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PREMIUM RATES VARYING BY POLICY SIZE

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ONE DIMENSION: PLAN

IN THE primitive days of life insurance, premium rates did not vary by age. They took account only of the conditions of the insurance, *viz.*, the plan. Referring to " 'The Cambridge Tables for renewing of Leases and purchasing Liens,' a standard work in England, with the certificate of Sir Isaac Newton to its accuracy," certified Sept. 10, 1685, the Encyclopaedia Britannica, 11th Edition, article—Insurance, states that "no distinction of ages was recognized, and the results . . . are worthless," and then makes the comment:

> Thus the foremost minds of the world had as yet no apprehension of a true method of reasoning on the subject.

In his presidential address, *TASA* XXI, Mr. W. A. Hutcheson on page 345 refers to the founding in London of the Amicable in 1706 and of two stock companies, the London Assurance and the Royal Exchange, both in 1720 and both in active business to this day. He states that the latter two companies guaranteed a definite sum at death but that:

> These two corporations, however, did not, in their early days, charge premiums dependent upon the age of the insured, so that they were in this respect in the same category as the Amicable.

and that their life insurance was written "at the same premium rates irrespective of the age of the insured."

TWO DIMENSIONS: PLAN, AGE

It was just 200 years ago that life insurance in its evolution came to recognize a second independent variable, the age of the insured, and life insurance passed into a realm of two dimensions. The premium rate no longer varied only according to the plan of insurance, but also varied for each plan according to the age of the insured.

This brilliant advance ushered in the scientific era of life insurance. The *Institute of Actuaries Year Book*, 1955–1956, page 2, states:

> Not until 1756 was scientific life assurance shown to be practicable, with premiums properly graduated according to age, for to that year can be dated with tolerable accuracy the unpublished "First Lecture on Insurances" by James Dodson, F.R.S. His vision of mutual life assurance has had a profound influence on British life assurance to this day. Though he did not live to see it, the foundation stone of modern life assurance was laid when the Equitable Society was established in 1762 on his plan.

Life insurance now was on a sound basis, and in the succeeding two centuries it has developed into the important place that it occupies in the lives of individuals and in the national economy with which we are all familiar.

It appears that for 150 years the two-variable concept was regarded as the ultimate and final. The rate for lives of the same class depended only on plan and age and the premium for a policy was always that rate multiplied by the amount of the policy.

THREE DIMENSIONS: PLAN, AGE, SIZE

About fifty years ago it began to be realized that there is a third independent variable that demands recognition, *viz.*, the relative amount insured, referred to as policy size.

Size has been a consideration from time immemorial in the commercial world. I have no doubt that the hieroglyphic records show merchants allowing quantity discounts. These merely are realistic pricing: there are economies resulting from quantity.

As is well known, the time-honored philosophy of government in Great Britain in relation to business has been minimum regulation with maximum publicity. Insurance rates are not restricted by law but are left to be determined by competition.

With such free interplay of economic forces it seems surprising that scientific life insurance operated 150 years with rates untouched by the recognition of size so common in commercial pricing. Perhaps in those days both the range of policy size was more limited, and the clerical and other per policy expense more modest, than what we now deal with. On the other hand, mortality was then high and is now low. Hence the effect of size may at that time have been trivial in proportion to the premium, whereas now it has become important.

Dr. Arthur Hunter tells me that in his early days in the British companies around 1895 there was no practice of varying the premium rate by policy size. Mr. Arthur Coburn informs me that in connection with the reinsurance accepted by his company in Great Britain fifteen years later, around 1910, he saw the practice of recognizing policy size in the rates being quoted by a number of companies.

The earliest reference I have been able to find to the subject is the paper "Analysis and Apportionment of the Expenses of Management of a Life Office with a view to ascertaining the Office Premium Loadings," by H. J. Rietschel, *JIA* XLIV, 415, read and discussed at a meeting of the Institute in April, 1910. The author includes "Method D, which allocates the same proportion of the expense to each policy irrespective of the sum assured." After some discussion he says: "Under all the circumstances Method D must I think be dismissed as impracticable."

On page 421, Mr. Rietschel further makes the statement:

It is at present the universal practice to charge definite rates of premium per-cent of the sum assured regardless of the amount of the policy.

In the discussion, Mr. J. F. Little took exception to this statement because "he knew of at least one office that charged a lower premium per-cent for policies of £500 and upwards than for those below that amount."

Thus it is seen that policy size was on the threshold of recognition, as a third factor to take its place along with plan and age, about fifty years ago.

Bourne's *Insurance Year Book*, 1926–27, 30 years ago shows 10 British companies varying the premium rate by policy size and the subsequent editions show how the practice has spread. Its acceptance has been so general that now for many years in Great Britain it has been regarded as the normal practice.

Information from Dr. Hans Zwingli of the Swiss Reinsurance Company states that the practice is also followed by most Swiss life insurance companies and is common in certain other European countries such as Austria, France, Germany and Holland.

This recognition of policy size has obviously come in obedience to economic law.

In America, with elaborate governmental regulation of life insurance, the business has been less responsive to natural forces, and recognition of policy size as an independent variable in premium rates has been retarded.

In Canada, the Dominion Insurance Act expressly provided that premium rates per $1,000 were applicable pro rata, until this limitation was omitted from the law in 1932. Upon such omission, however, there remained doubt on the point because of vagueness in the discrimination provision of the Criminal Code. However, this latter hindrance was removed a year or two ago, and at the beginning of 1955 the London Life Insurance Company became the first to give full recognition on this continent to policy size as a factor in its rates. Several other Canadian companies are now also operating on that basis.

In the United States there has not been a specific legal prohibition as there was in Canada, but doubts have been raised under the provisions against discrimination. In most states where such doubt existed, these have now been resolved by insurance department rulings.

Furthermore, at its meeting in St. Louis, Missouri, May 1956, the National Association of Insurance Commissioners approved the following conclusion:

It is in the best interest of the insuring public to recognize that life insurance companies, which desire to do so, may graduate their premiums or dividends by policy size for the principal plans of insurance, subject to the responsi-

bility on their part to show that any system of groupings of premium rates or dividend classifications is reasonable, equitable and non-discriminatory.

THEORY

Let P be the gross annual premium for a policy such that

$$P = Sa + b \,, \tag{1}$$

where S is the amount of the policy and a and b are constant with respect to S.

Thus part of the premium is proportionate to policy size and part is independent of size. This provides for recognition of the factor of policy size through the final addition of an amount which may be thought of as a fee. Thus formula (1) expresses a system of policy fee.

Formula (1) is readily transformed to produce

$$P = S\,(a + b/S) \,, \tag{2}$$

where a and b have the same values as before. Here the premium is found by applying the amount insured to a rate of which a portion varies inversely with policy size. Hence, if R_s is the rate for policy size S, we have

$$R_s = a + b/S \,. \tag{3}$$

Recognition of the size factor is in fixing a pattern of rates, starting with a rate for the initial size and then successively reducing the rate by amounts of discount according to successive increasing policy sizes. Thus, formulas (2) and (3) express a system of quantity discount.

For example, let

a = \$25.00
b = 7.50 (= Policy Fee)

S	Policy Fee System	Quantity Discount System	
	Premium $Sa+b$	Rate $a+b/S$	Premium (1)×(3)
(1)	(2)	(3)	(4)
\$ 1,000	\$ 32.50	\$32.50	\$ 32.50
2,000	57.50	28.75	57.50
3,000	82.50	27.50	82.50
4,000	107.50	26.88	107.52
5,000	132.50	26.50	132.50
10,000	257.50	25.75	257.50
15,000	382.50	25.50	382.50
20,000	507.50	25.38	507.60
50,000	1,257.50	25.15	1,257.50
100,000	2,507.50	25.08	2,508.00
200,000	5,007.50	25.04	5,008.00

Note.—(2) and (4) are equivalent, discrepancies being due to rounding.

Pivot

In the quantity discount system, the rate may be stated for a pivotal amount, T, of the amount insured, with a difference, ${}_sU_t$, to be added or subtracted to obtain the rate for S, the amount of insurance in question. To show this, we deduce from formula (3) that

$$R_s = R_t + b/S - b/T$$

or

$$R_s = R_t + {}_sU_t\,, \tag{4}$$

where

$${}_sU_t = b/S - b/T\,. \tag{5}$$

Thus, in accordance with the foregoing example:

Pivot = \$5,000 = T; rate for pivot = \$26.50 = R_t

S	${}_sU_t$	Rate
\$ 1,000	+\$6.00	\$32.50
2,000	+ 2.25	28.75
3,000	+ 1.00	27.50
4,000	+ .38	26.88
5,000	.00	26.50
10,000	− .75	25.75
15,000	− 1.00	25.50
20,000	− 1.12	25.38
50,000	− 1.35	25.15
100,000	− 1.42	25.08
200,000	− 1.46	25.04

Policy Fee System

This has the advantage of simplicity. Also it has the mathematical appeal of continuity in the premium rate: whatever increase is made in S, the effective rate as exemplified in the third column of the above tables continues to decrease, if ever so little.

However, this unceasing movement of the rate may be a disadvantage in practice. Whatever the amounts may be of the insured's existing policy and of his application for a new policy, if the policies were combined the charge for size in his premium would be reduced. Thus the company is exposed over the whole amount range to possible pressure to consolidate insurance on a life to get a lower rate. This fact might be used as a selling point and so the question might be continually agitated.

Furthermore, the policy fee system tends to produce prohibitive premiums for small amounts, so that its simplicity and consistency may be marred by the necessity of modification.

The opinion has been expressed that it is objectionable to show an explicit policy fee. Objection has also been made to the fact that the fee would be in the form of an additional charge; there seems to be no satisfactory method of turning the fee around into a deduction.

Quantity Discount System

Formula (3) is of the form

$$y = a + b/x$$

$$x(y - a) = b .$$

Hence, from analytical geometry it is seen that R_s is represented by an equilateral hyperbola with asymptote $R_s = a$, the value approached as S is increased indefinitely.

The variation of R_s diminishes rapidly with increase in S, as will be clear from the following table extracted from the foregoing example:

S	Difference in S	Rate	Difference in Rate
\$ 3,000	\$ 2,000	\$27.50	\$1.00
5,000	10,000	26.50	1.00
15,000	∞	25.50	.50
∞		25.00	

Thus, for the \$2,000 rise in amount from \$3,000 to \$5,000, the drop in rate is \$1.00, the same as the drop for the \$10,000 rise in amount from \$5,000 to \$15,000, which in turn is *double* the maximum possible further drop of 50¢, in this example, for an infinite rise in amount. This rapid tapering off of the rate with increase in policy size suggests division of the amount range into a small number of bands with stepped rates each constant for its band. This is the form in which the quantity discount method is used in practice.

In practice it becomes a simple matter to provide a suitable modification as needed for small amounts. A second advantage is that pressure to consolidate insurance to get a lower rate arises only where the amount crosses a step point. Also, the presence of steps is an incentive to a reasonable upgrading of new insurance. On the other hand, quantity discount tends to become complicated if many bands are used.

Because of the step in rate, the premium for the amount of insurance at that point will be less than for slightly smaller amounts of insurance just below the step point. Reference to Appendix 1, B, reveals that this anomaly is also present in the method used for small policies by some of the companies employing the fee system for their larger policies.

Except that special rules are called for in correcting error in age, this anomaly apparently presents no difficulty in practice. Applications for the slightly smaller amounts of insurance referred to would automatically be issued for the amount at the step point.

As regards error in age, and assuming that the policy has the common provision:

> If the age of the insured has been misstated, the amount payable shall be such as the premium paid would have purchased at the correct age.

the anomaly requires special consideration in cases where the age was understated and the premium paid is less than the required premium for the minimum amount of insurance in the size band in which the policy was issued. Without such special consideration, the discontinuity in premium rate at the step point would result in penalizing such cases out of proportion with all other error in age cases. It is accordingly felt that where through error in age the premium paid is found to be less than the required premium for the minimum amount of insurance in the size band in which the policy was issued, then (*a*) if the correction is made after the policy becomes a death claim, the correction should be on the basis of the correct premium rate in the size band in which the policy was issued; and (*b*) if the correction is made before death of the insured, suitable adjustment for underpayment of premium should be permitted with no change in the amount of the policy.

Nondiscriminatory

It has been pointed out that a rate of, say, $25.00 per $1,000 plus, say, $7.50 per policy is the same rate for everyone. Thus, the policy fee system is clearly nondiscriminatory.

It follows that the quantity discount system is equally nondiscriminatory, because the systems are essentially equivalent as shown by the equivalence of formulas (1) and (2). In either case, the insured is being charged a rate per $1,000 plus a rate per policy, which is essentially the same rate for everyone.

Costs Inversely Proportional to Policy Size

The analysis has been in terms of premiums which are in part directly proportional to policy size, and in part constant, viz., $Sa + b$. Theoretically the premiums should also recognize costs which are inversely proportional to policy size, adding a third term, c/S. Thus the rate for policy size S would be $a + b/S + c/S^2$.

The third term rapidly becomes insignificant with increase of S, because the second power of S is involved, and a, b and c are ordinarily in decreasing order of amount. The effect is therefore to increase the varia-

tion of the rate in the small policy range with no significant change thereafter.

By the policy fee method this would mean a varying fee which rapidly decreases to an ultimate constant. By the quantity discount method, the desired recognition would readily be taken into account in fixing the rate pattern, increasing the rate differential in the small policy range only.

Thus, costs that vary inversely with policy size become part of the general considerations taken into account in fixing the rates in the small policy range, and in practice do not otherwise affect the variation in the rate with policy size.

PRACTICE IN GREAT BRITAIN

Examination of "Stone and Cox Ordinary Branch Life Assurance Tables" shows 97 life offices operating in Great Britain.

