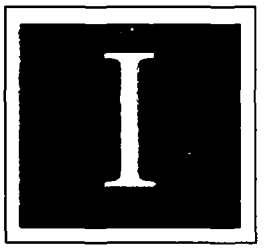


A New Mortality Basis for Annuities

Wilmer A. Jenkins and Edward A. Lew

I. Introduction

That a new mortality basis for annuity premiums and reserves and pension calculations is needed has become increasingly apparent during the last several years. This need has arisen from a growing appreciation of the necessity for appropriate and adequate recognition of (1) changes in mortality rates which have occurred during the years since the 1937 Standard Annuity Table was prepared, and (2) the probability that mortality rates will, as they have for many past decades, follow a long continued, though gradual and frequently irregular, trend to lower levels. The possibility that the age incidence of this trend will change does not diminish its importance.

In its 1948 report[1] on the mortality experienced under individual immediate annuities, the Joint Mortality Committee concluded that the 1937 Standard Annuity Table "is not entirely satisfactory for computing the net annuity values at individual ages because it does not accurately portray the variations in such values by age and sex" and "is unsatisfactory as a standard for mortality investigations without extended analysis." In its 1947 report[2] on the mortality experienced under life income settlement options and individual deferred annuities, the Committee brought out that the 1937 Standard Annuity Table was a more or less deficient measure of current mortality under these contracts. While the use of this table with age setbacks has tended to offset the latter shortcoming in the aggregate, this expedient has resulted in greater distortion of equities by age and sex. It is clear, moreover, that further resort to setbacks does not provide a satisfactory method of allowing for future improvements in mortality.

In the infancy of American annuity business, annuity mortality tables not only made no provision for probable future decreases in mortality, but were usually somewhat out of date and, therefore, probably somewhat inadequate even when published and adopted. Apparently the 1937 Standard Annuity Table was constructed with the objective of injecting some sort of safety margin, which was an advance in actuarial thought.[3] However, studies made by the authors have convinced them that the nature of the problem calls for a more explicit forecasting of future mortality rates. To this end, past mortality trends and their future possibilities and probabilities should be discussed by actuaries, and in the calculation of annuity values there should be included a specific provision for future mortality decreases.

The authors offer this fundamental proposition: the actuary must heed both past history and potential developments, and calculate his annuity values so as to give full recognition not only to the long continued trends in the past towards lower mortality levels but also to the impact of probable further advances in medicine and public health, and to other influences operating to increase longevity. For premium and reserve purposes his annuity values must also include an appropriate and adequate safety margin for mortality fluctuations and like contingencies. He should not and probably cannot avoid revision of his annuity premium rates under new contracts every 5 or 10 years, more or less, even though at the time each scale of rates is adopted proper provision is made for the two factors just mentioned. He should bend every effort and act promptly if, at any time, he realizes that his rates do not make proper provision for these factors; failure to do so can involve and, on occasion, has resulted in large losses.

This paper is written in an attempt to furnish the actuary with a more satisfactory basis for annuity premiums and reserves.

Because no one can read the future and because opinions as to the possible or probable magnitude of future mortality changes doubtless differ, this paper does not propose a new single mortality table; it offers, instead, an up-to-date mortality table together with several sets of adjustments to be used with this table—some necessary but alternate in form and some optional. The alternate adjustments make specific allowance for future decreases in mortality and are, therefore, essential; the actuary may increase or decrease them in his judgment, but they must be included to some extent. The optional adjustments allow for differences in mortality by type of annuity and may not only be varied, but even disregarded, in the actuary's discretion. Because this mechanism differs from the usual mortality table, reference is made in this paper to a new mortality "basis" rather than to a "table."

This paper concerns all kinds of annuities—individual nonrefund and refund, immediate and deferred, annuities; life income settlement options available at death or maturity of life insurance policies and elected by the payee or otherwise; and group annuities before and after retirement. The rather complicated mechanics necessary to apply the mortality basis presented in this paper to group annuities are not developed fully.

Grateful acknowledgment is due the Joint Mortality Committee for furnishing much of their data and to Messrs. Charles M. Sternhell and Tapp S. Taves of the Metropolitan Life Insurance Company for their invaluable assistance with the extensive calculations.

II. Brief Description of New Mortality Basis

Before discussing in detail the new mortality basis offered in this paper, a brief description of its construction may be helpful.

As a starting point, a mortality table was prepared for each sex; these two tables taken together are called the "1943 Experience Table" (Section III). At ages 60 and over this table was based on the Joint Mortality Committee experience under immediate nonrefund annuities, by number of contracts, from 1941 to 1946 anniversaries.[4] At ages 55 and under in the case of males and at ages 50 and under for females this table was based on the intercompany active lives experience under group annuity contracts covering predominantly clerical employees, by lives, for the calendar years 1939, 1940, 1946, and 1947, with an allowance for deaths among "ill-health terminations." A Makeham curve was fitted by the method of moments to the immediate annuity experience at ages 60 to 99, excluding the first policy year, and the group annuity experience was graduated graphically so as to provide a smooth junction at age 60 with the Makehamized mortality rates. First policy year select mortality rates were then added. This table was intended to represent the mortality level existing about 1943 under the kinds of annuities upon which it was based, without any conservatism or allowance for future

mortality decreases. Without adjustment this table is, therefore, unsuitable for other than historical or analytical purposes.

The ultimate death rates in the 1943 Experience Table were then decreased by percentages varying by age and sex, determined so as to reflect conservatively changes in mortality rates that occurred between 1943 and 1949 (Sections IV and V). The ultimate mortality rates so obtained were graduated by Makeham's formula, with modification of the constant A below age 60 for males and below age 50 for females, and first policy year death rates were then added. This pair of graduated tables—one for each sex—is called the "Annuity Table for 1949" and is the principal mortality table on which the new mortality basis rests. It is intended to be a conservative representation of current mortality under the kinds of annuities comprising the basic data of the table. The conservatism involved in this table was not designed to cover probable future decreases in mortality rates and is insufficient to do so for most types of annuity. It is, instead, in the nature of a safety margin of the kind needed, beginning immediately, to cover mortality fluctuations and like contingencies. Proper allowance for future mortality decreases requires, therefore, adjustment of this table.

Alternate adjustments to be applied to the Annuity Table for 1949 in order to introduce an allowance for future mortality decreases are then developed (Sections VI, VII, VIII, IX, X). The more important of these consist of two alternate sets of projection factors. Each set of factors varies by age, by the time that will elapse before annuity payments begin, and by sex. While these projection factors may be adjusted upward or downward by the actuary, as his judgment may dictate, they should not be disregarded unless the actuary decides to assume that the long continued downward trend of annuity mortality rates in the past will not continue and that advances in medicine, public health, and other fields will not operate to decrease future mortality rates. In the opinion of the authors, such a decision would be unwise and likely to prove costly.

Another kind of adjustment, which is optional, can be used to reflect the somewhat different levels of mortality prevailing under different kinds of annuities (Section XI). These adjustments are relatively small except for retired lives under group annuities.

Methods for calculating joint annuities are presented (Section XIV).

Two kinds of adjustment are not dealt with: (1) those which would inject the additional conservatism necessary for participating as contrasted with nonparticipating contracts, and (2) those which would reflect a situation in which the annuity mortality experience of the individual company is known to depart materially from the averages represented by the intercompany experiences upon which the Annuity Table for 1949 was based. The latter are discussed briefly in Section XI.

This paper does not go beyond presentation of the essential tables which, it is hoped, will permit the actuary to calculate what, in his opinion, are proper values of the several types of annuities beginning in various future years. It does not attempt to show the details of precisely how, in practice, these annuity values can be used, in particular for contracts such as retirement income insurances, for life income settlement options, and other special forms of annuities. Thus, instead of presenting a "ready made" suit of clothes, the authors offer a bolt of cloth, shears, needles, etc., with which the actuary can fashion a suit designed to satisfy his requirements. This arrangement leaves, of course, the major decisions in the matter of determining premium rates and reserves up to the individual actuary.

III. 1943 Experience Table

The data presented in the reports of the Joint Mortality Committee in 1947 and 1948[5] provide the most reliable, suitable, and recent information as to the mortality experienced under individual annuities and life income settlements in United States and Canadian life insurance companies. The particular type of annuity selected for the base mortality table is of little importance if reasonably accurate adjustment factors are developed to translate the experience under one type of annuity into that under another. Following tradition, the authors chose the immediate nonrefund annuity experience as a starting point.

The Joint Committee experience on immediate nonrefund annuities runs back to 1931, but it was decided to use only the experience from 1941 to 1946 anniversaries because the volume of data in this experience was deemed sufficient for the purpose at hand and because the experience covered a relatively short period of time. December 31, 1943 being the center of this period, the data may be considered as approximately representative

of the mortality level prevailing in 1943. The experience by number of contracts was selected because the excess of mortality by amounts of annual income was rather small and because the experience by amounts showed considerable fluctuation by age; its reliability is, therefore, less than that of the experience by number of contracts.

For several purposes it seemed desirable, if not essential, that the table extend to ages much younger than those for which there were sufficient data in the Joint Committee immediate annuity experience. In the judgment of the authors, the most suitable material at the younger ages was that derived from the intercompany active lives experience under group annuity contracts covering predominantly clerical employees. This material is by lives. Group annuities provide the only substantial annuity experience available at the younger ages; and it is available separately for each sex. The mortality rates derived from the experience under group annuity contracts covering predominantly clerical employees appeared to be reasonable in relation to both population and insurance death rates. At ages under 30 the mortality rates were quite low in all of the experiences considered by the authors and the differences between them were not large.

It was, of course, desirable that the data at the younger ages, chosen as the basis of the 1943 Experience Table, should cover the same period of years as that represented by the immediate nonrefund annuity experience selected as the basis of the table at the older ages. However, the experience of active lives under group annuity contracts was not prepared for the war years and, therefore, the experience for the calendar years 1939, 1940, 1946, and 1947, centering about the year 1943, was selected as the most suitable alternative.[6]

It was the authors' judgment that, for the purpose of preparing the 1943 Experience Table, the mortality rates among active lives under group annuity contracts should be calculated so as to include the extra mortality that would have been experienced if the "ill-health terminations" had remained in the experience after termination. After trial of several methods of making this allowance, the authors concluded that the mortality among ill-health terminations after termination could reasonably be assumed to follow that which has been observed under group conversions and that an appropriate method for making this allowance was to add the following proportions of ill-health terminations as actual deaths:

Central Attained Age	Percentage of Ill-Health Terminations Included as Deaths*	Central Attained Age	Percentage of Ill-Health Terminations Included as Deaths*
18.....	2.7%	38.....	5.5%
23.....	2.9	43.....	7.1
28.....	3.4	48.....	9.2
33.....	4.3	53.....	13.5

* Derived from the Metropolitan Life Insurance Company's experience under group conversions.

The volume of data at ages under 60 in the Joint Committee experience under immediate nonrefund annuities was not large. In the case of males in the age group 50-59, the mortality was moderately elevated in comparison with the level at neighboring ages. Somewhat the same characteristic has been noted previously in some other annuity experiences.[7] It should be noted, however, that in the experience from 1941 to 1946 anniversaries (see Table 2) the excess deaths at ages 50-59 occurred in only the first three policy years, the somewhat larger experience at durations 4 and over yielding mortality rates which are in conformity with those at neighboring ages. Consequently, the authors decided that the 1943 Experience Table should be based on the immediate nonrefund annuity data at ages 60 and over. Because of the disturbing effects of varying retirement practices, the active lives group annuity experience was not considered to provide a suitable base for the 1943 table at ages over 55 for males or over 50 for females. Accordingly, the 1943 table was based on the active lives group annuity experience below these ages. The gap between the two experiences was smoothly bridged by graphic graduation.

The minimum age of the 1943 table was taken as 15 because the active lives group annuity experience sheds no light on death rates below that age. The table was terminated by arbitrarily increasing the death rates at age 109 from approximately 0.9 to 1.

The 1943 Experience Table is a select and ultimate table with a one-year select period and is presented as Table A in the Appendix. Charts 1 and 2 and Table 1 compare the ultimate mortality rates in this table with those of the 1937 Standard Annuity Table.

