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Defined Contribution Plans and Equitable Distribution

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As a consulting actuary who works with attorneys in matrimonial actions, I am frequently confronted with the problem of computing the account balance in a defined contribution plan where the marriage occurred subsequent to the date of entry into the plan. Typically in this situation, the pre-marital contributions plus investment earnings thereon must be excluded from the value of the participant's account balance for equitable distribution purposes.

Before embarking on any computations in the above situation, I try to explain to the attorney that there are two ways to compute the value of this marital asset.

In my judgment, the theoretically correct method is to ascertain the actual contributions over the marriage up to the cut-off date and then add investment earnings at the actual rates earned by the plan over the intervening period. Alternatively (and this should give the same result) we can subtract from the account balance at the cut-off date, the account balance at the date of the marriage brought forward to the cut-off date at the actual investment rates earned by the plan in this intervening period (this, of course, assumes loans are disregarded). This approach is simple to describe and understand, but often difficult (or impossible) to accomplish because the approach requires all the account statements between the marriage date and the cut-off date in order to compute the actual earned investment rates. I call this the exact method.

The second method is simple to describe and understand and even easier to carry out and that is to multiply the account balance at the cut-off date by a coverture fraction—the numerator being service between the marriage date and the cut-off date with the denominator being all service while a plan participant as of the cut-off date (if there are loans outstanding, a decision must be made as to which of the parties is responsible for such loans). I call this the pro-rata method.

Having described both methods, the next question from the attorney is “which is more beneficial to my client?” My response is that I don’t know and in any case that is not a concept that enters my computations. I emphasize that I am an expert, not an advocate. If the past statements are available (and often they are), I will use what I consider to be the theoretically correct approach. If not, there is no choice but to use the pro-rata approach.

Recently I tried to look at the mathematics of each method to see how they compare. My simplified analysis is as follows:

Definitions and Notations:

Salary at date of hire	=	S
Years from plan entry to cut-off date	=	N
Years from plan entry to marriage	=	n
Annual rate of pay raises	=	R
Annual rate of investment earnings	=	R + a
Rate of contribution-payable in a single amount at the beginning of each plan year	=	c %

Total accumulated value of contributions at cut-off date

$$\left(\frac{c}{100}\right)(S)\left[(1+R+\alpha)^{N}+(1+R)^{1}\ (1+R+\alpha)^{N-1}+(1+R)^{2}\ (1+R+\alpha)^{N-2}+\cdots+(1+R)^{N-1}\right]$$

$$=\left(\frac{c}{100}\right)(S)(1+R+\alpha)\left[(1+R+\alpha)^{N-1}+(1+R)^{1}(1+R+\alpha)^{N-2}+\cdots+(1+R)^{N-1}\right]$$

$$=\left(\frac{c}{100}\right)(S)(1+R+\alpha)\left[\frac{(1+R+\alpha)^{N}-(1+R)^{N}}{(1+R+\alpha)-(1+R)}\right]$$

$$=\left(\frac{c}{100}\right)(S)\left(\frac{1+R+\alpha}{\alpha}\right)\left[(1+R+\alpha)^{N}-(1+R)^{N}\right]$$

Therefore, on a pro-rata basis, the amount available for equitable distribution purposes =

$$\left(\frac{N-n}{N}\right)\left(\frac{C}{100}\right)(S)\left(\frac{1+R+\alpha}{\alpha}\right)\left[(1+R+\alpha)^N-(1+R)^N\right] \qquad \text{...I}$$

Accumulation of contributions over the marriage

$$\left(\frac{c}{100}\right)(S)\left[(1+R)^n(1+R+\alpha)^{N-n}+(1+R)^{n+1}(1+R+\alpha)^{N-n-1}\right.$$
$$\left.+(1+R)^{n+2}(1+R+\alpha)^{N-n-2}+\cdots+(1+R)^{N-1}(1+R+\alpha)\right]$$
$$=\left(\frac{c}{100}\right)(S)(1+R)^n(1+R+\alpha)\left[\frac{(1+R+\alpha)^{N-n}-(1+R)^{N-n}}{(1+R+\alpha)-(1+R)}\right]$$
$$=\left(\frac{c}{100}\right)(S)(1+R)^n\left(\frac{1+R+\alpha}{\alpha}\right)\left[(1+R+\alpha)^{N-n}-(1+R)^{N-n}\right] \qquad \text{...II}$$

We have two expressions for the value of this plan for equitable distribution purposes. The question now is how do they relate? Put another way, under what conditions does I exceed (II), (I) equals (II), (I) fall short of (II)?

First note that if a = 0 and R is a constant, then the two expressions are equal. Proof of this is to take the limit of each as a 0 using L' Hospital's rule or what is easier, evaluate the two series putting a $\longrightarrow$ 0.

