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JOINT-AND-SURVIVOR ANNUITIES FOR THE UNIFORMED SERVICES: LEGISLATIVE HISTORY AND APPLICATION OF UNIVAC TO ACTUARIAL PROBLEMS INVOLVED

ROBERT J. MYERS, EDWARD H. FRIEND,
AND FRANCES E. HOLBERTON*

THE rapid development of electronic computing devices in recent years has been of great interest to the life insurance industry. Actuaries of several companies have been following these developments closely, and the Society has had several meetings devoted to this subject. Recently enacted legislation making joint-and-survivor annuities available on an elective basis for retired members of the uniformed services is of interest not only because of the subject matter itself but also because the many factors required for the several complex options provided were computed by the UNIVAC. In fact, the rigid time schedule necessary to get the program into operation would never have been met, or even closely approximated, if it had not been for the UNIVAC. It is believed that this was the first application of large-scale electronic computing machinery to actuarial problems in the United States[1]—and, quite obviously, will not be the last.

This paper first discusses the history of the legislation and, in particular, its final form. Then it deals with the specific actuarial assumptions made and the formulas developed for applying these assumptions for ready adaptability to the UNIVAC. Finally, the specific programming procedures involved in using the UNIVAC are discussed.

I. LEGISLATIVE HISTORY, PROVISIONS, AND ACTUARIAL ASSUMPTIONS

Legislative History

The uniformed services (Army, Navy, Air Force, Marine Corps, Coast Guard, Coast and Geodetic Survey, and Public Health Service) have for many years had relatively simple noncontributory pension plans

* Mr. Myers prepared Part I; Lt. (j.g.) Friend of the U.S. Navy, a student of the Society, prepared Part II; and Mrs. Holberton, Systems Engineer for the Navy Department and not a member of the Society, prepared Part III.

[1] R. L. Michaelson, in his paper "Large-Scale Electronic Digital Computing Machines" (*JIA* LXXIX, 274), mentions that LEO, a much smaller machine than UNIVAC, was used to compute the two-life annuity values for the $a(55)$ tables, using seven different interest rates.

which, generally, contain similar provisions. The amount of the pension is $2\frac{1}{2}\%$ of base pay at time of retirement (*i.e.*, excluding allowances for dependents, living expenses, hazardous duty, etc.) per year of service not in excess of 30 years. In general, nondisability retirement is not dependent upon age but rather upon length of service (at least 20 years is required but, in effect, voluntary retirement on the part of the individual cannot occur until 30 years). Disability retirement can occur at any age or length of service; the amount of the pension is the $2\frac{1}{2}\%$ formula or the percentage of disability (at least 30% but not more that 75%) times base pay, whichever is larger.

As to survivor benefits prior to this legislation, a variety of forms existed, including a gratuity of 6 months' pay for death in active service, the life insurance (maximum of $10,000) under the Veterans Administration programs (United States Government Life Insurance for World War I, National Service Life Insurance for World War II, and Servicemen's Indemnity insurance since the Korean conflict), the military service wage credits under the Old-Age and Survivors Insurance program for service in and after World War II, and the Veterans Administration compensation for surviving dependents. In addition, reserve personnel during peacetime have the protection of the Federal Employees Compensation Act which is, in effect, workmen's compensation for Federal civilian employees. For personnel retired in good health who have Servicemen's Indemnity insurance (rather than USGLI or NSLI), survivor benefits cease at the date of retirement except for small and highly restricted Veterans Administration pensions and except for the possibility of converting SI to a nonconvertible term plan at relatively high premium rates.

Thus, survivor benefits for the uniformed services, considering both those in active service and those who have retired, are not integrated. Except for the FECA benefits for reserves, the monthly survivor benefits are uniform amounts and bear no relation to the previous earnings of the deceased member or, in some instances, to the number and type of dependent survivors. In 1947, this matter was considered by an interservice committee, and in 1948 a bill (H.R. 6953) was introduced. Legislative action was, however, held in abeyance because of appointment of the Hook Commission, which did not, as it turned out, consider this subject. In 1950, another bill (H.R. 8035) was introduced and hearings held thereon, but no action was taken because of doubts as to actuarial soundness as well as the complexity of the provisions. In 1951, revised bills (H.R. 5169 and H.R. 5594) were introduced and hearings held.

All these bills would have provided (1) contributory benefits on a very broad type of 1-year term, group insurance basis for members in active

service and (2) joint-and-survivor annuity options for retired members. At the same time certain existing survivor benefits which do not properly meet the dependency problems arising (*i.e.*, the FECA benefits applicable to reserves and National Service Life Insurance) would have been eliminated. The active service benefits would have been paid for completely by the members and would have supplemented the veterans compensation benefits (paid for by the Government) so as to produce reasonably adequate survivor protection varying by pay level and dependency status. The active service benefits would have consisted of small lump-sum death payments and monthly survivor benefits for widows, children, and dependent parents, the amounts thereof being based on the grade of the deceased member. Contributions, too, would have varied with grade, an attempt being made to vary them so that, considering age, marital, and dependency composition, each grade would be self-supporting to about the same relative degree.

Concurrently, however, other survivor benefit legislation was being enacted, such as providing, without charge to servicemen, the $10,000 of insurance under the Veterans Administration in place of National Service Life Insurance, extending the OASI wage credits from the end of World War II through 1953, and increasing veterans pensions and compensation. As a result, much of the justification for active duty survivor benefits as contained in these bills was removed, and the Congressional committees concerned then limited their attention to providing joint-and-survivor options for retired personnel. To assure that the test of actuarial soundness would be met, consultations were held in 1952 with representatives of various life insurance organizations, including the actuarial committee of the Life Insurance Association of America. Finally, in 1953, a revised version was introduced as H.R. 2521, and then somewhat modified in H.R. 5304, on which public hearings were held. Descriptive committee reports were prepared as the legislation was being enacted by Congress (House Report No. 496 and Senate Report No. 672, 83rd Congress, 1st session).

Provisions of Final Legislation

The Uniformed Services Contingency Option Act of 1953 (Public Law No. 239, 83rd Congress, 1st session) was approved by President Eisenhower on August 8. Under this act, personnel of the uniformed services may, by electing a reduced amount of retired pay during their lifetime, provide benefits for their surviving widow and children.

A variety of options, both as to amount and beneficiaries selected, is available. The member may elect to have his retired pay reduced so that

the survivor annuity may equal $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ of such reduced amount and may also decide how it is to be divided. Under Option 1 he may elect to protect his wife, who will, on his death, receive an annuity until she remarries or dies. He may elect to protect his children (Option 2), who will be paid the annuity until all of them reach age 18, or marry, or die; the annuity is payable beyond age 18 to a mentally defective or physically handicapped child who has been continuously in that condition since age 18. Under Option 3, the member would protect his family—wife and children—and the annuity would be payable as long as there is an eligible person in the family. Under these three options the reduction in retired pay continues for the life of the retired member.