CHART 1
COMPARISON OF MORTALITY RATES—MALES

- - - 1937 STANDARD ANNUITY TABLE
— — 1943 EXPERIENCE TABLE ULTIMATE
—— ANNUITY TABLE FOR 1949 ULTIMATE

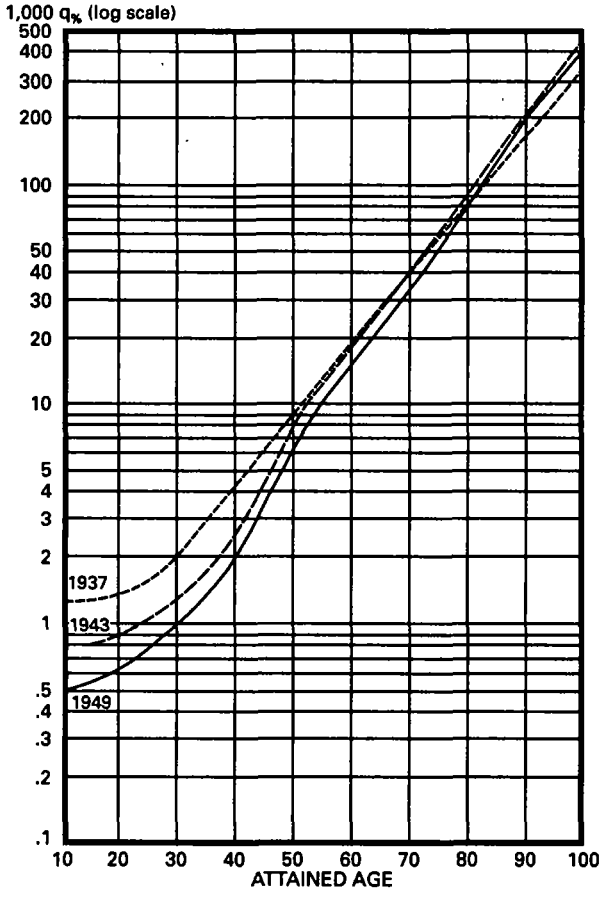


CHART 2
COMPARISON OF MORTALITY RATES—FEMALES

- - - 1937 STANDARD ANNUITY TABLE
— — 1943 EXPERIENCE TABLE ULTIMATE
—— ANNUITY TABLE FOR 1949 ULTIMATE

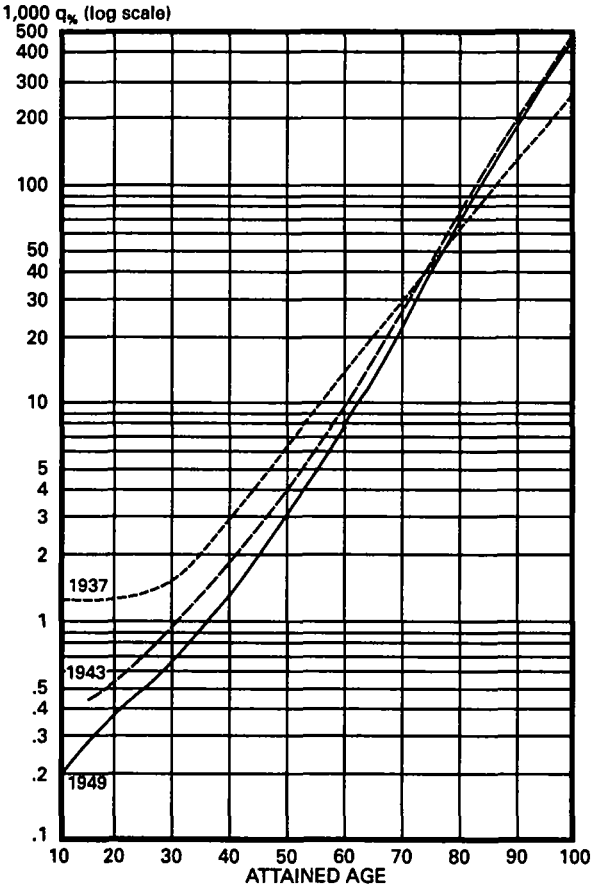


TABLE 1
COMPARISON OF MORTALITY RATES
1943 EXPERIENCE TABLE AND 1937 STANDARD
ANNUITY TABLE

Age x	Males			Females		
	1943 Table Ultimate 1000 q_x	1937 Standard Annuity Table 1000 q_x	Percentage 1943 Table Ultimate q_x Is of 1937 Standard Annuity q_x	1943 Table Ultimate 1000 q_x	1937 Standard Annuity Table 1000 q_x	Percentage 1943 Table Ultimate q_x Is of 1937 Standard Annuity q_x
15	.800	1.262	63.4%	.432	1.257	34.4%
25	1.034	1.561	66.2	.719	1.331	54.0
35	1.779	2.981	59.7	1.266	2.065	61.3
45	4.489	6.362	70.6	2.689	4.356	61.7
55	12.876	13.554	95.0	5.920	9.288	63.7
65	26.959	28.751	93.8	14.940	19.753	75.6
75	60.248	60.464	99.6	41.267	41.758	98.8
85	143.268	124.837	114.8	114.487	87.161	131.4
95	332.413	248.059	134.0	300.501	177.138	169.6

As to the select period, the Joint Mortality Committee showed that initial selection under immediate nonrefund annuities affects mortality rates significantly for two or three years at most. This can be gauged from Table 2, which shows the immediate nonrefund annuity experience from anniversaries in 1941 to anniversaries in 1946 for the first, second, third, and fourth and subsequent policy years on the basis of the 1943 Experience Table (Ultimate). Because the effect of initial selection is most marked during the first policy year and is less pronounced in the second and third policy years and because a one-year select period is very desirable for purposes of calculation, the authors decided that a one-year select period would serve as a happy compromise between fidelity to actual data and practical considerations. In the interests of further simplicity, first policy year mortality rates in the 1943 Experience Table were, at all ages, taken as 75% of the ultimate mortality rates in the case of males and as 50% of the ultimate mortality rates in the case of females. These percentages are indicated as reasonable by Table 2.

The Joint Committee experience under immediate nonrefund annuities from 1941 to 1946 anniversaries brought out clearly that a single mortality table, with an age differential of four or five years at all ages, does not represent closely enough the mortality rates of both sexes. Fully appreciating the great convenience of such an assumption, the authors concluded, nevertheless, that its use for the 1943 Experience Table would distort the data unduly. The extent of this distortion is indicated in Tables 3 and 4. Accordingly, the 1943 Experience Table is presented in the form of two separate tables—one for each sex.

The 1943 Experience Table (Ultimate) was graduated by Makeham's formula at ages 60 and over and graphically below age 60. The Makeham constants for $\text{Colog}_e(p_x) = A + Bc^x$ were as follows:

	Male Table	Female Table
1000 A.....	6.2434455	1.0043832
1000 B.....	.037385118	.013028780
$\text{Log}_{10} c$.....	.042327	.046657

These constants were obtained by equating the first three moments of the actual and expected deaths.

TABLE 2
JOINT COMMITTEE 1941-46 EXPERIENCE UNDER IMMEDIATE NONREFUND ANNUITIES. FIRST, SECOND, THIRD, FOURTH AND SUBSEQUENT POLICY YEARS ON 1943 EXPERIENCE TABLE (ULTIMATE)

Age Group	First Policy Year		Second Policy Year		Third Policy Year		Fourth And Subsequent Policy Years	
	Male	Female	Male	Female	Male	Female	Male	Female
	Ratios of Actual to Expected Mortality (by Number of Contracts) on 1943 Experience Table (Ultimate)							
Under 60.....	115%	74%	170%	96%	263%	118%	93%	93%
60-64.....	89	45	65	79	72	101	104	112
65-69.....	96	56	118	92	83	84	104	98
70-74.....	73	48	92	76	104	77	96	101
75-79.....	83	54	94	68	94	91	103	100
80 and over.....	56	44	72	68	119	79	100	103
All ages.....	80%	51%	94%	77%	106%	86%	100%	102%
	Actual Deaths—Number of Contracts							
Under 60.....	12	13	17	17	25	20	100	127
60-64.....	15	15	11	28	12	37	211	354
65-69.....	35	34	45	64	31	61	460	882
70-74.....	36	36	48	67	55	76	803	1,767
75-79.....	36	33	44	50	48	80	973	2,134
80 and over.....	21	22	33	44	67	65	1,477	3,130
All ages.....	155	153	198	270	238	339	4,024	8,394

This method of graduation produces a good fit to the data and was chosen after a great deal of experimentation. Tests of the graduation are given in Table B of the Appendix. While a Gompertz or Makeham graduation with the same value of c for both sexes would have been of considerable practical advantage in computing joint annuities, trial graduations indicated that the assumption of the same value of c for both sexes did too great violence to the data. Moreover, all Gompertz curves seemed quite unsuitable.

The select annuity values on the 1943 Experience Table in the age range from 50 to 75 are within about 1% of the corresponding experience annuity values calculated by the Joint Mortality Committee from the 1941–46 select nonrefund experience and published in the committee's 1948 report.

Tables 5 and 6 present comparisons of annuity values based on the 1943 Experience Table and the 1937 Standard Annuity Table, the latter with no setback and with setbacks of 1, 2, and 3 years of age.

As previously indicated, the 1943 Experience Table was constructed as a starting point in the authors' investigation of annuitant mortality and was not designed for and should not be used for other than historical or analytical purposes.

TABLE 3
COMPARISON OF MALE AND FEMALE MORTALITY RATES
1943 EXPERIENCE TABLE

Age x	Male 1000 q_x Ultimate	Female 1000 q_{x+5} Ultimate	Percentage Female q_{x+5} Is of Male q_x Ultimate
15.....	800	.557	69.6%
25.....	1.034	.939	90.8
35.....	1.779	1.807	101.6
45.....	4.489	4.042	90.0
55.....	12.876	9.172	71.2
65.....	26.959	24.733	91.7
75.....	60.248	68.927	114.4
85.....	143.268	187.654	131.0
95.....	332.413	464.139	139.6

TABLE 4
COMPARISON OF MALE AND FEMALE ANNUITY VALUES
1943 EXPERIENCE TABLE AT 2½% INTEREST

Age x	Male a_x Ultimate	Female a_{x+5} Ultimate	Percentage Female a_{x+5} Is of Male a_x Ultimate
15.....	29.412	29.624	100.7%
25.....	26.733	26.948	100.8
35.....	23.393	23.657	101.1
45.....	19.380	19.734	101.8
55.....	15.124	15.232	100.7
65.....	10.964	10.464	95.4
75.....	6.980	6.115	87.6
85.....	3.734	2.867	76.8
95.....	1.576	.956	60.7

IV. Statistics on Recent Decreases in Mortality Rates

A table conservatively representing mortality rates prevailing in 1949 was desired as a basis upon which could be superimposed appropriate allowances for future decreases in mortality. While these future decreases must necessarily remain a matter of conjecture, the mortality trends recently experienced can be ascertained, or at least estimated, as a matter of fact. The authors accordingly concluded that, instead of projecting future trends from 1943, it would be preferable to consider first the changes in mortality that have already occurred, separately from the problem of projecting future mortality trends.

Accordingly, a measure was needed of the changes which have occurred in mortality among annuitants over the period from 1943 to 1949. No satisfactory statistics of this kind exist since published mortality data either do not relate to annuitants or do not, with one exception, extend beyond 1947. However, considerable collateral or related data yielding measures of mortality changes through 1947 are available and, on the basis of this information and other unpublished data, estimates of the changes in mortality through 1949 can be made by analogy.

Table 7 summarizes the data obtained by the authors which are most pertinent to the question of recent decreases in annuity mortality rates. This table shows for a number of recent experiences the average percentage per year by which the death rates at various ages have fallen over the periods of years indicated. These figures were calculated on the "geometrical basis." As might be expected, the data in Table 7 are somewhat irregular, but the table shows clearly that the percentage rate of decrease in mortality has diminished with advancing age. In a few instances at the younger ages the effects of war mortality render the data meaningless. Nevertheless, it must be concluded, the authors believe, that substantial decreases in mortality occurred between 1943 and 1947.

TABLE 5
COMPARISON OF ANNUITY VALUES AT 2½% INTEREST—MALES
1943 EXPERIENCE TABLE AND 1937 STANDARD ANNUITY TABLE

Age x	1943 Table Ultimate a_x	1943 Table Select $a_{[x]}$	1937 Standard Annuity Table a_x	Percentage 1943 Table Ultimate a_x Is of Standard Annuity Table a_x	Percentage 1943 Table Select $a_{[x]}$ Is of Standard Annuity Table a_x
15.....	29.412	29.417	28.870	101.9%	101.9%
25.....	26.733	26.740	26.180	102.1	102.1
35.....	23.393	23.404	22.887	102.2	102.3
45.....	19.380	19.402	19.121	101.4	101.5
55.....	15.124	15.173	15.065	100.4	100.7
65.....	10.964	11.040	11.013	99.6	100.2
75.....	6.980	7.093	7.344	95.0	96.6
85.....	3.734	3.890	4.387	85.1	88.7
95.....	1.576	.....	2.259	69.8	.....

TABLE 5—*Continued*
COMPARISON OF ANNUITY VALUES AT 2½% INTEREST—FEMALES
1943 EXPERIENCE TABLE AND 1937 STANDARD ANNUITY TABLE

Age x	1943 Table Ultimate a_x	1943 Table Select $a_{[x]}$	1937 Standard Annuity Table a_x	Percentage 1943 Table Ultimate a_x Is of Standard Annuity Table a_x	Percentage 1943 Table Select $a_{[x]}$ Is of Standard Annuity Table a_x
15.....	30.759	30.766	29.986	102.6%	102.6%
25.....	28.358	28.368	27.604	102.7	102.8
35.....	25.383	25.400	24.602	103.2	103.2
45.....	21.770	21.799	21.054	103.4	103.5
55.....	17.549	17.601	17.114	102.5	102.8
65.....	12.843	12.940	13.016	98.7	99.4
75.....	8.189	8.365	9.107	89.9	91.9
85.....	4.323	4.602	5.761	75.0	79.9
95.....	1.755	.....	3.228	54.4	.....

TABLE 6
COMPARISON OF ANNUITY VALUES AT 2½% INTEREST
1943 EXPERIENCE TABLE AND 1937 STANDARD
ANNUITY TABLE SET BACK

Age x	Males			Females		
	Percentage 1943 Table Select $a_{[x]}$ Is of Standard Annuity Table a_x Set Back			Percentage 1943 Table Select $a_{[x]}$ Is of Standard Annuity Table a_x Set Back		
	1 Year	2 Years	3 Years	1 Year	2 Years	3 Years
15.....	101.1%	100.3%	99.5%	101.9%	101.2%	100.6%
25.....	101.0	99.9	98.8	101.8	100.8	99.9
35.....	100.7	99.2	97.8	101.9	100.6	99.3
45.....	99.4	97.5	95.6	101.7	100.0	98.3
55.....	98.0	95.5	93.1	100.5	98.2	96.0
65.....	96.8	93.5	90.4	96.4	93.6	90.9
75.....	92.3	88.3	84.6	88.3	84.9	81.7
85.....	83.8	79.2	75.0	75.9	72.2	68.8

The only mortality data shown in Table 7 which extend beyond 1947 are those reflecting the experience among white policyholders insured under weekly premium industrial policies in the Metropolitan Life Insurance Company. This shows a greater rate of decrease through 1948 than is indicated by the various experiences extending through 1947, and an even greater rate of decrease can be surmised from corresponding figures through the first half of 1949. Other indirect evidence considered by the authors also suggests that the average rate of reduction in mortality between 1947 and 1949 has been at least as great as that between 1943 and 1947. In the authors' opinion, the only safe conclusion that may be drawn with regard to recent mortality rates among annuitants is that they, too, have decreased in conformity with the trends observed in the general population and among insured lives.

