases [(Step (2) + 1) / (Step (3) + 1)] - 1 |  |  |  |  |  |
| Initial: 0-4 | 2.4\% | 4.0\% | 4.5\% | 3.5\% | 4.0\% |
| 5-9 | (0.5\%) | 1.0\% | 0.5\% | 1.5\% | 1.0\% |
| 10+ | 4.0\% | 0.5\% | 0.7\% | 0.2\% | 0.5\% |
| Adjusted: 0-4 | 24 | 4.0\% | 4.5\% | 3.5\% | 4.0\% |
| 5-9 | - | 1.0\% | 0.5\% | 1.5\% | 1.0\% |
| 10+ | - | 0.5\% | 0.7\% | 0.2\% | 0.5\% |

## Special Considerations

## Additional Information

When conducting studies of historic experience, additional information may become available that may influence other assumptions or methods. For example if in studying historic merit/longevity increases considerable variation from year to year in items such as bonuses or overtime is noted, consideration should be given to introducing some smoothing of these components or of total pay in the valuation process. If this is not done, the actuary is implicitly assuming that the bonus level and/or overtime pay at the time of a given valuation will stay at the same levels in the future.

## Aggregation of Components

Adjustments may be made after individual components are aggregated. For instance:

- Wage increases in current bargaining agreements should be reflected for years remaining in the bargaining agreement.
- Final post-retirement cost-of-living increase assumptions must be adjusted for any caps included in the plan provisions.

[^15]
## Social Security Taxable Wage Base

For this assumption productivity increases at the national level should be used.

## Financial Market Expectations

When setting economic assumptions, the actuary also has available the financial market expectations for future rates as embedded in the prices of financial instruments. Thus, an inflation assumption based on past experience can be compared for reasonability to the difference between current yields on Treasury bonds and TIPS (Treasury Inflation Protected Securities) of similar maturities.

## Forecast Information

Forecast information is often helpful in setting these assumptions. Sources for forecast economic information are shown below by source.

## Philadelphia Federal Reserve Board

The Livingston Survey provides 10-year forecasts of GDP, productivity, CPI, nominal and real equity returns, nominal and real 10-year Treasury rates, and nominal and real 3-month Treasury bills.

## Office of the Chief Actuary of the Social Security Administration

The Annual Trustees Report provides low, medium, and high projections for long-term annual increases in CPI, productivity, real wage differential (i.e., productivity reflected in wages), and GDP.

[^16]
## Chapter 8 - Economic Assumptions - Choosing the Discount Rate

## Overview

A discount rate, its structure, and derivation will vary based upon the nature of the calculation and the types of securities upon which the discount rate is assumed to be based. For instance, for calculation of minimum required and maximum tax-deductible funding levels for single-employer plans under the Pension Protection Act of 2006, discount rates are based on interest rates published by the Department of Treasury that reflect a yield curve of investment-grade corporate bonds. For other purposes, such as for multi-employer plans and for employers' funding policies relative to single-employer plans, a single discount rate may be used that is based on investment returns anticipated to be earned on the plan's trust fund. Discount rates also may vary by plan year - for example, to recognize short-term market trends. In some cases where payments of pensions are assumed to be assured, a discount rate based on risk-free rates (as estimated by the return on government securities) may be used to provide a "market price" for pension obligations based on the tenets of "financial economics ${ }^{25}$."

Discount rates are usually set with reference to the types of assets contained in the plan's trust fund or in a hypothetical fund having certain desired characteristics. The discount rate is then set as the anticipated investment return on the fund's assets or the hypothetical fund's investments for the period during which benefits are anticipated to be paid from the plan. A variety of methods may be used to derive this discount rate including building block methods that separate anticipated investment returns into components and cash-flow methods that directly take into account the anticipated pattern of future benefit payments.

Three different approaches to the derivation of discount rates will be discussed in the sections that follow:

- Building block approach
- Yield curve approach
- Cash-flow matching method.

[^17]
## Building Block Approach

## Basic Approach

In the prior chapter we saw that under the Building Block Approach the discount rate could be modeled as the combination of two components: inflation and real rate of return.

The real rate of return assumption is the weighted average (generally based on a real or hypothetical target asset allocation) of the anticipated long-term rates of return net of inflation for each asset class included in a real or hypothetical investment portfolio.

The derivation of the inflation assumption was discussed in the previous chapter. The sources of data below (or other sources available to the actuary) may be used to develop the assumed real rate of return. Note that:

- Geometric means should be used in calculating averages, to be consistent with the application of compound interest; and
- Historic data underlying the calculations may be edited to remove periods that due to their nature are not believed to be predictive of future experience in the actuary's judgment, (for instance, periods for which available information is not considered reliable or to have been derived in a manner inconsistent with current methods, or periods prior to a major shift in the economic or fiscal policy, for example moving from the gold standard). ${ }^{26}$


## Sources of Rates of Return on Securities by Asset Class

Sources for rates of return by asset class include:

- the Stock, Bonds, Bills and Inflation Yearbook produced by Morningstar, Inc. ${ }^{27}$
- the Society of Actuaries Pension Section Investment Statistics web site, and
- the FRED database (St. Louis Federal Reserve http://research.stlouisfed.org/fred2/).

Morningstar's Stock, Bonds, Bills and Inflation Yearbook contains historic data starting in 1926 for broad investment categories: large company stocks, small company stocks, long-term corporate bonds, longterm government bonds, intermediate-term government bonds and treasury bills. Total returns are broken down between income and capital appreciation. The book is updated annually and may be found in many public libraries.

[^18]The Society of Actuaries Pension Section Investment Statistics web site provides historic rates of return for a variety of asset classes including indices for domestic and international equities, domestic and international corporate bonds, US government securities, and real estate investment trusts. Data starts in 1970 or later for most indices and varies by index. (While this data is only available to SOA Pension Section members, membership in the Pension Section is open to non-SOA members.)

Along with historic information for Treasury Securities (including inflation-indexed securities (TIPS)), the FRED database contains yields for Moody's Aaa and Baa Corporate Bonds dating from 1919 as well as short-term CD yields dating from 1965.

## Calculation of Real Rates of Return by Asset Class

Both the Morningstar yearbook and the Pension Section web site mentioned previously include real rates of return (i.e. returns net of inflation). Real rates of return may also be calculated directly by the formula: [(1+ rate of return ${ }_{\mathrm{t}}\left(1+\right.$ inflation rate $\left.\left.\mathrm{e}_{\mathrm{t}}\right)\right]-1$ where $t$ is the year of the return

Example (Calculation of Real Rates of Return):
Starting with the following hypothetical 1-year rates of return:
Stocks: 10.40\%
Long Term Corporate Bonds: 5.90\%
Long-Term Government Bonds: 5.40\%
3-month Treasury Bills: 3.70\%
Inflation: 3.00\%
Then real rates of return would be:
Stocks: 1.1040 / $1.03-1=7.18 \%$
Long Term Corporate Bonds: 1.059 / $1.03-1=2.82 \%$
Long-Term Government Bonds: 1.054 / 1.03 - 1 = 2.33\%
3-month Treasury Bills: 1.037 / 1.03 - 1 = .68\%.