Three other options are also available. Annuities under so-called Options 1-4, 2-4, and 3-4 are payable under the same terms and conditions as in Options 1, 2, and 3 respectively, with the additional provision that no further deductions are to be made from the member's retired pay after there is no longer an eligible beneficiary remaining.

The member may elect any one option or else Option 1 or 1-4 combined with Option 2 or 2-4. Options 1 and 2, when combined, produce a different result from that under Option 3, and of course similarly for Options 1-4 and 2-4 combined as compared with Option 3-4. Under Options 1 and 2 combined, the total survivor annuity payments are less when the widow is no longer eligible, since payments then are only on the portion paid under Option 2; the total payments are also reduced when no child is eligible, since payments then are only under Option 1. On the other hand, under Option 3, the same amount is payable as long as there is any survivor eligible to receive an annuity.

The amount of the reduction in retired pay to provide for these benefits is to be determined by the actuarial equivalent method—that is, the deductions made from the individual's retired pay are intended to be enough, on the average, to meet the cost of the benefits accruing to his survivor. The congressional intent, therefore, is that no additional cost to the Government should be involved in the aggregate (other than administrative expense), although there may be a different incidence of cost. No separate funds or accounts will be established for these survivor benefits, but rather the appropriations requested from Congress to meet the cost of retired pay will reflect the effects of this law. Such appropriations will be lower by the amount of the reductions made in the retired pay of those electing an option and will be higher by the amount of the survivor annuities payable.

The new law establishes a Board of Actuaries to select the appropriate mortality and remarriage tables to be used, which can be changed from

time to time (the changed tables applicable, of course, only to retirements occurring after the change). This Board, by law, consists of the Government Actuary in the Treasury Department, the Chief Actuary of the Social Security Administration, and an actuary appointed by the President of the United States from the membership of the Society of Actuaries[2] (Walter Klem was so appointed). The Board also will advise in the administration of the law. Each uniformed service will administer the provisions for its own retired members, using the same reduction factors. The interest rate is specified by law to be 3% or such other rate[3] as the Secretary of the Treasury may specify in the future on the basis of considering the average yield on marketable long-term obligations of the United States.

The reduction in the member's retired pay varies with the proportion that the survivor annuity is of the retired pay and with his age and the ages of the wife and youngest child. In addition, the reduction varies with whether the annuity is to depend upon the survival of the wife, or of the children, or of both the wife and children, and whether the full amount of the retired pay is to be restored when there is no longer an eligible beneficiary.

Different factors have been prescribed for disability and nondisability retirants, with a further differentiation between the nondisability retirants on the roll on the effective date (November 1, 1953) or within 6 months thereof and those coming on the roll in the future, who must make an advance election.

Consider the operation of these provisions for a "typical" case of an individual aged 55 who is a future nondisability retirant and who has a wife aged 50 for whom he has elected a survivor annuity of $\frac{1}{2}$ the reduced retired pay. Under Option 1 the reduction would be about 14.0%—the man would receive 86% of full retired pay, and the surviving widow 43%. Under Option 1-4—the "restoration" option—the reduction would be 15.2%, or only slightly larger. If he also has a child aged 10, Option 3, providing for both the widow and child, would effect a reduction amounting to only about 0.03% more than that under Option 1. Under Option 2, providing only for the child, the reduction would be only 1.1%. If the member chooses a survivor annuity of $\frac{1}{4}$ or $\frac{1}{8}$, the reduction would be correspondingly less. For example, under Option 1 with a survivor annuity of $\frac{1}{8}$, his retired pay would be reduced about 4%, so that he would receive about 96% of full retired pay, and his widow about 12%. For disability

[2] This is the first time that the Society has been referred to in Federal legislation.

[3] As will be indicated later, changes in the interest rate have a relatively small effect on the reduction factors.

retirants the reductions are somewhat greater than for nondisability retirants. For those nondisability retirants on the roll before May 1954, the reductions fall between those for the other two categories. Thus, if the typical case described above were a disability retirant, under Option 1 the reduction would be 21.1%; if a nondisability retirant, on the roll before May 1954, the reduction would be 18.6%.

In general, the election must be made before the individual completes 18 years of service and is effective only if there is one or more of the designated types of beneficiaries living on the date of his subsequent retirement. Thus, an unmarried individual upon attainment of 18 years of service could elect Option 1, but this election would have no effect if he was not married when he retired. Similarly, for a man with a wife and children when he elected Option 3, but whose children at the time of his retirement were all over age 18, the election would then be under Option 1. Members retired for disability before they have had 18 years of service may make the election at the time of retirement. Those in active service for more than 18 years and those already retired on the effective date had to make their election before May 1954.[4] An election once made may be modified or revoked before retirement, but the action is effective only if the member does not retire within the next 5 years after the modification or revocation has been requested. A revocation, once made, cannot itself be revoked and is final after the expiration of the 5-year period.

The advance-election provisions are designed to avoid the adverse selection that would occur if all individuals were permitted to make the election at the time of retirement. The usual practice (such as under group annuities) with this type of benefit is either to require election 5 years before the individual reaches retirement age or else to require proof of good health at retirement. Such procedure is not practical here because retirement from the uniformed services is influenced more by length of service than by attainment of a fixed minimum age, such as 65.

When an individual's retired pay is suspended because of return to active duty, civilian Federal employment, or election to take veterans benefits, the amount of the reduction in retired pay because of election of an option must be deposited monthly in the Treasury. Of course, during this time the survivor protection continues. When a member retired for disability is removed from the roll because of recovery (within 5 years), a refund is made of the excess (if any) of the total reduction in retired pay over the cost of the term insurance protection he had.

[4] Public Law No. 346, 83rd Congress, 2nd session changed this provision to permit those in active service for more than 18 years on the effective date to make the election at any time before December 1954.

Actuarial Assumptions Made by Board of Actuaries

Two separate sets of reduction factors were developed for nondisability retirants. Definite antiselection can be expected for those on the roll before May 1954 because no advance election is required, whereas for those coming on the roll thereafter there will be an adequate advance election period. For disability retirants no such separation seemed necessary. In all instances, age nearest birthday at the time the member retires is to be used, except that for those on the roll before May 1954, age nearest birthday at effective date of election is used. As specified by law, the interest rate used is 3%. It was decided that the actuarial formulas would be developed on a continuous basis since, although monthly payments are involved, this would facilitate the work and would involve little loss of accuracy.

Mortality data available included a table for regular army officers retired on account of disability, based on experience in 1936–42 (prepared by E. A. Lew) and a study of naval officers retired in 1946, with the experience carried through 1952. The mortality rates of the American Experience Table gave a reasonably close fit for the disability retirants. The military experiences were not comparable to the life insurance experience of about a century ago underlying the American Experience Table, but as it turns out, the military experience comes close to the rates of that table, which can, as a matter of expediency, be used.