The long-term trends in mortality rates are discussed later in Sections VI, VII, VIII, and IX.

V. Annuity Table for 1949

The Annuity Table for 1949 was obtained by adjusting the 1943 Experience Table for the changes in mortality estimated to have occurred between 1943 and 1949. This 1949 table is in the same form as the 1943 table and was obtained from the 1943 table in three steps.

First, the 1943 ultimate death rates were decreased by percentages approximately equal to those shown in Table 8. These percentages represent, in the authors' opinion, conservative estimates of the decreases in annuity mortality rates which occurred between 1943 and 1949. The degree of conservatism can be ascertained by comparing Tables 7 and 8. It cannot, however, be measured exactly and reduced to figures. In the opinion of the authors, it is no more than sufficient to provide a moderate safety margin (for mortality fluctuations and like contingencies) of the kind required for premium and reserve purposes. In Section X projection factors are presented which make allowance for probable future mortality decreases. Provision for such decreases is quite apart from and in addition to the safety margin included in the Annuity Table for 1949. The former will be needed only as the years pass; the latter is needed beginning immediately.

TABLE 7

RECENT DECREASES IN MORTALITY RATES: AVERAGE RATE OF DECREASE PER YEAR (GEOMETRICAL BASIS)

Age Group	U.S. White Population		Intercompany Group Annuities Active Lives, Predominantly Clerical Groups* (By Number of Lives)	N.Y. State Retirement Plan, Clerical Employees Active Lives† (By Number of Lives)	Joint Committee Settlement Options‡ (By Number of Contracts)	White Industrial Policyholders Met. Life Ins. Co. (By Number of Lives)
	From 1943 to 1947	From 1939-40 to 1947	From 1939-40 to 1946-47	From 1935-40 to 1940-45	From 1934-40 to 1940-45	From 1939-40 to 1948
Males:						
15-24.....	8.3%	2.3%	.....	.....	.....	3.1%
25-34.....	6.6	3.6	–0.9%§	–7.6%§	.....	4.6
35-44.....	3.1	2.2	0.4	6.2	.....	2.6
45-54.....	1.6	0.9	1.3	5.5	–0.3%	1.3
55-64.....	1.1	0.6	0.9	4.8	2.4	1.3
65-74.....	0.9	0.4	.....	.....	1.2	1.6
75-84.....	1.9	1.4	.....	.....	.....	.....
Females:						
15-24.....	6.7%	5.8%	.....	.....	.....	7.5%
25-34.....	6.9	5.9	7.0%	7.6%	.....	5.9
35-44.....	4.8	3.9	4.8	0.6	.....	4.0
45-54.....	3.7	2.9	1.2	–2.3	3.0%	3.0
55-64.....	3.5	2.8	.....	6.3	1.4	3.1
65-74.....	2.3	1.7	.....	.....	0.6	3.0
75-84.....	2.7	2.0	.....	.....	.....	.....

Age Group	Intercompany Group Life Insurance Predominantly Clerical Groups¶ (By Number of Lives)	Joint Committee Ordinary Ultimate Experience# (By Amounts of Insurance)	
	From 1939-40 to 1946-47	From 1942-43 to 1946-47	From 1939-40 to 1946-47
Both Sexes Combined:			
15-24.....	3.9%	.....	.....
25-34.....	4.8	9.3%	6.7%
35-44.....	3.3	3.2	4.2
45-54.....	1.4	2.2	2.7
55-64.....	1.6	1.1	2.1
65-74.....	2.3	2.7	1.6
75-84.....	2.0	2.4	0.9

* Experience for 1939, 1940, 1946, and 1947 furnished by Committee to Prepare Mortality Studies on Group Annuities; actual deaths increased by an allowance for deaths among ill-health terminations as explained on p. 374.

† Based on the 20th and 25th reports of N.Y. State Controller on the Operation of the State Employees' Retirement System.

‡ Experience for 1934-40 from *TASA*, XLII, 172; experience for 1940-45 from *TASA*, XLVIII, 133.

§ Affected by war deaths.

¶ Experience for 1939-40 from 1939 and 1941 Reports of Committee on Group Mortality Investigations; experience for 1946-47 from *TASA*, XLIX, 477.

\# Experience for 1939-40 from TASA, XLII, 140; experience for 1942-43 from *TASA*, XLV, 404; experience for 1946-47 from *TASA*, XLIX, 468.

TABLE 8
COMPARISON OF MORTALITY RATES
ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION)
AND 1943 EXPERIENCE TABLE

Age x	Males				Females			
	1949 Table Ultimate $1000q_x$	1943 Table Ultimate $1000q_x$	Decrease in $1000q_x$	Percentage Decrease	1949 Table Ultimate $1000q_x$	1943 Table Ultimate $1000q_x$	Decrease in $1000q_x$	Percentage Decrease
15.....	.537	.800	.263	32.9%	.278	.432	.154	35.6%
20.....	.624	.888	.264	29.7	.376	.557	.181	32.5
25.....	.768	1.034	.266	25.7	.501	.719	.218	30.3
30.....	1.004	1.318	.314	23.8	.677	.939	.262	27.9
35.....	1.391	1.779	.388	21.8	.942	1.266	.324	25.6
40.....	2.025	2.587	.562	21.7	1.355	1.807	.452	25.0
45.....	3.625	4.489	.864	19.2	2.019	2.689	.670	24.9
50.....	6.557	7.993	1.436	18.0	3.109	4.042	.933	23.1
55.....	10.565	12.876	2.311	17.9	4.705	5.920	1.215	20.5
60.....	15.662	19.013	3.351	17.6	7.504	9.172	1.668	18.2
65.....	23.066	26.959	3.893	14.4	12.406	14.940	2.534	17.0
70.....	35.092	39.760	4.668	11.7	20.964	24.733	3.769	15.2
75.....	54.501	60.248	5.747	9.5	35.829	41.267	5.438	13.2
80.....	85.503	92.700	7.197	7.8	61.415	68.927	7.512	10.9
85.....	134.178	143.268	9.090	6.3	104.760	114.487	9.727	8.5
90.....	208.485	220.091	11.606	5.3	176.161	187.654	11.493	6.1
95.....	316.834	332.413	15.579	4.7	288.153	300.501	12.348	4.1
100.....	463.415	487.766	24.351	5.0	449.400	464.139	14.739	3.2

The second step in constructing the Annuity Table for 1949 was to graduate by Makeham's formula the ultimate death rates obtained in the manner described above. The constants for $\text{Colog}_e(p_x) = A + Bc^x$ are as follows:

	Ages	Male Table	Female Table
$1000A$.....	60 up	4.00	1.00
	51-60	$4.00-.0009(60-x)^2(x-30)$	1.00
	41-50	$4.00-.0009(60-x)^2(x-30)$	$1.00-.04(50-x)+.000008(50-x)^2(50+x)$
	40 down	0.40	$1.00-.04(50-x)+.000008(50-x)^2(50+x)$
$1000B$.....	All	.031	.0075
$\text{Log}_{10}c$.....	All	.043	.049

These constants produced reasonable mortality rates at ages 10 to 14 which were, therefore, added to the table.

Even though the male and female values of c differ and the values of A vary with age, a reasonably simple method of obtaining joint annuity values can be used, as indicated in Section XIV.

The third step in the construction of the Annuity Table for 1949 was to obtain first policy year mortality rates by applying to the corresponding ultimate rates the same percentages as were used in the case of the 1943 Experience Table.

Table 9 presents the Annuity Table for 1949 and shows for all ages the elementary ultimate functions, select mortality rates, ultimate and select life annuity values, and ultimate commutation columns at 2½% interest. The ultimate death rates of the Annuity Table for 1949 are compared with a number of recent experiences in Table 10. There is also shown in Charts 1 and 2 a comparison of the death rates in the 1937 Standard Annuity Table and the Annuity Table for 1949. Tables 11, 12, and 13 provide a detailed comparison between the Annuity Table for 1949 and the 1937 Standard Annuity Table both as to ultimate mortality rates and as to ultimate and select annuity values at 2½% interest. The comparison of annuity values is also presented in Chart 3.

VI. Statistics on Long-Term Mortality Decreases

A large amount of data is available regarding past mortality trends in the United States and Canada, but most of it is not directly applicable to the problem of ascertaining the "long-term" trends in mortality among American annuitants, considering "long- term" to span a period of not less than twenty and preferably more than thirty years.

The authors reviewed the principal United States and Canadian mortality statistics that extend over considerable periods of time, with respect to the following classes of lives:

(1) Annuitants and pensioners.

(2) Population.

(3) Insured lives.

Consideration was also given to foreign mortality statistics, notably the British experiences among annuitants and the Scandinavian and British population data, which cover very long periods. However, the authors concluded that these experiences are not sufficiently pertinent to justify discussion here.

On the basis of the more pertinent American experiences, average rates of decrease per year were calculated for the mortality rates by decennial age groups. These long-term rates of decrease, shown in Table 14, were calculated on the same geometrical basis as used in Table 7.

The long-term decreases in mortality among annuitants and pensioners are, of course, the most significant for the purpose at hand. Unfortunately, the annuity experiences on which the comparisons in Table 14 were based were affected by changing class selection, which tends to mask the underlying trends in some degree. At some ages, too, these comparisons are affected by the methods of graduation used in the construction of the annuity tables involved. Moreover, the data are limited largely to the age range from 65 to 85. Despite these shortcomings, the annuitant mortality statistics presented in Table 14 are of considerable value.

TABLE 9

ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION)

ELEMENTARY FUNCTIONS AND ANNUITY VALUES

Age x	Males					
	Ultimate			First Year $1000q_{[x]}$	a_x at 2½% Interest	$a_{[x]}$ at 2½% Interest
	l_x	d_x	$1000q_x$			
10.....	1000.0000	.4830	.483	.362	31.028	31.033
11.....	999.5170	.4918	.492	.369	30.820	30.824
12.....	999.0252	.5015	.502	.376	30.606	30.609
13.....	998.5237	.5112	.512	.384	30.386	30.390
14.....	998.0125	.5230	.524	.393	30.162	30.166
15.....	997.4895	.5357	.537	.403	29.932	29.936
16.....	996.9538	.5493	.551	.413	29.697	29.701
17.....	996.4045	.5650	.567	.425	29.456	29.461
18.....	995.8395	.5816	.584	.438	29.210	29.215
19.....	995.2579	.6001	.603	.452	28.958	28.963
20.....	994.6578	.6207	.624	.468	28.700	28.704
21.....	994.0371	.6441	.648	.486	28.435	28.440
22.....	993.3930	.6695	.674	.506	28.165	28.170
23.....	992.7235	.6969	.702	.526	27.889	27.894
24.....	992.0266	.7272	.733	.550	27.606	27.611
25.....	991.2994	.7613	.768	.576	27.317	27.322
26.....	990.5381	.7984	.806	.604	27.021	27.027
27.....	989.7397	.8403	.849	.637	26.719	26.725
28.....	988.8994	.8861	.896	.672	26.411	26.416
29.....	988.0133	.9356	.947	.710	26.095	26.101
30.....	987.0777	.9910	1.004	.753	25.773	25.780
31.....	986.0867	1.0522	1.067	.800	25.444	25.451
32.....	985.0345	1.1190	1.136	.852	25.108	25.115
33.....	983.9155	1.1935	1.213	.910	24.765	24.773
34.....	982.7220	1.2746	1.297	.973	24.415	24.422
35.....	981.4474	1.3652	1.391	1.043	24.057	24.066
36.....	980.0822	1.4642	1.494	1.120	23.693	23.702
37.....	978.6180	1.5726	1.607	1.205	23.322	23.331
38.....	977.0454	1.6932	1.733	1.300	22.943	22.954
39.....	975.3522	1.8259	1.872	1.404	22.558	22.568
40.....	973.5263	1.9714	2.025	1.519	22.165	22.176
41.....	971.5549	2.1569	2.220	1.665	21.765	21.777
42.....	969.3980	2.4051	2.481	1.861	21.359	21.372
43.....	966.9929	2.7114	2.804	2.103	20.947	20.963
44.....	964.2815	3.0732	3.187	2.390	20.532	20.548
45.....	961.2083	3.4844	3.625	2.719	20.112	20.130
46.....	957.7239	3.9420	4.116	3.087	19.690	19.711
47.....	953.7819	4.4418	4.657	3.493	19.266	19.288
48.....	949.3401	4.9802	5.246	3.934	18.840	18.864
49.....	944.3599	5.5528	5.880	4.410	18.412	18.439
50.....	938.8071	6.1558	6.557	4.918	17.984	18.014
51.....	932.6513	6.7869	7.277	5.458	17.556	17.587
52.....	925.8644	7.4421	8.038	6.028	17.126	17.161
53.....	918.4223	8.1189	8.840	6.630	16.697	16.734
54.....	910.3034	8.8136	9.682	7.262	16.267	16.307
55.....	901.4898	9.5242	10.565	7.924	15.837	15.879
56.....	891.9656	10.2496	11.491	8.618	15.406	15.451
57.....	881.7160	10.9862	12.460	9.345	14.975	15.022
58.....	870.7298	11.7340	13.476	10.107	14.543	14.592