For years $\mathrm{t}=1$ through n , the geometric mean real rates of return may be calculated as:
$\left[\Pi\left(1+\text { real rate of return } n_{t}\right)\right]^{(1 / n)}-1$, where $n$ is the number of years.
Or alternatively as:
$\left[\Pi\left(1+\text { rate of return }_{t}\right) / \Pi\left(1+\text { inflation }_{t}\right)\right]^{(1 / n)}-1$, where n is the number of years.

## Calculation of Real Rates of Return for Portfolio

The real rate of return for the portfolio is the weighted average (generally based on a real or hypothetical target asset allocation) of the anticipated long-term rates of return for each asset class included in a real or hypothetical investment portfolio.

## Example (Calculation of Portfolio Real Rate of Return)

Assuming that the final assumed real rates of return by asset class are as shown below (based on periods of historic returns considered appropriate by the actuary, adjusted for risks, and taking into account other information such as forecasts), the total real rate of return for the portfolio is calculated below assuming the asset allocation shown:

|  | Real Rate of Return | Allocation | Product |
| :--- | :---: | :---: | :---: |
| Stocks | $6.6 \%$ | $60 \%$ | $3.96 \%$ |
| Long Term Corporate Bonds | $2.4 \%$ | $15 \%$ | $0.36 \%$ |
| Long-Term Government <br> Bonds | $2.1 \%$ | $20 \%$ | $0.42 \%$ |
| 3-month Treasury Bills | $.6 \%$. | $5 \%$ | $0.03 \%$ |
| Portfolio Real Rate of Return |  |  | $4.77 \%$ |

## Correlations

The correlation between returns of various asset classes contained in a portfolio may affect anticipated returns over time positively or negatively. Correlations between broad asset classes are contained in the Morningstar yearbook. Correlations between the various asset classes of a particular plan may often be obtained from the plan's investment manager. The actuary should note that historical correlations between asset classes may not continue into the future. Changes in the US economy, for example the emergence of global competition and the post-industrial economy, may change the relationship of asset class returns in the future.

The impact of these correlations on portfolio investment returns is usually measured by using Monte Carlo simulation techniques where a series of random trials (generally 1,000 to 10,000 ) is used to simulate returns taking into account the interaction between the various asset classes.

## Plan's Investment Manager

Projected rates of return for the various asset classes held by a particular plan may often be obtained from the plan's investment manager. However, it should be noted that the time-horizon for projected rates of return will often be shorter than an actuary's time-horizon. Also, if nominal rates are provided, the inflation rate underlying the projections should be considered since the investment manager's and actuary's inflation assumptions may differ.

## Caveats

Financial Economics Framework: Under the financial economics framework, investment policy and funding policy would not impact the assumptions used and a discount rate based on a hypothetical portfolio of US government securities or high-grade corporate bonds is considered most appropriate. Proponents of this approach believe that investment gains and losses due to equity premiums should not be anticipated, but should be recognized as they occur (as part of the actuarial gain loss process).

Generally under this framework, current market rates are used in the development of the discount rate.

Risks: Returns should be reduced as necessary in the actuary's judgment to take into account the impact of the risks inherent in the types of investments held in the investment portfolio (for instance, the default risk on the bond portfolio).

Range of Assumptions: To develop a range of assumptions that is more likely than not, a series of n-year averages may be used to obtain a distribution of returns (for instance the 67 15-year periods between 1926 and 2006 or the 5230 -year periods between 1926 and 2006). The $25^{\text {th }}$ and $75^{\text {th }}$ percentiles of this distribution may then be used as the range that is more likely than not.

If the software capability is available, Monte Carlo simulation techniques (taking into account relevant information such as target asset allocations, distributions of assumed returns for each asset class, and the correlation of returns between the various asset classes) may be used to derive a distribution of possible returns and the resulting percentiles.

## Extended Approach

The process of deriving the real rate of return can be further broken down using the Building Block Approach as illustrated below, where the individual components are based on periods of historic returns considered appropriate by the actuary, adjusted for risks, and taking into account other information such as forecasts.

|  |  | Bond Default Premium |  |
| :---: | :---: | :---: | :---: |
|  | Bond Horizon Premium | Bond Horizon Premium | Equity Risk Premium |
| Real Risk-Free Rate of Return | Real Risk-Free Rate of Return | Real Risk-Free <br> Rate of Return | Real Risk-Free Rate of Return |
| Short-term <br> Treasury Bills | Long-term Treasury Bonds | Corporate Bonds | Equities |

Example:
Assume the following component assumptions:
Real Risk-Free Rate of Return: 0.6\%
Bond Horizon Premium: 1.5\%
Bond Default Premium: 0.3\%
Equity Risk Premium: 6.0\%
Then real rates of return are as follows:
Short-term Treasury Bills $=0.6 \%$
Long-term Treasury Bonds $=[1.006 \times 1.015]-1=2.1 \%$
Corporate Bonds[1.006 x $1.015 \times 1.003]-1=2.4 \%$
Equities $=[1.006 \times 1.06]-1=6.6 \%$

Set out below are definitions of the various components referred to above.

## Real Risk-free Rate of Return

The risk-free rate of return is the price paid for the use of funds with a $100 \%$ certitude that the funds will be repaid. The real risk-free rate of return is the risk-free rate of return net of inflation.

## Risk Premiums

Risk premiums may be established for different asset classes based on the relationship of total return to the risk-free rate or to the total return of another asset class. Typical risk premiums include:

## Bond Horizon Premium:

This premium compensates for the impact of changes in interest rates after purchase. It may be measured by comparing the total return on long term government bonds to the return on 3-month Treasury bills.

## Bond Default Premium:

This premium compensates for the impact of default after purchase. It may be measured by comparing the total return on investment-grade corporate bonds to the return on government bonds of similar duration ${ }^{28}$. It should be noted that this premium has not been reduced for the impact of actual bond defaults; neither has it been reduced for the reinvestment risk associated with the exercise of corporate bond call provisions.

## Equity Risk Premium:

This premium compensates for the additional risk of equity exposure. Its value depends on the comparison base (for instance, investment grade corporate bonds, 3-month Treasury bills, or inflation). It should be noted that this premium has not been reduced for the various risks inherent in the investment in equities.

## Sources of Data for Extended Approach

## Risk-free Rate of Return

There is, of course, no perfect measure of the risk-free rate. However, 3 -month Treasury bills are often used as a surrogate because they are assumed not to be subject to default risk. In addition they are zerocoupon bonds (thus having no interest payments subject to reinvestment risk and no associated risk premium.) Also they are less susceptible to purchasing power risks due to variations in inflation rates due to their short maturity. Historic 3-month Treasury bill yields for constant maturities may be found on the Federal Reserve web site as part of the H .15 series at

[^19]http://www.federalreserve.gov/releases/h15/data.htm. Other sources (mentioned previously) include the FRED database on the St. Louis Federal Reserve web site, the Morningstar Yearbook, and the Society of Actuaries Pension Section web site.