For nondisability retirants coming on the roll after April 1954, the Annuity Table for 1949 was made applicable. Current mortality investigations, such as the one mentioned previously, indicated very low mortality for this group (in fact, as much as 25–30% lower); because of the advance election generally necessary for this category it was believed that it would be safe to use this table. However, for nondisability retirants through April 1954 antiselection is likely to be prevalent, so that a mortality basis considerably more stringent was used. Up to age 50, the a-1949 Table was selected as being reasonable. At and after age 75, the same basis as for disability retirants (namely, the American Experience Table) was used, since little reason seemed to exist for differentiating between persons at the same advanced age now on the roll as to whether some years before they had been classified as disability or nondisability retirants. Between ages 50 and 75, there was proportional grading in between these two tables (*i.e.*, the mortality rate for age 51 is 24/25 of a-1949 and 1/25 of American Experience, etc., until at age 74 the mortality rate is 1/25 of a-1949 and 24/25 of American Experience).

For the relatively few female retired members who might elect an

option, a 5-year rate-down is used, along with a corresponding 5-year rate-up for the male survivor beneficiary.

The reduction factors as finally presented are rounded to the nearest .01%. Furthermore, a minimum is prescribed for the reduction factor (applicable, of course, only for options involving a child but not the spouse), namely, .0040 or 0.40% for the $\frac{1}{2}$ option, with the minimums for the $\frac{1}{4}$ and $\frac{1}{8}$ options deriving therefrom.

The reduction factor for age 90 is used for retired members over age 90. Where the wife is more than 15 years older than the retired member, the reduction factor is the same as for wife 15 years older. Where the wife is more than 25 years younger than the retired member, the reduction factor is obtained by second difference extrapolation (using the values for 25, 24, and 23 years), but in no case is the reduction factor to be less than for wife 25 years younger.

No differentiation in mortality was made for wives and widows for the three retirement categories. In theory, perhaps, wives of those on the roll at enactment should have been assumed to have lower mortality than future wives and widows because of the antiselection possible as a result of the election being an immediate one. This element, however, is offset to an appreciable extent, particularly for the nondisability retirants, by conservatism in the mortality bases used for retired individuals. The *a*-1949 Table for females was first considered, but it was decided that this would be too conservative a basis since the women involved would not be as select lives as the men retiring in the future for nondisability reasons. In addition, it was believed that some of the safety factor in the *a*-1949 Table should properly be eliminated. Adjustment factors were derived from the figures in the last column of Table 8 of the Jenkins-Lew paper (*TSA* I, 382). The decrease in mortality for the *a*-1949 Table as against the 1943 Experience Table ranges downward from about 30% at the youngest ages to 15% at age 70, with a further decrease thereafter to roughly 5% at the highest ages. The 15% factor was used for ages 70 and under, and an empirically decreasing factor after age 70 which would vanish at age 85.

As to remarriage rates for widows, no military experience upon which to make a decision was obtainable. Under such experiences as are available (such as OASI, Railroad Retirement, and workmen's compensation), the recent experience has been considerably higher than the basic rates of the American Remarriage Table. Furthermore, a general qualitative consideration of the termination of widows' benefits payable by the Veterans Administration seems to indicate high remarriage rates among widows of retired military personnel. The basic rates of the American Remarriage

Table were empirically adjusted upward by 35% at attained age 15, decreasing this factor by 1% for each older age, until for ages 50 and over no adjustment was made. The American Remarriage Table begins at age 18 but was extended by taking the same basic remarriage rates for ages 15–17 at widowhood as for age 18 at widowhood. The factors applied to the basic remarriage rates are by attained age; the factor of 130% for age 20 is applied not only for the first year for a woman widowed at age 20, but also for the second year for a woman widowed at age 19, and for the third year for a woman widowed at age 18.

For children, marriage and mortality probabilities are ignored since these would have relatively little effect. Accordingly, the number and ages of all the children in the family need not be considered, but only the youngest child. This omission is an approximate offset against the cost of paying benefits to children over age 18 (when mentally incapacitated or physically disabled). A number of methods were considered to allow for this latter element where the child is over age 18 or nearly age 18 when the member retires (such as a fixed minimum percent for the reduction factor, or some grading up of the resulting factors). No appropriate method was believed possible for determining reduction factors where there is an eligible child over age 18 at the time the individual retires, because of the very sharp boundary that would be set up. Thus an individual with a child aged 17 normally would receive a very small reduction; there would be no possible way of finding out then if this child would be eligible for payments beyond age 18, in which case a much larger reduction theoretically should be made. Thus as a practical solution it was deemed necessary to consider all cases involving a child aged $16\frac{1}{2}$ or more as being exactly age 17 (and thus generally subject only to the minimum reduction factor). Accordingly, a "free ride" is, of necessity, given where a disabled child aged 18 or over is present at time of retirement.

The Board of Actuaries, in considering the reduction factors developed by UNIVAC, noted what at first seemed to be an anomaly. For a disability retirant aged 55, with a wife aged 50 and a child aged 10, the reduction factor for Option 3-4 for the $\frac{1}{2}$ basis is slightly *lower* than for Option 1-4 (.2223 versus .2232). This occurs despite the fact that under the former the survivor annuity might be paid longer (namely, to the child after the death of both parents). This additional possibility of survivor benefits is, however, more than offset by the possibility that the reduction in retired pay (restored under both cases if the member outlives his beneficiaries) would continue longer (until the child attains age 18), and in a sizable amount (as compared with Option 2-4), after the wife dies and only the child is eligible—and eligible at that for benefits

of only a relatively small value. Of course, as would logically be expected, the reduction factor for Option 3 in this case is larger than for Option 1 (.2116 versus .2111).

Comparison of Reduction Factors

A brief comparison of the reduction factors under Option 1 (widow only) will now be made with factors developed from other bases.

Table 1 makes a comparison with the factors used in the actuarial equivalent bases formerly in use under Railroad Retirement and Civil Service Retirement. On the whole, the Uniformed Services figures are consistent with the available figures from the other two plans. For non-

TABLE 1

COMPARISON OF REDUCTION FACTORS UNDER VARIOUS BASES FOR OPTION 1, UNDER ½ BASIS

Male Age	Uniformed Services			Combined Annuity*	McClintock†
	Disability	Nondisability through April 1954	Nondisability after April 1954		
	Wife Same Age				
30	.106	.060	.050	‡	‡
40	.128	.088	.071	.088	‡
50	.157	.128	.098	.115	‡
60	.200	.180	.125	.145	.160
	Wife 5 Years Younger				
30	.114	.072	.060	‡	‡
40	.144	.105	.086	.106	‡
50	.185	.155	.121	.140	‡
60	.241	.221	.159	.182	.191
	Wife 10 Years Younger				
30	.119	.082	.068	‡	‡
40	.158	.120	.099	.124	‡
50	.209	.179	.142	.166	‡
60	.278	.258	.192	.218	.220

* Formerly used by Railroad Retirement.