TABLE 9—*Continued*

Age x	Males					
	Ultimate			First Year $1000q_{[x]}$	a_x at 2½% Interest	$a_{[x]}$ at 2½% Interest
	l_x	d_x	$1000q_x$			
59	858.9958	12.4915	14.542	10.906	14.110	14.162
60	846.5043	13.2580	15.662	11.746	13.676	13.730
61	833.2463	14.0560	16.869	12.652	13.241	13.298
62	819.1903	14.9084	18.199	13.649	12.805	12.864
63	804.2819	15.8170	19.666	14.750	12.368	12.430
64	788.4649	16.7809	21.283	15.962	11.932	11.997
65	771.6840	17.7997	23.066	17.300	11.496	11.564
66	753.8843	18.8697	25.030	18.772	11.062	11.132
67	735.0146	19.9873	27.193	20.395	10.629	10.703
68	715.0273	21.1484	29.577	22.183	10.199	10.277
69	693.8789	22.3443	32.202	24.152	9.773	9.855
70	671.5346	23.5655	35.092	26.319	9.351	9.436
71	647.9691	24.7991	38.272	28.704	8.933	9.022
72	623.1700	26.0304	41.771	31.328	8.521	8.614
73	597.1396	27.2415	45.620	34.215	8.115	8.212
74	569.8981	28.4106	49.852	37.389	7.715	7.816
75	541.4875	29.5116	54.501	40.876	7.323	7.428
76	511.9759	30.5184	59.609	44.707	6.938	7.049
77	481.4575	31.3987	65.216	48.912	6.563	6.677
78	450.0588	32.1198	71.368	53.526	6.196	6.315
79	417.9390	32.6465	78.113	58.585	5.839	5.963
80	385.2925	32.9437	85.503	64.127	5.492	5.621
81	352.3488	32.9774	93.593	70.195	5.156	5.289
82	319.3714	32.7174	102.443	76.832	4.830	4.968
83	286.6540	32.1376	112.113	84.085	4.516	4.659
84	254.5164	31.2213	122.669	92.002	4.214	4.361
85	223.2951	29.9613	134.178	100.634	3.923	4.075
86	193.3338	28.3638	146.709	.....	3.644	.....
87	164.9700	26.4501	160.333	.....	3.378	.....
88	138.5199	24.2582	175.124	.....	3.123	.....
89	114.2617	21.8412	191.151	.....	2.881	.....
90	92.42050	19.26829	208.485	.....	2.651	.....
91	73.15221	16.61960	227.192	.....	2.433	.....
92	56.53261	13.98232	247.332	.....	2.226	.....
93	42.55029	11.44433	268.960	.....	2.032	.....
94	31.10596	9.08661	292.118	.....	1.849	.....
95	22.01935	6.97648	316.834	.....	1.677	.....
96	15.04287	5.16154	343.122	.....	1.517	.....
97	9.881330	3.665707	370.973	.....	1.366	.....
98	6.215623	2.488437	400.352	.....	1.227	.....
99	3.727186	1.607159	431.199	.....	1.097	.....
100	2.120027	.982452	463.415	.....	.977	.....
101	1.137575	.565227	496.870	.....	.865	.....
102	.5723480	.3041394	531.389	.....	.763	.....
103	.2682086	.1520091	566.757	.....	.669	.....
104	.1161995	.0700351	602.714	.....	.583	.....
105	.04616440	.02949702	638.956	.....	.503	.....
106	.01666738	.01125286	675.143	.....	.428	.....
107	.00541452	.00384917	710.898	.....	.352	.....
108	.00156535	.00116747	745.822	.....	.248	.....
109	.00039788	.00039788	1000.000	.....	0	.....

TABLE 9—*Continued*

ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION)

ELEMENTARY FUNCTIONS AND ANNUITY VALUES

Age x	Females					
	Ultimate			First Year $1000q_{[x]}$	a_x at 2½% Interest	$a_{[x]}$ at 2½% Interest
	l_x	d_x	$1000q_x$			
10.....	1000.0000	.1910	.191	.096	32.208	32.211
11.....	999.8090	.2080	.208	.104	32.019	32.022
12.....	999.6010	.2249	.225	.112	31.826	31.830
13.....	999.3761	.2418	.242	.121	31.629	31.633
14.....	999.1343	.2598	.260	.130	31.428	31.432
15.....	998.8745	.2777	.278	.139	31.222	31.227
16.....	998.5968	.2956	.296	.148	31.012	31.016
17.....	998.3012	.3145	.315	.158	30.796	30.801
18.....	997.9867	.3333	.334	.167	30.576	30.581
19.....	997.6534	.3532	.354	.177	30.351	30.357
20.....	997.3002	.3750	.376	.188	30.121	30.126
21.....	996.9252	.3968	.398	.199	29.885	29.892
22.....	996.5284	.4195	.421	.210	29.645	29.651
23.....	996.1089	.4443	.446	.223	29.399	29.405
24.....	995.6646	.4709	.473	.236	29.147	29.154
25.....	995.1937	.4986	.501	.250	28.890	28.897
26.....	994.6951	.5282	.531	.266	28.627	28.634
27.....	994.1669	.5597	.563	.282	28.358	28.367
28.....	993.6072	.5942	.598	.299	28.084	28.092
29.....	993.0130	.6316	.636	.318	27.803	27.812
30.....	992.3814	.6718	.677	.338	27.516	27.525
31.....	991.7096	.7150	.721	.360	27.223	27.233
32.....	990.9946	.7631	.770	.385	26.924	26.934
33.....	990.2315	.8140	.822	.411	26.618	26.629
34.....	989.4175	.8697	.879	.440	26.306	26.318
35.....	988.5478	.9312	.942	.471	25.988	25.999
36.....	987.6166	.9975	1.010	.505	25.662	25.675
37.....	986.6191	1.0705	1.085	.542	25.330	25.344
38.....	985.5486	1.1501	1.167	.584	24.992	25.007
39.....	984.3985	1.2364	1.256	.628	24.647	24.663
40.....	983.1621	1.3322	1.355	.678	24.295	24.311
41.....	981.8299	1.4374	1.464	.732	23.936	23.953
42.....	980.3925	1.5520	1.583	.792	23.570	23.589
43.....	978.8405	1.6787	1.715	.858	23.198	23.217
44.....	977.1618	1.8165	1.859	.930	22.818	22.840
45.....	975.3453	1.9692	2.019	1.010	22.433	22.455
46.....	973.3761	2.1375	2.196	1.098	22.040	22.065
47.....	971.2386	2.3222	2.391	1.196	21.641	21.667
48.....	968.9164	2.5250	2.606	1.303	21.235	21.262
49.....	966.3914	2.7494	2.845	1.422	20.822	20.852
50.....	963.6420	2.9960	3.109	1.554	20.404	20.436
51.....	960.6460	3.2287	3.361	1.680	19.979	20.013
52.....	957.4173	3.4869	3.642	1.821	19.548	19.584
53.....	953.9304	3.7747	3.957	1.978	19.110	19.147
54.....	950.1557	4.0952	4.310	2.155	18.665	18.706
55.....	946.0605	4.4512	4.705	2.352	18.215	18.257
56.....	941.6093	4.8455	5.146	2.573	17.758	17.804
57.....	936.7638	5.2833	5.640	2.820	17.296	17.345
58.....	931.4805	5.7687	6.193	3.096	16.829	16.882

TABLE 9—*Continued*

Age x	Females					
	Ultimate			First Year	a_x at 2½%	$a_{[x]}$ at 2½%
	l_x	d_x	$1000q_x$	$1000q_{[x]}$	Interest	Interest
59	925.7118	6.3059	6.812	3.406	16.358	16.414
60	919.4059	6.8992	7.504	3.752	15.882	15.942
61	912.5067	7.5537	8.278	4.139	15.402	15.466
62	904.9530	8.2749	9.144	4.572	14.919	14.988
63	896.6781	9.0672	10.112	5.056	14.433	14.507
64	887.6109	9.9368	11.195	5.598	13.945	14.023
65	877.6741	10.8884	12.406	6.203	13.455	13.540
66	866.7857	11.9261	13.759	6.880	12.965	13.055
67	854.8596	13.0554	15.272	7.636	12.474	12.571
68	841.8042	14.2795	16.963	8.482	11.984	12.088
69	827.5247	15.6013	18.853	9.426	11.496	11.607
70	811.9234	17.0212	20.964	10.482	11.010	11.128
71	794.9022	18.5379	23.321	11.660	10.527	10.652
72	776.3643	20.1498	25.954	12.977	10.047	10.181
73	756.2145	21.8485	28.892	14.446	9.573	9.715
74	734.3660	23.6253	32.171	16.086	9.104	9.256
75	710.7407	25.4651	35.829	17.914	8.642	8.802
76	685.2756	27.3473	39.907	19.954	8.187	8.358
77	657.9283	29.2456	44.451	22.226	7.741	7.920
78	628.6827	31.1280	49.513	24.756	7.303	7.494
79	597.5547	32.9533	55.147	27.574	6.876	7.076
80	564.6014	34.6750	61.415	30.708	6.459	6.671
81	529.9264	36.2380	68.383	34.192	6.054	6.276
82	493.6884	37.5801	76.121	38.060	5.661	5.894
83	456.1083	38.6356	84.707	42.354	5.280	5.524
84	417.4727	39.3359	94.224	47.112	4.913	5.169
85	378.1368	39.6136	104.760	52.380	4.560	4.827
86	338.5232	39.4071	116.409	.....	4.221	.....
87	299.1161	38.6667	129.270	.....	3.896	.....
88	260.4494	37.3602	143.445	.....	3.586	.....
89	223.0892	35.4801	159.040	.....	3.292	.....
90	187.6091	33.0494	176.161	.....	3.012	.....
91	154.5597	30.1257	194.913	.....	2.747	.....
92	124.4340	26.8030	215.399	.....	2.498	.....
93	97.63100	23.20826	237.714	.....	2.263	.....
94	74.42274	19.49452	261.943	.....	2.043	.....
95	54.92822	15.82773	288.153	.....	1.838	.....
96	39.10049	12.37104	316.391	.....	1.646	.....
97	26.72945	9.26641	346.674	.....	1.468	.....
98	17.46304	6.61825	378.986	.....	1.304	.....
99	10.84479	4.48178	413.266	.....	1.152	.....
100	6.363010	2.859537	449.400	.....	1.012	.....
101	3.503473	1.706948	487.216	.....	.884	.....
102	1.796525	.945829	526.477	.....	.768	.....
103	.8506960	.4822357	566.872	.....	.661	.....
104	.3684603	.2240301	608.017	.....	.565	.....
105	.1444302	.0938015	649.459	.....	.478	.....
106	.05062870	.03496793	690.674	.....	.399	.....
107	.01566077	.01144946	731.092	.....	.321	.....
108	.00421131	.00324315	770.105	.....	.224	.....
109	.00096816	.00096816	1000.000	.....	0	.....

TABLE 9—*Continued*

ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION) ULTIMATE COMMUTATION COLUMNS AT 2½% INTEREST

Age x	Males				
	D_x	N_x	C_x	M_x	R_x
10	781.1984	25020.5925	.36812	170.94003	9956.66328
11	761.7767	24239.3941	.36568	170.57191	9785.72325
12	742.8311	23477.6174	.36380	170.20623	9615.15134
13	724.3494	22734.7863	.36179	169.84243	9444.94511
14	706.3206	22010.4369	.36111	169.48064	9275.10268
15	688.7321	21304.1163	.36086	169.11953	9105.62204
16	671.5729	20615.3842	.36100	168.75867	8936.50251
17	654.8321	19943.8113	.36226	168.39767	8767.74384
18	638.4983	19288.9792	.36381	168.03541	8599.34617
19	622.5614	18650.4809	.36622	167.67160	8431.31076
20	607.0108	18027.9195	.36956	167.30538	8263.63916
21	591.8361	17420.9087	.37413	166.93582	8096.33378
22	577.0269	16829.0726	.37940	166.56169	7929.39796
23	562.5737	16252.0457	.38530	166.18229	7762.83627
24	548.4671	15689.4720	.39224	165.79699	7596.65398
25	534.6976	15141.0049	.40062	165.40475	7430.85699
26	521.2555	14606.3073	.40990	165.00413	7265.45224
27	508.1321	14085.0518	.42089	164.59423	7100.44811
28	495.3177	13576.9197	.43300	164.17334	6935.85388
29	482.8038	13081.6020	.44604	163.74034	6771.68054
30	470.5821	12598.7982	.46093	163.29430	6607.94020
31	458.6435	12128.2161	.47746	162.83337	6444.64590
32	446.9796	11669.5726	.49538	162.35591	6281.81253
33	435.5823	11222.5930	.51548	161.86053	6119.45662
34	424.4429	10787.0107	.53708	161.34505	5957.59609
35	413.5535	10362.5678	.56123	160.80797	5796.25104
36	402.9056	9949.0143	.58724	160.24674	5635.44307
37	392.4914	9546.1087	.61533	159.65950	5475.19633
38	382.3031	9153.6173	.64636	159.04417	5315.53683
39	372.3323	8771.3142	.68002	158.39781	5156.49266
40	362.5710	8398.9819	.71630	157.71779	4998.09485
41	353.0115	8036.4109	.76459	157.00149	4840.37706
42	343.6369	7683.3994	.83178	156.23690	4683.37557
43	334.4237	7339.7625	.91484	155.40512	4527.13867
44	325.3522	7005.3388	1.01162	154.49028	4371.73355
45	316.4052	6679.9866	1.11900	153.47866	4217.24327
46	307.5690	6363.5814	1.23508	152.35966	4063.76461
47	298.8322	6056.0124	1.35773	151.12458	3911.40495
48	290.1859	5757.1802	1.48517	149.76685	3760.28037
49	281.6230	5466.9943	1.61554	148.28168	3610.51352
50	273.1386	5185.3713	1.74730	146.66614	3462.23184
51	264.7294	4912.2327	1.87945	144.91884	3315.56570
52	256.3931	4647.5033	2.01062	143.03939	3170.64686
53	248.1290	4391.1102	2.13997	141.02877	3027.60747
54	239.9371	4142.9812	2.26642	138.88880	2886.57870
55	231.8186	3903.0441	2.38942	136.62238	2747.68990
56	223.7750	3671.2255	2.50869	134.23296	2611.06752
57	215.8084	3447.4505	2.62339	131.72427	2476.83456
58	207.9214	3231.6421	2.73362	129.10088	2345.11029