The risk-free rate is sometimes based on securities with maturities over 3 months. In this case the bond horizon premium would be included in the risk-free rate.

## Risk Premiums

Calculation of the various components of risk premiums is included in the Morningstar Yearbook based on data starting in 1926. Alternatively, the various premiums may also be calculated directly from other data sources using the formula:
$\left[\left(1+\right.\right.$ return $\left._{t}\right) /\left(1+\right.$ base return $\left.\left._{t}\right)\right]-1$, where $t$ is the year of the return and where return ${ }_{t}$ and base return ${ }_{\mathrm{t}}$ are as described for each type of premium on the prior page.

Example:
Starting with the following hypothetical 1-year rates of return:
Stocks: 10.40\%
Long Term Corporate Bonds: 5.90\%
Long-Term Government Bonds: 5.40\%
3-month Treasury Bills: 3.70\%
Inflation: 3.00\%
Then:
Risk-free real rate of return: $1.037 / 1.03-1=0.68 \%$
Horizon Premium: 1.054 / $1.037-1=1.64 \%$
Default Premium: 1.059 / 1.054-1 = 0.47\%
Equity Risk Premium: $1.104 / 1.037-1=6.46 \%$.

For a series of years, the geometric mean of the premium rates may be calculated for years $t=1$ through n as:
$\left[\Pi\left(1+\text { premium }_{t}\right)\right]^{(1 / n)}-1$, where $n$ is the number of years.
Or alternatively as:
$\left[\Pi\left(1+\text { return }_{t}\right) / \Pi\left(1+\text { base return }_{t}\right)\right]^{(1 / n)}-1$, where n is the number of years.

## Yield Curve Approach

Yield curves display the relationship between yield to maturity and term (period to maturity) of fixed income securities as of a point in time (i.e. fixed date). Thus, they provide a basis of comparison of the impact of term (period to maturity) on yield rates. In order to be most meaningful they must be based on a large number of securities and these securities must be liquid, of similar credit quality, and subject to similar taxation. Commonly referenced yield curves are based on Treasury Securities and/or various investment grades of corporate bonds. As an example of a yield curve, the first 25 years of the February 2009 yield curve developed by the Department of Treasury pursuant to the Pension Protection Act of 2006 (PPA) is shown below.


The previous sections addressed the derivation of a single discount rate assumption. When calculating present values based on a hypothetical portfolio of fixed income securities (for instance when a marketbased approach is being used), it is more accurate to use a series of discount rates that reflects yield to expected payment dates. This can be achieved by using a series of discount rates derived from a suitable yield curve.

The impact of the yield curve approach (versus the single discount rate approach) will depend on the shape of the yield curve and the pattern of expected benefit cash flows. For instance, if the yield curve is increasing, with lower discount rates at the shorter terms, liabilities for plans with higher expected benefit
payments in the near-term (for instance, those with a larger proportion of retirees relative to actives) will tend to be higher.

## Spot Rates

The PPA yield curve is comprised of spot rates. Spot rates are yield rates on hypothetical zero-coupon bonds (bonds that have a single payment, at maturity ${ }^{29}$ ). The use of spot rates results in a clearer representation of the impact of term on rates, as opposed to coupon bonds that have payments prior to maturity. Since interest accumulates to maturity, the use of spot rates also eliminates reinvestment risk on payments prior to maturity.

When calculating present values, yield rates from a spot yield curve may be applied directly to discount payments per the following formula.

Let:
$\mathrm{s}_{\mathrm{t}}=$ spot rate with term t (i.e. term from time zero to maturity at time t )
$\mathrm{bp}_{\mathrm{t}}=$ benefit payment at time t
then $\mathrm{PVB}=\sum_{t=0}^{\infty}\left(\frac{1}{\left(1+\mathrm{s}_{\mathrm{t}}\right)}\right)^{t} \times \mathrm{bp}_{\mathrm{t}}$

## Forward Rates

It is also possible to use a curve of 1-year term forward rates to calculate present values. In this context forward rates are spot rates starting at future dates that are implied by current bond prices. Let the forward rate from time j to time k be denoted by $\mathrm{f}_{\mathrm{j}, \mathrm{k}}$ then:
$\mathrm{f}_{\mathrm{j}, \mathrm{k}}=\left[\left(1+\mathrm{s}_{\mathrm{k}}\right)^{\mathrm{k}} /\left(1+\mathrm{s}_{\mathrm{j}}\right)^{\mathrm{j}}\right]^{[1 /(\mathrm{k}-\mathrm{j})]}-1$
conversely,
$\mathrm{s}_{\mathrm{k}}=\left[\left(1+\mathrm{s}_{\mathrm{j}}\right) \mathrm{x}\left(1+\mathrm{f}_{\mathrm{j}, \mathrm{k})}\right)\right]^{(1 / \mathrm{k})}-1$

In particular, given a series of 1-year forward rates, spot rates may be calculated as:
$s_{k}=\left[\prod_{j=0}^{k-1}\left(1+f_{j, j+1}\right)\right]^{(1 / k)}-1$, where $f_{0,1}=s_{1}$
Forward rates may be used to calculate present values by projecting benefit payments for each year and then discounting them back year by year using forward rates.

[^20]
## Examples

Simplified examples of present value calculations using spot and forward rates are shown below. For these examples it has been assumed that benefit payments are payable at maturity dates and that no benefit payments are paid at time 0 . Note that as would be expected, ignoring differences due to rounding, the same present value results from using either approach.

Present Value Using Spot Rates

|  | (A) | (B) | (C) <br> Term | Spot Rate |
| :---: | :---: | :---: | :---: | :---: |

Present Value Using Implied Forward Rates

|  | (A) <br> 1-Year Term <br> Forward Rate* | (B) | (C) | (D) <br> Discounted | Discounted <br> Benefit Payments <br> (Formula) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | $2.00 \%$ | 100,000 | 1.0200 | $\mathbf{9 2 8 , 6 6 5}$ | (D2+B1)/C1 |
| 1 | $2.10 \%$ | 150,000 | 1.0210 | 847,238 | (D3+B2)/C2 |
| 2 | $2.20 \%$ | 200,000 | 1.0220 | 715,030 | (D4+B3)/C3 |
| 3 | $2.30 \%$ | 250,000 | 1.0230 | 530,761 | (D5+B4)/C4 |
| 4 | $2.40 \%$ | 300,000 | 1.0240 | 292,969 | B5/C5 |
| Total Present Value |  |  | $\mathbf{9 2 8 , 6 6 5}$ |  |  |

* Can be derived from spot rates: $2.10 \%=\left(1.0205^{\wedge} 2\right) / 1.02,2.20 \%=(1.0210)^{\wedge} 3 /(1.0205)^{\wedge} 2$, etc.


## Construction

A variety of different techniques are used to derive yield curves and forward rates. A modeling process may be used based on current price for selected securities, with adjustments for items such as creditworthiness, call features and other embedded options.