† Formerly used by Civil Service Retirement.

‡ Not available.

disability retirants after April 1954, the Uniformed Services factors are more favorable (*i.e.*, a lower reduction). This is to be expected because the male life under the Uniformed Services is assumed to have relatively lower mortality than the female life, whereas for the other two plans 100% of both the male and female rates were used in conjuction with each other. For the nondisability retirants through April 1954, the Uniformed Services factors are somewhat higher, while of course for disability retirants there is even more of a difference.

Table 2 makes a comparison of the Uniformed Services reduction factors with those computed under various other actuarial assumptions.

TABLE 2

COMPARISON OF REDUCTION FACTORS UNDER VARIOUS BASES FOR OPTION 1, UNDER ½ BASIS, HUSBAND 5 YEARS OLDER THAN WIFE

BASIS				MALE AGE		
Male Mortality	Female Mortality	Remarriage	Interest	40	50	60
Combined Annuity	Combined Annuity		3%	.106	.140	.182
Standard Annuity	Standard Annuity		3%	.105	.138	.177
Standard Annuity	Standard Annuity		2¼%	.117	.150	.189
Standard Annuity	Standard Annuity		2½%	.113	.146	.185
Standard Annuity	Standard Annuity	100% American	2½%	.106	.140	.180
U.S. White 1939–41	U.S. White 1939–41	150% American	3%	.099	.135	.174
U.S. White 1939–41	Standard Annuity	100% American	3%	.126	.169	.221
U.S. White 1939–41	Standard Annuity	100% American	2½%	.136	.179	.230
300% U.S. White 1939–41	Standard Annuity	100% American	2½%	.279	.364	.457
American Experience	Standard Annuity	100% American	2½%	.147	.186	.243
Uniformed Services*						
Disability				.144	.185	.241
Nondisability through April 1954				.105	.155	.221
Nondisability after April 1954				.086	.121	.159

* See text for specific mortality, remarriage, and interest bases.

Joint-and-survivor annuity options under many group annuities are currently based on the 1937 Standard Annuity Table for both lives (sometimes with a 1-year setback, which produces slightly smaller reduction factors) with either 2¼% or 2½% interest. For nondisability retirants after April 1954, the factors are always somewhat lower than under any other basis (for reasons given previously). For nondisability retirants through April 1954, the factors are somewhat higher than the "group annuity" ones but are close to those for several tables which either do not include remarriage or else use somewhat higher relative mortality

for men than for women. The factors for the disability retirants compare reasonably closely with those on several of the bases where higher relative mortality is used for men, but they are by no means as large as in the one instance involving very high mortality for men (namely, 300% of population mortality). Where all elements except interest are the same, the reduction factors vary only slightly (by about 1 percentage point for an interest differential of $\frac{1}{2}\%$), being slightly lower for a higher interest rate.

Where all elements are identical except for the use of remarriage rates, the reduction factors vary only slightly, there being more effect, of course, for the younger ages—especially so for the youngest ages (not shown), which are applicable to disability retirants only.

II. EXPLOITATION OF THE UNIVAC

The UNIVAC Enters the Picture

According to a prominent representative of Remington Rand, the UNIVAC probably caused more controversy and received more publicity from having succumbed to human error on Election Night 1952 than it would have if it had been successful. The publicity paid off through the foresightedness of Navy Captain Joseph B. Hoyt who was given responsibility for carrying out the legislative provisions of the new law. He remembered the UNIVAC and realized that it would be the answer to the mammoth job of developing the necessary reduction factors.

The calculations would undoubtedly have consumed five man-years of work utilizing desk machines and sophisticated actuarial methods. After four weeks of programming preparation UNIVAC generated the calculations in forty hours of brute force computing time, employing first principle concepts only. Final output was in the form of 120 tables with a total of approximately 400,000 four-character entries. Forty of these tables were for all physical disability retirants, forty for nondisability retirants through April 1954, and forty for subsequent nondisability retirants.

A notable fact is that programming preparation which consumed the four-week period preceding the actual computer run was all but completed when the mortality and remarriage assumptions were submitted by the Board of Actuaries.

The Act had been approved on August 8, 1953, but by September 1, with the third member of the Board not yet appointed by President Eisenhower and the official tables not available, it became apparent that even a large staff of clerks with desk machines could not be depended upon to complete the calculations by the October 15 deadline (to allow sufficient time for the plan to go into effect in November). The evaluation

job was turned over at this time to the Applied Mathematics Laboratory of the Navy's David Taylor Model Basin in Washington, D.C., which had just received a UNIVAC. A staff of four programmers was assigned to the task.

Flow charting and coding commenced immediately and by September 21 the first programming routines were ready. Official tables were still not available and consequently the U.S. White Life Tables, 1939–41 and the 150% American Remarriage Table were used as dummy inputs to the machine for the purpose of "proving" the programming routines. Results were checked against key desk computations worked out at the Model Basin and against a limited number of commutation functions and annuity values which had been calculated a month earlier in Capt. Hoyt's office.

The Board convened on September 30 and authorized *three* sets of tables, one for each of the three retirement groups. This tripled the original job with regard to input and tabular output but required no additional programming. It was a simple matter to prepare three different sets of input tapes and run the complete job step by step in three different sections.

On October 1, the very next day, reduction factors for all three categories of Option 2 had been calculated and printed. As UNIVAC continued to finish the job, several entries were calculated at the Equitable Life Assurance Society independently and each agreed exactly with its equivalent computed value. All 120 tables, including two sets of conversion factors (see next section), were delivered on October 13, two days ahead of schedule.

Derivation of Reduction Factors and Conversion Equation

Stated in symbolic form the retiring member is provided with the right to elect in lieu of a full annual retirement annuity of 1, a reduced annuity, $1 - W$. The withheld annuity per unit of full retired pay, or the reduction factor, W, then purchases for a spouse (Option 1 and 1-4), youngest child (Option 2 and 2-4), or spouse and youngest child (Option 3 and 3-4) a reversionary annuity in the amount of a fractional portion of $1 - W$. The annual deduction, W, is effective either for the life of the retired member (Options 1, 2, and 3) or until the termination of eligibility of the reversionary annuitants (Options 1-4, 2-4, and 3-4).

In general for all six options:

$$W_K \cdot F = K(1 - W_K) \cdot G \qquad (1)$$

where F is the unit present value of the withheld annuity; G, the unit present value of the reversionary annuity; and K, the fraction applied

to $1 - W_K$ for determination of the amount of the annual reversionary annuity payments. For any one of the six options, $K = \frac{1}{2}, \frac{1}{4}$, or $\frac{1}{8}$. Consequently a total of 18 choices are provided.

For exposition purposes let x indicate the age of the male retiring member, y the age of his spouse, and z the age of his youngest child. All primed annuity values will indicate that spouse remarriage decrements as well as life decrements are considered. Childhood mortality and marriage rates are neglected. The composition of F and G is as follows for each of the six options.