Seventeen of these do not vary the premium rate for policy size, and examination shows that in these offices there are special circumstances as follows:

Offices	
10	Friendly and fraternal societies
3	Foreign companies
1	Largely fire insurance
1	Largely sickness insurance
1	New company with low limit
1	Specialized business associated with another office
17	

The remaining 80 offices, comprising all the usual companies, vary the premium rate for policy size, as follows:

General Method Used	Number of Offices
Quantity Discount	71
Policy Fee	7
Not stated, or complicated	2
	80

From the foregoing facts, it is a fair statement to say that variation of premium rates by policy size has become the usual practice in Great Britain.

In comparison with previous studies it is also found that there has been some movement recently toward the fee method, whereas in the past this method appears to have been little used. One company, which years

ago moved away from the fee method, has recently returned to it. However, it is apparent from the table that quantity discount is by far the most popular method.

The same is true of other European companies, according to my information. I am informed that numerous companies there use quantity discount, the only use of the fee method reported on the Continent being by two Swiss companies.

In Appendix 1 is given a tabulation of the patterns for quantity discount and of the premium formulas for policy fee used by 79 of the 81 British companies referred to.

DETERMINATION OF b: EXPENSE CONSTANT PER POLICY

The problem of allocation of expense is one of especial difficulty and there can be a great many different thoughts on this subject. At the same time the introduction of premium rates varying by policy size calls for a more fundamental approach to the determination of the expense which is constant per policy than has usually been made heretofore. The following is offered as a method which is felt to be satisfactory.

The quantity nb is to be understood as the portion of the total company expense which is proportionate to the number (n) of policies regardless of their size and which should therefore be charged at the rate of b to each policy.

In order to investigate the amount of nb, let us regard n as an independent variable according to the principles of the differential calculus, and let us endeavor to determine what portion of the company expense is a function of that variable. To do so, we must assume that in all other respects the company remains constant while only n varies.

It should be emphasized that we are not dealing here with merely the breakdown of expenses as commonly made per policy, per $1,000 insured, per $100 of premium, etc. Nor are we concerned with the formula by which expenses are customarily paid or assessed. Our problem is the broader one of recognizing the expense that is in reality a function of the number of policies, even though the relation may be obscured. An example is field expense such as salaries of office managers and clerks, rents, office equipment, etc. In branch office companies these are explicit. In general agency companies these are partly explicit in vouchered general agency allowances and partly implicit as included along with compensation of general agents contained in commissions. A further example is compensation for field supervision, inasmuch as general agents and field supervisors exist solely because of the fragmentation of the insurance in force into so many policies. If the same amount of insurance and premium revenue were

achieved with but a comparatively few mammoth policies, the same solicitors' commissions would presumably be paid but the field organization would become negligible.

The circumstance that general agents' compensation and expense provision are contained largely, if not exclusively, in commission margins based on premium revenue should not obscure the fact that they nevertheless stem from the number of policies. The method of payment is only a superficial formula, and admittedly a realistic one; although subject to lag, it makes a real response to any significant change in its fundamental relation to number of policies. For example, if for some reason all policies were divided into halves with twice as many to handle, with the same amount insured and premium revenue as now, does anyone doubt that general agents' margins for expense provision would soon be expanded to reflect that fact, no doubt on their prompt initiative? Likewise, in case of important change in the contrary direction, company management would necessarily bring about a like recognition in due course, mindful of the pressure of competition to reduce costs to policyholders.

The Question

Accordingly we have the question:

If all other factors of a company remain unchanged, but the insurance is in a smaller number of policies of correspondingly increased average amount, how would the expense differ from now?

In answer to this question, it will be observed that the investment expense would be affected only within the area of policy loans. The policy loan expense is of concern here if the total loan interest is a fixed amount for a given total of principal regardless of number of policies, which is the usual situation. However, if variation in size of loan is recognized by variation in interest rate, policy loan interest is eliminated as a factor here. Also much of the insurance expense would remain the same, *e.g.*, premium taxes, and soliciting agents' compensation to the extent that the terms of agents' contracts are independent of policy size. Settlement option expense tends to increase with policy size and can therefore be disregarded because this is a study of expense which is constant per policy. There would be a significant change in such items as rent, salaries, medical and inspection fees, postage, printing, furniture, etc.

Thus if, with exactly the same amount and distribution of insurance, the number of policies were reduced to 1/2, 1/10, or 1/100, a portion of the total expense would continue the same and the residue would vary in some relation to the 1/2, 1/10, or 1/100 level. Our interest is limited to

this residue which is a function of the number of policies, and it is the amount of this residue that we desire to determine.

The residue with which we are concerned is accordingly contained or taken into account in Column 3 of Page 5 of the Annual Statement as portions of:

Item 21. Commissions on premiums
Item 23. General insurance expenses
Item 24. Taxes, licenses and fees
Item 4. Policy loan expenses excluded from net investment income

An example of the determination of the residue is given in Appendix 2 for a company writing ordinary business, but not industrial or group.

Invariant

The foregoing residue is the portion of company expenses which is some function of the number of policies. It is fairly obvious that in its variation with the number of policies, the amount of insurance being constant, this residue would not be expected to approach zero as n became very small. However, it is reasonable to establish an invariant amount, such that the remaining portion of the foregoing residue, referred to as the variant, may be regarded as varying in direct proportion to the number of policies, thus:

$$\text{Residue} - \text{Invariant} = \text{Variant}.$$

The deduction for the invariant is from consideration of the following hypothetical example:

	Company A	Company B	Company C
Insurance in force	$5,000,000,000	$5,000,000,000	$50,000,000
Policy liability	2,000,000,000	2,000,000,000	20,000,000
Premium revenue	150,000,000	150,000,000	1,500,000
Number of policies	1,000,000	10,000	10,000
	Staff for insurance and policy loan operations, excluding other investments		
Officers	50	?	10
Clerks	2,000	?	40

Companies A, B, and C have the same distribution of business with average reserve $400 per $1,000 and average premium $30 per $1,000. A and C have the same average policy, $5,000. B has the same insurance as

A but in only the same number of policies as C, or an average policy of $500,000.

The staffs of A and C may be regarded as typical. C handles 10,000 policies of $5,000 average amount with a staff of 10 officers and 40 clerks. A, with 1,000,000 policies of the same average amount, requires a much larger staff of 2,000 clerks led by 50 officers, of whom some are executives and the remainder are in intimate contact with the work. The average salary of officers will be higher than that of clerks.

The question is: What staff will B require?

A fair answer is that C is a measure of the number of work units experienced by B in the operations referred to and that B's staff above would therefore be expected to be a total of 50, the same as for C.

There is, however, the important difference that B's transactions involve average policies of $500,000 calling for a higher calibre of work than in C with $5,000 average policy. Therefore B's staff of 50 would be at a higher average salary than C's.

It would be a reasonable view to consider that the 50 persons required by B are the 50 officers seen in A. In other words, B is the same as A except that the 1,000,000 policies requiring 2,000 clerks to assist the officers in A have shrunk to 10,000 policies such that the 50 officers can do the work themselves with only a nominal clerical staff. Because of the $500,000 average policy, the demands of the work would accord with salaries at the officer level. In any organization the work is primarily performed by the officers, overflowing to clerical help as volume dictates.

The foregoing suggests the reasonable view that B's total salaries may be taken as an amount equal to the salaries of A's officers. Thus this salary amount remains invariant as the number of policies is reduced from A to B. This invariant is merely the recognition that with $5,000,000,000 insurance in force with $150,000,000 premium revenue, there will obviously be a "hard core" of salary expense due to that fact, regardless of whether the business is in 1,000,000 or in 10,000 or any other number of contracts.

The study in Appendix 2 is with actual figures for 1955 for a certain company. Information from that company shows that the ratio of officers' salaries to total salaries, taking account only of insurance operations, and excluding investments, is about 20%, and this is taken as the invariant ratio for that company. For other companies the ratio may differ somewhat from 20%. The invariant ratio is applicable to salaries and employees' benefits. It is regarded as applicable also to home office rent, on the ground that relative importance of individuals in the organization is ordinarily reflected in like degree by salary and value of space assigned to them.

Variant

The variant, as determined in Appendix 2, is to be regarded as being in direct proportion to the number of policies. Thus if, instead of the actual number of policies, the company had the same amount and distribution of insurance but in only 1/2, 1/10, or 1/100 as many policies, the variant would correspondingly be only 1/2, 1/10, or 1/100 the foregoing amount. Hence the ratio, variant/number of policies, is a constant. This constant is the portion of the company's actual expense attributable to each policy because of its entity as a policy. That is to say, it is the expense per policy that is the same regardless of the number of policies.

Value of b

It is well known that first year expense is higher than that of subsequent years. Investigation shows that, for the company to which the figures in this study relate, medical and inspection fees, together with salaries and rent attributable to selection of risks, represent 13% of the amount of the above variant. For other companies the ratio may differ somewhat from 13%.

Let: T = Mean total number of premium paying policies in force for the year.

N = Number of new policies paid for in the year.

V = The variant as determined above, of which 13% applies to first year only and 87% applies to both first year and renewal years.

b = The first year and renewal expense constant per premium paying policy.

b' = The first year additional expense constant per premium paying policy.

Then:

$$b = \frac{.87V}{T}$$

$$b' = \frac{.13V}{N}.$$

For the company to which the figures in this study relate, we have for 1955:

$$T = 1{,}375{,}576$$

$$N = 69{,}686$$

$$V = \$12{,}357{,}468$$

Hence:

$$b = \frac{\$12{,}357{,}468 \times .87}{1{,}375{,}576} = \$\ 7.82$$

$$b' = \frac{\$12{,}357{,}468 \times .13}{69{,}686} = \$23.05.$$

In considering what variation is to be made in premium or dividend rates in recognition of variation in the expense rate according to policy size, it is suggested that the first year and renewal value b, as above, should be used in all premium paying years, including the first, and that the first year excess b', as above, be disregarded for this purpose. This is for the reason that annual flat variation in premium or dividend rates by policy size should preferably be justified currently by recurring margins.

It has been suggested that instead of the first year excess b' being disregarded, it might possibly be allowed for through an additional quantity discount over a limited number of years, which might or might not include the first year. This would have obvious practical objections.

Hence, the expense which is constant per policy, *i.e.*, the constant b as found for the company in question, is $7.82 on the basis of 1955 operations.

Following are the corresponding values found for each year from 1940:

1940	$4.59	1948	$5.64
1941	4.72	1949	5.48
1942	4.68	1950	5.63
1943	4.58	1951	6.16
1944	4.61	1952	6.81
1945	4.88	1953	7.12
1946	5.33	1954	7.55
1947	5.84	1955	7.82

This tabulation reflects the increases in expense rates which have occurred since the close of World War II, followed by the Korean conflict.

The value of b has been determined herein with an effort to base it upon a sound theory. In so doing, the result has been held down by about 8% in eliminating the invariant and by about 15% in omitting extra first year expense through discarding of b'.

As explained above, the determination of b requires more detailed information than appears in the Annual Statement. An attempt was made

at a rough approximation for several large companies which was found to support the above 1955 result of about $7.50.

It should be made clear that there are doubtless various methods that can be followed in determining b. The author offers the foregoing simply as one method that appears to be logical.

Additional Benefits

Some expenses are related, not to the number of all policies, but to the number of policies containing additional benefits, such as waiver of premium benefit. These have been excluded in the foregoing development and in Appendix 2. Theoretically the extra premiums for such benefits should vary with policy size as well as the basic premiums, although probably the variation would be too small to be worth while.

DETERMINATION OF PREMIUM RATES

The foregoing determination of b is directed to establishing a value that can be justified, as a guide to management in fixing the actual value to be used. The latter is a matter of judgment depending on various considerations.

It will be sufficient here to consider only formula (2), quantity discount. As previously stated, this will in practice result in stepped premiums for a small number of amount bands, the rate for a given band being constant for all amounts in that band.

Having settled upon a value of b to be used, the ranges of the bands are likewise a matter of judgment, depending to some extent upon the value of b adopted. The steps in rate are found from the value of b by using the average sizes of policy in the respective bands. Inasmuch as the average sizes referred to must be found prospectively, they also call for judgment in drawing conclusions from past data.

Thus the exercise of judgment arises on three scores: fixing the value of b to be used, fixing of number and limits of bands, and estimate of average policy sizes expected in the respective bands for use in determining rate steps. Therefore it is not surprising that a considerable variety of patterns are in use, as will be observed from Appendix 1.

Illustrative Pattern 1

One conclusion that might be reached from the foregoing "Determination of b" for one company, including the yearly values shown for 1940 to 1955, might be to fix the value of b as $7.50.

Adopting $7.50 as the value of b, examination of the example given following formula (2) suggests bands and steps as follows, resulting in premi-

um rates as indicated, where R is the rate per \$1,000 for a policy of \$5,000, taken here as the pivot:

Band of S	Step	Premium Rate	Departure (U)
\$ 1,000–\$2,999.....	\$1.00	R+\$2.00	+\$2.00
3,000– 4,999.....	1.00	R+ 1.00	+ 1.00
5,000– 9,999.....	1.00	R	0
10,000 and over.....		R– 1.00	– 1.00

The step between the first and second bands departs from the example referred to; it is suggested from practical considerations. For the second and third bands, it is assumed in this particular illustration that the effect of upgrading and the known tendency toward heaping may be expected to result in average policy amounts not differing materially from the initial amounts of \$3,000 and \$5,000 respectively, and for the fourth band in an average policy amount of about \$15,000. Round values for steps between the second, third and fourth bands are accordingly taken as suggested by these average amounts as they appear in the example referred to. This may be compared with Pattern 7 in Appendix 1, A.

Illustrative Pattern 2

Instead of \$7.50 as in Illustrative Pattern 1, quite a different decision might be reached from the same data given in the "Determination of b" with yearly values for 1940 to 1955.