TABLE 9—*Continued*

Age x	Males				
	D_x	N_x	C_x	M_x	R_x
59.....	200.1165	3023.7207	2.83911	126.36726	2216.00941
60.....	192.3965	2823.6042	2.93983	123.52815	2089.64215
61.....	184.7641	2631.2077	3.04076	120.58832	1966.11400
62.....	177.2169	2446.4436	3.14650	117.54756	1845.52568
63.....	169.7480	2269.2267	3.25684	114.40106	1727.97812
64.....	162.3510	2099.4787	3.37104	111.14422	1613.57706
65.....	155.0202	1937.1277	3.48849	107.77318	1502.43284
66.....	147.7507	1782.1075	3.60800	104.28469	1394.65966
67.....	140.5390	1634.3568	3.72848	100.67669	1290.37497
68.....	133.3828	1493.8178	3.84885	96.94821	1189.69828
69.....	126.2807	1360.4350	3.96731	93.09936	1092.75007
70.....	119.2334	1234.1543	4.08209	89.13205	999.65071
71.....	112.2432	1114.9209	4.19100	85.04996	910.51866
72.....	105.3145	1002.6777	4.29179	80.85896	825.46870
73.....	98.45408	897.36319	4.381925	76.567170	744.609738
74.....	91.67083	798.90911	4.458518	72.185245	668.042568
75.....	84.97644	707.23828	4.518341	67.726727	595.857323
76.....	78.38550	622.26184	4.558523	63.208386	528.130596
77.....	71.91514	543.87634	4.575622	58.649863	464.922210
78.....	65.58549	471.96120	4.566542	54.074241	406.272347
79.....	59.41930	406.37571	4.528219	49.507699	352.198106
80.....	53.44183	346.95641	4.457992	44.979480	302.690407
81.....	47.68038	293.51458	4.353710	40.521488	257.710927
82.....	42.16373	245.83420	4.214033	36.167778	217.189439
83.....	36.92132	203.67047	4.038394	31.953745	181.021661
84.....	31.98240	166.74915	3.827564	27.915351	149.067916
85.....	27.37478	134.76675	3.583507	24.087787	121.152565
86.....	23.12360	107.39197	3.309696	20.504280	97.064778
87.....	19.24991	84.26837	3.011114	17.194584	76.560498
88.....	15.76929	65.01846	2.694229	14.183470	59.365914
89.....	12.69044	49.24917	2.366620	11.489241	45.182444
90.....	10.01430	36.55873	2.036908	9.122621	33.693203
91.....	7.733137	26.544427	1.714056	7.085713	24.570582
92.....	5.830468	18.811290	1.406889	5.371657	17.484869
93.....	4.281373	12.980822	1.123433	3.964768	12.113212
94.....	3.053517	8.699449	.8702313	2.8413348	8.1484445
95.....	2.108809	5.645932	.6518464	1.9711035	5.3071097
96.....	1.405528	3.537123	.4705051	1.3192571	3.3360062
97.....	.9007421	2.1315946	.3260010	.8487520	2.0167491
98.....	.5527718	1.2308525	.2159056	.5227510	1.1679971
99.....	.3233839	.6780807	.1360418	.3068454	.6452461
100.....	.1794547	.3546968	.08113364	.17080355	.33840066
101.....	.09394413	.17524213	.04553954	.08966991	.16759711
102.....	.04611327	.08129800	.02390642	.04413037	.07792720
103.....	.02108213	.03518473	.01165702	.02022395	.03379683
104.....	.00891091	.01410260	.00523974	.00856693	.01357288
105.....	.00345383	.00519169	.00215302	.00332719	.00500595
106.....	.00121657	.00173786	.00080133	.00117417	.00167876
107.....	.00038557	.00052129	.00026740	.00037284	.00050459
108.....	.00010875	.00013572	.00007913	.00010544	.00013175
109.....	.00002697	.00002697	.00002631	.00002631	.00002631

TABLE 9—*Continued*

ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION) ULTIMATE COMMUTATION COLUMNS AT 2½% INTEREST

Age x	Females				
	D_x	N_x	C_x	M_x	R_x
10	781.1984	25941.7892	.14557	148.47186	9542.56683
11	761.9992	25160.5908	.15466	148.32629	9394.09497
12	743.2592	24398.5916	.16315	148.17163	9245.76868
13	724.9678	23655.3324	.17113	148.00848	9097.59705
14	707.1145	22930.3646	.17938	147.83735	8949.58857
15	689.6884	22223.2501	.18707	147.65797	8801.75122
16	672.6797	21533.5617	.19427	147.47090	8654.09325
17	656.0786	20860.8820	.20165	147.27663	8506.62235
18	639.8751	20204.8034	.20849	147.07498	8359.34572
19	624.0599	19564.9283	.21555	146.86649	8212.27074
20	608.6233	18940.8684	.22327	146.65894	8065.40425
21	593.5556	18332.2451	.23049	146.42767	7918.75331
22	578.8481	17738.6895	.23773	146.19718	7772.32564
23	564.4922	17159.8414	.24564	145.95945	7626.12846
24	550.4784	16595.3492	.25400	145.71381	7480.16901
25	536.7981	16044.8708	.26238	145.45981	7334.45520
26	523.4431	15508.0727	.27118	145.19743	7188.99539
27	510.4050	14984.6296	.28034	144.92625	7043.79796
28	497.6758	14474.2246	.29036	144.64591	6898.87171
29	485.2470	13976.5488	.30111	144.35555	6754.22580
30	473.1106	13491.3018	.31246	144.05444	6609.87025
31	461.2588	13018.1912	.32445	143.74198	6465.81581
32	449.6842	12556.9324	.33783	143.41753	6322.07383
33	438.3784	12107.2482	.35157	143.07970	6178.65630
34	427.3347	11668.8698	.36647	142.72813	6035.57660
35	416.5454	11241.5351	.38281	142.36166	5892.84847
36	406.0030	10824.9897	.40006	141.97885	5750.48681
37	395.7004	10418.9867	.41887	141.57879	5608.50796
38	385.6303	10023.2863	.43904	141.15992	5466.92917
39	375.7857	9637.6560	.46047	140.72088	5325.76925
40	366.1597	9261.8703	.48405	140.26041	5185.04837
41	356.7449	8895.7106	.50954	139.77636	5044.78796
42	347.5343	8538.9657	.53674	139.26682	4905.01160
43	338.5211	8191.4314	.56640	138.73008	4765.74478
44	329.6981	7852.9103	.59795	138.16368	4627.01470
45	321.0587	7523.2122	.63240	137.56573	4488.85102
46	312.5956	7202.1535	.66971	136.93333	4351.28529
47	304.3016	6889.5579	.70983	136.26362	4214.35196
48	296.1698	6585.2563	.75299	135.55379	4078.08834
49	288.1931	6289.0865	.79992	134.80080	3942.53455
50	280.3641	6000.8934	.85040	134.00088	3807.73375
51	272.6756	5720.5293	.89410	133.15048	3673.73287
52	265.1309	5447.8537	.94205	132.25638	3540.58239
53	257.7222	5182.7228	.99493	131.31433	3408.32601
54	250.4414	4925.0006	1.05308	130.31940	3277.01168
55	243.2799	4674.5592	1.11671	129.26632	3146.69228
56	236.2296	4431.2793	1.18598	128.14961	3017.42596
57	229.2819	4195.0497	1.26160	126.96363	2889.27635
58	222.4281	3965.7678	1.34391	125.70203	2762.31272

TABLE 9—*Continued*

Age x	Females				
	D_x	N_x	C_x	M_x	R_x
59.....	215.6591	3743.3397	1.43323	124.35812	2636.61069
60.....	208.9659	3527.6806	1.52983	122.92489	2512.25257
61.....	202.3393	3318.7147	1.63411	121.39506	2389.32768
62.....	195.7701	3116.3754	1.74646	119.76095	2267.93262
63.....	189.2488	2920.6053	1.86701	118.01449	2148.17167
64.....	182.7659	2731.3565	1.99616	116.14748	2030.15718
65.....	176.3121	2548.5906	2.13397	114.15132	1914.00970
66.....	169.8778	2372.2785	2.28034	112.01735	1799.85838
67.....	163.4541	2202.4007	2.43538	109.73701	1687.84103
68.....	157.0320	2038.9466	2.59876	107.30163	1578.10402
69.....	150.6032	1881.9146	2.77007	104.70287	1470.80239
70.....	144.1599	1731.3114	2.94846	101.93280	1366.09952
71.....	137.6953	1587.1515	3.13287	98.98434	1264.16672
72.....	131.2041	1449.4562	3.32222	95.85147	1165.18238
73.....	124.6817	1318.2521	3.51444	92.52925	1069.33091
74.....	118.1263	1193.5704	3.70755	89.01481	976.80166
75.....	111.5376	1075.4441	3.89881	85.30726	887.78685
76.....	104.9184	963.9065	4.08486	81.40845	802.47959
77.....	98.27452	858.98806	4.261859	77.323592	721.071142
78.....	91.61572	760.71354	4.425536	73.061733	643.747550
79.....	84.95565	669.09782	4.570773	68.636197	570.685817
80.....	78.31279	584.14217	4.692274	64.065424	502.049620
81.....	71.71045	505.82938	4.784177	59.373150	437.984196
82.....	65.17724	434.11893	4.840354	54.588973	378.611046
83.....	58.74720	368.94169	4.854930	49.748619	324.022073
84.....	52.45941	310.19449	4.822370	44.893689	274.273454
85.....	46.35754	257.73508	4.737966	40.071319	229.379765
86.....	40.48890	211.37754	4.598309	35.333353	189.308446
87.....	34.90306	170.88864	4.401867	30.735044	153.975093
88.....	29.64990	135.98558	4.149399	26.333177	123.240049
89.....	24.77733	106.33568	3.844474	22.183778	96.906872
90.....	20.32853	81.55835	3.493750	18.339304	74.723094
91.....	16.33896	61.22982	3.107002	14.845554	56.383790
92.....	12.83345	44.89086	2.696894	11.738552	41.538236
93.....	9.823545	32.057414	2.278239	9.041658	29.799684
94.....	7.305708	22.233869	1.867004	6.763419	20.758026
95.....	5.260516	14.928161	1.478862	4.896415	13.994607
96.....	3.653349	9.667645	1.127694	3.417553	9.098192
97.....	2.436549	6.014296	.8240863	2.2898585	5.6806390
98.....	1.553034	3.577747	.5742229	1.4657722	3.3907805
99.....	.9409325	2.0247128	.3793709	.8915493	1.9250083
100.....	.5386121	1.0837803	.2361486	.5121784	1.0334590
101.....	.2893266	.5451682	.1375264	.2760298	.5212806
102.....	.1447435	.2558416	.07434547	.13850340	.24525083
103.....	.06686768	.11109808	.03698089	.06415793	.10674743
104.....	.02825587	.04423040	.01676102	.02717704	.04258950
105.....	.01080568	.01597453	.00684668	.01041602	.01541246
106.....	.00369545	.00516885	.00249010	.00356934	.00499644
107.....	.00111522	.00147340	.00079540	.00107924	.00142710
108.....	.00029256	.00035818	.00021982	.00028384	.00034786
109.....	.00006562	.00006562	.00006402	.00006402	.00006402

TABLE 10
COMPARISON OF MORTALITY RATES
ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION)
AND OTHER RECENT EXPERIENCES

Age Group	1949 Annuity Table Ultimate (Central Age)	Jt. Committee Ultimate Ord. Experience 1946/47	Intercompany Group Life Ins. (Predominantly Clerical) 1946/47	White Industrial Policyholders Met. Life Ins. Co. 1948	U.S. White Population 1947	Canada Population* 1945
	$1000q_x$					
Males:		(Males and Females Combined)	(Males and Females Combined)			
10-14.....	.502	.....	.....	.60	.77	1.1
15-19.....	.567	.....	.98	1.13	1.43	1.8
20-24.....	.674	.....	1.23	1.59	1.92	2.0
25-29.....	.849	1.08	1.15	1.60	1.96	2.1
30-34.....	1.136	1.73	1.50	2.33	2.36	2.2
35-39.....	1.607	1.97	2.35	3.56	3.41	3.2
40-44.....	2.481	3.13	3.60	6.18	5.31	4.3
45-49.....	4.657	5.85	6.02	9.40	8.26	6.8
50-54.....	8.038	9.47	10.44	14.96	13.17	10.0
55-59.....	12.460	15.93	15.59	21.45	19.39	15.1
60-64.....	18.199	25.04	22.87	31.93	29.22	23.6
65-69.....	27.193	35.64	33.80	47.15	43.66	36.6
70-74.....	41.771	54.08	52.19	70.70	63.61	54.9
75-79.....	65.216	86.21	77.07	.....	.....	89.3
80-84.....	102.443	124.12	116.14	.....	.....	136.0
Females:						
10-14.....	.225	.....		.38	.47	.8
15-19.....	.315	.....		.52	.79	1.2
20-24.....	.421	.....		.81	1.02	1.6
25-29.....	.563	.....		1.16	1.24	1.9
30-34.....	.770	.....		1.40	1.62	2.3
35-39.....	1.085	.....		2.09	2.30	3.0
40-44.....	1.583	.....		3.40	3.29	3.6
45-49.....	2.391	.....		5.15	4.94	5.3
50-54.....	3.642	.....		7.58	7.42	7.6
55-59.....	5.640	.....		11.69	10.96	11.1
60-64.....	9.144	.....		18.60	17.48	17.5
65-69.....	15.272	.....		29.90	28.92	27.1
70-74.....	25.954	.....		51.27	46.57	45.1
75-79.....	44.451	.....		.....	.....	74.0
80-84.....	76.121	.....		.....	.....	118.9

* Vital Statistics 75th Annual Report.