Option 1

$F = \bar{a}_x$ (Deduction period is for life of retired member) (2)

$G = \bar{a}'_{x|y}$ (Beneficiary is spouse) (3)

Option 1-4

$F = \bar{a}_{xy}$ (Deduction period is for joint life of retired member and spouse) (4)

$G = \bar{a}'_{xy}$ (Beneficiary is spouse) (3)

Option 2

$F = \bar{a}_x$ (Deduction period is for life of retired member) (2)

$G = \bar{a}_{\overline{18-z}|} - \bar{a}_{x:\overline{18-z}|}$ (Beneficiary is child) (5)

Option 2-4

$F = \bar{a}_{x:\overline{18-z}|}$ (Deduction period is for life of retired member or until child loses eligibility) (6)

$G = \bar{a}_{\overline{18-z}|} - \bar{a}_{x:\overline{18-z}|}$ (Beneficiary is child) (5)

Option 3

$F = \bar{a}_x$ (Deduction period is for life of retired member) (2)

$G = \bar{a}_{\overline{18-z}|} - \bar{a}_{x:\overline{18-z}|} + \bar{a}'_{x|y} - \bar{a}'_{(x|y):\overline{18-z}|}$ (7)

(Beneficiaries are child or spouse as long as at least one is eligible)

Option 3-4

$F = \bar{a}_{xy} + \bar{a}_{(y|x):\overline{18-z}|}$ (8)

(Deduction period is for life of retired member or until both child and spouse lose eligibility)

$G = \bar{a}_{\overline{18-z}|} - \bar{a}_{x:\overline{18-z}|} + \bar{a}'_{x|y} - \bar{a}'_{(x|y):\overline{18-z}|}$ (7)

(Beneficiaries are child or spouse as long as at least one is eligible)

Inserting into formula (1) the appropriate values of F and G for each of the six options and solving for $W_{1/2}$, we have the results found in Chart I.

It should further be noted that formula (1) leads to

$$W_K = \frac{2W_{1/2}}{\frac{1}{K} - \left(\frac{1}{K} - 2\right)W_{1/2}}. \tag{9}$$

For any option, this conversion equation permits expedient derivation of values of $W_{1/4}$ and $W_{1/8}$ from those for $W_{1/2}$.

Application of UNIVAC to the Problem

The center of the UNIVAC system is a high-speed electronic digital computer which will do arithmetical or logical operations such as multiplying numbers from its 1,000 "word" (12 characters per word) memory

CHART I

(Parentheses are employed to indicate identical expressions in numerator and denominator.)

Option 1:

$$^{1}W_{1/2} = \frac{\alpha}{\beta + \alpha}$$

Option 1-4:

$$^{1\text{-}4}W_{1/2} = \frac{\alpha}{\gamma + \alpha}$$

Option 2:

$$^{2}W_{1/2} = \frac{(\delta - \phi)}{\beta + (\delta - \phi)}$$

Option 2-4:

$$^{2\text{-}4}W_{1/2} = \frac{\delta - \phi}{\delta + \phi}$$

Option 3:

$$^{3}W_{1/2} = \frac{(\delta - \phi + \alpha - \eta)}{\beta + (\delta - \phi + \alpha - \eta)}$$

Option 3-4:

$$^{3\text{-}4}W_{1/2} = \frac{(\delta + \alpha - \eta) - \phi}{\gamma - \theta + (\delta + \alpha - \eta) + \phi}$$

The Greek letters are defined as follows:

$$\delta = \bar{a}_{\overline{18-x}} \qquad \alpha = \bar{a}'_{xy}$$

$$\beta = 2\bar{a}_x \qquad \theta = 2\bar{a}_{xy:\overline{18-x}}$$

$$\phi = \bar{a}_{x:\overline{18-x}} \qquad \eta = \bar{a}'_{(xy):\overline{18-x}}$$

$$\gamma = 2\bar{a}_{xy}$$

at the rate of 465 per second or make comparisons at the rate of 2,740 per second. Further, the system utilizes up to ten independent magnetic tape units, called Uniservos, for input to and output from the Central Computer's memory at speeds of 720 words per second. One tape may be feeding input to, and another accepting output from, the Central Computer while internal operations are in progress. Conversions of information to and from magnetic tapes are processed by peripheral equipment not under control of the Central Computer. Tapes are prepared for Uniservos by Card-to-Tape Converters or by Unitypers which operate much like standard typewriters. For the reverse procedure the system employs Uniprinters which will convert edited magnetic tapes to printed matter.

In general, to complete any type of problem on the UNIVAC, excluding peripheral processing, (1) a program or instruction tape is read into the Central Computer, (2) one or more input data tapes are read into the Central Computer, (3) the data are processed internally, (4) output (which may later be input) is sent back to magnetic tape coincidently with more of the activity described in (1), (2), and/or (3) in an amount depending on the type of problem and design of the program.

For computer activity involving considerable data handling with small amounts of internal processing, a great deal of time is consumed locating and reading required data into the high speed memory from tape. Consequently, for the most efficient solution of a problem it becomes important to utilize that approach which will necessitate a minimum of tape handling even at the expense of a large increase in internal operations.

Recognizing this principle and the fact that seven specific actuarial functions are sufficient to express all of the options, it became apparent that the most expedient approach would be to (1) evaluate these seven functions independently for all necessary values of x, y, and z by first principles, (2) place the evaluations of each of the seven functions on tapes in the arrangements and positions that would permit the algebraic calculations of $W_{1/2}$ to be effected in a desirable order for final tabular output with a minimum of tape movement, (3) using these function tapes, complete the calculations for each option independently, and (4) edit the results for conversion to printed form. Output from the Uniprinters was planned to be acceptable in quality and format for photographic reproduction and subsequent distribution to field accounting and disbursing offices.

In planning the attack on this problem, as on any other problem involving massive data output, it was essential to consider the desired end result first and plan backwards to the beginning. It would be difficult to

evaluate the seven functions in an optimum manner without determining the best arrangement of the function evaluations for later use in assembling the option calculations. The programming staff was asked to arrange its work so that printed tabular output would appear in formats as shown in Chart II. Accordingly, Greek letter data and reduction factors were

CHART II

FORMAT I

Age of Spouse Minus Age of Member

Age of Member at Retirement	−25	−24	−23	. . . . .	−1	0	1	2	. . . .	15
18										
19										
20										
21										
.										
.										
.										
.										
.										
.										
.										
95										
96										

To display Options 1, 1-4, 3, and 3-4, thirty-eight tables with Format I are necessary. Options 1 and 1-4 call for one table each but Options 3 and 3-4 call for eighteen tables each (one for every age of child between 0 and 17).

FORMAT II

Child's Age at Retirement

Age of Member at Retirement	0	1	2	3	4	5	. . . . .	17
18								
19								
20								
21								
.								
.								
.								
.								
.								
.								
.								
95								
96								

Two tables with Format II will display Options 2 and 2-4.

computed and assembled on magnetic tape to accommodate this restriction (see Part III—*Detailed Analysis*).