Yield curves can be developed more directly from Treasuries. Since zero-coupon bonds are a rarity (there are no zero-coupon Treasuries with maturities over 1 year), spot rates are developed using theoretical
processes. STRIPS (Separate Trading of Registered Interest and Principal of Securities), which separate out repayment of principal and each coupon payment, have been constructed since 1985 from Treasury securities by the Treasury Department and are sold on the market by financial institutions. For Treasuriesbased yield curves, STRIPS provide values at more terms and, unlike bonds with coupons, have terms equal to durations ${ }^{30}$. However, concerns about the STRIP market have also lead to construction of Treasury yield curves using other methods.

## Extension of Yield Curves

Actuaries often price liabilities due far beyond those of most bonds. Even though there may be a few available rates for very long issues, they tend to be single bond issues that may not be actively traded, so the yield may not represent current market information. Some of the more common methods for extending the yield curve under these circumstances are listed below. None is perfect; the actuary should carefully consider which method best meets the situation at hand.

- Extend the yield rate using the rate for the longest market-priced security. A rationale for using this method is that projecting any upward or downward trend would imply something the market hasn't suggested. This method works better when there are well-defined rates at longer issues (e.g. for the Treasury yield curve, where 30-year Treasury bonds are issued fairly regularly and in reasonable quantity).
- Extrapolate the shape of the yield curve into the future. Extrapolating yield curves into the future can be difficult and misleading if the extrapolation is based on a few data points, and may infer a trend in market pricing that is not logical. In addition, it is possible, for instance in the case of the 30 -year Treasuries, that rates may be distorted by the fact that the 30 -year bond is the longest available Treasury bond. However, some degree of extrapolation might be prudent if the market index doesn't include many longer issues. In this case, basing the extrapolation on the shape of on an existing curve (e.g. Treasury yield curve) may be prudent.
- Extrapolate forward rates


## Sources of Yield Rates

PPA yield rates are published in IRS notices. Currently this yield curve is based on AAA, AA and A rated corporate bonds that meet certain criteria including criteria related to size and call provisions.

[^21]The Citigroup Pension Discount Curve, provided on the Society of Actuaries’ web site at http://www.soa.org/files/xls/pen-discount-curve.xls and referenced at http://www.dol.gov/ebsa/pdf/treasurywhitepaper.pdf

This is a spot-rate curve based on theoretical zero-coupon bond yields to maturity for Grade AA corporate bonds.

Current and historic daily Treasury yield curve rates, based on the H. 15 constant maturity series, and real yield curve rates, based on TIPS (Treasury Inflation Protected Securities), may be found on the Department of Treasury web site at http://www.treas.gov/offices/domestic-finance/debt-management/interest-rate/. The difference between rates at the same maturities gives an idea of investors’ inflation expectations.

## Cash-Flow Matching Method

This method builds on a base rate of return derived by matching a hypothetical or actual bond portfolio to projected benefit payments. This base rate is then adjusted as necessary to derive the discount rate.

The hypothetical portfolio may be based on investment grade corporate bonds or Treasury securities. Instead of constructing a hypothetical portfolio, a similar result can be obtained by applying a yield curve to the benefit and expense streams to derive the base rate of return. For instance, the Citigroup Pension Discount Curve or the yield curve supplied by Treasury under the PPA minimum funding requirements might be used. This approach is simpler to apply and, depending on the yield curve chosen, may automatically provide yield rates to apply to payment streams beyond the range of typical bond maturities.

The base rate is then adjusted as necessary to derive the discount rate. Adjustments may be necessary for:

- Reinvestment risk resulting from the mismatch of the bond portfolio and projected benefit payments, including an approach for payments that extend beyond typical bond maturity dates or to maturity dates where the bond market is thin
- Reflection of the actual asset allocation (for example, to reflect equity premiums for the equities contained in the portfolio)

The cash-flow matching method approach reflects a timeframe based on anticipated benefit payments to current plan participants. It also reflects market discount rates that are based on the current market's estimation of inflation rates over the benefit payment period. Care should be given to assuring that the recognition of inflation in the resulting discount rate and the recognition of inflation in the other economic
assumptions (such as salary increases) are consistent. A potential method for estimating the inflation assumptions inherent in the derivation of the cash-flow method discount rates is to compare yield rates for government bonds with yield rates of TIPS as discussed in Chapter 7.

## Information That Can Affect the Choice of Discount Rate

## Duration of Liabilities

Duration is a measure of the sensitivity of the present value of a stream of payments to changes in discount rate. It was originally developed as a tool for comparing the interest rate sensitivity of the prices of bonds with different coupon and maturity provisions. The price of bonds (or the present value of other streams of payments) with the same duration would generally be expected to have similar sensitivity to changes in interest rates.

Specifically, if PV is the present value of a stream of payments discounted at a single interest rate i, duration is:
(1) the negative of the rate of change (derivative) of PV with respect to interest rate i, divided by PV. Since present value decreases as interest rate increases, duration will be positive. Since a single interest rate is being used, we are assuming a flat yield curve with a shift to a parallel yield curve.

To the extent that durations differ between retirement plan liabilities and assets, changes in interest rates will affect the level of liabilities and assets differently and increased volatility in contribution rates and measures of funded status will tend to result.

Different formulas exist for estimating duration. Two formulas are shown below that are based on (1) above.

## Formula 1:

Let:
$\mathrm{i}=$ interest rate
$\mathrm{v}=1 /(1+\mathrm{i})$
$\mathrm{k}=$ compounding per year
$\mathrm{t}=$ years until payment
$\mathrm{P}_{\mathrm{t}}=$ payment at time t ,

Modified Duration $=\left(\sum_{t=0}^{t=\infty} t P_{t} v^{t} / \sum_{t=0}^{t=\infty} P_{t} v^{t}\right) /(1+\mathrm{i} / \mathrm{k})$

The numerator of this formula, $\sum_{t=0}^{t=\infty} t P_{t} v^{t} / \sum_{t=0}^{t=\infty} P_{t} v^{t}$ is the Macaulay formula for duration. In this form, duration may be interpreted as the weighted average of terms to payment dates, with weights equaling the present value of payments at each term.

Formula 2
A formula useful in estimating durations of retirement plan liabilities is shown below.

Let $\mathrm{PVB}_{\mathrm{i}}$ equal the Present Value of Benefits at interest rate i.
Letting interest rate i vary by $.1 \%$ the duration of retirement plan liabilities may be estimated as ${ }^{31}$ :

$$
\begin{aligned}
\text { Duration } & =-\left[\left(\mathrm{PVB}_{(i+0.1 \%)}-\mathrm{PVB}_{\mathrm{i}}\right) / 0.1 \%\right] /\left(\mathrm{PVB}_{\mathrm{i}}\right) \\
& =\left(\mathrm{PVB}_{\mathrm{i}}-\mathrm{PVB}_{(i+0.1 \%)}\right) /\left(0.1 \% \times \mathrm{PVB}_{\mathrm{i}}\right)
\end{aligned}
$$

## Expenses

Unless already taken into account in the data used to derive real rates of return, investment management expenses paid from the trust must be taken into account to the extent permitted under statutory, accounting or other requirements, and to the degree that the actuary does not feel that they are being made up by superior investment performance. Fees for investment management expenses are generally related to fund size so recognition of these expenses generally takes the form of a decrease in the assumed discount rate where permitted for the particular type of calculation.