It might be argued that the upward trend of the yearly values may not be the indication of a further rise, but that on the contrary they are about to reach a crest in the curve, to be followed by a decline. Perhaps electronic equipment will soon operate significantly to reduce the costs. Furthermore, conservatism may suggest a cautious move in introducing the innovation of recognition of size for various practical considerations, such as its effect on the market as between small and large policies. Thus it might be concluded, from the same data as before, to fix the value of b at only \$2.50 and to use but three bands, as follows, taking \$5,000 as the pivot:

Band of S	Step	Premium Rate	Departure (U)
\$ 1,000–\$4,999.....	\$.85	R+\$.85	+\$.85
5,000– 9,999.....	.20	R	0
10,000 and over....		R– .20	– .20

In this illustration, the smallness of the $2.50 value is judged to warrant but three bands. Also, the sizes of the steps result from the judgment, in the case of the particular company assumed to be illustrated, that the average size policies in the respective bands would be about $2,000, $6,250 and $12,500. (It should be noted that the average policies for respective bands would undoubtedly vary a great deal between companies, even though the ranges of the bands are alike. This is to be expected because of company differences with respect to such factors as agency representation and type of business solicited.)

This may be compared with Patterns 21 and 25 (second variation) in Appendix 1, A.

For a given division in bands of a particular distribution of business, the sizes of the rate steps theoretically would vary in direct proportion to the value used for b. This follows from formula (3), in which R_s is seen to vary directly with b, if a is constant. In practice, however, the stipulation "a particular distribution of business" could probably never be met when comparing rate patterns in actual use by various companies. Even though the ranges of the bands are alike, there would no doubt always be differences in amount distributions within the respective bands and therefore differences in policy weights. Hence, even for identical bands in actual rate patterns for different companies, the comparative sizes of the steps can be expected to be only approximately in proportion to the respective values of b used.

Single Premium and Paid-up

The foregoing development contemplates premium paying insurance, including first year of single premium insurance, but not paid-up. A reasonable practice for single premium insurance would be to recognize size in the same way as for annual premium but at steps four times as large.

As shown by the values of b and b' found in the foregoing study for a particular company, $b + b'$ roughly equals $4b$; *i.e.*, $\$7.82 + \$23.05 = \$30.87$, approximately equal to $\$7.82 \times 4$. Since in the case of single premium there is but the one premium paying policy year, the reason previously stated for discarding b' does not apply and $b + b'$ should be used.

Also, steps of four times the amount applicable to annual premiums give a good relation between single premium and limited payment plans, of which the shortest may probably be five years. Thus in the above Illustrative Pattern 1 the $1.00 steps would at the end of the premium period in the case of 5 payment life amount to $5, which is comparable to the $4 suggested for single premium plans.

Paid-up insurance is considered not eligible for policy size recognition; there are no premiums to be varied. There are practical objections in attempting such recognition through the dividends. Expenses on paid-up are low because (*a*) much of the insurance accounting and valuation is by attained age grouping, (*b*) there is no premium collection or accounting expense, and (*c*) there is usually no dividend accounting expense under extended insurance.

Retirement Income

For retirement income insurance plans, size recognition would naturally be according to the policy face amount of insurance, disregarding the increase in amount of insurance in later policy years which culminates in the maturity value.

Supplemental Term Rider

For policies with supplemental term insurance, size recognition might be by finding the premium as $A + B$, where A is the premium for the basic policy, with the same size recognition as for a like policy without rider, and B is the premium for the rider, allowing 50% of the size recognition that would apply to like term insurance if in a separate policy. This 50% factor reduces the rate steps for riders to one-half of those for policies, on the assumption that expense which is constant per rider is one-half of that per policy; expense charges for supplemental insurance are normally lower than for regular insurance. It should be noted that in the case of decreasing term insurance, the amount of insurance for purposes of size recognition will ordinarily be taken as one-half the initial amount.

The Standard Nonforfeiture Law as commonly interpreted treats the total coverage under a policy with level supplemental term rider as a single policy. This has the indirect effect of putting a maximum limit on the supplemental term premium, if minimum values on the basic policy after the term period are not to exceed the values which would be contained in a similar policy without supplemental term (see paper by J. E. Hoskins, *RAIA* XXXV, 235). This arises from the fact that the adjusted premiums during and after the term period, which define the minimum nonforfeiture values, are proportionate to the gross premiums with and without supplemental term. If premiums are varied by policy size, then the relationship of the premiums with or without supplemental term varies with each size group. This would greatly increase the difficulty of determining supplemental term premiums which comply with the Standard Nonforfeiture Law, unless the latter is suitably amended.

Pivotal Rate

Having decided upon a rate pattern, it remains to determine the pivotal rates to be used.

It is understood that the basis of the customary premium rates that do not vary by policy size has been determined, and that we are here concerned only with introducing the size variation.

Let C be the customary premium rate not varying by policy size, but of course varying by plan and age. Then the total premium revenue by customary rates is

$$\Sigma SC .$$

Let s' designate the size band in which S falls and t' designate the size band adopted as the pivot, such that $R_{s't'}$ and ${}_{s'}U_{t'}$ are flat amounts of R_{st} and ${}_{s}U_{t}$ for the respective bands.

Then, to introduce the size variation without alteration of the total premium revenue we must have

$$\Sigma SR_{s't'} = \Sigma SC .$$

Also by application of formula (4), we have

$$\Sigma SR_{s't'} = \Sigma SR_{t't'} + \Sigma S\,{}_{s'}U_{t'} .$$

Hence

$$\Sigma SC = \Sigma SR_{t't'} + \Sigma S\,{}_{s'}U_{t'} .$$

Determine D with reference to the pivotal size t' adopted, such that

$$\Sigma S\,{}_{s'}U_{t'} = D\Sigma S .$$

Then

$$\Sigma SC = \Sigma SR_{t't'} + D\Sigma S$$

$$= \Sigma S\,(R_{t't'} + D) .$$

$$\Sigma S\,(C - D) = \Sigma SR_{t't'} .$$

This condition is satisfied if

$$R_{t't'} = C - D$$

for all plans and ages. Accordingly the pivotal premium rates for all plans and ages are found by simply subtracting from the customary premium rates the constant value D found as above with reference to the pivotal size adopted. Of course, D may be positive or negative.

D may be large or small according to the selection made of t'. However D cannot necessarily be reduced to zero, or even to an insignificant amount. This is for the reason that $R_{st'}$, being limited to discrete values, is not a continuous function; and furthermore the designations of s' and t' will usually be in terms of round amounts of insurance.

DIVIDENDS

In participating insurance, recognition of the size factor may be through the premium or through the dividend.

If the recognition is through the premium, we have as follows: The total loadings less dividend expense charges per $1,000 in the case of customary premiums is

$$\Sigma SP' - \Sigma SP - \Sigma Se$$

(where P′ and P are gross and net premium and e is expense charge, respectively)

which now becomes

$$\Sigma SR + \Sigma SU - \Sigma SP - \Sigma Se$$

(where R is the pivotal premium rate and U is the premium rate departure from R for policy size)

and, rearranged in the previous order, is

$$(\Sigma SR - \Sigma SP - \Sigma Se) + \Sigma SU .$$

This shows that the dividends are to be calculated by the customary method, but based on the pivotal premium rate instead of the customary premium rate, if different. However, in determining the total loading surplus available for distribution, ΣSU is to be added, being the additional loading collected in the actual premiums over the premiums at the pivotal rate. This additional ΣSU may be a positive or negative substantial amount or it may be negligible, depending on the location of the pivot.

Thus, if the size recognition is through the premium, the dividend rates will be constant without regard to size. This, among other advantages, has the important advantage of simplifying illustrative dividends for purposes of solicitation of new business.

If no rate adjustment for size is made in premiums, and such adjustment is left to be made through the dividend, the rate bands will still be the same as they otherwise would be for the premium, but the rate steps will, of course, be of opposite sign, being increased dividend instead of decreased premium and vice versa. However, the rate steps will tend to be smaller for dividends than for premiums because deprived of certain savings of expense that is a percentage of premium.

TRANSITION

In participating insurance, when starting to recognize size through the premium for new insurance, a transition problem arises with respect to the existing business.

Inasmuch as the existing business was written on the prior theory of not

varying premium rates by policy size, it might be argued that such business should continue to be administered according to the prior theory.

This argument is plain as regards the premiums for the smaller policies, because it is not possible to increase such premiums. Also, from this fact, it is a reasonable position to take that therefore neither shall the premiums for the larger policies be reduced, because the company cannot be expected to alter some of the premiums if it cannot alter the others.

Plainly also the view can be maintained, if desired, that dividends on the existing business should continue to be determined according to the prior theory of not varying rates by policy size. Existing policyholders are entitled to some consideration of the fact that their expectations were based on the prior method. Clearly the adoption of a new theory for new business does not invalidate continuance of the prior theory for existing business. Both are equitable.

On the other hand, because of its own characteristics a company may feel that the new method should apply to the dividends on a recent portion, or possibly all, of its existing business. An object would be to discourage substitution of new insurance for size advantage; age and settlement differences are factors minimizing advantages in such substitution.

POLICY CHANGE FOR SIZE ADVANTAGE

If the insured has several policies, a size advantage in rate may result from combining policies. This would occur only if the amount crossed into another rate band. The company has an offset through savings due to fewer policy units, but it is confronted with the cost of handling the change.

The most obvious case is consecutively numbered identical policies with different beneficiaries; these can readily be merged in one policy with divided beneficial interest. Policies which are alike, except that they have different dates reasonably close to each other, present a problem of redating which may be feasible. Difference in plan would need to be resolved before merger would be possible.

With the advent of electronic data processing machines, it may become practicable to handle the insured's policies as an account without disturbing their identity, and to recognize size through a fee per account.

SUBSTITUTION

There may be some tendency to substitute new insurance for existing insurance to obtain a size advantage. This could hardly be resisted if, when the practice of size recognition commences, the existing insurance is so recent that its premium is higher than for a new policy. This anomaly is cured with passage of time by the age differential in premiums.

COMPANIES IN UNITED STATES AND CANADA FULLY RECOGNIZING POLICY SIZE

In so far as I have been able to ascertain, the companies in the United States and Canada (including fraternal societies and assessment associations) which have commenced the practice of giving full recognition to policy size in their rates, at least for permanent plans, are as listed below. For ease of comparison, I have shown the premium rate patterns in terms of a uniform $5,000 pivot. Thus R is the premium rate per $1,000 for a $5,000 policy where size is recognized through the premium. However, where the size recognition is through the dividend, I have used a $1,000 pivot, D being the dividend for a $1,000 policy.

Where S, the amount insured in thousands, appears, the item is the amount of premium according to the policy fee method. The remaining items are rates per $1,000 according to the quantity discount method.

FULL RECOGNITION IN PREMIUMS

Commenced	Company	Territory	Premium Rate Pattern
January 1955...	London Life (Canada)	Canada	$ 2,000–$ 4,999 R+$1.25 5,000– 9,999 R 10,000 and over R– .50
June 1955...	Toronto Mutual	Canada	$ 1,000–$ 4,999 R+$1.92 5,000 and over R
January 1956...	Companion Life	New York	$ 1,000–$ 2,499 R+$1.00 2,500– 4,999 R+ .50 5,000– 9,999 R 10,000 and over R– .25
January 1956...	Monarch Life (Canada)	Canada	$ 1,000–$ 2,999 R+$2.25 3,000– 9,999 R 10,000 and over R– .75
January 1956...	Mutual Life (Canada)	Canada	$ 1,000–$ 2,999 R+$2.00 3,000– 4,999 R+ .80 5,000– 9,999 R 10,000 and over R– .50
January 1956...	Sovereign Life (Canada)	Canada	$ 1,000–$ 4,999 R+$2.60 5,000– 9,999 R 10,000 and over R– .75
February 1956...	Prudential (England)	Canada	$ 1,000–$ 1,999 R+$3.00 2,000– 4,999 R+ 1.50 5,000– 9,999 R 10,000– 24,999 R– .50 25,000 and over R– 1.00
July 1956...	Standard (Oregon)*	California Idaho Oregon, Utah Washington Hawaii	$ 1,000–$ 2,499 R+$1.50 2,500– 4,999 R+ 1.00 5,000– 9,999 R 10,000– 24,999 R– 1.00 25,000 and over R– 1.75
September 1956...	West Coast Life†	Alaska, Ariz., Cal., Colo., Hawaii, Idaho, Mont., Nev., New Mex., Ore., Utah, Wash., Wyo.	$ 1,000–$ 4,999 SR+$3.00 5,000– 9,999 R 10,000– 19,999 R– .50 20,000 and over R– 1.00

FULL RECOGNITION IN DIVIDENDS

Commenced	Company	Territory	Dividend Rate Pattern
January 1956....	Teachers Ins. & Ann.	New York (U.S. and Canada by correspondence)	$ 1,000–$ 4,999 D 5,000– 9,999 D+$1.00 10,000– 14,999 D+ 1.50 15,000 and over D+ 1.75

* For life and endowment plans only. † Pattern shown for adult policies only.

APPENDIX 1

RECOGNITION OF POLICY SIZE IN GREAT BRITAIN

The stated practice applies to life and endowment plans; in many cases it also applies to term plans and in other cases the practice for term plans is different.

The British practice has been translated into dollars, using the traditional round amount £1 = $5. This is higher than the current exchange rate of about $2.80, which expresses trade relations, because for our purpose we are also interested in an adjustment for relative standards of living and wage levels.* For ease of comparison a uniform $5,000 pivot has been used, *i.e.*, *R*, the pivotal rate, is the premium rate per $1,000 for a policy of $5,000. The number of companies is given in italics over the adjustment.