The question of whether or not to make use of commutation functions in evaluating the seven Greek letters had to be faced almost at once. Three factors led to a quick decision in favor of the "first principles" approach.

First, computing commutation functions would put an extra burden on the programming staff, since one more step would have to be flow charted and coded. There would be more control problems, more chances for logical error, and consequently more valuable time lost. It should be clear that it is just as easy for the programmer to code *multiplications* for successive summation evaluations as to code *additions* for successive summation evaluations. Furthermore, it is sometimes easier to program a complicated set of summations of products than to program a "table look-up" of a set of answers from an externally stored mass of data.

Second, even if commutation functions were utilized and the programming burden out of the way, it would not be surprising to find that the time consumed by computer control and calculation of the extra step, along with the time consumed by the inevitable extra tape movement for the associated table look-up, would exceed that consumed by a first principles approach.

Third, the required additional tapes would provide more opportunity for operator error in tape handling and manipulation. This is a very delicate matter, and experience has shown that it is most desirable to have as little human intervention in the running of a problem as possible.

Since the programming staff knew very little about actuarial mathematics, it was necessary to state the problem of Greek letter evaluation in basic mathematical terminology. Chart III shows a reproduction of the mathematical definition of η as presented to the programmer. This was easily the most complicated of all the Greek letter functions.

A final significant job, indirectly related to the previous computations, was initiated since the aforementioned reduction factors applied only for $K = \frac{1}{2}$ (see preceding section). A routine was programmed utilizing formula (9), and as a result conversion factors were made available for converting from $K = \frac{1}{2}$ to $K = \frac{1}{4}$ and $K = \frac{1}{8}$ with four-place accuracy.

III. PROGRAMMING PROCEDURES

General Approach

Limitations on time suggested the partitioning of the problem into easily programmed routines so that members of the David Taylor Model Basin programming staff would be able to work independently on flow

charting, coding, and code-checking. The coding was kept simple to facilitate proving routines on the UNIVAC and kept flexible to handle anticipated alterations in the range of the variables or unforeseen additions to the scope of the program.

Since the UNIVAC is self-checking, it was not necessary to program computational checks; but to insure against errors in operator handling

CHART III

$$\eta = \bar{a}'_{(x|y):\overline{18-z}|}$$

$$= \frac{v^{1/2}}{l_x l_y} \sum_{t=0}^{18-z-1} v^t d_{x+t} l_{y+t+1/2} \bar{a}'_{[y+t+1/2]:\overline{18-z-t-1/2}|}$$

where

$$\bar{a}'_{[y+t+1/2]:\overline{18-z-t-1/2}|} = \tfrac{1}{2}\left(\bar{a}'_{[y+t]:\overline{18-z-t}|} + \bar{a}'_{[y+t+1]:\overline{18-z-t-1}|}\right)$$

where

$$\bar{a}'_{[y+t]:\overline{18-z-t}|} \overset{①}{=} \tfrac{1}{2}\left(1 + v^{18-z-t} \frac{l'_{[y+t]+18-z-t}}{l'_{[y+t]}} \overset{②}{}\right) \overset{③}{} + \frac{1}{l'_{[y+t]}}\left(\sum_{s=1}^{4} v^s l'_{[y+t]+s} + \sum_{s=5}^{18-z-t-1} v^s l'_{y+t+s}\right) \overset{④}{}$$

where

$$l_{y+t+1/2} = \tfrac{1}{2}\left(l_{y+t} + l_{y+t+1}\right)$$

where

$$d_{x+t} = l_{x+t} - l_{x+t+1}$$

where

$$v = \frac{1}{1.03}, \qquad 18 \le x \le 96, \qquad 15 \le y \le 96,$$

$$15 \le [y] \le 73, \qquad 0 \le z \le 17$$

and $l'_{[y]}$, $l'_{[y]+1}$, $l'_{[y]+2}$, $l'_{[y]+3}$, $l'_{[y]+4}$, l'_y, l_y, and l_x are given in tabular form

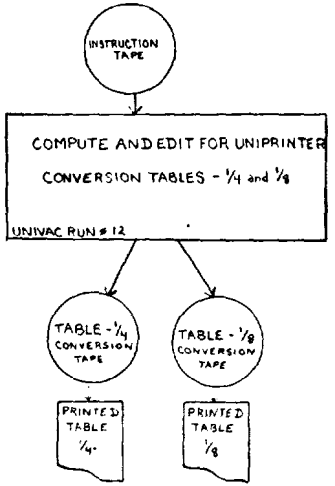
① for $18 - z - t \le 0$, $\bar{a}'_{[y+t]:\overline{18-z-t}|} = 0$

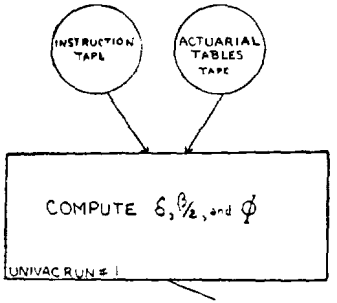
② for $s \ge 5$, $l'_{[y+t]+s} = l'_{y+t+s}$

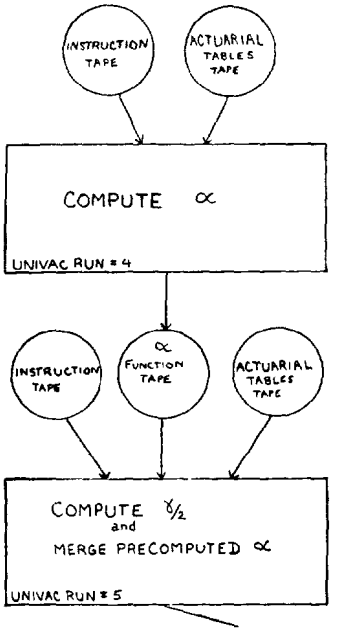
③ for $[y + t] \ge 74$, $l'_{[y+t]} = l'_{y+t}$, $l'_{[y+t]+1} = l'_{y+t+1}$, etc.