TABLE 11
COMPARISON OF MORTALITY RATES
ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION)
AND 1937 STANDARD ANNUITY TABLE

Age x	Males			Females		
	1949 Table Ultimate $1000q_x$	1937 Standard Annuity Table $1000q_x$	Percentage 1949 Table Ultimate q_x Is of 1937 Standard Annuity q_x	1949 Table Ultimate $1000q_x$	1937 Standard Annuity Table $1000q_x$	Percentage 1949 Table Ultimate q_x Is of 1937 Standard Annuity q_x
10	.483	1.257	38.4%	.191	1.234	15.5%
15	.537	1.262	42.6	.278	1.257	22.1
20	.624	1.331	46.9	.376	1.262	29.8
25	.768	1.561	49.2	.501	1.331	37.6
30	1.004	2.065	48.6	.677	1.561	43.4
35	1.391	2.981	46.7	.942	2.065	45.6
40	2.025	4.356	46.5	1.355	2.981	45.5
45	3.625	6.362	57.0	2.019	4.356	46.3
50	6.557	9.288	70.6	3.109	6.362	48.9
55	10.565	13.554	77.9	4.705	9.288	50.7
60	15.662	19.753	79.3	7.504	13.554	55.4
65	23.066	28.751	80.2	12.406	19.753	62.8
70	35.092	41.758	84.0	20.964	28.751	72.9
75	54.501	60.464	90.1	35.829	41.758	85.8
80	85.503	87.161	98.1	61.415	60.464	101.6
85	134.178	124.837	107.5	104.760	87.161	120.2
90	208.485	177.138	117.7	176.161	124.837	141.1
95	316.834	248.059	127.7	288.153	177.138	162.7
100	463.415	362.122	128.0	449.400	248.059	181.2

Mortality data for American annuitants began really with the American Annuitants Tables[8] because most of the lives on which the McClintock Tables[9] were based were foreign. The United States Annuitants Tables[10] supply information as to annuitant mortality between 1918 and 1927. The reports of the Joint Mortality Committee,[11] covering the years 1931 to 1946, provide data as to recent annuitant mortality. From the American Annuitants Tables, which were based on experiences through 1918—centering about the year 1910—to the latest report of the Joint Committee, upon which the 1943 Experience Table was based in part, there is an average span of about 33 years over which immediate non-refund annuities can be observed. From the United States Annuitants Tables to the 1943 table there is an average span of about 20 years.

Mortality statistics derived from life income settlement options under life insurance may be said to begin with a study[12] which presented the experience of five large companies during the period 1924 to 1934. More recent experiences, covering the years from 1934 to 1940[13] and from 1940 to 1945,[14] were published by the Joint Mortality Committee. From the earliest to the latest of these sets of data there is an average span of only about 14 years over which life income settlements (payee-elections and non-payee-elections combined) can be observed.

TABLE 12

COMPARISON OF ANNUITY VALUES AT 2½% INTEREST—MALES ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION) AND 1937 STANDARD ANNUITY TABLE

Age x	1949 Table Ultimate a_x	1949 Table Select $a_{[x]}$	1937 Standard Annuity Table a_x	Percentage 1949 Table Ultimate q_x Is of Standard Annuity Table a_x	Percentage 1949 Table Select $a_{[x]}$ Is of Standard Annuity Table a_x
15.....	29.932	29.936	28.870	103.7%	103.7%
20.....	28.700	28.704	27.604	104.0	104.0
25.....	27.317	27.322	26.180	104.3	104.4
30.....	25.773	25.780	24.602	104.8	104.8
35.....	24.057	24.066	22.887	105.1	105.2
40.....	22.165	22.176	21.054	105.3	105.3
45.....	20.112	20.130	19.121	105.2	105.3
50.....	17.984	18.014	17.114	105.1	105.3
55.....	15.837	15.879	15.065	105.1	105.4
60.....	13.676	13.730	13.016	105.1	105.5
65.....	11.496	11.564	11.013	104.4	105.0
70.....	9.351	9.436	9.107	102.7	103.6
75.....	7.323	7.428	7.344	99.7	101.1
80.....	5.492	5.621	5.761	95.3	97.6
85.....	3.923	4.075	4.387	89.4	92.9
90.....	2.651	.....	3.228	82.1	.....
95.....	1.677	.....	2.259	74.2	.....
100.....	.977	.....	1.356	72.1	.....

A number of developments in the use of annuities and life insurance settlement options, which have taken place mainly since the depression of the 1930's, have in varying degrees changed the classes of persons covered under these contracts. Increased use of settlement options has apparently been accompanied by increased selection against the companies, operating to produce lower mortality rates. This may have been due to the increasing proportion of payee-elections or to increasing discrimination in the choice of options, or both. On the other hand, a tendency towards the purchase of annuities for investment or tax purposes and to supplement income for early retirements may have tended to increase somewhat the mortality rates under immediate annuities, but apparently chiefly at ages under 65 and on refund contracts. In the authors' opinion, the mortality under immediate nonrefund annuities at ages 65 and over has probably been least affected by changing class selection.

If this view is correct, it can be concluded from Table 14 that there has been a substantial long-term decrease in mortality rates under immediate nonrefund annuities in the important age-range from 65 to 84. At ages 65 to 74 this decrease has been of the order of 1% per year in the case of males and 1.6% per year among females. A decrease of 0.6% per year may be similarly predicated for males in the age-range from 75 to 84. If allowance is made for the effects of the graduation of the female data at advanced ages in the American Annuitants Tables, a long-term decrease in mortality of over 1% per year can be assumed to have occurred among females at ages 75 to 84.

For the reasons cited above and in Section III, the figures for annuitants at ages 55 to 64, appearing in Table 14, probably underestimate the actual long-term mortality decreases, and in any event do not seem to be sufficiently reliable to permit definite conclusions as to the magnitude of the long-term decreases in mortality at these ages.

TABLE 12—*Continued*
COMPARISON OF ANNUITY VALUES AT 2½% INTEREST—FEMALES
ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION)
AND 1937 STANDARD ANNUITY TABLE

Age x	1949 Table Ultimate a_x	1949 Table Select $a_{[x]}$	1937 Standard Annuity Table a_x	Percentage 1949 Table Ultimate a_x Is of Standard Annuity Table a_x	Percentage 1949 Table Select $a_{[x]}$ Is of Standard Annuity Table a_x
15.....	31.222	31.227	29.986	104.1%	104.1%
20.....	30.121	30.126	28.870	104.3	104.4
25.....	28.890	28.897	27.604	104.7	104.7
30.....	27.516	27.525	26.180	105.1	105.1
35.....	25.988	25.999	24.602	105.6	105.7
40.....	24.295	24.311	22.887	106.2	106.2
45.....	22.433	22.455	21.054	106.5	106.7
50.....	20.404	20.436	19.121	106.7	106.9
55.....	18.215	18.257	17.114	106.4	106.7
60.....	15.882	15.942	15.065	105.4	105.8
65.....	13.455	13.540	13.016	103.4	104.0
70.....	11.010	11.128	11.013	100.0	101.0
75.....	8.642	8.802	9.107	94.9	96.7
80.....	6.459	6.671	7.344	87.9	90.8
85.....	4.560	4.827	5.761	79.2	83.8
90.....	3.012	.....	4.387	68.7	.....
95.....	1.838	.....	3.228	56.9	.....
100.....	1.012	.....	2.259	44.8	.....

The much larger rates of decrease in mortality indicated by Table 14 for life income settlement options are believed to have been affected very substantially by the changing class selection mentioned above. In the authors' opinion, they are, therefore, of only general value for ascertaining long-term trends.

For further detail of the long-term decreases, as measured between the current experiences and the American Annuitants Tables, the United States Annuitants Tables, and the 1924-34 Life Income Settlement Option experience, the reader is referred to Chart 5.

The mortality experience of one very large corporation (predominantly nonhazardous occupations) among its retired lives, included in Table 14, shows long-term decreases in mortality of 1.1% per year in the age-range from 65 to 74 for both sexes, and very little change at ages 75 to 84. While these figures may be considered as supporting, in a general way, the credibility of the data for immediate nonrefund annuities, the experience of this corporation was affected to a degree by specialized selection arising from eligibility, retirement, and other rules.

The long-term decreases that have occurred in population mortality rates shed considerable light on the question under consideration. When analyzed by cause of death, the population figures show why the underlying trends have changed, and when examined from decade to decade in a long historical sequence, they provide a basis for judging the relative merits of the "generation" and "year of exposure" hypotheses[15] of mortality changes. At the same time, it is recognized that class selection has, without doubt, caused annuitants, as a class, to be somewhat different mortality-wise from the class consisting of the entire population. This difference, as measured by mortality rates at a given time, is easily shown and has been frequently demonstrated. However, the degree to which this class difference has changed over the years is difficult if not impossible to determine and the effects of this changing class difference on the rates of decrease in annuitant mortality cannot be separated from the underlying mortality trends. The long-term population mortality trends are believed to furnish the best available indication of the long-term trends underlying the mortality decreases among annuitants.

TABLE 13

COMPARISON OF ANNUITY VALUES AT 2½% INTEREST ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION) AND 1937 STANDARD ANNUITY TABLE SET BACK

Age x	Males			Females		
	Percentage 1949 Table Select $a_{[x]}$ Is of Standard Annuity Table a_x Set Back			Percentage 1949 Table Select $a_{[x]}$ Is of Standard Annuity Table a_x Set Back		
	1 Year	2 Years	3 Years	1 Year	2 Years	3 Years
15.....	102.9%	102.1%	101.3%	103.4%	102.7%	102.1%
25.....	103.2	102.1	101.0	103.7	102.7	101.8
35.....	103.6	102.0	100.6	104.3	103.0	101.7
45.....	103.2	101.1	99.2	104.8	103.0	101.3
55.....	102.6	99.9	97.4	104.2	101.9	99.6
65.....	101.4	98.0	94.7	100.9	97.9	95.1
75.....	96.7	92.5	88.6	92.9	89.3	86.0
85.....	87.7	83.0	78.6	79.6	75.8	72.2

CHART 3

PERCENTAGE BY WHICH ANNUITY VALUE ON 1937 STANDARD ANNUITY TABLE AT 2½% DIFFERS FROM SELECT ANNUITY VALUE ON ANNUITY TABLE FOR 1949 AT 2½%

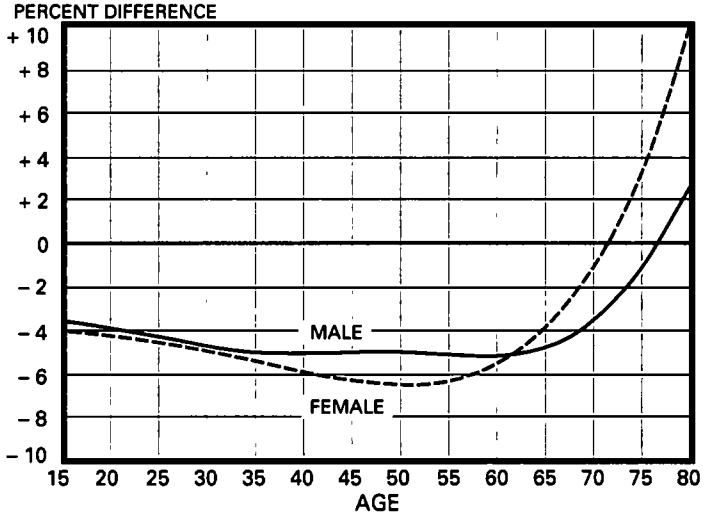


TABLE 14

LONG-TERM DECREASES IN MORTALITY RATES

AVERAGE RATE OF DECREASE PER YEAR (GEOMETRICAL BASIS)

Age Group	Immediate Annuities* (By Number of Contracts)		Joint Committee Settlement Options† (By Number of Contracts)	Large Industrial Corporation Retired Lives (By Number of Lives)	N.Y. State Retirement Plan Clerical Employees Active Lives‡ (By Number of Lives)	U.S. White Population§	Ordinary Policy Holders Met. Life Ins. Co. (By Amounts of Insurance)
	From 1910 to 1943	From 1923 to 1943	From 1924-34 to 1940-45	From 1913-28 to 1944-48	From 1921-25 to 1940-45	From 1920 to 1947	From 1911 to 1939
Males:							
15-24.....	.....	.....	.....	.....	.....	3.4%	2.6%
25-34.....	.....	.....	.....	.....	6.2%	3.7	3.1
35-44.....	.....	.....	.....	.....	2.9	2.1	2.0
45-54.....	.....	.....	0.4%	.....	0.1	0.4	1.0
55-64.....	1.0%	0.3%	2.1	.....	0.7	0.1	0.8
65-74.....	0.9	1.1	1.7	1.1%	.....	0.1	0.6
75-84.....	0.6	0.7	.....	0.1	.....	0.3	.....
Females:							
15-24.....	.....	.....	.....	.....	.....	5.5%	4.0%
25-34.....	.....	.....	.....	.....	4.0%	5.5	3.1
35-44.....	.....	.....	.....	.....	4.4	3.5	2.0
45-54.....	.....	.....	3.9%	.....	3.9	2.1	–0.6
55-64.....	2.4%	0.7%	4.1	.....	–0.3	1.6	1.1
65-74.....	1.6	1.7	2.8	1.1%	.....	1.1	1.2
75-84.....	0.7	1.7	.....	–0.2	.....	0.8	.....

Both Sexes Combined:	Intercompany Group Life Insurance Predominantly Clerical Lives¶ (By Number Of Lives)	Ordinary Insurance Experience# (By Amounts Of Insurance)
	From 1932-35 to 1946-47	From 1900-15 to 1946-47
15-24.....	4.1%	.....
25-34.....	4.6	2.9%
35-44.....	3.0	2.1
45-54.....	1.1	1.0
55-64.....	0.8	0.6
65-74.....	2.2	0.7
75-84.....	1.8	0.6

* From 1910 to 1943: American Annuitants Ultimate Table compared with 1943 Experience Table Ult. From 1923 to 1943: U.S. Annuitants Tables compared with 1943 Experience Table Ult.

† Experience for 1924-34 from *TASA*, XXXIX, 8; experience for 1940-45 from *TASA*, XLVIII, 133.