* If a different conversion rate is used for translating the British practice into dollars, only the limits of the size bands and the policy fees, if any, are affected. There is no change in the rates of premium, because these, of course, are per unit of the sum insured, no matter what the currency may be. For comparison, the pattern listed as No. 10 is shown as follows on two bases:

(*a*) At Current Exchange Rate £1=$2.80			(*b*) At Traditional Round Equivalent £1=$5		
Under	$ 1,400	*R*+$1.50	Under	$ 2,500	*R*+$1.50
$ 1,400–	2,799	*R*+ .50	$ 2,500–	4,999	*R*+ .50
2,800–	5,599	*R*	5,000–	9,999	*R*
5,600–	11,199	*R*– .50	10,000–	19,999	*R*– .50
11,200 and over		*R*– 1.00	20,000 and over		*R*– 1.00

Comment: (i) The bands in (*a*) are narrower than in (*b*), but with the same steps in rate; consequently, the rate begins to decrease at a lower amount in (*a*) than in (*b*); likewise, the minimum rate begins to apply at a lower amount in (*a*) than in (*b*).

(ii) The inference is that with the pound standing at a low dollar exchange value, insurance purchases would continue normal in pounds, shrinking in dollar equivalent. However, in such unsettled economic conditions it appears probable that insurance purchases may correspondingly increase in pounds, maintaining the traditional round amount dollar equivalent.

(iii) The main interest in the pattern is the number and size of steps, which are seen to be independent of the conversion basis. Likewise, when viewed internally from the British standpoint the limits of the bands are not affected by the vagaries of foreign exchange rates. It is only when viewed externally through conversion at fluctuating exchange rates that the bands appear to expand and contract as the exchange rate goes up and down. Being thus subject to varying distortion, the effect is somewhat of an illusion, and in the author's opinion the traditional round equivalent is the best basis that can be used for the present purpose of translating the British practice—which is stable in the British economy—into American terms correspondingly stable in the American economy.

A. Premium Rate Patterns of 71 Companies Using Quantity Discount Method

1.

	One Co.
Under $ 1,250	*R*+$1.50
$ 1,250– 2,499	*R*+ 1.00
2,500– 4,999	*R*+ .50
5,000– 12,499	*R*
12,500 and over	*R*– .50

2.

	One Co.
Under $ 1,250	*R*+$1.00
$ 1,250– 2,499	*R*+ .50
2,500– 9,999	*R*
10,000 and over	*R*– .50

3.

	One Co.
Under $ 1,250	*R*+$1.25
$ 1,250– 12,499	*R*
12,500– 24,999	*R*– .50
25,000 and over	*R*– 1.00

4.

	One Co.
Under $ 2,500	*R*+$1.00
$ 2,500 and over	*R*

5.

	One Co.	*One*	*One*
Under $ 2,500	*R*+$ .50	$ .75	$1.00
$ 2,500– 4,999	*R*+ .25	.25	.50
5,000 and over	*R*		

6.

	One Co.
Under $ 2,500	*R*+$1.00
$ 2,500– 4,999	*R*+ .50
5,000* and over	*R*

* For Endt. use *R*–$.50.

7.

	One Co.
Under $ 2,500	*R*+$3.00
$ 2,500– 4,999	*R*+ 1.50
5,000– 7,499	*R*
7,500 and over	*R*– 1.50

8.

	One Co.
Under $ 2,500	*R*+$ .75
$ 2,500– 4,999	*R*+ .25
5,000– 7,499	*R*
7,500– 9,999	*R*– .25
10,000– 12,499	*R*– .50
12,500 and over	*R*– 1.00

9.

	One Co.
Under $ 2,500	*R*+$1.00
$ 2,500– 4,999	*R*+ .50
5,000– 7,499	*R*
7,500– 9,999	*R*– .50
10,000– 24,999	*R*– 1.00
25,000 and over	*R*– 1.50

10.

	One Co.
Under $ 2,500	*R*+$1.50
$ 2,500– 4,999	*R*+ .50
5,000– 9,999	*R*
10,000– 19,999	*R*– .50
20,000 and over	*R*– 1.00

11.

	One Co.
Under $ 2,500	*R*+$1.25
$ 2,500– 4,999	*R*+ .50
5,000– 9,999	*R*
10,000– 24,999	*R*– .75
25,000– 49,999	*R*– 1.25
50,000 and over	*R*– 2.00

12.

	One Co.
Under $ 2,500	*R*+$1.00
$ 2,500– 4,999	*R*+ .50
5,000– 12,499	*R*
12,500 and over	*R*– .50

13.

	One Co.
Under $ 2,500	*R*+$2.00
$ 2,500– 4,999	*R*+ 1.00
5,000– 12,499	*R*
12,500– 24,999	*R*– .50
25,000 and over	*R*– 1.00

14.

	One Co.
Under $ 2,500	*R*+$1.00
$ 2,500– 4,999	*R*+ .50
5,000– 12,499	*R*
12,500– 24,999	*R*– .50
25,000– 49,999	*R*– 1.00
50,000 and over	*R*– 1.50

15.

	One Co.
Under $ 2,500	*R*+$1.00
$ 2,500– 4,999	*R*+ .50
5,000– 12,499	*R*
12,500– 37,499	*R*– .50
37,500 and over	*R*– 1.00

16.

	One Co.
Under $ 2,500	*R*+$ .75
$2,500– 4,999	*R*+ .25
5,000– 12,499	*R*
12,500– 49,999	*R*– .50
50,000 and over	*R*– 1.00

17.

	One Co.
Under $ 2,500	*R*+$2.50
$ 2,500– 4,999	*R*+ 1.00
5,000– 14,999	*R*
15,000– 49,999	*R*– .50
50,000 and over	*R*– 1.00

18.

	One Co.
Under $ 2,500	*R*+$1.00
$ 2,500– 7,499	*R*
7,500– 12,499	*R*– .50
12,500– 24,999	*R*– .75
25,000 and over	*R*– 1.25

19.

	One Co.
Under $ 2,500	*R*+$ .50
$ 2,500– 9,999	*R*
10,000 and over	*R*– 1.00

20.

	One Co.
Under $ 2,500	*R*+$1.00
$ 2,500– 9,999	*R*
10,000– 24,999	*R*– .50
25,000 and over	*R*– .75

21.

	Four Cos.
Under $ 5,000	*R*+$ .50
$ 5,000 and over	*R*

22.

	One Co.
Under $ 5,000	*R*+$ .50
$ 5,000– 7,499	*R*
7,500– 12,499	*R*– .50
12,500 and over	*R*– 1.00

23.

	Two Cos.
Under $ 5,000	*R*+$ .50
$ 5,000– 9,999	*R*
10,000 and over	*R*– .50

A. Premium Rate Patterns of 71 Companies Using Quantity Discount Method—*Continued*

24.	*Two Cos.*
Under $ 5,000	R+$ .50
$ 5,000– 9,999	R
10,000– 24,999	R– .50
25,000– 49,999	R– 1.00
50,000 and over	R– 1.50

25.	*Four Cos.*	*Two*	*One*
Under $ 5,000	R+$ .50	$1.00	$1.00
$ 5,000– 12,499	R		
12,500 and over	R– .50	.50	1.00

26.	*One Co.*
Under $ 5,000	R+$ .50
$ 5,000– 12,499	R
12,500– 19,999	R– .50
20,000 and over	R– 1.00

27.	*One Co.*
Under $ 5,000	R+$ .50
$ 5,000– 12,499	R
12,500– 19,999	R– .50
20,000– 24,999	R– 1.00
25,000 and over	R– 1.50

28.	*One Co.*
Whole Life & Endt.	
Under $ 5,000	R+$ .50
$ 5,000– 12,499	R
12,500– 19,999	R– .50
20,000 and over	R– 1.00
25,000* and over	R– 1.50

* Whole Life only.

29.	*Four Cos.*	*One*	*One*
Under $ 5,000	R+$ .50	$ .50	$ .75
$ 5,000– 12,499	R		
12,500– 24,999	R– .50	1.00	.50
25,000 and over	R– 1.00	1.50	1.00

29 (*Cont.*)	*One Co.*	*One*	*One*
Under $ 5,000	R+$ .75	$1.50	$2.50
$ 5,000– 12,499	R		
12,500– 24,999	R– .75	.50	.50
25,000 and over	R– 1.75	1.00	1.00

30.	*One Co.*
Under $ 5,000	R+$ .50
$ 5,000– 12,499	R
12,500– 24,999	R– .50
25,000* and over	R– 1.00

* Except special rates for lim. pay life and term.

31.	*One Co.*
Under $ 5,000	R+$ .50
$ 5,000– 12,499	R
12,500– 24,999	R– .50
25,000– 37,499	R– 1.00
37,500 and over	R– 1.25

32.	*One Co.*	*One*	*Four*
Under $ 5,000	R+$ .50	$ .50	$ .50
$ 5,000– 12,499	R		
12,500– 24,999	R– .50	.50	.50
25,000– 49,999	R– .75	1.00	1.00
50,000 and over	R– 1.00	1.25	1.50

33.	*One Co.*
Under $ 5,000	R
$ 5,000– 24,999	R*
25,000 and over	R*–$.50

* First year reduced to (R–$10).

34.	*One Co.*
Under $10,000	R
$10,000– 24,999	R–$ .50
25,000 and over	R– 1.00

35.	*One Co.*
Under $10,000	R
$10,000– 24,999	R–$1.00
25,000– 49,999	R– 1.50
50,000 and over	R– 2.00

36.	*One Co.*
Under $12,500	R
$12,500 and over	R–$1.00

37.	*Three Cos.*
Under $12,500	R
$12,500– 24,999	R–$ .50
25,000 and over	R– 1.00

S = Amount Insured in Thousands

Item involving S is amount of premium; other items are rates per $1,000

38.	*One Co.*
$ 1,250–$ 2,499	S (R+$.50)+$2.50
2,500– 4,999	R+$ .50
5,000– 9,999	R
10,000 and over	R– 1.00

39.	*One Co.*
Under $ 2,500	S (R+$ .75)+$1.50
$ 2,500– 4,999	R+$ .75
5,000– 12,499	R
12,500– 24,999	R– .75
25,000 and over	R– 1.25

40.	*One Co.*
Under $ 2,500	SR+$1.00
$ 2,500– 7,499	R
7,500– 14,999	R– .50
15,000– 24,999	R– 1.00
25,000 and over	R– 1.25

41.	*One Co.*
Under $ 2,500	SR+$1.00
$ 2,500– 12,499	R
12,500– 24,999	R– .50
25,000 and over	R– 1.00

42.	*One Co.*	*One*
Under $ 5,000	SR+$2.00	$2.50
$ 5,000– 12,499	R	
12,500– 24,999	R– .50	.75
25,000 and over	R– 1.00	1.00

Number of Companies *71*

Note.—Patterns 38–42 are for six companies which modify the quantity discount method in the lowest amount range by introducing a policy fee.

B. Premium Formulas of 7 Companies Using Policy Fee Method

S = Amount Insured in Thousands

	Amount of Policy	Amount of Premium
1.		*One Co.*
	Under $2,500	$S(R+\$1.00)$
	$ 2,500 and over	$S(R-\$1.00)+\$5.00$
2.		*One Co.*
	Under $2,500	$S(R+\$1.50)$
	$ 2,500– 4,999	$S(R+\$ .50)$
	5,000 and over	$S(R-\$1.00)+\$5.00$
3.		*One Co.*
	Under $2,500	$S(R+\$1.50)$
	$ 2,500– 9,999	SR
	10,000 and over	$S(R-\$3.00)+\$30.00$
4.		*One Co.*
	Under $5,000	SR
	$ 5,000 and over	$S(R-\$2.50)+\$12.50$
5.		*One Co.*
	Under $5,000	SR
	$ 5,000 and over	$S(R-\$1.25)+\$6.25$
6.		*One Co.*
	Under $5,000	SR
	$ 5,000 and over	$S(R-\$1.00)+\$5.00$
7.		*One Co.*
	Under $5,000	$SR+\$1.25$
	$ 5,000 and over	$S(R-\$1.00)+\$5.00$

Number of Companies 7

Note.—Formulas 1–6 are for six companies which depart from the policy fee method in the low amount range. In formula 7, the policy fee method is adhered to in that range, but at a different figure combined with a change in rate.

APPENDIX 2

DETERMINATION OF THE PORTION OF COMPANY EXPENSE WHICH IS A FUNCTION OF THE NUMBER OF POLICIES

The allocation of expense is to some extent a matter of judgment and therefore expense rates can be found only approximately at best. Hence refinement of minor items is futile and they should simply be included along with the dominant items with which they are naturally associated.

We may consider that taxes, licenses and fees vary by policy size and therefore are not a function of the number of policies.

Also, we shall disregard soliciting agents' compensation, using only the remainder of Item 21 in Column 3 of Page 5 of the Annual Statement in the case of general agency companies. Such remainder of Item 21 will be zero for branch office operations; for general agency operations it will normally consist of general agents' compensation for services other than as soliciting agent, plus agency expense not covered by agency expense allowance, the required amounts being those contained in Column 3.

Also, we shall require details as appearing on Page 9, Exhibit 5, of the Annual Statement but only such portions thereof as are the components of Column 1, together with the policy loan expense included in Column 3, but in either case only to the extent that these are contained or taken into account on Page 5 in Column 3 for Items 23 and 4.

The components, as referred to, will usually be obtainable from company work sheets used in preparation of Pages 5 and 9. The work sheets also provide a basis for estimating the policy loan expense referred to, which in the following figures is by use of the factor .1533, the ratio of policy loan salaries to total investment salaries in the company and year for which the figures were determined. For other companies this factor may be different.

