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MINIMUM STATUTORY NONFORFEITURE VALUES FOR RETIREMENT ANNUITY CONTRACTS

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INTRODUCTION

SEVERAL excellent papers which discuss the application of the adjusted premium method in the calculation of minimum statutory cash values for insurance policies have been published. However, no papers to my knowledge have been published with respect to the calculation of minimum statutory nonforfeiture values for annuity contracts with stipulated annual payments. This paper is prepared with the thought that students and others interested in this subject might welcome a presentation of the application of the New York law to a form of retirement annuity contract which has the following characteristics:

(1) a level gross annual premium payable from the date of issue to the date of retirement;
(2) early optional retirement dates at which time the cash value may be applied under settlement options;
(3) a cash value scale defined in terms of the accumulation with interest only of assumed net premiums; and
(4) a death benefit scale which provides for the payment of the gross premiums paid or the cash value, whichever amount is the greater.

A perusal of the New York law as it applies to the level premium retirement annuity form of contract would indicate that, in part, it effectively provides:

(1) in event of default in any stipulated payment, the company must grant a paid-up nonforfeiture benefit;
(2) any cash surrender value allowed must be at least equal to the present value of any paid-up nonforfeiture benefit available on the date of default;
(3) any paid-up nonforfeiture benefit shall be such that its present value on the date of default shall not be less than the excess, if any, of the then present value of the future guaranteed benefits which would have been provided by the contract over the then present value of the adjusted stipulated payments;
(4) the present value on the date of issue of the adjusted stipulated payments shall be equal to the present value of the future guaranteed benefits, plus 20% of the adjusted stipulated payment for the first contract year, plus 2% of the adjusted stipulated payment for the first contract year for each year over five during which stipulated payments are payable;

(5) all present values and adjusted stipulated payments shall be computed on the basis of the 1937 Standard Annuity Mortality Table (or such table with the age of the life set back not more than three years as specified in the contract) with the rate of interest not exceeding $3\frac{1}{2}\%$ per annum as specified in the contract;

(6) in the determination of minimum values, annuity payments shall be deemed to commence at the latest date permitted by contract, provided however that such date shall not be later than the contract anniversary nearest the annuitant's seventieth birthday.

The points listed above indicate that the adjusted stipulated payment, which is to be used to define minimum nonforfeiture values, is a unique function which is dependent not only on the age at issue but also on the particular cash value scale under consideration since the particular scale determines the policy benefits.

STIPULATED ADJUSTED PAYMENT DEFINED

One essential problem confronting a company which desires to adopt a scale of cash values is to determine whether or not legal requirements will be satisfied. Let us consider the statutory formula which will define the adjusted stipulated payment for a particular form of retirement annuity contract with a level annual gross premium of $1, with an age at retirement not greater than 70 and with cash value and death benefit scales of the type heretofore discussed. We will further assume that the rate of interest is level, that the interval between the date of issue and the latest permissible date of retirement is a whole number of years and that contractual values are based on the 1937 Standard Annuity Table.

Let: x = age at issue;

n = number of years from date of issue to latest date of retirement;

b = number of years during which the death benefit is greater than the cash value;

${}_tu$ = contract cash value at end of tth contract year;

${}_tP$ = contract net premium assumed payable at the beginning of the tth contract year per $1 of level gross annual premium;

i = rate of interest ($\not> 3\frac{1}{2}\%$) stated in contract for purpose of defining the cash value;

P_x^a = adjusted stipulated payment for the particular issue age and scale of benefits under consideration;

${}_tu_x^a$ = minimum statutory cash value at end of tth contract year, hereinafter called the "adjusted" cash value;

E_n = .2 when $n \leq 5$, or $= .1 + .02n$ when $5 < n \leq 70 - x$.

Case I—When ${}_tP$ is a variable (the general case)

The stipulated adjusted payment P^a_x when the assumed contractual net premiums vary with respect to duration is defined as follows:

$$P^a_x(\ddot{a}_{x:\overline{n}|} - E_n) = {}_nu \cdot A_{x:\overline{n}|}^{\;\;1} + (IA)^{\;1}_{x:\overline{b}|} + \sum_{r=b+1}^{n} {}_ru \cdot \frac{C_{x+r-1}}{D_x} \qquad (1)$$

where

$${}_tu = \sum_{r=1}^{t} {}_rP(1+i)^{t-r+1}.$$

Case II—When ${}_tP = {}_2P$ for $t = 2$ to n (a more realistic case)

A company may wish to establish the tth year cash value equal to:

$${}_tu = {}_1P(1+i)^t + {}_2P \cdot \ddot{s}_{\overline{t-1}|} \qquad (2)$$

$$= {}_nu \cdot v^{n-t} - {}_2P \cdot \ddot{a}_{\overline{n-t}|}. \qquad (2a)$$

Equation ($2a$) expresses ${}_tu$ in the form of the present value of the maturity value (discounted with interest only) less the present value of the contract net level renewal payments. The value of P^a_x for this case emerges from the reduction of equation (1) to:

$$P^a_x(\ddot{a}_{x:\overline{n}|} - E_n) = (IA)^{\;1}_{x:b} + {}_bu \cdot A_{x:b}^{\;\;1} + {}_2P \cdot {}_{b|n-b}\ddot{a}_x. \qquad (3)$$

For the purpose of this paper, the definitions of the "adjusted" stipulated payment given above for Cases I and II suffice to provide illustrations of the application of the principles involved. The remainder of this paper will be limited to a more detailed examination of Case II.