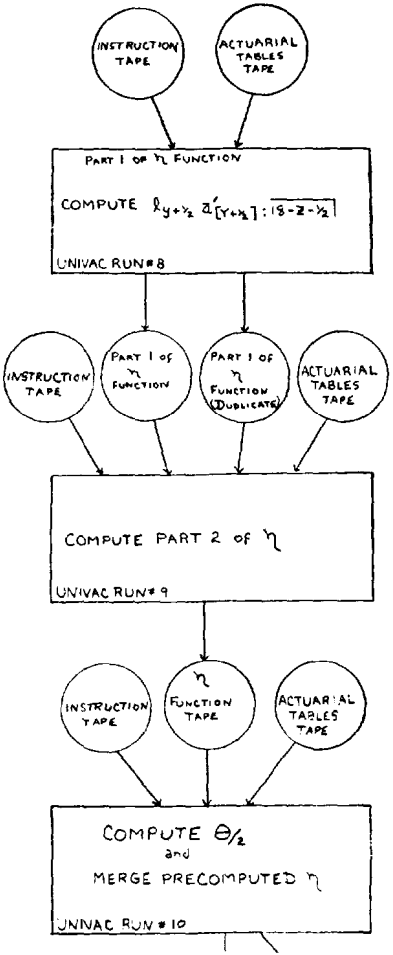
④ for $y + t + s$ and $[y + t] + s \ge 96$, l'_{y+t+s} and $l'_{[y+t]+s} = 0$
for $18 - z - t - 1 \le 4$, the second summation vanishes and the upper limit of the first summation becomes $17 - z - t$

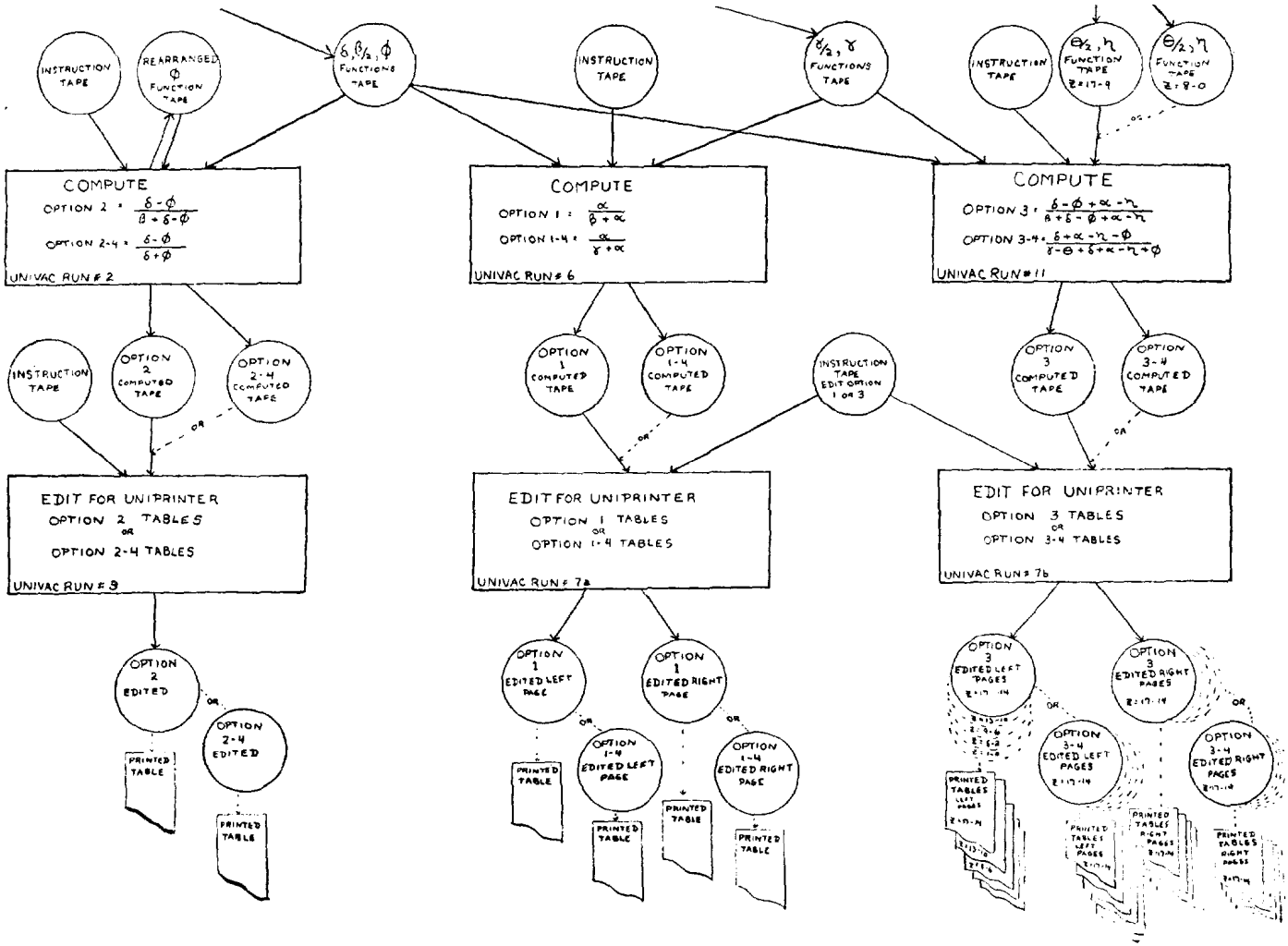
CHART IV


INSTRUCTION TAPE
COMPUTE AND EDIT FOR UNIPRINTER
CONVERSION TABLES - 1/4 and 1/8
UNIVAC RUN # 12
TABLE - 1/4 CONVERSION TAPE
TABLE - 1/8 CONVERSION TAPE
PRINTED TABLE 1/4
PRINTED TABLE 1/8


INSTRUCTION TAPE
ACTUARIAL TABLES TAPE
COMPUTE $\delta, \beta/2$, and ϕ
UNIVAC RUN # 1


INSTRUCTION TAPE
ACTUARIAL TABLES TAPE
COMPUTE α
UNIVAC RUN # 4
INSTRUCTION TAPE
α FUNCTION TAPE
ACTUARIAL TABLES TAPE
COMPUTE $\gamma/2$ and MERGE PRECOMPUTED α
UNIVAC RUN # 5


INSTRUCTION TAPE
ACTUARIAL TABLES TAPE
PART 1 OF η FUNCTION
COMPUTE $l_{y+\frac{1}{2}}\ \ddot{a}'_{[y+\frac{1}{2}]:\overline{18-2-\frac{1}{2}}}$
UNIVAC RUN # 8
INSTRUCTION TAPE
PART 1 OF η FUNCTION
PART 1 OF η FUNCTION (DUPLICATE)
ACTUARIAL TABLES TAPE
COMPUTE PART 2 OF η
UNIVAC RUN # 9
INSTRUCTION TAPE
η FUNCTION TAPE
ACTUARIAL TABLES TAPE
COMPUTE $\theta/2$ and MERGE PRECOMPUTED η
UNIVAC RUN # 10