Detail of Page 9, Exhibit 5	Annual Statement Page 9 Exhibit 5 General Expenses INSURANCE Life Col. (1) (i)	Component of (i) contained in Page 5, Analysis of Operations by Lines of Business ORDINARY —Life Insurance, Col. (3) Item 23 —General Insurance Expenses (ii)	Annual Statement Page 9 Exhibit 5 General Expenses INVESTMENT Col. (3) (iii)	Policy loan expense component of (iii) contained in Page 5, Analysis of Operations by Lines of Business ORDINARY —Life Insurance, Col. (3) Item 4—Net Investment Income (iv)
Items Treated as Not Involving Number of Policies				
Soliciting agents' benefits —#3.12, 3.22, 3.32	$ 528,397	×	$ ×	×
Fees and investigation expense—#4.1, 4.4, 4.5	43,449	×	22,755	×
Travel, advertising—#5.1, 5.2	561,169	×	202,089	×
Sundry—#6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7	251,543	×	155,418	×
Agency—#7.2, 7.3	60,000	×	×	×
Other investment expense —#9.1, 9.2	×	×	3,139,042	×
Adjustment for incurred—#11, 12	6,386	×	34,026	×
	$1,450,944	×	$3,553,330	×
Items Treated as Fully Varying with Number of Policies				
Medical and Inspection Fees—#4.2, 4.3	$ 801,942	$ 786,785	$ ×	$ ×
Postage, etc., printing, equipment—#5.3, 5.4, 5.5, 5.6	887,945	777,161	512,062	78,499
Agency expense allowance —#7.1	0	0	×	×
	$1,689,887	$1,563,946 (*a*)	$ 512,062	$ 78,499 (*b*)
Items Treated as Partly Varying with Number of Policies				
Rent—#1	$ 699,225	$ 611,987	$ 392,549	$ 60,178
Salaries and employee benefits—#2, 3.11, 3.21, 3.31	5,252,970	4,597,588	3,276,541	502,294
	$5,952,195	$5,209,575 (*c*)	$3,669,090	$562,472 (*d*)
Total—Page 9, Item 13	$9,093,026		$7,734,482	
Invariant @ 20% (see *Invariant*)		$1,041,915 (*i*)		

Commissions on premiums (Page 5, Col. 3, Item 21)	$22,568,950	
Add: Agency expense allowance No. 7.1 (above)	0	
	$22,568,950	
General agents' compensation and voucherable expense as general agent		
—taken as 29% of total commissions plus agency expense allowance, if any	$ 6,545,000	
Deduct: Agency expense allowance above	0	
	$ 6,545,000	(e)
Deduction for paid-up insurance, other than nonparticipating extended insurance, at 60¢ per $1,000	$ 560,109	(f)

Recapitulation:

(a)	$ 1,563,946	
(b)	78,499	
(c)	5,209,575	
(d)	562,472	
(e)	6,545,000	
	$13,959,492	
−(f)	560,109	
	$13,399,383	Residue
−(i)	1,041,915	Invariant
	$12,357,468	Variant

DISCUSSION OF PRECEDING PAPER

CHARLES A. ORMSBY:

In his fine paper on the grading of gross premiums by policy size, Mr. Fassel has covered the subject with his usual thoroughness and insight into many of the practical aspects. The subject matter of his paper is no doubt destined to be of major historical significance to life insurance through its impact on competition within the industry and the insuring public's reaction to more scientific pricing.

Mr. Fassel first presents a historical background, tracing the development of premiums from one dimension, plan, to the emerging three dimensions, plan, age, and policy size. There may be those who would prefer to include a fourth dimension, namely, underwriting classification, which dimension was employed before policy size in determining both gross and net premiums. It might also be recalled that policy size has for many years been given some recognition in that the premiums for Term plans have been based on a higher average size than those for permanent plans, and in a few instances at least the rates for juvenile insurance have been modified to reflect in part their distinctly lower average size. In fact, a number of stock companies have for years been observing a differentiation in policy size for both issue age and plan groupings.

The factor of policy size has also been a determinant in causing premium rates in the industry to vary significantly from company to company. In other words, other things being equal, a company with a high average size showing relatively little variation in its amounts of insurance from the average has in effect been making available to the public the savings associated with larger purchases. Finally, we all know that the principle of quantity discount has for some time now been applied to calculate premiums for casualty lines as well as premiums in the field of Group Life insurance.

One of the obvious and welcome implications of grading premiums by policy size is that a company will be less vulnerable to shifts in the average size policy with respect to either issue age or plan of insurance, the extent of this immunity depending on the width of the bands adopted. This circumstance could be particularly helpful to a new company because it does not know in advance what its over-all average size will be. Viewing the industry as a whole, a possible unwelcome result is the added impetus that may be given to the trend toward fewer but larger policies.

Although the distinct movement toward premium gradation by policy size is gaining momentum, and is no doubt a real forward step in the evolution of the life insurance business, we might well ask ourselves whether the adoption of this additional dimension to a greater extent than is now practiced will raise a number of related questions of considerable importance. One of these has to do with our premium margins for profits and/or contingencies, which have traditionally been invariant for each unit of face amount at a given age, for a given plan, and in a specified underwriting classification. It seems pertinent at this point to consider the possibility that our traditional position with respect to premium margins may be less defensible after we have decided to recognize policy size on a "band" or "policy fee" basis. Whether there is any concern or not over this possible derivative of introducing another variable in our premium formulas, it seems clear that circumstances will thus be created under which it will become more feasible to reduce the unit margins with increase in size and to employ additional refinements in setting premium rates. Secondly, will recognition of policy size lend support to the argument that commissions themselves should be broadly graded by size of policy? It is not inconceivable that the idea of quantity discount will encourage some of our critics to argue that the same concept should be applied to the agent's compensation, in part because a larger purchase does not necessarily entail proportionately greater effort. As is well known, the grading of commissions is a firmly entrenched practice of long standing in the Group field.

Making life insurance rates a function of policy size has at least two interesting implications with respect to nonparticipating as opposed to participating insurance. We are all familiar with the difficulties posed over the past few years by deficiency reserve requirements in connection with plans having high minimum amounts. It appears likely that policy size gradation of premium rates will accentuate this problem and possibly result in greater discrimination against nonparticipating insurance in favor of participating. A second corollary of size gradation is that the principle can more easily be applied retroactively to the participating business in force than to the nonparticipating. However, I am inclined to agree with the arguments Mr. Fassel clearly stated in his paper for administering the existing business "according to the prior theory."

In applying the principle of quantity discount, a company will face the practical problem of determining the optimum pattern of rate bands, taking into consideration not only the level of its own expenses which are a function of the number of policies but also the administrative and sales

aspects. Because of the low premiums in relation to per policy expenses, it may be necessary to adopt a set of bands for short-term Term plans different from that for permanent plans. In addition, the probability of substantial changes in per policy expenses with the passage of time may be such that in future rate revisions a different set of bands may be desirable. This contingency might be accorded some recognition in selecting the bands initially in order to minimize, where practicable, radical changes at a later date in the over-all pattern of rate bands. Some companies may also want to be influenced by the higher mortality on the larger amounts in the past and the absence of assurance that such higher loss ratios will not reappear, at least partly, in subsequent investigations.

One of the key sections of the paper is that which deals with the determination of the "residue" of expenses and its separation into "variant" and "invariant." Starting with a process of general reasoning, Mr. Fassel eventually arrives at a figure of $7.50 as that part of the variant which applies to both first and renewal years, after deducting from the residue an invariant equal to 20% of salaries and rent. While many may agree that these results are broadly representative, it may be of interest to compare them with those of another company which made such an investigation at the time it last revised its nonparticipating rates. (I am indebted to Ward Hart of my Company for the information he furnished me on the variant and invariant for our Company and which is presented here.)

Wholly apart from the question of varying premiums by policy size, a nonparticipating company before revising rates will want to separate its expenses into those assessed per policy and those assessed per thousand, in order to distribute the expenses equitably between ages and plans. At the time of our last investigation, the various unit costs (other than investment expense) entering into our premium calculations were:

a) Expenses which appear only in the first year, separated into (i) expenses as a percentage of premium, and (ii) expenses per policy.
b) Renewal expenses as a percentage of premium.
c) First and renewal expenses per policy.
d) First and renewal expenses per thousand.

In making such an expense analysis, one objective is to keep individual judgment to a minimum by studying each department of the company and each officer's salary either by time studies or by interviews, and to allocate all expenses to one of the above categories. Then each ledger account, other than salaries, is studied and suitably allocated or, in some instances, pro-rated according to salaries.

Typical items included in (*d*) are salaries for actuarial research and annual statement compilation, certain legal salaries, directors' fees, and a considerable portion of general executive salaries. Examples of expenses other than salaries most of which should be charged per thousand and not per policy are membership dues in trade associations and contributions to civic and welfare organizations.

It would appear that our item (*c*) above is a proper figure to compare with Mr. Fassel's $7.50 and likewise that the ratio of the expense in (*d*) to the total of those in (*c*) and (*d*) (or at least the salary portions thereof) would be a proper ratio to compare with his 20% as the value of the invariant.

Since in making the comparison we arrive at a figure of less than $5.00 per policy instead of his $7.50, the conclusion seems inescapable that we have included many expenses in (*a*) and (*b*) which are in his residue. Our ratio of first year underwriting and issue expenses to the total of such expenses plus handling expenses (including home office and applicable field expenses) is almost 40% rather than the 13% used in the paper. We also allocate to "Home Office Sales" an amount which is of the same general order of magnitude as that allocated to first year issue and underwriting. However, we do regard "Home Office Sales" expense as the type of expense which should be charged per dollar of premium or per $1,000 of insurance rather than per policy. In any event, the percentage of the variant applicable to the first year only can vary appreciably with the rate of growth of a company as well as with other factors.

The ratio referred to above for determining the invariant works out at 22% if based only on salaries, and at 31% if expenses other than salaries are also included in both numerator and denominator of the ratio. In other words, Mr. Fassel's figure of 20% is more or less confirmed if we accept his premise that the invariant should be confined to salaries and rent. It seems to us, however, that certain expenses other than salaries might also enter into the thinking on this subject, and it may be that 30% will be considered by some companies a more appropriate figure.

It should be stressed that the figures quoted for comparison relate to the expenses of one company only and should be viewed with due recognition of the fact that cost accounting at best is not an exact science and that the individual judgment of the person making the cost accounting analysis can vary the final results within a sizable range.

Our actuarial literature on gross premiums needed this contribution by Mr. Fassel. He is to be congratulated for his excellent treatment of a timely subject.

ROBERT J. RANDALL:

Mr. Fassel has written an excellent paper setting forth the historical, theoretical, and practical aspects of premium and dividend gradation by policy size. Obviously expenses and philosophies of expense allocation will vary between companies. Nevertheless, I believe we are fortunate in having in the record this logical and well-written exposition of one company's approach to this question, since grading of premiums by policy size promises to gain increasing acceptance in the future.

TIAA decided to use the dividend approach mainly because we felt we could maintain the equities more satisfactorily in later policy years as expense patterns change. We used $6.00 as an appropriate amount assessable per policy. The average policy sizes shown below are approximately equal to our actual average sizes:

Amount Group	Average Policy Size	Charge per $1,000 of Average Policy Size	Dividend Credit
$ 0–$ 4,999.....	$ 3,000	$2.00 (*A*)	0
$ 5,000–$ 9,999.....	6,000	1.00 (*B*)	*A* − *B* = $1.00
$10,000–$14,999.....	12,000	.50 (*C*)	*A* − *C* = 1.50
$15,000 and over.....	24,000	.25 (*D*)	*A* − *D* = 1.75

Mr. Fassel mentions two methods, the policy fee method and the quantity discount method. One disadvantage of the policy fee system is that it results, unless modified in some way, in very high premiums per $1,000 for small policies. A disadvantage of the quantity discount method is that the premium for a policy in areas just below the points at which steps in rate are made is greater than the premium for a larger policy just above the rate step point. For example, a policy for $4,500 might cost more than a policy for $5,000. A third method is to charge one premium rate per $1,000 for the first, say, $3,000, of insurance on any policy, and a lower rate per thousand on the excess, if any, over that amount. Upon analysis, it will be seen that this is equivalent to the quantity discount method for smaller policies, and the policy fee method for larger policies, the policy fee being equal to the amount of insurance at the breaking point times the difference in the two premium rates. In fact, this is the method used by six of the British companies listed by Mr. Fassel as using the policy fee system. It was also recently adopted for a new special plan issued by a large American company. The following table compares dividends and

net costs under TIAA's quantity discount dividend method with net costs under the modified policy fee system described above.

Amount	Quantity Discount		Modified Policy Fee	
	Dividend	Net Cost	Dividend	Net Cost
$ 3,000.....	$ 15.00	$ 75.00	$ 15.00	$ 75.00
$ 4,900.....	24.50	122.50	28.30	118.70
$ 5,000.....	30.00	120.00	29.00	121.00
$ 6,000.....	36.00	144.00	36.00	144.00
$12,000.....	78.00	282.00	78.00	282.00
$15,000.....	101.25	348.75	99.00	351.00
$50,000.....	337.50	1,162.50	342.00	1,158.00

Basis: *Premium—$30 per $1,000*
Dividends per $1,000:

Quantity Discount—$5.00 + { $ 0 for $ 0–$ 4,999; $1.00 for $ 5,000–$ 9,999; $1.50 for $10,000–$14,999; $1.75 for $15,000 and over }

Modified Policy Fee—$5.00 on first $3,000 + $7.00 on excess over $3,000.

It will be seen that the modified policy fee method gives very close agreement with the quantity discount method and does not produce the inconsistency of a $4,900 policy costing more than a $5,000 policy.