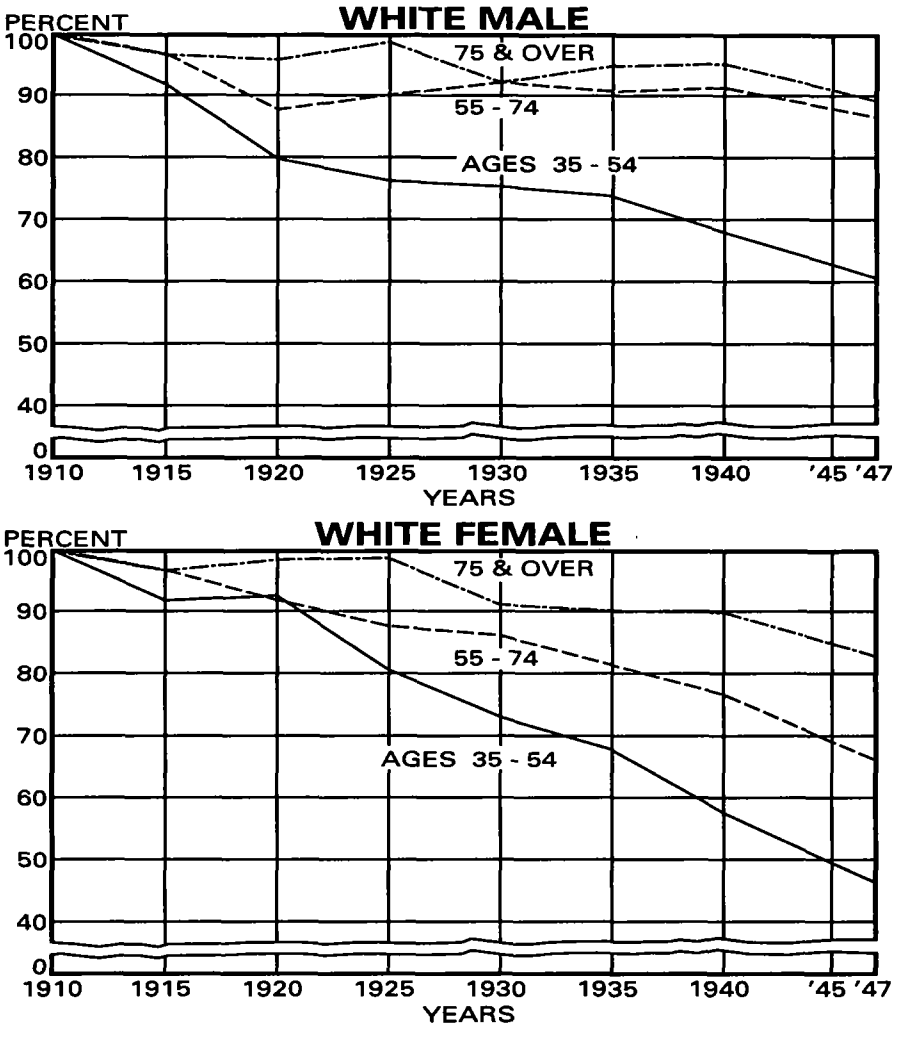
‡ Based on the 5th and 25th reports of New York State Controller on the Operation of the State Employees' Retirement System.

§ Expanding Registration States.

¶ Experience for 1932-35 from the 1936 report of the Committee on Group Mortality Investigations; experience for 1946-47 from *TASA*, XLIX, 477.

\# American Men Ultimate Table compared with joint Committee ultimate experience from anniversaries in 1946 to anniversaries in 1947 from *TASA*, XLIX, 468.

CHART 4
LONG-TERM DECREASES IN U.S. WHITE POPULATION MORTALITY* RATES
PERCENT THAT MORTALITY RATE IN SPECIFIED YEAR
IS OF MORTALITY RATE IN 1910



* Expanding Registration State, 1910–47

TABLE 15*
LONG-TERM AND SHORT-TERM DECREASES IN POPULATION MORTALITY BY CAUSE OF DEATH—U.S.† WHITE MALES AND WHITE FEMALES 1911–46 AND 1939–46

Age Group	Cardiovascular-Renal					Cancer (All Forms)				
	Death Rates per 100,000			Average Rate of Decrease per Year (Geometrical Basis)		Death Rates per 100,000			Average Rate of Decrease per Year (Geometrical Basis)	
	1920‡	1939	1946	1920-46	1939-46	1911	1939	1946	1911-46	1939-46
Male:										
35-44...	104.7	131.4	131.9	–0.9%	–0.1%	29.3	38.8	37.7	–0.7%	0.4%
45-54...	307.4	446.5	479.5	–1.7	–1.0	106.3	130.9	140.8	–0.8	–1.1
55-64...	957.5	1197.7	1220.7	–0.9	–0.3	261.4	348.9	370.1	–1.0	–0.9
65-74...	2626.4	2965.8	2902.6	–0.4	0.3	469.9	737.0	788.3	–1.5	–0.9
75 and over...	.....	8123.2	7546.1	.....	1.1	.....	1349.3	1417.4	.....	-0.7
Female:										
35-44...	121.5	87.9	70.2	2.1	3.2	87.0	79.2	74.1	0.5	1.0
45-54...	323.3	269.4	228.8	1.3	2.3	222.5	198.7	191.5	0.4	0.5
55-64...	884.3	770.0	653.8	1.2	2.3	386.5	391.7	363.5	0.2	1.1
65-74...	2444.1	2286.3	2054.8	0.7	1.5	574.4	669.7	659.2	–0.4	0.2
75 and over....	.....	7478.4	6845.6	.....	1.3	.....	1152.1	1141.3	.....	0.1
	Influenza And Pneumonia					Accidents				
	Death Rates per 100,000			Average Rate of Decrease per Year (Geometrical Basis)		Death Rates per 100,000			Average Rate of Decrease per Year (Geometrical Basis)	
	1911	1939	1946	1911-46	1939-46	1911	1939	1946	1911-46	1939-46
Male:										
35-44...	97.2	34.8	15.4	5.1%	11.0%	149.3	84.9	77.8	1.8%	1.2%
45-54...	153.9	64.8	32.1	4.4	9.6	167.3	106.5	92.8	1.7	2.0
55-64...	263.4	123.3	65.3	3.9	8.7	185.2	143.4	125.1	1.1	1.9
65-74...	522.7	264.3	150.1	3.5	7.8	222.7	212.8	203.5	0.3	0.6
75 and over....	.....	911.1	575.5	.....	6.4	.....	540.5	521.9	.....	0.5
Female:										
35-44...	57.3	23.9	9.2	5.1	12.8	19.3	16.6	16.4	0.5	0.2
45-54...	102.1	38.9	15.6	5.2	12.2	31.1	24.8	21.8	1.0	1.8
55-64...	229.4	77.9	33.7	5.3	11.3	52 2	48 8	36.7	1.0	4.0
65-74...	582.4	213.3	106.4	4.7	9.5	124.5	138.4	111.7	0.3	3.0
75 and over....	.....	887.5	535.5	.....	7.0	.....	721.1	637.1	.....	1.8

* Basic data from various publications of the Bureau of the Census.
† Expanding Registration States.
‡ Strictly comparable data for cardiovascular-renal diseases not available prior to 1920.

TABLE 15—*Continued*

Age Group	Tuberculosis (All Forms)					Syphilis				
	Death Rates per 100,000			Average Rate of Decrease per Year (Geometrical Basis)		Death Rates per 100,000			Average Rate of Decrease per Year (Geometrical Basis)	
	1911	1939	1946	1911-46	1939-46	1911	1939	1946	1911-46	1939-46
Male:										
35-44...	258.3	60.9	46.1	4.8%	3.9%	29.6	15.6	7.2	4.0%	10.5%
45-54...	245.1	86.5	74.1	3.4	2.2	44.9	31.0	19.8	2.3	6.2
55-64...	259.5	97.3	95.6	2.8	0.3	57.3	53.1	33.6	1.5	6.3
65-74...	247.7	101.4	102.9	2.5	-0.2	65.5	56.5	43.5	1.2	3.7
75 and over....	.....	91.1	91.0	.....	0.0	.....	48.7	37.5	.....	3.7
Female:										
35-44...	179.5	35.3	24.8	5.5	4.9	9.5	5.6	2.8	3.4	9.4
45-54...	136.7	31.9	21.1	5.2	5.7	14.4	8.9	5.2	2.9	7.4
55-64...	145.0	40.4	25.5	4.8	6.4	20.3	13.1	7.4	2.8	7.8
65-74...	184.4	58.6	42.1	4.1	4.6	31.3	16.0	11.8	2.8	4.3
75 and over....	.....	73.5	54.5	.....	4.2	.....	18.3	13.1	.....	4.7
	Diabetes					Appendicitis				
	Death Rates per 100,000			Average Rate of Decrease per Year (Geometrical Basis)		Death Rates per 100,000			Average Rate of Decrease per Year (Geometrical Basis)	
	1911	1939	1946	1911-46	1939-46	1911	1939	1946	1911-46	1939-46
Male:										
35-44...	9.0	4.7	4.8	1.8%	–0.3%	14.8	12.6	3.8	3.8%	15.7%
45-54...	20.2	17.7	15.8	0.7	1.6	14.2	16.9	6.0	2.4	13.8
55-64...	58.2	57.7	49.4	0.5	2.2	14.2	20.5	10.0	1.0	9.8
65-74...	92.3	140.1	126.1	-0.9	1.5	12.4	24.5	12.3	0.0	9.4
75 and over....	.....	233.6	209.8	.....	1.5	.....	21.9	15.0	.....	5.3
Female:										
35-44...	8.5	6.0	4.0	2.1	5.6	9.9	6.2	2.1	4.3	14.3
45-54...	24.3	30.0	21.6	0.3	4.6	9.8	9.5	3.0	3.3	15.2
55-64...	70.9	111.0	92.7	-0.8	2.5	10.8	14.0	4.4	2.5	15.2
65-74...	120.3	243.11	223.7	-1.8	1.2	10.4	15.9	7.7	0.9	9.8
75 and over....	.....	294.8	302.8	.....	-0.4	.....	21.9	11.7	.....	8.6

TABLE 16

DECREASES IN POPULATION MORTALITY RATES

AVERAGE RATE OF DECREASE PER YEAR OVER THE PERIODS INDICATED (GEOMETRICAL BASIS)

Sex	Age Group	1870-80 Mass. (1)*	1880-90 Mass. (1)*	1890-1901 Mass. (2)	1901-10		1910-20		1920-30		1930-40		1940-47 U.S White (8)
					Mass. (2)	U.S. White (6)	Mass. (2)(3)	U.S. White (6)(7)	Mass. (3)(4)	U.S. White (7)	Mass. (4)(5)	U.S. White (8)	
Males...	10-19	.6%	.3%	2.9%	2.4%	1.8%	.5%	.2%	4.2%	-.1%	4.6%	4.0%	3.2%
	20-29	1.0	1.2	2.4	2.8	2.4	.8	1.2	4.7	-.3	4.8	4.1	3.1
	30-39	.3	-.1	1.9	1.2	1.2	1.5	2.3	3.6	-.1	3.5	3.3	3.2
	40-49	.2	-1.4	.9	.5	. 1	2.2	3.1	1.0	.1	1.1	1.9	1.7
	50-59	- .5	-1.4	.2	.0	-.3	2.0	2.5	-.2	.2	.1	.6	.8
	60-69	-1.2	-1.0	-1.2	-.8	-.6	1.7	2.0	-.0	.1	-.1	.5	.5
	70-79	- .6	- .4	- .7	- .7	- .5	.7	1.2	.0	.1	.2	.4	.9†
	80 and over	-.8	.6	-.6	.0	.0	.4	.7	-1.2	.0	.5	.2	
Females...	10-19			4.2%	2.5%	2.6%	.8%	.4%	4.9%	-.1%	6.8%	5.3%	5.8%
	20-29			2.9	3.0	2.9	-1.3	-.5	5.9	- .2	6.5	6.0	6.3
	30-39			2.5	1.6	2.0	.3	.9	4.9	- . 1	4.8	4.4	4.8
	40-49			1.5	1.1	1.0	1.3	1.9	2.3	.0	2.6	2.9	3.3
	50-59			-.0	.7	.3	1.3	1.8	1.2	.2	1.5	2.0	2.8
	60-69			-.6	- .6	- .5	1.1	1.6	.7	.1	1.1	1.6	2.2
	70-79			-1.0	- .2	- .4	.4	.9	.3	.1	.6	.9	1.5†
	80 and over			-.3	-.1	-.3	.3	.6	.2	.0	1	.4	

* Males and Females combined.
† Ages 70 and over.
(1) Massachusetts 1870-1880,1880-1890, males and females combined, data from 70th Report of Births, Marriages, and Deaths in Massachusetts, 1911
(2) Massachusetts 1890-1901, 1901-1910, and 1910, data from U.S. Life Tables 1890, 1901, 1910, and 1901-10
(3) Massachusetts 1920 from U.S. Abridged Life Tables, 1919-1920
(4) Massachusetts 1930 from Population Statistics, National Resources Committee, 1937
(5) Massachusetts 1940 from State and Regional Life Tables 1939-41
(6) U.S. 1901-10 and 1910 original registration states
(7) U.S. 1920 and 1920-30 Registration States of 1920
(8) U.S. 1930-40 and 1940-47 Continental U.S. based on U.S. Life Tables 1930, U.S. Life Tables 1939-41 and Federal Security Agency Release of January 30, 1949

The long-term decreases in mortality rates of the white population of the United States are shown in Table 14 for the period from 1920 through 1947. The corresponding rates of decrease among the Canadian population during the period from 1931 through 1945 (not shown in Table 14 because of the relatively short timespan) were somewhat smaller. Both sets of population data show the characteristic reduction in the rate of decrease in mortality with advancing age, a feature of all experiences examined which covered both young and old ages. This characteristic is illustrated in Chart 4.

Table 15 presents long-term and short-term rates of decrease in the mortality rates from the principal causes of death, at ages 35 and over by decennial age groups, for the white population of the United States. By considering the death rates from all cardiovascular-renal diseases combined, a more meaningful picture is obtained of the trend of mortality rates from heart disease and allied conditions, since the effects of differences in terminology and reporting at various times are thereby minimized. The long-term mortality decreases among males at ages 45 to 64 are seen to have occurred because the declines in death rates from tuberculosis, influenza, pneumonia, accidents, and syphilis more than offset the increases in mortality from cardiovascular-renal diseases and cancer. Among males at ages 65 and over, the long-term mortality trends have not been as favorable mainly because of relatively smaller declines in mortality from influenza, pneumonia, and accidents, greater increases in mortality from cancer and increased mortality from diabetes. Among females at ages 55 and over, the long-term over-all decreases in mortality

resulted because increases in death rates from cancer and diabetes were offset by decreases from the other major causes of death considered in Table 15.