Administrative expenses paid from the trust may be recognized in a variety of ways. See details under Administrative Expenses at the end of Chapter 5.

## Investment Policy

The investment policy will generally include a target asset allocation. In most cases, this target allocation will be used in setting the asset-class weights used to set the discount rate. When setting a discount rate based on anticipated trust fund performance, this target allocation should be compared to the fund's actual

[^22]allocation (taking into account anticipated changes) and any material differences considered. If current differences are anticipated to be relatively short-term in nature, the target allocation would continue to be used in setting the discount rate.

If a portion of the plan's obligations (for instance a portion of retiree liabilities) is cash-flow matched with a portfolio of fixed income securities and these securities are valued at market, an investment return assumption reflective of anticipated returns on the securities may be used in certain situations (for instance for contribution rate forecasting). However, this assumption would not be appropriate for minimum and maximum funding purposes for single-employer plan because of mandated discount rates set in the statute.

## Funding Policy

Funding policy will affect cash flow and thus liquidity needs. Higher levels of prefunding may increase the possible holding period for securities, thus potentially increasing rates of investment return.

## Types of Investments

Similarly to Funding Policy, the types of investments held by the plan may affect cash flow - investments that produce income (such as bonds with coupon payments or equities with dividends) may allow the plan to increase the holding period for securities as well.

## Lump Sum Availability

The availability of lump sums will affect liquidity and, potentially, investment return. Also, to the extent that the interest basis used to calculate the lump sum payment does not match the plan's investment portfolio, the plan may experience additional volatility in contributions and funded status.

## Cash Flow

The pattern of anticipated future cash flows may influence the choice of discount rate. For instance, if assets are being liquidated to make benefit payments more liquid investments may be necessary which would generally be expected to lower anticipated investment returns.

## Special Considerations

## Confirmation Using Financial Market Expectations

When setting discount rates the actuary also has available the financial markets’ expectations for future rates as embedded in the prices of financial instruments. For example, an assumption regarding the future total returns on the bond portion of a portfolio may be compared for reasonableness to current yields for similar grades of bonds.

## Comparisons to Assumptions used for Comparable Plans

In order to check assumptions for reasonableness, assumptions for comparable plans may be reviewed. Surveys are often available regarding economic assumptions.

Survey data provides a source of peer information; however, economic assumptions must be chosen based on theory and the circumstances surrounding each case.

## Annuities

In the case of plans where annuities are purchased at retirement, anticipated annuity purchase prices at the time of retirement should be taken into account in setting the post-retirement discount rate except where not permitted by law. The resulting post-retirement discount rate should be consistent in perspective with the other economic assumptions, in particular the pre-retirement discount rate. For example, when the pre-retirement discount rate is based on best-estimate assumptions, this assumption should be set based on expected long-term annuity prices. However, when the pre-retirement discount rate is based on current market rates, as would generally be the case when using a financial economics perspective, the discount rate assumption would be based on current annuity purchase prices.

## Special Derivations

Interest rates used in calculating benefits such as lump sum options and interest credited on cash balance accounts may be based on special criteria that are different from the discount rate. For instance, interest rates used in calculating lump sums may be tied in part to rates specified in the Internal Revenue Code. When this occurs, assumptions should be derived that appropriately reflect the criteria involved. Again, these should be consistent in outlook with the other economic assumptions, including the discount rate.

## Chapter 9: General Considerations in Setting Assumptions

## Internal Consistency

## Across Assumptions

Not only should each assumption be considered individually, but it should be considered for consistency with the remainder of the assumptions.

For instance, all economic assumptions for a given measurement should be based on a consistent future economic environment. In particular, they generally should anticipate the same future inflation environment and the same expectations for GDP growth. Similarly, expected rates of return for bonds and equities should include consistent risk premiums.

Demographic assumptions should generally be consistent among the various valuations done for a plan or for different plans for the same population, unless constrained by statute. While in some cases statute may specify a particular set of assumptions for a particular purpose, the actuary should attempt to use consistent mortality, termination, disability and rates of retirement for a given population.

## Within Assumptions

Final assumptions should be checked for internal consistency. An example relates to applying mortality improvement scales that vary by age, such as Scale AA, to retiree mortality tables. After applying the scale, the results should be checked for reasonable progression (i.e., generally mortality rates should increase somewhat smoothly with age). Also to the extent that one assumption is projected, consider whether other assumptions should be projected as well to keep the set of assumptions internally consistent. For example, if mortality improvement is projected, does the actuary think that rates of disablement (morbidity) will improve or stay the same?

## Purpose

The purpose of the study may impact the choice of assumptions as discussed below. In some cases it will call for additional assumptions. In others it may impact the margins and conservatism used.

## Funding Valuation

Plan funding can be done looking at three sets of values: the minimum required contribution, the maximum tax-deductible contribution, and the employer's funding policy contribution. Minimum required and maximum tax-deductible contributions are largely determined by statute. Funding policy
valuations, on the other hand, often use a traditional actuarial funding method that projects future pay improvements, and has as its goal a strategy of setting a funding policy contribution as a consistent amount (e.g., dollar or percent of pay) to facilitate budgeting by the plan sponsor.

In setting funding policy contribution rates, a degree of conservatism may be appropriate in order to provide additional security to plan participants. This conservatism is considered particularly appropriate, depending on the plan's asset allocation strategy, if the employer's tolerance for contribution rate volatility is low.

On the other hand in plans of for-profit entities plan surplus cannot be accessed (excise taxes effectively tax any surplus away), so the sponsor may not wish to accumulate surplus funds in excess of the value required to fund the plan, as that takes capital away from the core business. In this case, funding policy could be to ensure that the plan is funded at a level that does not exceed the amount needed to be fully solvent on a termination basis at all times.

## Plan Termination / Frozen Plans

In setting assumptions for plan termination studies including testing for sufficiency, possible adverse investment and other experience between the calculation date and the actual distribution of funds should be considered and assumptions adjusted accordingly. Also, depending on the reasons for plan termination, rates of termination and retirement may increase, changing the expected pattern of payments from the plans.

For frozen plans the likelihood of plan termination may be increased. The plan sponsor may also wish to terminate the plan and therefore may want the actuary to set assumptions to mimic what the eventual annuity purchase price might be for the plan. This may be different than the value for the ongoing plan, as the insurer will charge margins for profitability, contract initiation and administration costs, and include reserves for risks and adverse deviation.