Each becomes: (N-n) $\left(\frac{c}{100}\right)$ (S) $(1+R)^N$.

If $\alpha > 0$ and R is a constant, then I > II

And $\alpha < 0$ and R is a constant, then I < II.

Proof is as follows:

$$\text{I} \gtrless \text{II if } \left(\frac{N-n}{N}\right)\left(\frac{c}{100}\right)(S)\left(\frac{1+R+\alpha}{\alpha}\right)\left[(1+R+\alpha)^N-(1+R)^N\right]$$
$$\gtrless \left(\frac{c}{100}\right)(S)(1+R)^n\left(\frac{1+R+\alpha}{\alpha}\right)(1+R+\alpha)^{N-n}-(1+R)^{N-n}$$

or $$\frac{1}{N}\left[\frac{(1+R+\alpha)^{N}-(1+R)^{N}}{\alpha}\right] \gtrless \frac{(1+R)^{n}}{N-n}\left[\frac{(1+R+\alpha)^{N-n}-(1+R)^{N-n}}{\alpha}\right]$$

Note that the left hand side is the arithmetic mean of the following N quantities:

$(1 + R + a)^{N-1}$, $(1 + R + a\lambda)^{N-2}$ $(1 + R)$, $(1 + R + a\lambda)^{N-3}$ $(1+R)^{2}$,

…. $(1 + R + a)^{1}$ $(1 + R)^{N-2}$, $(1 + R)^{N-1}$

The right hand side is the arithmetic mean of the following (N – n) , quantities:

$(1 + R + a)^{N-n-1}$ $(1 + R)^{n}$, $(1 + R + a)^{N-n-2}$ $(1 + R)^{n+1}$, $(1+ R + a)^{N-n-3}$ $(1 + R)^{n+2}$,

…. , $(1 + R + a)^{1}$ $(1 + R)^{N-2}$, $(1 + R)^{N-1}$

Note that (N-n) quantities in the right hand side are the last (N-n) quantities in the left hand side. What are left over are the first n quantities in the left hand side.

These are: $(1 + R +a)^{N-1}$, $(1+ R + a)^{N-2}$ $(1 + R)$, …. , $(1 + R)^{N-n}$ $(1 + R + a)^{n-1}$

Now if $a > 0$ these quantities will produce a larger arithmetic mean on the left hand side than the right hand side with the converse if $a < 0$.

The conclusion is that if $a > 0$ then I > II.

And if $a < 0$ then I < II.

Two examples will illustrate this:

	Example A	**Example B**
S	$40,000	$40,000
N	9	9
n	2	2
R	4%	4%
α	3%	-2%
c	15%	15%
Expression I (Pro-Rata)	$69,098.98	$51,787.99.
Expression II (Exact)	$67,089.31	$53,265.42

Of course R is never a constant from year to year, and a can vary quite wildly.

Here is a better proof showing that the Pro-Rata Method gives a larger value than the exact method if a > 0 and vice-versa.

Pro-Rata gives

$$\left(\frac{N-n}{N}\right)\left(\frac{c}{100}\right)(S)\left(\frac{1+R+\alpha}{\alpha}\right)\left[(1+R+\alpha)^N-(1+R)^N\right] \qquad \text{...I}$$

$$\left(\frac{c}{100}\right)(S)\left(\frac{1+R+\alpha}{\alpha}\right)(1+R)^n\left[(1+R+\alpha)^{N-n}-(1+R)^{N-n}\right] \qquad \text{...II}$$

Exact Method gives

We want to show that for a > 0, I > II.

$$f(\alpha)=\frac{N-n}{n}\left[(1+R+\alpha)^N-(1+R)^N\right]-(1+R)^n\left[(1+R+\alpha)^{N-n}-(1+R)^{N-n}\right]$$

$$f'(\alpha)=(N-n)(1+R+\alpha)^{N-1}-(N-n)(1+R)^n(1+R+\alpha)^{N-n-1}$$

$$=(N-n)(1+R+\alpha)^{N-n-1}\left[(1+R+\alpha)^n-(1+R)^n\right]$$

therefore, $f'(\alpha)=0$ if $\alpha=0$

$$f''(\alpha)=(N-n)\left[(N-1)(1+R+\alpha)^{N-2}-(N-n-1)(1+R)^n(1+R+\alpha)^{N-n-2}\right]$$

at $\alpha=0$ $\quad f''(\alpha)=(N-n)\left[(N-1)(1+R)^{N-2}-(N-n-1)(1+R)^{N-2}\right]$

$$=(N-n)\left[n(1+R)^{N-2}\right]$$

$$>0$$

Define f (a) as:

Therefore, a = 0 gives a minimum value for f (a) and this minimum is zero.

Therefore, for a > 0, f (a) > 0 and this tells us that for a > 0, the Pro Rata Method gives a larger value than the Exact Method.