The area just below each step point in the quantity discount system where a policy costs more than a larger policy just above the step point is generally quite small but it does become larger for low premium plans. Mathematically, if

t = amount of insurance at step point,
d = difference in premium rate,
P = rate per $1,000 for smaller policies and
t' = largest amount below t with total premium not greater than premium for t,

then it can be shown that

$$t' = t - \frac{td}{\mathrm{P}}.$$

The table on the following page sets forth values of t' under TIAA's system of quantity discounts and for premiums of $10, $30, and $50.

Because most policies are issued for round amounts such as $5,000, the existence of the areas described above has not created any practical problem.

We have not given any policy size credit for paid-up insurance, for the reasons given by Mr. Fassel.

For Family Income policies, since our credit is given through the dividend, the Standard Nonforfeiture Law did not affect our approach. We based the extra size credit on the average extra amount of insurance during the Family Income period. A large proportion of our Family Income policies are written for exactly $5,000 basic face amount. Two methods

t	d	P	t'
$ 5,000.....	$1.00	$10	$ 4,500
		30	4,833
		50	4,900
10,000.....	.50	10	9,500
		30	9,833
		50	9,900
		10	14,625
15,000.....	1.25	30	14,875
		50	14,925

of defining the average extra amount of insurance were considered, one of which put a $5,000 Family Income policy just above a step point and the other just below. It just so happened that the more logical definition placed a $5,000 Family Income policy in the higher dividend class.

Since adopting our new dividend method, we have had relatively few requests for combinations of existing policies and have tried to approve them whenever feasible. For example, we approve combination of policies with issue dates within a year, and also changes of plan to be followed by combination with another policy on the new plan. In one interesting case, the policyholder had seven Term to 70 policies, involving $13,000 of insurance, and expiring on different dates in the year he would attain age 70. We offered to combine them in such a way that the period of coverage was shortened for $5,000 of insurance, left unchanged for $5,000, and increased for $3,000. The policyholder wrote back objecting that his average period of coverage was being decreased, but when we replied, giving the amount of the increased dividend, he decided to go ahead with the change. In another case, a man had six fairly large policies, and combined them in pairs into three policies since any further combination would not have added to his dividends.

This question of combining policies naturally suggests the question: Why were separate policies issued in the first place? In the great majority of the cases that have been combined, it was in order to spread premium

payments throughout the year without paying the extra charge for fractional premium payment. Our scale of extra dividends completely cancels out the advantage of this type of policy splitting for many plans and issue ages and substantially reduces it for all others.

One final comment: As a result of the introduction of our new scale, our sales of individual life insurance have increased sharply, both in number and in amount per policy.

DANIEL J. LYONS:

We are indebted to Mr. Fassel for his stimulating paper.

A very substantial part of the per policy expense illustrated by Mr. Fassel results from considering overriding commissions to general agents as a per policy expense. The variant of $12,357,468 in Appendix 2 includes general agents' overriding commissions of $6,545,000 and this is $4.76 of the $7.50 per policy used. It seems to me that Mr. Fassel's theory can be sustained only if the $6,545,000 of overriding commissions actually varies with size of policy.

It may be that Mr. Fassel does not have in mind a schedule of general agents' overriding commissions which vary with size of policy. He refers to the method of payment as being only a superficial formula. Underlying the superficial formula is one which presumably is based on size of policy. If this reasoning is accepted, it seems to me that we are getting into an area fraught with difficulties. A company anxious for larger quantity discounts than Mr. Fassel's may argue that agents' commissions vary by size according to some hypothetical formula underlying the superficial formula. Furthermore, the theory would break down completely where the increase in average size results from inflation.

It would appear that the only satisfactory method for determining quantity discounts is to include in the variant only those items which do actually vary with size—that is, where we can see a reduction per thousand if, for example, the average policy increases from $10,000 to $20,000. Medicals, inspections, cost of underwriting, cost of issue, cost of premium collection and many other items are likely to be less per thousand when we have such an increase in the average policy. General agents' overriding commissions, however, will not be less unless the agency contract so provides.

If the foregoing is correct, Mr. Fassel must anticipate that the general agents' contract will be written with bands showing higher commission rates on small policies and lower rates on large policies. It would seem that, under the theory outlined, higher commissions should be paid on smaller policies since more than half of the additional premiums to be

charged the small policyholders is for the purpose of paying such higher commissions. A company which is already paying the New York limit will, of course, find it impossible to pay higher commissions on the small policies even though it can, of course, pay lower commissions on the large policies. This would seem to be a problem which many companies will encounter and the practical solution may be simply to reduce commissions on the large policies. However, this does violence to Mr. Fassel's theory.

The general agent's overriding commissions, as Mr. Fassel properly points out, cover both compensation for the general agent's work as manager of the agency and part or all of the expense of running the agency. Many general agents may question Mr. Fassel's assumed savings in expenses on large policies. While it is true that the premium collection function in the agency office would cost less per thousand for large policies, service expenses are likely to be very much higher. The net result might indeed be more expense rather than less. As regards compensation of the general agent, Mr. Fassel feels that the only reason for such compensation is "fragmentation of insurance in force into so many policies." General agents exist primarily to get new business. While some servicing functions are carried out in the agency office, if it were not for new business, an entirely different type of individual would be in charge of the agency office and his compensation would be at the clerical level. The general agent's compensation is at the executive level.

Thus it would seem that most of the general agent's compensation is for securing new business and he may resent earning less if his average policy is $20,000 rather than $10,000. He may feel that he worked very hard to achieve this result. It is true that some companies at present pay reduced commissions on certain minimum size and preferred risk policies, but the reductions in overriding commissions are small compared with Mr. Fassel's proposition that over half the quantity discount is justified by the variation in general agents' overriding commissions based on size.

While it appears that Mr. Fassel would substantially reduce general agents' commissions on the larger policies, he proposes no reduction for soliciting agents. It would seem to me that the case for a reduction in soliciting agents' compensation is at least as strong as for general agents' compensation. However, Mr. Fassel dismisses this question by merely saying "the same solicitors' commission would presumably be paid." It may be observed that in Group insurance there is a reduction for size for both the agent and the general agent. I suspect that Mr. Fassel proposes no variation in soliciting agents' commissions because of the practical difficulties. I believe similar difficulties would be encountered with the general agents' reductions.

DATON GILBERT:

Mr. Fassel's typically thorough paper presents much helpful information and comment regarding a seemingly significant new trend in our industry. His paper shows the fundamental role played by expense analysis in any realistic approach to this problem. My comments fall under three broad headings in this area:

1. Basic forces bearing on the necessary cost studies.
2. Problems posed by different types of companies.
3. Comments on certain phases of the field expense allocation problem.

Basic Forces

As we approach a new development such as this, we should be reminded of the social service aspect of our business and of the importance, therefore, of maintaining equitable treatment for all policyholders. Thus, the NAIC statement suggested that companies have a responsibility "to show that any system of groupings of premium rates or dividend classifications is reasonable, equitable, and nondiscriminatory."

Also, there is another force at work. Recent years have brought a rapid increase in competitive pressures, particularly in the large policy market. This force will press toward maximizing the reduction in premium as size of policy increases.

Problems of Different Types of Companies

Relatively young companies and even older companies going through a period of rapid change and expansion will tend to have relatively high expense rates. Values of Mr. Fassel's expense constant per policy (*i.e.*, b) determined during such high expense periods may prove to be quite inappropriate for use under conditions where future reductions in expense rates seem likely. Thus, in many situations conservatism is indicated in fixing the value of b, along the lines mentioned by Mr. Fassel in discussing his "Illustrative Pattern 2."

An important portion of expense in the typical company arises from organizing and maintaining a network of agency offices. Here we have the two different systems of general agencies and branch offices. In recent years these two types of agency operation, with some exceptions, have approached each other more closely. Many differences have come to be matters of accounting and of form. Such conditions would seem to justify an approach to the analysis of field expenses which will treat general agency and branch office companies similarly in determining the value of b. Mr. Fassel's approach to this problem has been to obtain such equality

by treating general agents' compensation and voucherable expense allowances as "fully varying with number of policies."

Although his objective is in line with my previous comments, it seems doubtful to me that his simple treatment is in keeping with reality.

Field Expense Allocation

From a broad viewpoint, field expenses can be considered as including (1) compensation for agency management and (2) other expenditures (such as clerical salaries, rent, etc.) incurred in operating an agency office.

In a general agency company, compensation for agency management arises primarily from the balance of overriding commissions and collection fees remaining after taking care of any operating expenses not covered by allowances. Clearly overriding commissions and collection fees, considered without regard for the nature of underlying functions made possible by their payment, are not *constant per policy*. Disregarding any slight gradation by plan of insurance, they vary directly with the amount of premium.

As to the formulas used in determining expense allowances to general agents, only rarely do they include a factor based on number of policies. A 1953 report of the LIAMA Agency Costs Committee summarizes the expense allowance formulas used by 28 general agency companies. Only two companies include any factor relating directly to number of policies. With only such minor exceptions, companies are using factors based on premiums, amounts of insurance, and commissions.

However, as some of Mr. Fassel's comments suggest, we should look beneath the formula or the method on which payments to general agents are based. What are the broad functional activities made possible by the payment of overriding commissions and expense allowances to general agents? Presumably, these underlying functional activities are very similar in general agencies and branch offices. Are the expenditures which might reasonably be related to such functional activities the type which can be properly considered as constant per policy? Let us keep these questions in mind as we turn to the branch office companies.

One such broad functional activity is represented by the *agency management activities* of the branch manager. In my opinion, to classify managerial compensation as "constant per policy" is at variance with the normal facts. Seemingly, most companies want managers who will build paid-for business, insurance in force, and premium income rather than merely numbers of policies. An inspection of a typical managerial compensation plan will reveal the tremendous weight assigned to amounts of insurance, premiums, and commissions. If two managers build agencies of equivalent

size when measured by numbers of new policies and of policies in force, their compensation nevertheless can be expected to differ greatly if their average size policies so differ.

Turning to *other branch office expenditures*, at least three broad underlying functional activities are normally present: (1) new business development activities, (2) routine clerical work on new policies issued, and (3) servicing and premium collections for insurance in force. Items (2) and (3) clearly tend to be the "constant per policy" type. On the other hand, new business development activities, as with agency management activities, are usually concerned more with building amounts of insurance and premium income than with building numbers of policies. From this viewpoint, it is difficult to understand the treatment of development expenses on a "constant per policy" basis.

Mr. Fassel states: "If the same amount of insurance and premium revenue were achieved with but a comparatively few mammoth policies, the same solicitors' commissions would presumably be paid but the field organization would become negligible." Another approach might be to consider this matter within the scope of current-day reality. In some companies the average size new policy in individual agencies may range from a low of, say, $4,000 to a high of some $25,000. Corresponding but smaller differences can be expected in the average size policy in force. It hardly seems that a study of management compensation and development costs for agencies with average policies at such actual extremes would reveal any support for treating such expenses as constant per policy.

Conclusion

Mr. Fassel's illustrative figures show over $6,500,000 of management compensation and other agency office expenses out of his total so-called "variant" item of $12,350,000. In my opinion, a substantial part of this $6,500,000 should be excluded in determining his constant b. Perhaps in advancing his "Illustrative Pattern 2" this possibility was one of several underlying considerations. As he suggests, some conservatism is advisable in determining this constant, and field expenses present a logical area for such caution.

JAMES A. CAMPBELL:

We are indebted to Mr. Fassel for a paper on a topic which is of particular interest at the present time. He has given us an outline of the general theory underlying the variation of premium rates by amounts of insurance and has put forward a method of arriving at the proper differentials. His comments on certain problems which are created by adoption of premium rates varying by size are useful and it will be convenient to

have available the summary of the methods which are being used by companies in Great Britain.

The London Life is mentioned as being the first company in North America to give full recognition to policy size in its rates. While we believe this to be true, there are several Canadian companies which have for a number of years issued policies in different amount classes at different rates of premium but without necessarily including all plans in each class. Our only claim is to have been the first to make the variation applicable to all plans and to make the variation for participating policies in premiums alone, with the same scale of dividends applying to all amounts of insurance.

In determining our premium differentials we had available the methods of a number of English companies and the differentials which were used by some of the Canadian companies. An examination of our own expenses was made, but our methods were somewhat different from those proposed by Mr. Fassel. The range of the amounts of insurance issued by a combination company is so great that the cost of medical and inspection fees cannot be considered to vary entirely with the number of policies. Small amounts are issued on nonmedical applications and in many cases without inspection reports. Furthermore, the handling of small amounts in the Underwriting Department is much simpler and the records maintained for Debit business are very much less extensive than those for Regular Ordinary. We therefore assessed all medical and inspection expenses on the basis of the amount of insurance.

In estimating the part of the expenses which should properly be said to vary by number of policies, we made use of time studies for branch office and head office work. The general principle we followed was to allocate by amounts of insurance all expenses which appeared to be unaffected by the actual number of policies in force. In this category would fall branch office salaries and rent connected with acquisition expenses and the space occupied by salesmen. In head office the salaries and other costs of the Agency Department and the Advertising Department were so assigned. Half of the policy issue costs and three-quarters of the time of underwriters were charged on a per $1,000 basis, as well as general expenses of the Accounting Department, Mathematical Division, and Claims Department. The remaining expenses in all categories were assumed to vary with the number of policies.

As an offset to the per policy expenses, some allowance should be made for the additional loading in fractional premiums. Part of this additional loading is, of course, for loss of interest and the mortality cost involved

in waiving the remaining fractional premiums in the policy year of death. However, a substantial part of the additional loading is intended to offset the collection costs and it is reasonable to make some deduction from the cost per policy. In our own case we arrived at a figure of $5.46 as the annual cost per policy and this was reduced to $4.87 by crediting the part of the additional loading for fractional premiums which we thought was applicable to per policy expense.