## Accounting Valuation

Publicly traded corporations are subject to Statement of Financial Accounting Standards No. 87 as amended by Statement of Financial Accounting Standards No. 158. Requirements regarding actuarial assumptions include the following:

1. The best estimate criteria apply to each "significant" assumption individually.
2. Current annuity purchase rates and currently available returns on high quality fixed-income investments serve as guides in setting the discount rate used to calculate the projected benefit
obligation, service cost, and interest cost. (Interest cost pertains to the cost of interest during the year on the projected benefit obligation.)
3. The expected long-term rate of return on plan assets is to be representative of the average investment return expected on funds to be used to provide the benefits included in the projected benefit obligation as of the valuation date. "Returns being earned by the plan assets in the fund and the rates of return expected to be available for reinvestment"32 are to be considered in setting this assumption. (The expected long-term rate of return on plan assets is applied to the market-related value of plan assets to calculate the expected return on plan assets. This expected return offsets the interest cost above. The market-related value of plan assets is "either fair [market] value or a calculated value that recognizes changes in fair value in a systematic and rational manner over not more than five years" ${ }^{33}$ )
4. Assumptions must be consistent in reflecting future economic conditions, including inflation.
5. Assumptions should be based on an ongoing plan "in the absence of evidence" that the plan will not continue.

FAS 158 amended FAS 87 partly to clarify the intent of the Board regarding the discount rate assumption. The clarification language is reproduced below.
"Pursuant to paragraph 44, an employer may look to rates of return on high-quality fixed-income investments in determining assumed discount rates. The objective of selecting assumed discount rates using that method is to measure the single amount that, if invested at the measurement date in a portfolio of high-quality debt instruments, would provide the necessary future cash flows to pay the pension benefits when due. Notionally, that single amount, the projected benefit obligation, would equal the current market value of a portfolio of high-quality zero coupon bonds whose maturity dates and amounts would be the same as the timing and amount of the expected future benefit payments. Because cash inflows would equal cash outflows in timing and amount, there would be no reinvestment risk in the yields to maturity of the portfolio. However, in other than a zero coupon portfolio, such as a portfolio of long-term debt instruments that pay semiannual interest payments or whose maturities do not extend far enough into the future to meet expected benefit payments, the assumed discount rates (the yield to maturity) need to incorporate expected reinvestment rates available in the future. Those rates shall be extrapolated from the existing yield curve at the measurement date. The determination of the assumed discount rate is separate from the determination of the expected rate of return on plan assets whenever the actual portfolio differs from the hypothetical portfolio above. Assumed discount rates shall be reevaluated

[^23]at each measurement date. If the general level of interest rates rises or declines, the assumed discount rates shall change in a similar manner."

The Citigroup Pension Discount Curve on the SOA Pension Section web site is an example of a yield curve that can be used to comply with this approach. This curve approximates the yield curve for constructed zero-coupon bonds with AA credit ratings.

## Forecast Valuations and Simplified Cash-Flow Projections

Forecast valuations, which project valuation results (liabilities and assets) in future years by preparing valuations in future years using specially designed software, will require consideration of additional assumptions, including:

- Workforce growth and new entrant demographic profiles,
- A set of experience assumptions (used to calculate actuarial gains and losses and the impact of the plan's experience on future contribution levels) based on individual best estimates of the plan's actual experience,
- Projection of changes in future valuation assumptions as necessary and
- Election of optional forms of payment for cash-flow purposes.

Forecast valuations may also incorporate assumptions as to future benefit improvements (such as ad hoc cost-of-living increases to retirees, increases in fixed dollar benefit formula amounts, and updates of career-average plans).

For simple projections of trust fund cash flows, materiality to the purpose at hand should be considered. For these projections simplifying assumptions and methods are often used, particularly in projecting future contribution levels. However when projecting benefit payments, careful attention should be given to assumptions regarding election of optional forms of benefits - in particular cashouts.

## Chapter 10 - Assumptions Prescribed By Statute / Regulation

A brief summary of requirements of various government bodies that impact the assumption-setting process is provided below. The material is based on statutes, regulations and other guidance as of the time of writing.

## Pension Protection Act of 2006 (PPA)

The Pension Protection Act of 2006 (PPA) changed the assumptions used for calculating minimum required and maximum tax-deductible contributions for single-employer plans, minimum present values for lump sums, and PBGC Variable Premiums. The new requirements are outlined below. With the exception of the prohibition regarding the use of implicit assumptions and the update of mortality tables used to calculate the full-funding limit based on current liability, the requirements regarding actuarial assumptions for minimum required and maximum tax-deductible contributions for multi-employer plans were not changed.

## Discount Rate:

The discount rate is changed from a single discount rate to a yield curve basis.

## Yield Curve:

The PPA yield curve is composed of spot rates and is calculated each business day of the month.

Bonds included in the derivation of the daily yield curves generally have the following characteristics:

- Issued by US corporations and denominated in US dollars
- In the top three quality levels available (i.e. rated AAA, AA, or A)
- Outstanding par value exceeds $\$ 250$ million
- Maximum maturity 30 years
- Pay fixed nominal semiannual coupons and the principal upon maturity
- Generally do not include special features such as embedded options.

In 2007 about 1,400 bonds were in each daily set of bonds used in construction of the yield curve.

A model is used to produce a spot rate yield curve using the price, cash flows, and ratings of the daily bond set. The model is in part based on generating instantaneous forward rates for each point in the future. Beyond 30 years this instantaneous forward rate is set to its average value at terms 15 through 30 . The yield curve extends to a term of 100 years.

Monthly yield curves are calculated by averaging the daily spot rates for a given month.

Segment Rates: The monthly full yield curve spot rates are grouped to produce three "segment rates":

- Segment 1: a short-term rate based on average yield curve rates for maturities of 0-5 years,
- Segment 2: a medium-term rate based on average yield curve rates for maturities of 5-20 years, and
- Segment 3: a long-term rate based on average yield curve rates for maturities of 20+years.


## Minimum and Maximum Contributions:

In this case 24-month segment rates are the default option for discount rates. These segment rates are calculated by averaging spot rates from monthly yield curves for the most recent 24 months.
Alternatively, the sponsor may use the full yield curve rather than the segment rates - in which case 24 month averaging does not apply.

## Lump Sums and Variable PBGC Premiums

The Pension Protection Act of 2006 also changed the assumptions used for calculating minimum lump sums and variable PBGC premiums to the three segment rate basis. In this case, the monthly segment rates (rather than the 24-month average of segment rates) must be used.

## Transition Provisions

For minimum and maximum funding the new discount rate provisions are effective in 2008 and are phased in over 3 years unless an election is made to the contrary. For lump sums the new provisions are phased in starting in 2008 and ending in 2012.

## Sources

The Internal Revenue Service publishes Notices containing monthly full yield curve, segment rates, and transition rates data.

## Mortality Rates:

## Minimum and Maximum Contributions

The Department of Treasury is required to prescribe post-retirement mortality tables based on "actual experience of pension plans and projected trends in such experience ${ }^{34}$." Treasury is to update these tables every 10 years.

Under final regulations issued in July 2008, the RP-2000 Table is required for non-disabled lives. While retirement benefits are being received, the Healthy Annuitant Table must be used. Prior to that time when benefits are not being received (i.e. while employed or while benefits are being deferred) the Employee Table is used. Plans with fewer than 500 participants are allowed to use one table (the Combined Healthy Table) both before and after benefit receipt.

Mortality improvement must be taken into account using Projection Scale AA. Either the Generational Method or a Fixed Years Projection (Static) Method must be used. If the Fixed Years Projection (Static) Method is used, mortality improvement is first projected to the valuation date. Next, for nonannuitants it is projected 15 years beyond the valuation date and for annuitants it is projected 7 years beyond the valuation date. Tables reflecting these adjustments are supplied in IRS guidance.