INSTRUCTION TAPE
REARRANGED Φ FUNCTION TAPE
δ, β/2, Φ FUNCTIONS TAPE
COMPUTE
OPTION 2 = (δ - Φ)/(β + δ - Φ)
OPTION 2-4 = (δ - Φ)/(δ + Φ)
UNIVAC RUN # 2
OPTION 2 COMPUTED TAPE
OPTION 2-4 COMPUTED TAPE
OR
EDIT FOR UNIPRINTER
OPTION 2 TABLES
OR
OPTION 2-4 TABLES
UNIVAC RUN # 8
OPTION 2 EDITED
OPTION 2-4 EDITED
PRINTED TABLE
INSTRUCTION TAPE
α/2, γ FUNCTIONS TAPE
COMPUTE
OPTION 1 = α/(β + α)
OPTION 1-4 = α/(γ + α)
UNIVAC RUN # 6
OPTION 1 COMPUTED TAPE
OPTION 1-4 COMPUTED TAPE
INSTRUCTION TAPE EDIT OPTION 1 OR 3
EDIT FOR UNIPRINTER
OPTION 1 TABLES
OR
OPTION 1-4 TABLES
UNIVAC RUN # 7a
OPTION 1 EDITED LEFT PAGE
OPTION 1 EDITED RIGHT PAGE
OPTION 1-4 EDITED LEFT PAGE
OPTION 1-4 EDITED RIGHT PAGE
PRINTED TABLE
INSTRUCTION TAPE
θ/2, η FUNCTION TAPE
COMPUTE
OPTION 3 = (δ - Φ + α - η)/(β + δ - Φ + α - η)
OPTION 3-4 = (δ + α - η - Φ)/(δ - Θ + δ + α - η + Φ)
UNIVAC RUN # 11
OPTION 3 COMPUTED TAPE
OPTION 3-4 COMPUTED TAPE
EDIT FOR UNIPRINTER
OPTION 3 TABLES
OR
OPTION 3-4 TABLES
UNIVAC RUN # 7b
OPTION 3 EDITED LEFT PAGES
OPTION 3 EDITED RIGHT PAGES
OPTION 3-4 EDITED LEFT PAGES
OPTION 3-4 EDITED RIGHT PAGES
PRINTED TABLES LEFT PAGES
PRINTED TABLES RIGHT PAGES

of tapes, an elaborate set of controls was included in all routines. Each block of words on tape, either utilized or produced by the UNIVAC system, contained identifying information which the computer could examine and verify as correct or incorrect for the particular operation at hand.

As programmers began flow charting and coding some of the Greek letter routines, a process chart or block diagram of the project (see Chart IV) was developed. The problem was separated into twelve routines. Six routines were required to produce the seven Greek letter functions, three routines to compute the six options, two routines to edit the computed data for the Uniprinter, and one routine to compute the two conversion tables. (Further detailed analysis of the first nine routines will be found in the next section.)

The block diagram led to a tentative schedule designed for efficient organization of the staff in flow charting, coding, and code-checking the routines. Considering each routine in order of desirability for proving on the machine, the schedule showed: flow chart starting date, assigned analyst; starting date for coding, assigned programmer; starting date for code-checking, assigned programmer; and code-check completion date. For optimum results it was decided that four members of the staff should do most of the job. Although the schedule was not always accurate and the time for proving routines on UNIVAC not predictable, the estimates were sufficiently close to stay within required time limits.

During the process of proving each routine on the UNIVAC, three and sometimes four persons were busy at the computer to reduce the real time. A UNIVAC operator handled the controls at the console, while the cognizant analyst, programmer, and code-checker together tracked down errors in the routines.

The actual production running of the problem required four persons at the computer constantly. A UNIVAC operator handled the controls and helped change tapes. Two others handled all output tapes and managed the files so that the correct input tapes were placed on the Uniservos for each run. A supervisor was required to direct the operations and keep a detailed log of the progress.

The cost for completing this project was approximately $15,000. Table 3 demonstrates the time expended in carrying it through. It does not include the liaison work, statement of the problem, and other contributions as discussed in Part II of this paper.

Detailed Analysis

To avoid any sorting, rearrangement, or unnecessary scanning of tapes for computed values, the seven Greek letter functions were computed

and stored on tapes in a prearranged sequence dictated by the formats of the final tables (see Chart II).

For all Greek letter functions contributing to reduction factors displayed in Format I, the major classification for sequencing of computed values was the child's age, z; the intermediate classification, the member's age, x; and the minor classification, the spouse's age, y. Further, the range of the minor classification, y, was necessarily dependent on the intermediate classification, x, since $x - 25 \leq y \leq x + 15$ for all tabular entries.

TABLE 3

TIME BREAKDOWN OF PROJECT

Distribution of Total Project Man-Hours*

Operation Performed	Percentage of Total Man-Hours
Block Diagram Analysis and Detailed Flow Charting	10%
Coding and Code-Checking of Routines	33
Proving Routines on UNIVAC	16
Hand Computations for Check Values	4
Production Operation	12
Unityper Preparation of Instruction Tapes and Data Tapes	2
Feeding Two Uniprinters for Final Tabular Output	8
Administration, Report Writing, and Reviewing of Printed Tables	15
Total	100%

Distribution of UNIVAC System Operation Time

Item		Hours
Central Computer and Uniservos		104
Production	($41\frac{1}{2}$)	
Proving Routines	($61\frac{1}{2}$)	
Consolidation of Instruction Tapes for File	(1)	
Unityper		26
Uniprinters		144
Total		274

* Approximately 1300 man-hours between September 1 and October 13, 1953.

Consequently for evaluations of η ($= \bar{a}'_{(x:y):\overline{18-z}|}$) and θ ($= 2\bar{a}_{xy:\overline{18-z}|}$), y varied while x was held fixed, and then x varied while z was held fixed. Similarly, for α ($= \bar{a}'_{x|y}$) and γ ($= 2\bar{a}_{xy}$), y varied while x was held fixed. Since ϕ ($= \bar{a}_{x:\overline{18-z}|}$) was used in both Format I and Format II, it was arranged so that x varied while z was held fixed and then rearranged so that z varied while x was held fixed. In order to take advantage of partial sums, all functions involving z were assembled with z descending from 17 to 0.

Since all seven Greek letter functions were used in the calculation of Options 3 and 3-4, it was necessary to store more than one function on each tape. Holding one of the eight Uniservos in reserve for emergency use, the operation was completed by grouping the data in an expedient manner. The seven functions were separated into three groups as follows: (1) both three-variable functions (*i.e.*, η and θ), (2) both functions involving x and y (*i.e.*, α and γ), and (3) the remaining functions (*i.e.*, δ, β, and ϕ).

The η and α functions were computed independently and the results stored on tape. During the calculation of the θ function, the previously computed values of η were merged with the θ results so that two adjacent entries on the tape, one for η and the other for θ, were related to the same values of x, y, and z. This procedure was repeated for the merging of the α and γ functions.

Earlier, the computation of the η function had been separated into two phases because of its complex nature. A table of appropriate values of $l_{y+1/2}\,\bar{a}'_{[y+1/2]:\overline{18-z-1/2}\rceil}$ had been computed and duplicated many times on two different tapes so that no tape time would be consumed in rewinding and searching during the completion of the second phase.

The δ, β, and ϕ functions had been calculated in a single routine. Since the δ and β functions occupied less than 120 storage locations, they were held in the Central Computer during pertinent computations and the ϕ function values entered into the computer as needed.