It was this figure of $4.87 which confirmed our idea that about $5.00 per policy was a reasonable factor to use in calculating differentials.

Mr. Fassel's method of determining a pivotal rate assumes that the premium revenue before and after the change will be the same. This will be true only if the distribution of business by amounts remains relatively unchanged—and such a result is not too probable. Providing discounts for size through the whole range of amounts gives our salesmen a new incentive to upgrade amounts of insurance being applied for into the next higher classification, and if they are successful in doing so the total premium revenue for a given volume of business may decline.

Our basic rate is the one for $10,000 and over; thus it is not a pivotal rate of the type Mr. Fassel has described. The new $10,000 premium rate was lower than the rate we previously used for Preferred policies of $5,000 and over, but the dividend formula was essentially the same, so that the net payment on the new $10,000 level was only slightly below our previous preferred scale. The effect of our method of calculation, therefore, was to provide us with some additional expense loading for policies in the group $5,000 to $9,999. This had become necessary because of the very large number of policies for exactly $5,000 which had been and are still being issued by our Debit salesmen on the Monthly Debit Ordinary plan.

This higher expense loading in policies of $5,000 to $9,999 applies only to issues under the new plans. Policies of these amounts issued before 1955 participate in dividends on the basis of the same formula as policies of $10,000 and over issued from 1955 on. This has minimized the transition problem and we have had no real difficulties with applications for consolidation of existing policies.

Mr. Fassel suggests methods of dealing with changes of age which seem quite satisfactory and his proposal that the differential in supplemental term riders should be at only one-half the scale normally used is probably justified. In our own case we used the same differential of 50¢ per $1,000 between $5,000 and $10,000 riders, without thinking too much of the matter, and probably on the basis that a differential of less than 50¢ per $1,000 might seem to be not worth while.

The only comment I have to make about the variation of dividends by size is that for a company which issues both participating and nonparticipating business, differentials in the premiums are essential. However, we have always had it in mind that if, at some time in the future, costs of operation were such that the differential by policy size became smaller than that in the premiums we might find it necessary to vary dividends also. We hope that this will not be required, but over a long period of time it is certainly a possibility.

FREDERIC P. CHAPMAN:

Mr. Fassel has presented a timely paper on a broad and complicated subject. There are many phases of this subject which will require a great deal of thought and study before we can expect to reach solid conclusions. There will, I am sure, be many variations of opinion.

The paper is confined to the effect of expense. While other phases, such as mortality, may also be involved, expense undoubtedly constitutes the major part of the problem.

The basic problem to which Mr. Fassel addresses himself seems to be to determine as accurately as possible what expense would actually be incurred if a company issued all of its policies in one or another of various size groups. There are, however, several assumptions in the paper that warrant close scrutiny. The first involves the author's apparent abandonment of the usual methods of expense distribution as commonly made per policy, per $1,000 insured, per $100 premiums, etc.

He states: "Our problem is the broader one of recognizing the expense that is in reality a function of the number of policies even though the relation may be obscured." Mr. Fassel then goes on to outline a proposal for determining the amount of the expense which he considers to be a function of the number of policies and which undoubtedly differs considerably from the figure he would arrive at on the basis of the usual methods of expense distribution. It seems to me, however, that the whole purpose of "the breakdown of expenses as commonly made per policy, per $1,000 insured, per $100 of premium, etc." is to recognize and measure, in so far as is practical, the expenses that are really a function of number of policies, amount of insurance, amount of premiums, etc. If the usual breakdowns do not accomplish this purpose, for what reason would they be made? It would be just as improper to use unrealistic breakdowns for distributing expenses by plan and age as it would be to use them in distributing expenses among policies in various size groups.

At the beginning of his paper, Mr. Fassel mentions that in the early days premium rates varied only by plan (one dimension) then by plan

and age (two dimensions) and he now considers the third dimension, that is, size. It seemed possible at first that Mr. Fassel might be suggesting that the method of expense distribution he outlines be used for all three dimensions. However, the paper indicates that he plans to retain "the basis of the customary premium rates that do not vary by policy size" and "dividends . . . calculated by the customary method." In other words, he first distributes all expenses by plan and age, following customary methods, and then makes an adjustment by size that is completely independent of the expenses that have been actually charged to each plan and age. It seems to me that this procedure is quite likely to result in a final expense charge on some particular plan, age and size class that is completely unreasonable. For example, this procedure could result in a credit for size on large policies with low premiums per thousand which is greater than the amount of the "variant" expense charged to the policy on the basis of the plan and age.

It would be quite interesting to see what his proposals as to the distribution of expense by size would do if applied to the basic pattern of premiums by plan and age. I think we would find a rather startling difference from the current pattern.

In arriving at the amount of expense considered to be a function of policies, the author assumes that for certain items the formula by which the expense is customarily paid or assessed need not be followed in distributing such expense. I do not question this conclusion where it can be demonstrated that the particular expense is not really a function of the way it is paid or assessed. However, where expense is actually incurred in direct relation to the volume of premium, for example, there would appear to be a substantial burden of proof on the company to support a theory that such expense was in fact distributable on a per policy basis.

The author uses compensation for field supervision to illustrate this type of expense, stating that general agents and field supervisors exist solely because of the fragmentation of the insurance in force into so many policies. In developing his figures, the author then includes the entire compensation of general agents, other than soliciting agents' commissions, in determining the expense assumed to be the same for each policy for each year the policy is in force. This does not appear to be very realistic. In the first place, it would seem that a major part of field supervision would relate to sales rather than maintenance, and hence that a major part of this expense should be applicable only at issue. Moreover, it seems questionable to assume that compensation for field supervision is really a function of policies. Compensation is paid in return for services rendered. Are the services of the agency manager the same for all policies regardless

of size? Certainly the question of programming would be more important on the larger policies; settlement options would be more complicated; changes in plan and in settlement arrangements are much more likely to be made where larger amounts of insurance are involved. The very fact that the larger policies constitute a progressively decreasing proportion of the total number inevitably results in questions arising that cannot be handled by normal procedures and which require special attention by the supervisory staff. If the author's assumption is correct, wouldn't it be general practice for companies to pay lower commission rates to those agencies which concentrate on and produce the larger policies? I doubt very much if this is done, yet there must be a great deal of difference in the average policy produced by different agencies. Admittedly, it is difficult to determine the relation between the services which the agency manager performs and size of policy, but on the other hand, it seems clear that one cannot simply assume that there is no variation by size whatsoever.

A somewhat similar situation occurs in connection with home office expense, except that in this case the author assumes that 20% of home office expense (the ratio of officers' salaries to total salaries) is excluded from his variant and presumably charged per thousand dollars of insurance. This helps to take into account the fact that the home office expense of handling one small policy must, of course, be considerably less than that for handling one of the larger policies. Smaller policies do not demand, nor do they receive, the home office service that large policies do. I am not prepared to say whether the basis on which he arrives at his figures is appropriate, or whether the resulting figure of 20% of home office expenses is large enough to adequately take this fact into account. Each company's experience will differ in this respect. For my company I am sure that the ratio of officers' salaries to total salaries would not be appropriate, and that a figure of 20%, however obtained, would not be large enough to produce the correct result.

In analyzing home office expense, the author calculates a figure of $23.05 which comprises medical and inspection fees, together with salaries and rent attributable to selection of risks. While he sets this figure aside and does not use it in determining the variation in expense to be used for the various size groups, he nevertheless implies that this figure would be constant regardless of size. Wouldn't it be true in most companies that these items actually vary considerably by size of policy? Don't the larger policies require more and costlier medical examinations? Isn't the cost of inspection reports higher for the larger policies? Aren't higher grade underwriters used in the home office on the larger than on the smaller

policies? Several years ago we made a detailed analysis of medical and inspection fees, together with all home office costs of underwriting, and found that the cost per policy for this expense varied considerably by size groups. Using the cost per policy in the $5,000 to $10,000 group as a base, the cost was only about two-thirds of the base for policies of less than $5,000, two and one-half times the base for policies from $15,000 to $25,000, and nearly five times the base for policies of $50,000 and over. Thus, if a company should decide to include underwriting costs in determining their size differentials, it would not be appropriate, according to our experience, to use one figure per policy for all size groups. We do not do so.

Toward the end of his paper, Mr. Fassel comes to the conclusion that if size recognition is through the premium, the dividend rates would be constant without regard to size. It might be noted that when a different breakdown of expense is used for determining premiums by plan and age than is used in determining size differentials, this conclusion would be valid only if dividends were calculated for all size classes taken together. If they were calculated for each size class taken by itself, dividends in the various size classes would differ. Furthermore, the conclusion would be valid only if future experience is not significantly different from the original assumptions. The paper itself includes a table showing the values of the per policy expense from 1940 through 1955 which illustrates the substantial change that has actually occurred in a relatively short period.

If, as seems to be indicated by the consideration noted above, the methods used by the author overstate the expense which is actually a function of policies, this would mean that the appropriate premium (or dividend) differentials between various size groups of policies would be smaller than those presented in the paper. While the results will, of course, differ materially from company to company, a very careful analysis of our own expense, relating to policies only those specific items of expense which in our best judgment are actually a function of policies, etc., and including issue and underwriting as well as maintenance expense, results in theoretical premium variations by size which are considerably smaller than those shown in the paper.

We have issued a special $5,000 minimum Whole Life policy since 1909. When this policy was first issued and for a number of years thereafter, it was severely criticized by our competitors on the grounds that they considered its cost to be lower than could be justified on reasonable assumptions as to distribution of expense, etc. If we had analyzed our expense on the basis used by Mr. Fassel, the already low cost of this policy would have been substantially lower.

ARTHUR PEDOE:

I want to pay tribute to the actuarial erudition displayed in the paper. One expects this from Mr. Fassel. However, I do feel that an incorrect impression may be obtained on account of the weight attached to the practice in Britain. It is only proper to point out that the Metropolitan of New York has been selling a Special $5,000 Ordinary Life policy in Canada for over 45 years and, to my knowledge, such special plans have been in operation throughout Canada for over 30 years. The legal prohibition in Canada referred to by Mr. Fassel had been circumvented for years, without protest on anyone's part, by referring to the regular plan as Endowment at 85 or Life to 85 to distinguish it from the plan with special minimum sum assured and special low rates.

The impression might also be formed that the procedure of my Company (The Prudential of England) as outlined in the paper is merely an extension of my Company's practice in Britain. This is not so. In my investigations into life insurance costs in Canada, which formed the subject of a paper in 1952 published in the *Transactions* (Vol. IV), I had become increasingly aware of the increasing cost per policy irrespective of sum assured (particularly first-year costs). It is this trend in recent years which is the reason for the adoption of the step rate plan by my Company in Canada, namely to give effect to this inflation in costs. Mr. Fassel arrives at a first year expense constant of $30.87 per premium paying policy and does not pursue the matter further. If in Mr. Fassel's company the figure is $30.87, then the corresponding figure of $40.00 which I hesitatingly put in my paper in 1952 is not only substantiated but I would say is now exceeded by many companies. However, the corresponding renewal expense cost per premium paying policy which Mr. Fassel gives as exceeding $7.00 at the present time seems to me to be too high.

The value of the paper is that it draws attention again to the inflation of life insurance costs and this is the fundamental reason for the step rate plan. Prior to the introduction of the step rate plan by my own Company in Canada we had in operation, for some years, several plans with varying minimum sums assured of $5,000 and $10,000 and a step rate on two plans at the $25,000 level. In my opinion the introduction of the step rate plan was a welcome simplification. Although it has been a tradition in my Company to put service to the smaller policy buyer above all other considerations, we do feel that the adjustment indicated in our step rate plan is more equitable than previously. In our present setup the smaller policy buyer still does not pay as much as he would be obliged to pay if the full theoretical charge were made and if we did not make every effort to reduce

costs on smaller policies and to cut down the commission on them. In other words, a substantially higher differential could be justified.

DALLAS H. FEAY:

Beginning with new issues dated January 3, 1956 our Company revised its gross rates to incorporate the "reduction by size" philosophy in the premium scale. The purpose of this revision was two-fold. First, a comparison of our rates with other nonparticipating companies indicated we were a little high and not in a good competitive position for the larger risks. Second, we wanted to improve the quality of our business through an increase in the average amount per policy. Our desire, then, was to stimulate our agents in the hope that we would increase our production and at the same time improve the quality of the business.

The change we made was an across-the-board reduction for all plans of insurance, except Family Income Riders, 5 Year Term Insurance and Mortgage Protection Plans. These plans were excluded from the reduction because their manual rates assumed a somewhat higher average size than the other plans. No reduction was made for single premium insurance, or immediate or deferred annuities. The change, since we continued the manual rate on the lowest size class and adopted reductions for the larger policies, results in an over-all reduction in our premium income.

The reduction in premium adopted was as shown below:

Size of Policy	Premium per $1,000
$1,000–$2,499	Manual Rate
$2,500–$4,999	Manual Rate minus $0.50
$5,000–$9,999	Manual Rate minus $1.00
$10,000 and over	Manual Rate minus $1.25

In determining the class for a policy with a term rider only the basic policy is considered. The reserves, nonforfeiture values, substandard extras, policy contract forms, and the rate of agent's compensation are the same for all four rate classifications.