Sponsors of plans with significant amounts of credible experience, as prescribed by regulation, may request to use an alternate mortality table based on their own experience, but must reflect "projected trends in general mortality experience.,35

The Department of Treasury also is required to prescribe mortality tables for disability retirees. For "individuals whose disabilities occur in plan years beginning after December 31, 1994"36 they will only apply to those meeting the Social Security definition of disablement. As of this writing (per IRS Notice 2008-29) actuaries may continue to rely on the disability mortality rates in Revenue Ruling 967 until further guidance is issued.

## Salary Increases:

Salary increases beyond the current year are not taken into account when calculating minimum required contributions for single-employer plans. (Note that projected increases in compensation during the current year are taken into account for determining the target normal cost used to determine

[^24]the minimum required contribution.) However, projected future compensation increases are taken into account when determining maximum deductible contributions.

Special Assumptions for "plans in at-risk status"-: The Pension Protection Act requires special actuarial assumptions for calculating liabilities for plans in at-risk status." For these plans, calculations must be based on assumptions applicable to plans in at-risk status which include the assumption that all plan participants elect the most valuable benefit form and that all participants eligible to retire in the current and next 10 years retire when first eligible after the current year, unless they are expected to retire during the year containing the valuation date. These assumptions may be used in calculating one component of the maximum deductible contribution even if the plan is not "at risk."

## Lump Sums

Mortality tables are provided in IRS guidance and are based on the RP-2000 Combined Mortality Table on a 50/50 unisex basis with the Static projection described above that is used for minimum funding. Generational mortality projections may not be used.

## Variable Premiums

Under the Pension Protection Act of 2006, variable premiums will generally be based on the mortality assumptions used to determine minimum and maximum contributions (including "at-risk" assumptions where applicable).

## Pension Benefit Guaranty Corporation

## Sufficiency of Terminating Plans

Sufficiency is based on settlement of the obligations by annuity purchase and/or lump sum payouts. If the plan is not sufficient, the PBGC will value the plan using special PBGC assumptions in the valuation, including prescribed discount rates, mortality tables, and assumed retirement ages.

## Securities and Exchange Commission (SEC)

For the most part, the SEC requires registrants to comply with FASB standards in filings under federal securities laws. However, it retains the right to modify disclosure requirements as necessary and weighs in on issues that concern it - such as prior to the issuance of FAS 158, on the subject of the selection of discount rate.

## Chapter 11: Monitoring Assumptions

The assumption-setting process is based on the use of historic experience that is subject to statistical variation. In addition, it reflects available knowledge about the plan and its participants as of a specific point in time. Thus, it becomes necessary to adjust assumptions periodically to reflect both the impact of changing circumstances and the availability of additional data. This involves an awareness of any changes in the plan's environment that may impact assumptions and also the monitoring of the plan's experience in a systematic fashion.

## Monitoring the Plan's Environment

Changes in areas such as the plan provisions and administration, the employment environment, or the broader environment in which the plan and employer operate may impact actuarial assumptions.

Plan Provisions and Administration: Where changes in plan provisions are expected to impact experience, and the impact can be reasonably estimated and is material, assumptions should be adjusted on an estimated basis when the provision becomes effective. In any case, if assumptions are likely to be materially impacted, experience should be carefully monitored. Examples of plan changes that may impact assumptions are:

1. plan provisions regarding early retirement subsidies and retiree medical benefits will potentially impact retirement rates;
2. acquisitions, mergers and spinoffs may change the demographic profile of the employee group in both the short and long term, potentially influencing all demographic assumptions;
3. changes in vesting schedules will potentially impact termination rates; and
4. rates of disablement, disabled mortality and recovery rates may be impacted by changes in administrative practices regarding determination of eligibility for benefits and definitions of disablement, and also changes in plan provisions and administrative provisions related to continued eligibility for benefit payments.

Employment environment: Rates of retirement, termination and disablement and as well as salary increases may be impacted by changes in the outlook for the industry and the employer's practices in areas such as compensation and phased retirement.

Broader Environment: Broader changes such as changes in the economy or in societal norms (such as expectations as to retirement age or coordination of careers with child rearing) may also impact
assumptions. Macro changes to social conditions such as these tend to affect assumptions more slowly; experience should be monitored and adjusted as any effect of broader social trends works its way into plan experience.

## Monitoring the Plan's Experience

Gains and losses due to variation of actual experience from assumptions will be recognized as part of the actuarial funding process. From an assumption-setting view, the purpose of monitoring experience is to sort out the causes of gains and losses and find systematic sources of deviation that necessitate changes in assumptions. Thus, gains and losses due to random fluctuations and those due to temporary changes in experience (such as spikes" in rates of termination and retirement due to mergers, acquisitions and spinoffs that are then followed by a short period of depressed rates) would be differentiated and excluded from those believed to be due to long-term trends that would necessitate changes in assumptions.

Different methods can be used to monitor the plan's experience including:

- Gain / loss analysis,
- Comparison of actual and expected experience, and
- Full experience studies

The method (or methods) used will depend on the size of the plan and the materiality of the assumption.

## Gain / Loss Analysis

For some plans, detailed gain / loss analysis of liabilities by source may be performed annually as part of the valuation process. These studies will alert the actuary to any consistent material pattern of gains or losses for a particular assumption that can then be reviewed.

Even if gain / loss analysis is limited in scope, an unexplained pattern of consistent material liability gains or losses can alert the actuary to the possible need for an experience study.

Generally the most significant routine sources of gains and losses are relative to economic assumptions, such as those regarding salary increases and, for most plans with significant investments in equities, on asset performance. Here again, one must endeavor to differentiate between changes in long-term trends and more temporary phenomena.

In analyzing the causes of decrement gains and losses for actives, it should be remembered that the magnitude of gains and losses is dependent on the size of actuarial accrued liabilities and thus tends to be
more heavily influenced by deviations in experience for longer service and older employees. Thus losses or gains may indicate that the structure of an assumption rather than the number of expected decrements is inappropriate. For example, for a given age the expected number of terminations may be close to the actual number of terminations. However, if terminations for shorter service employees are significantly higher than for longer service employees, actuarial losses will be incurred unless the termination assumption is restructured to increase the number of terminations expected for shorter-service employees.

Plan provisions should also be carefully considered. For instance, a loss on termination rates may be an indication that actual termination rates are higher than expected for a cash balance plan when the interest crediting rate is lower than the discount rate - while for a final average pay plan a termination rate loss may be an indication that actual termination rates are lower than expected.

## Comparison of Actual to Expected Experience

Where a full gain / loss analysis is not performed, comparisons of actual to expected experience may be conducted in order to monitor assumptions or provide further analysis of liability gains or losses. Periodic comparisons of actual to expected experience may be conducted for demographic assumptions, and broken down for relevant age/gender/service groupings. To test salary increase assumptions, average pay increases for employees actively employed in consecutive valuations may be compared. These calculations may aid in explaining patterns of gain and loss. (Comparisons of averages of total payroll in consecutive valuations give an indication of the level of general pay increases (inflation plus productivity) but do not include changes in merit or longevity increases because employees leaving employment and new entrants are included.)