EXCESS INITIAL EXPENSE MARGIN FOR CASE II

Let us attempt to answer this query: "What is the minimum value of ${}_1P$ which may be used for the assumed contract first year net premium to define the tth year contract cash value ${}_tu = {}_1P(1+i)^t + {}_2P \cdot \ddot{s}_{\overline{t-1}|}$ by a company which desires to use a selected value of ${}_2P$ for the assumed contractual level renewal net premiums?"

First consider durations $t > b$ where the "adjusted" cash value is defined prospectively as:

$${}_tu^a_x = {}_nu \cdot A_{x+t:\overline{n-t}|}^{\;\;\;\;1} + \sum_{r=t+1}^{n} {}_ru \cdot \frac{C_{x+r-1}}{D_{x+t}} - P^a_x \cdot \ddot{a}_{x+t:\overline{n-t}|}. \qquad (4)$$

If we set ${}_ru = {}_nu \cdot v^{n-r} - {}_2P \cdot \ddot{a}_{n-r}$, equation (4) reduces to

$${}_tu^a_x = {}_tu - (P^a_x - {}_2P)\,\ddot{a}_{x+t:\overline{n-t}|}. \qquad (4a)$$

Equation (4a) reveals that the contract cash value ${}_tu$ will be at least equal to the "adjusted" cash value ${}_tu_x^a$ provided ${}_2\mathrm{P} \ngtr \mathrm{P}_x^a$. Hence, if P_x^a is to be equal to the selected value of ${}_2\mathrm{P}$ then the minimum value of ${}_1\mathrm{P} = {}_1\mathrm{P}_x^{\min}$ for $b < t \leq n$ will emerge from equation (3) as:

$$ {}_1\mathrm{P}_x^{\min} = \frac{{}_2\mathrm{P}\{\ddot{a}_{x:\overline{b}|} - E_n - \ddot{s}_{\overline{b-1}|}\cdot \mathrm{A}^{\;1}_{x:\overline{b}|}\} - (\mathrm{IA})^{\;1}_{x:\overline{b}|}}{(1+i)^b \mathrm{A}^{\;1}_{x:\overline{b}|}}. \tag{5} $$

Consider now durations $0 < t \leq b$ where the "adjusted" cash value, which is defined prospectively in the law, may be expressed retrospectively as follows:

$$ {}_tu_x^a = \frac{\mathrm{P}_x^a(\mathrm{N}_x - \mathrm{N}_{x+t} - E_n\mathrm{D}_x) - (\mathrm{R}_x - \mathrm{R}_{x+t} - t\mathrm{M}_{x+t})}{\mathrm{D}_{x+t}}. \tag{6} $$

If ${}_tu_x^a$ is set equal to ${}_tu = {}_1\mathrm{P}(1+i)^t + {}_2\mathrm{P}\cdot \ddot{s}_{\overline{t-1}|}$ we will obtain the following value of P_x^a from equation (6):

$$ \mathrm{P}_x^a = \frac{\{{}_1\mathrm{P}(1+i)^t + {}_2\mathrm{P}\cdot \ddot{s}_{\overline{t-1}|}\}\mathrm{D}_{x+t} + (\mathrm{R}_x - \mathrm{R}_{x+t} - t\mathrm{M}_{x+t})}{\mathrm{N}_x - \mathrm{N}_{x+t} - E_n\mathrm{D}_x}. \tag{7} $$

If we then equate the value of P_x^a in equations (3) and (7), we will obtain the minimum value of ${}_1\mathrm{P} = {}_1^t\mathrm{P}_x$ for $0 < t \leq b$ to be the following:

$$ {}_1^t\mathrm{P}_x = {}_2\mathrm{P}\cdot {}_t^bF_x^n + {}_t^bG_x^n \tag{8} $$

where:

$$ {}_t^bF_x^n = \frac{(\ddot{s}_{\overline{b-1}|}\mathrm{D}_{x+b} + \mathrm{N}_{x+b} - \mathrm{N}_{x+n})\,{}_tf_x^n - \ddot{s}_{\overline{t-1}|}\mathrm{D}_{x+t}\cdot {}_nf_x^n}{{}_t^bH_x^n} $$

$$ {}_t^bG_x^n = \frac{(\mathrm{R}_x - \mathrm{R}_{x+b} - b\mathrm{M}_{x+b})\,{}_tf_x^n - (\mathrm{R}_x - \mathrm{R}_{x+t} - t\mathrm{M}_{x+t})\,{}_nf_x^n}{{}_t^bH_x^n} $$

$$ {}_t^bH_x^n = (1+i)^t\mathrm{D}_{x+t}\cdot {}_nf_x^n - (1+i)^b\mathrm{D}_{x+b}\cdot {}_tf_x^n $$

$$ {}_tf_x^n = \mathrm{N}_x - \mathrm{N}_{x+t} - E_n\mathrm{D}_x\,. $$

It may be demonstrated that equation (8) conveniently reduces to equation (5) when $t = b$. Therefore, if a company wishes to use a selected value of ${}_2\mathrm{P}$, it must use a value of ${}_1\mathrm{P}$ not less than the maximum value of ${}_1^{t}P_x$ as determined from equation (8) for $0 < t \leq b$ within the range of issue ages for which a common set of cash values is to be granted.

CONCLUSION

In closing, the writer wishes to remind the reader that the foregoing discussion is directed primarily to a form of annuity under which contractual values are based on the 1937 Standard Annuity Table. If contractual values are to be based on any other table, further considerations are introduced in the determination of minimum nonforfeiture values.

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