Our problem was a little different from the one which Mr. Fassel solved, as we were seeking to derive and justify an adjustment in an existing premium scale for variations in size. Consequently, our approach differed from that of Mr. Fassel. To determine the adjustment for size the Company made an analysis of its issue and renewal expense, excluding commissions, agency expense allowances, premium taxes, advertising expense, executive salaries, and all items which appear to vary by amount of insurance. We were not hesitant about omitting or understating any item, as our aim was to take an extremely conservative view in determining the expected annual savings attributable to size. Fortunately, none of the

gross premiums resulting from these reductions is lower than our net valuation premium and the problem of deficiency reserves did not arise. The method which we used to make this analysis may be illustrated in the following manner:

First a study was made of the issue expense, and for this illustration this expense is assumed to be $15.00 per policy made up of:

Item	Average Cost
1. Medical examination	$ 3.15
2. Urinalysis	1.35
3. Inspection and M.I.B. reports	4.50
4. Salary of underwriter, policy typist and rate clerks	5.50
5. Policy forms, photostats, postage	.50
Total	$15.00

Similarly the annual cost per policy for premium collection and premium accounting was determined, and for the purpose of this illustration such cost is assumed to be $1.50 per year per policy, consisting of:

Item	Average Cost
1. Salaries	$1.30
2. Postage and supplies	.20
Total	$1.50

Assuming the Linton "A" Termination Rates, CSO Mortality, and 3% interest, a set of annuity values was computed. The value for a life annuity to a person age 40 was found to be $12.08, giving $1.24 as the yearly cost for amortization of the issue expense. Adding the annual administration expense of $1.50 gives a total annual cost of $2.74 per policy. This was translated into annual savings per $1,000 of issue as follows:

Size of Policy	Assumed Average	Annual Expense per $1,000	Annual Savings per $1,000
$ 1,000–$2,499	$ 1,750	$1.56	
$ 2,500–$4,999	2,600	1.05	$ .51
$ 5,000–$9,999	5,000	.55	1.01
$10,000 and over	10,000	.27	1.29

The above savings per thousand are applicable to ordinary life and long term endowments. Similar tables were worked up for the other plans of insurance, except that for the term plans we used Linton "C" Termination rate.

Our agents have been slow in taking full advantage of the graduated premium scale, and because of this and the short period of time since the

plan was put into effect I hesitate to make a comparison of the results. However, for what they may be worth, I quote the following statistics, derived from a comparison of the paid-for business placed during the first 9 months of 1955 and 1956.

Class	By Number of Policies		By Amount of Insurance	
	1955	1956	1955	1956
$ 1,000–$2,499	61.3%	61.7%	27.2%	27.3%
$ 2,500–$4,999	23.7	19.5	26.1	20.7
$ 5,000–$9,999	11.3	14.3	24.1	29.6
$10,000 and over	3.7	4.5	22.6	22.4

For the period covered by these data the number of policies placed in 1956 was 5% higher than 1955 as compared to an increase of 10% in the amount of insurance, resulting in an increase of 4½% in the average amount per policy.

For each of the two periods the number of policies not taken was 8.5% of the number written, with the following percentages by size:

Size	1955	1956
$ 1,000–$2,499	5.4%	5.9%
$ 2,500–$4,999	15.3	15.1
$ 5,000–$9,999	8.9	10.4
$10,000 and over	8.6	6.8

Judging from these results, we experience the highest not-taken rate in the $2,500 to $10,000 class, but because of the small numbers involved considerable chance fluctuation may be present.

WILLIAM B. WAUGH:

Mr. Fassel has presented a very timely and informative paper. This subject is becoming increasingly important, and I must congratulate Mr. Fassel on having the courage to set up his ideas as a target on such a controversial matter. In my discussion I will follow the headings in the paper.

Theory

Instead of a quantity discount based on sum assured, a quantity discount based on size of premium is quite possible. If an annual policy fee of $5.00, say, is required, and if the company's average annual premium

per policy is $100.00 then a charge of 5% of premium would recover the amount of money required for per policy expenses. A policyholder paying a $20.00 premium would contribute only $1.00 of the $5.00 required, but a policyholder with a $1,000.00 premium would pay $50.00, so the expenses would average out. Premium quantity discount would be given by varying the loading for per policy expenses by size of premium, *e.g.*, load premiums under $100 by 10%, premiums of $100 to $299 by 5%, and $300 and over by 2%. The Canada Life sells a retirement annuity contract without insurance where the unit policy is for $100 of annual premium, and we expect to introduce, in Canada only, a premium quantity discount for policies on this plan.

Determination of b: Expense Constant per Policy

Mr. Fassel emphasizes that he is finding the expense that is a function of number of policies, even though the relation may be obscured, and that he is not dealing with the breakdown of expenses as commonly made per policy, per $1,000 of sum assured, per $100 of premium, etc. However, he later introduces a premium rate C not varying by policy size, and to get it he likely used an expense per policy, and divided it by some average sum assured. This average sum assured might have been by plan, or by age, or an over-all average. I have inferred that the per policy expense he used here is different from b.

It seems to me that quantity discount essentially separates policies into groups by amount of sum assured, and the average sum assured in each amount group is strictly analogous to the average sum assured by plan or by age which has traditionally been recognized in rate making. It would seem natural to divide the usual expenses per policy by the average sum assured to arrive at the expense charges for each group. In practice, variations by plan and age have been averaged out, but I think that theoretically they should exist.

I therefore question that it is theoretically necessary to determine an expense constant different from the customary one.

Retirement Income Assurance

Since the sum assured increases in the later years, there is the question as to whether some special treatment is required. One company has adopted a different series of differentials for this plan, although the differential is based on the initial sum assured as Mr. Fassel has suggested.

Supplemental Term

The proposed quantity discounts are designed to avoid any change in the nonforfeiture values of the rider. The solution is very ingenious but I

doubt that it is practical to have the premium for the rider varying according to policy plan to which it is added. I question that any discounts for size should be given on additional benefits added to a basic policy because of the reduced expense margins in such premiums. The Standard Nonforfeiture Law was drafted without consideration of quantity discounts, and its unfortunate effect on policies with variable premiums should be eliminated. I believe that legislative action should be taken to remove this unforeseen difficulty.

Pivotal Rate

Mr. Fassel has explained how the pivotal rate is arrived at from an ungraded rate existing before quantity discounts. However, an alternative approach may be desirable if a change in rate structure is to be made at the same time as the quantity discount is introduced. The method might be best explained by an example.

Suppose we intend to introduce quantity discounts at the $5,000 and $10,000 level, that our annual policy expense is $5.00, and that we expect 60% of our business to be sold in the lowest group, 25% in the second group and 15% in the highest group. We estimate that the average sum assured in the three groups will be $1,250, $6,000 and $15,000, and we intend to have a reduction of $2.00 per $1,000 in premium in passing from the first to the second group and a further 50¢ in going to the highest group. We will calculate all premiums using an average sum assured of $5,000, and these premiums will be used for Group II after the adjustment calculated below.

The adjustment to the $5,000 rate to give the pivotal would be obtained by completing the table below:

Group	Sum Assured (1)	Average Sum Assured (2)	$5.00+(2)×1,000 (3)	Rate Differential (4)	(3)−(4) −(3) for Calc. Rate (5)	Percentage of Business (6)	Margin −(5)×(6)
I.....	$ 1,000–$4,999	$ 1,250	$4.00	+$2.00	+$1.00	60%	−$.60
II....	$ 5,000–$9,999	6,000	.83	0	− .17	25	+ .04
III...	$10,000 and over	15,000	.33	− .50	− .17	15	+ .03
Calculated rate.....		5,000	1.00	.....	.....	.....	.....
							−$.53

We must then increase our calculated rates by 53¢ to get the pivotal rates. If we want the calculated rates to be the pivotal rates, a corre-

sponding charge would be made in the dividend formula. This charge would differ slightly from 53¢ because of the effect of expenses that vary as a percentage of premium, and because of differences in payment dates between premiums and dividends.

The method outlined above gives a convenient way of testing whether a radical shift in distribution of business by size has improved the company's margins. For example if it turned out that the proportion of sums assured in Groups I, II and III respectively changed to 30%, 40%, 30%, and the average policies dropped to $1,100, $5,200 and $12,500, a similar calculation would show that the margin per $1,000 had improved by 11¢.

W. MURDOCH STEWART:

Mr. Fassel has done a very complete job in reviewing the various facets of this subject. In line with his research I think it might be well to consider a few of the forces leading to the adoption of a variation in rate by size of policy.

In the first instance I believe we may dwell profitably on the history of this extension of our business. Twenty-five years ago, in the larger companies at least, I believe that $1,000 was the minimum for an Ordinary policy. Apparently, many of the companies which have adopted the new system retained the $1,000 minimum. In fact, the new tool may have enabled them to do so. At the same time the number of policies for larger amounts was not nearly so great as today. Consequently, it would appear that today Ordinary companies are attempting to cover a wider range in size of policy than formerly. This in itself would tend to lead to a variation in price depending on size.

There are many more insurance companies operating today than there were 25 years ago. It is my feeling for this and other reasons that the competition among salesmen is greater. At least this is what our sales force tells us. The public, furthermore, is more insurance-minded than at any time in the past. Therefore, it would seem that greater competition and a more alert public are now more potent forces in demanding greater equity in the price of insurance.

Mr. Fassel has made reference to the fact that expenses per policy are higher than formerly and that mortality is much lower than formerly. Perhaps this point could be emphasized to a greater extent. At age 35 for an Ordinary life policy the net level premium on the American Experience table and 2½% interest is $22.37 per thousand. On the basis of the CSO Table and the same rate of interest the corresponding figure is $20.50. On the basis of the X_{14} Table the figure is $17.31. The percentage reduction between the American Experience and the X_{14} is 27%. On the other hand

we may refer to Mr. Fassel's paper to observe the increase in the overhead cost per thousand over recent years. In 1940 his figure per thousand is $4.59 and in 1955 the corresponding figure is $7.82. This is an increase of 70%. A point worth noting in this comparison is that the increase in expenses is of much more relative importance than the effect of decrease in mortality, at least for permanent plans. In any event these forces act in opposite directions and again tend to enhance the desirability for variation of price depending on size of policy.

It is interesting to observe that the development under consideration apparently has been perfected in countries which are not recognized as being so insurance-minded as they are in America. In our case it is an example of where arbitrary governmental restrictions, even though implied, apparently operated to the detriment of the industry's service to the public.

From an examination of the table in Mr. Fassel's paper I get the impression that the policy fee method is used when only a limited number of so-called bands are desired. There are fewer bands shown for companies listed in section B of Appendix 1. Also, as shown at the end of section A, where a company uses both the policy fee method and the quantity discount method the former is used for the smaller sized policies. This would seem to lead to the conclusion that the policy fee method is satisfactory only when a company is dealing primarily in smaller sized policies.

It is of interest to note the way in which the quantity discount method is being operated. It is set up so that there is no variation in rate for the medium sized policy. The smaller ones have to pay an extra amount and on the other hand the mass buyer of larger amounts gets a recognized advantage. I suppose there is some psychology to this. No one if he can avoid it likes to pay "something extra." Therefore, this would seem to be an incentive to raise the sights in the case of the buyers of smaller amounts.

Mr. Fassel in his paper arrives at an average value for b of $7.50 per thousand. In the past year we had to make similar calculations in the preparation of our new nonparticipating rates. This figure for the average plan is consistent with those which we used. Inasmuch as this figure can vary substantially by plan, some companies may wish to recognize this distinction.

I think it might be well to remember that we are not on uncharted ground. In this instance we have the previous experience of certain European countries which we can adapt to our situation. This aspect may be of particular help in approaching the problems associated with the transition from one method to the other.

In conclusion, it would seem that this is just another accomplishment of the industry toward providing greater service to the insuring public.

(AUTHOR'S REVIEW OF DISCUSSION)

ELGIN G. FASSEL:

The thorough discussion that my paper has received is gratifying and I wish to thank those who participated.

The paper deals with the principle of rate variation by policy size, showing its application objectively to expenses as they are. As for the subjective question of what the expenses ought to be, this is outside the scope of the paper. If, for example, commissions are graded by policy size, the expense assessment rates would be different than if commissions were not so graded, but the principles of the expense assessment would not be affected.

Regarding the comments as to Item (*e*) in Appendix 2, I may say that the figures given are for a general agency company which, so far as I know, is the only one that makes no agency expense allowances. For this company, which operates nationally with numerous general agencies providing both agency supervision and premium collection and other policy services, Line 23 of Annual Statement Page 5 (General insurance expenses), contains scarcely any field expense, the latter being almost entirely contained in Line 21 (Commissions). For branch office companies the field expense is included in Line 23. Averaging the five leading branch office companies which operate nationally with numerous branches, Annual Statement Exhibit 5 shows a ratio of branch office general expense to home office general expense of about 80% if agency supervision is included, and about 65% if agency supervision is excluded. These figures compare with 6,545,000/9,093,026 = 72%, from Appendix 2. This indicates that the method of Appendix 2 is reasonably successful in extracting from the commission item, for general agency companies, the field expense required for our analysis, which for branch office companies is more directly obtainable from the form of the Annual Statement.

As for the references in the paper to fragmentation of the insurance and to mammoth policies, we should note that it is because of the vastness of the number of policies making up a company's business that such a large force of agents is required. The home office agency department function is to generate the solicitation of new business, chargeable, of course, as new business expense along with solicitors' commissions. To generate the solicitation of new business means to recruit, train and activate solicitors; where the force of solicitors is large, it means to recruit,

train and activate supervisors in the field who in turn recruit, train and activate solicitors. In the assumed case of mammoth policies the force of solicitors is small and can be supervised directly in the exercise of the home office agency department function, eliminating the intermediate field (general agency) supervisory function. Thus, agency supervision in the field, which, of course, is part of the cost of procuring new business, is directly related to the policy size factor, and this relationship is bound in the long run to have its effect on costs.

I do not see how size recognition need result in unreasonable charges to some particular plan-age-size class. Is it not true that the existing practice is to charge a certain total expense to the total insurance in each particular plan-age class, according to a rate per $1,000 which is constant within the particular class? Assuming that the total now charged to each such class is correct, the proposal is merely that the rate for the class, instead of being constant, should vary with recognition of policy size. Of course, as suggested in the paper, there are practical considerations to be taken into account in the small policy range.