## Full Experience Studies

Experience studies may be conducted periodically or when determined to be necessary based on gain /loss analysis or periodic comparisons of actual to expected experience. In an experience study, actual historic experience will be compared to expected experience for all material assumptions. Experience from prior studies can be incorporated to study trends and improve credibility of data for decrements such as disability and mortality rates with low incidence.

## Updating Assumptions

Since the actuarial funding process automatically incorporates the impact of deviations in actual and expected experience into the funding process, some leeway is possible in the timing of assumption updates. Ideally assumption reviews and the resulting adjustments in assumptions, other than those necessitated by plan changes or unusual circumstances, can take place in periodic experience studies
when sufficient credible experience has been accumulated and all assumptions are adjusted at the same time. This will, of course, not always be possible - for instance when plan changes or unusual circumstances call for immediate adjustments to assumptions.

For demographic assumptions, the extent to which individual assumptions are adjusted will depend on whether the plan's experience is credible and differs materially from the current assumption. In order to minimize the risk of future backtracking, less than $100 \%$ credibility may be placed on the more recent experience data where the basis for a trend in experience is unclear.


[^0]:    ${ }^{1}$ Since pension benefits are considered part of employee compensation, various methods are used to allocate the cost of retirement benefits to the employee's period of employment. Some methods endeavor to spread costs evenly over the employee's career. Other methods focus on the amount of benefit earned in a given year. The new minimum required and maximum tax-deductible funding standards under the Pension Protection Act of 2006 for single-employer plans fall into the latter category. On the other hand, an employer's actual funding policy may be based on actuarial cost methods falling into the former category, so long as the applicable requirements of the law are satisfied.

[^1]:    ${ }^{2}$ So called because they are considered ancillary to the main purpose of the plan, which is to provide retirement benefits.

[^2]:    ${ }^{3}$ As defined in the Pension Protection Act of 2006.
    ${ }^{4}$ This ability to use implicit assumptions under prior law was moot to the extent that it conflicted with the actuarial standards of the time.

[^3]:    ${ }^{5}$ Which in this context (i.e. financial economics) generally is assumed to include the use of current market rates for both short and long-term Treasury securities.

[^4]:    ${ }^{6}$ For full scale see Table 7-3 in RP-2000 Mortality Table ,http://www.soa.org/files/pdf/rp00_mortalitytables.pdf
    ${ }^{7}$ Differences between Scale AA projection factors due to gender decline materially starting at ages in the mid-70's.

[^5]:    ${ }^{8}$ For further information and formulas for duration see the end of Chapter 8.

[^6]:    ${ }^{9}$ So called because mortality was not projected in this table. There are three 1994 Tables: The Basic Table (GAM94 Basic) includes no margins. The Static Table (GAM-94 Static) includes a 7\% margin, and the Reserving Table (GAR-94) includes both the 7\% margin and mortality improvement using a generational approach and is used for group annuity reserving by insurers. The UP-94 Table has been set to equal the GAM-94 Basic Table and thus does not contain the $7 \%$ margin.

[^7]:    ${ }^{10}$ See OASDI Trustees Report for additional information on Social Security disability provisions and assumed rates of disablement.

[^8]:    ${ }^{11}$ Salaried includes plans where the group is more than $90 \%$ salaried whether unionized or not.
    ${ }^{12}$ Hourly union includes plans where the group is more than $90 \%$ hourly and more than $90 \%$ union.
    ${ }^{13}$ Hourly non-union includes plans where the group is more than $90 \%$ hourly and more than $90 \%$ nonunion.
    ${ }^{14}$ A select and ultimate table is a table where rates vary by two variables during an initial period and then by only one variable thereafter. The initial period is referred to as the select period. Rates during the period which follows are referred to as ultimate rates. In this case rates were varied by age and service during the initial (select) period of

[^9]:    three years and then by age only thereafter. Thus, rates were developed by service ( 0 years, 1 year, 2 years) and age during the select period and during the ultimate period by age for those with 3 or more years of service.
    

[^10]:    ${ }^{16}$ The total margin in the table was 7\%, which included an additional 2\% margin to allow for mix of business and smaller blocks of business.

[^11]:    ${ }^{17}$ For purposes of this calculation, care should be taken to adjust for delays in reporting and benefit processing particularly with respect to death and disablement.

[^12]:    ${ }^{18}$ As used to project items such as automatic post-retirement cost-of-living increases and Social Security cost-ofliving increases. Note that many plans limit cost-of-living increases to specified maximums (e.g., 3\%). In such plans, it would not be reasonable to assume cost-of-living increases greater than the specified maximum.
    ${ }^{19}$ Examples are shown with components additive. Strictly speaking, this may be inconsistent with the derivation of the components, which may be multiplicative in some instances. However, for simplicity, final building blocks are usually expressed as additive.

[^13]:    ${ }^{20}$ Strictly speaking because the index is based on average W -2 earnings it is also influenced by changes in average hours worked. However, this is generally ignored.

[^14]:    ${ }^{21}$ Group where size and distributions of demographic characteristics such as age, service, and merit/longevity pay category remain the same from year to year. Such a group would be a mature group that has had over an extended period the same number and demography of new hires each year and constant rates of decrement.

[^15]:    ${ }^{22}$ Used instead of 2005-2008. See below.
    ${ }^{23}$ Input from employer based on budget for general pay increases, as well as bargaining agreements, indicates that $3 \%-3.5 \%$ general increases are more likely.

[^16]:    ${ }^{24}$ According to the plan sponsor, the unusual pattern of increases in 2005 was due to a special one-time market/equity adjustment to salaries for individuals in certain job classifications. For this reason 2005 data was considered atypical and the final assumptions were based on 2006-2008 data.

[^17]:    ${ }^{25}$ See end of Chapter 3.

[^18]:    ${ }^{26}$ It should be noted that new procedures may be necessary to appropriately reflect the recent market downturn of 2008/2009.
    ${ }^{27}$ Formerly this was produced by Ibbotson Associates

[^19]:    ${ }^{28}$ For further information and formulas for duration see page 57.

[^20]:    ${ }^{29}$ Zero coupon bonds are bought at discount and accumulate interest until maturity.

[^21]:    ${ }^{30}$ For further information and formulas for duration see the end of Chapter 8.

[^22]:    ${ }^{31}$ An Introduction to Duration for Pension Actuaries, Richard Daskais and David LaSueur, SOA Pension Forum, June. 1993 and Report for the RP-2000 Mortality Table

[^23]:    ${ }^{32}$ Statement No. 87, Employers' Accounting for Pensions
    ${ }^{33}$ Statement No. 87, Employers' Accounting for Pensions

[^24]:    ${ }^{34}$ IRC 430(h)(3)(B)
    ${ }^{35}$ IRC 430(h)(3)(C)
    ${ }^{36}$ IRC 430(h)(3)(D)