There was something of a "break" in the mortality trends by cause of death about the year 1939, as shown in Table 15. Death rates from cardiovascular-renal diseases, which had previously been increasing among males, have since the "break" shown a much smaller rate of increase at ages under age 65 and a decrease at ages 65 and over. Death rates from these diseases among females have shown a much greater rate of decrease since the "break." Similarly, cancer death rates have since that time shown a smaller increase among males and a sizable decline among females. The almost miraculous cures wrought by the "sulpha" drugs and, more recently, by penicillin are reflected in sharply declining death rates from influenza, pneumonia, and appendicitis. This "break" in mortality can thus be ascribed largely to chemotherapy, improvements in surgery, and progress in controlling the major constitutional diseases. The fact that mortality at all ages has been affected by these developments at about the same time throws considerable doubt on the applicability of the "generation" hypothesis to changes in mortality rates now in progress.

Some further light on the validity of the "generation" hypothesis for interpreting the changes in mortality rates of the white population in the United States is shed by Table 16. This table shows the average rate of decrease per year in mortality rates for decennial age-groups over 10-year or similar periods. It is difficult, if not impossible, to conclude that the long-term trends in this table are more discernible along the diagonals than horizontally. The record suggests rather that past mortality decreases have been concentrated in different age groups at different times, being probably associated with the hygienic, social, and medical advances characteristic of the various periods. In the authors' opinion, long-term mortality decreases, such as those indicated in Table 16, can more readily be represented as functions of attained age and calendar year of exposure.

The long-term morality decreases among lives insured under individual ordinary and group life insurance policies, shown in Table 14, adduce further data as to the pattern of mortality decreases at the younger ages, and over the entire age-range exhibit the characteristic reduction in the rate of mortality decreases with advancing age. The same pattern is indicated by the experience among clerical employees—active lives—in the New York State Retirement Plan, also shown in Table 14.

VII. Informed Opinion on Long-Term Mortality Decreases

The statistics on long-term decreases in mortality rates presented in Section VI naturally raise the question as to what may be expected in the future. The crucial question is perhaps whether future decreases in mortality will follow the pattern recorded in the past, and more particularly whether the relatively small and somewhat irregular reductions noted at the older ages can be regarded as a measure of their future trend. The issue may also be raised of whether future decreases in the mortality of annuitants are likely to resemble those anticipated for the general population or for groups of insured lives.

Many authorities in the fields of population, public health, geriatric medicine, specialists in heart disease and cancer, and other students of mortality at the older ages have in recent years expressed optimistic opinions as to the future course of mortality at the older ages. In considering the opinions of these experts, it is well to distinguish between those long-term decreases in mortality which might be expected on the basis of more intensive applications of existing knowledge, and those which have been visualized at greater range by assuming more far-reaching progress in our understanding of the diseases responsible for most of the deaths at the older ages.

Thus, P. K. Whelpton Associate Director of the Scripps Foundation for Research in Population Problems, made the following statement[16] in connection with his forecasts of the population of the United States for 1945-75:

> Although the degenerative diseases have not as yet been brought under control, there is continued hope for the future. Because certain damaging infectious diseases (*e.g.*, scarlet fever, diphtheria, and typhoid fever) have almost been eliminated, a substantial reduction should occur in the organic impairments and after-effects so common with such diseases. As these sequelae are reduced in frequency there should be a reduction in the number of organic breakdowns or a postponement of these breakdowns until later in life. Similar gains should result from the more recent campaigns to control venereal disease. For this reason and because of improved techniques for early diagnosis, there should be some reduction in the mortality from the degenerative diseases even without the discovery of better methods for their prevention

or treatment. In view of the great amount of research being done on the causes and control of cancer, it is quite possible that the number of deaths from this disease will be much lower before many years pass.

According to a very apt summary[17] by Dr. Sigismund Peller, practicing physician and outstanding biometrician:

The mortality of persons aged 50 years or more may be expected to fall further, even without any additional progress in the understanding of the degenerative diseases. An increase in general welfare and in the standard of living; eradication of slums; reduction of air pollution in the cities; the spread of garden cities; regulation of diet, especially for the middle-aged and old; action against alcoholism; reform of men's clothes; shorter working hours and fewer working days per week; paid annual vacations, combined with careful general check-ups to counteract the development of crippling diseases; social legislation diminishing the anxiety of aging persons; all these are bound to have a salutary effect upon health, duration of life and the mortality of persons in late maturity and old age.

In 1947 the eminent British surgeon, Sir John Conybeare, reviewed the effects on mortality of recent advances in treatment and concluded with this observation:[18]

Without doubt over the next twenty-five years there will be a very material decrease in mortality as a result of better provision for diagnosis and treatment, quite apart from further discoveries of new drugs or improvement in operative measures.

Dr. C. P. Rhoads, Director of Memorial Hospital in New York City and one of the foremost authorities in the field of cancer research, has indicated[19] that even within the limitations of present knowledge, there is room for considerable progress:

The contributions of wartime research to surgery in terms of better antibiotics and effective means of combating shock are too well recognized to need comment. The point should be made, however, that any improvement in general medical or surgical technics is promptly reflected in the cancer cure rate, since cancer is today a problem of surgery almost exclusively.

There can be no doubt that the cure of cancer by the means available at present is a cumbersome, expensive, and troublesome business. There is equally little doubt that by an adequately extensive program of education and study it can be materially improved, even though no magic potion or silver bullet be discovered. This is, perhaps, as cheering a conclusion as can be reached by those who wish to see immediate progress and are not too sanguine about the possibility of an immediate, revolutionary discovery.

There is a great deal of optimism among experts regarding current developments which promise increasing control over the diseases of old age. Dr. Louis I. Dublin, Second Vice-President and Statistician of the Metropolitan Life Insurance Company, recently pointed out that:[20]

A great many developments are going on which have a real bearing on the control of the diseases of old age. One is the resurgence of hospital ,construction. Another is the increasing help of health insurance in providing good medical care. The field of health education is becoming much better established and is developing its own techniques. There is a spirit of cooperation among the various volunteer official health services. All of these conditions are propitious for a successful mass attack on the diseases and conditions of old age.

In a similar vein, Dr. Rhoads, in discussing the perspectives in cancer research recently said:[21]

Of one thing we can be certain. Perhaps never before in history has a scientific effort been undertaken so extensive, and in such qualified hands, as the one now under way against cancer. The production in terms of new facts of general importance is assured. The result in terms of cancer control is equally certain but wholly unpredictable as to time.

The prospects for large reductions in mortality appear substantial if it is assumed that the best skills and techniques now known will be generally applied. In addition, some allowance must be made for probable further advances in our understanding of the degenerative diseases and for some new discoveries in prevention and treatment of these diseases.

Thus, in discussing the possibilities for control of heart disease, Dr. D. B. Armstrong, Second Vice-President of the Metropolitan Life Insurance Company and an authority in the field of preventive medicine said recently:[22]

The outlook for the control of heart disease is promising ... the prospect for heart disease patients appears to be better than ever before. The major requirements for further improvement in the picture are expansion of research activity and wider public education.

The fight against heart disease is only beginning. An increasing number of investigators are studying fundamental problems regarding the prevention and treatment of the diseases of the heart and arteries, notably those aided by the Life Insurance Medical Research Fund, recently established by 148 life insurance companies. These studies should result in new methods of prevention and treatment.

With special reference to the treatment of coronary thrombosis, Dr. I. S. Wright, a noted heart specialist, expressed himself as follows to the authors:

There are many factors which are obviously going on to prolong life in varying degrees so that our general conception of longevity will, in our opinion, have to be markedly changed. As an example, the above study[23] reveals that the death rate from coronary thrombosis may be reduced one third per attack with the use of anticoagulant therapy.

I believe there is now sufficient evidence at hand to make that statement valid. The result will be that, if sufficient persons with coronary thrombosis receive anticoagulant therapy (and this in a sense is dependent on newer and better anticoagulants which are now being developed) at least one third of those who have attacks will live longer than they would have previously. In a specific instance, no one can say how much longer.

The American Cancer Society has repeatedly gone on record to the effect that cancer mortality could be cut by a third under optimum conditions, with prompt diagnosis and proper application of the best techniques known today. This would increase the average cure rate for all forms of cancer to about 50%[24] from an estimated 25% at present. In commenting upon this, Dr. Louis I. Dublin has stated:[25]

> The estimate made by the American Cancer Society, that current cancer mortality can be cut one-third by application of present knowledge, does not overstate the case, and it should be the aim of every health officer to achieve that goal for his community.

Speaking more generally, Dr. Dublin observed that:[26]

> Other areas of the public health, heretofore neglected, will certainly receive more concentrated attention in the future. The gains in longevity of which we are so proud have greatly increased the numbers of older persons in the population and have given greater emphasis to the diseases of middle life and old age, such as heart disease, arteriosclerosis, cancer, and arthritis. We have been prone to consider these conditions the inevitable consequence of the aging process and as such beyond preventive or remedial measures. This view has been shortsighted and not at all in line with the most recent developments in medicine. But we have at last awakened to the urgency of the situation, and a vast amount of research is now going on to discover the causes of cancer and of the other degenerative processes. With all this activity, it is only a question of time before their vital secrets will be revealed. Once they are known, it should be possible to determine the measures best adapted to counteract, or at least to postpone, the afflictions of old age.

The question may be raised, of course, whether future mortality rates among annuitants are likely to respond to the same forces as and resemble those anticipated for the general population. In the past, annuitant mortality has to a degree followed that of the general population (see Table 14). Further indications of future trends may be deduced from an analysis of annuitant mortality by cause of death. Such an analysis is presented in Table 17, based on a limited experience under nonrefund and refund annuities in the Metropolitan Life Insurance Company (1941–46) and in the Teachers Insurance and Annuity Association (1939–48). The table brings out that among annuitants at ages 65 and over almost two-thirds of the deaths have been due to cardiovascular-renal diseases and that cancer has been responsible for about 10 percent of the deaths among male and about 15 percent of the deaths among female annuitants at these ages. It is difficult to avoid the conclusion that substantial reductions in population mortality from cardiovascular-renal diseases or cancer, particularly if due to better preventive or therapeutic measures, are bound to be reflected in reasonably comparable reductions in the mortality of annuitants from these causes. Influenza and pneumonia, tuberculosis, and accidents account for a large proportion of the deaths among annuitants from causes other than cardiovascular-renal diseases and cancer, and there is every reason to believe that mere continuance of past trends with respect to these conditions will result in further decreases in annuitant mortality.

Actuaries are apparently of this opinion. Mr. E. W. Marshall, Vice-President and Actuary of the Provident Mutual Life Insurance Company, recently expressed his views[27] in the matter as follows:

> We may reasonably expect that in due course the longevity of annuitants will be considerably greater than in recent years. Vigorous attacks are under way on important causes of death at the middle and older ages, and it seems likely that these attacks will be at least partly successful.

Mr. E. G. Fassel, Vice-President and Actuary of the Northwestern Mutual Life Insurance Company, recently commented[28] as follows:

> It seems to me that decreasing mortality is inherent in annuities. Remember the first law of nature—the instinct of self-preservation. The human race is constantly seeking means of living longer. In consequence we have meat for insurance and poison for annuities. Decreasing mortality has been a feature of the annuity business since it commenced and I am sure we will continue to have it for a long time.

Mr. R. D. Murphy, Vice-President and Actuary of the Equitable Life Assurance Society, had the following[29] to say in a recent paper:

> Every one is familiar with the steady and substantial decrease which has come about in the death rates among infants and younger adults as a result of the progress of medical science and public health measures in preventing and curing infectious and contagious disease. When life insurance companies point out that mortality rates have been decreasing at the older ages as well in their experience under annuities, sometimes surprise is expressed. In the general population, however, this lowering of mortality at older ages is also evident... Accordingly, it becomes clear that one of the essential problems that has to be solved for the successful management of the annuity business is the forecasting of future mortality at lower death rates than have been experienced in the past.

TABLE 17

ANNUITANT MORTALITY BY CAUSE OF DEATH

EXPERIENCE OF METROPOLITAN LIFE INSURANCE COMPANY AND TEACHERS INSURANCE AND ANNUITY ASSOCIATION UNDER IMMEDIATE NONREFUND AND REFUND ANNUITIES

Cause of Death	Age Group 65-74						Age Group 75-84						Age Group 85 And Over					
	MLICo 1941-46		TIAA 1939-48		Total		MLICo 1941-46		TIAA 1939-48		Total		MLICo 1941-46		TIAA 1939-48		Total	
	Deaths	% of Total	Deaths	% of Total	Deaths	% of Total	Deaths	% of Total	Deaths	% of Total	Deaths	% of Total	Deaths	% of Total	Deaths	% of Total	Deaths	% of Total
Males:																		
Cardiovascular-renal	110	65%	278	59%	388	61%	137	68%	189	59%	326	62%	51	80%	9	82%	60	80%
Cancer	21	12	53	11	74	12	13	6	36	11	49	9	2	3	0	0	2	3
Pneumonia and Influenza	11	7	26	6	37	6	18	9	21	7	39	8	3	5	1	9	4	5
Accidents	3	2	13	3	16	2	6	3	4	1	10	2	0	0	0	0	0	0
All Other Causes	23	14	101	21	124	19	28	14	70	22	98	19	8	12	1	9	9	12
Total	168	100%	471	100%	639	100%	202	100%	320	100%	522	100%	64	100%	11	100%	75	100%
Females:																		
Cardiovascular-renal	145	67%	63	50%	208	60%	179	71%	54	52%	233	65%	53	65%	4	37%	57	62%
Cancer	33	15	26	20	59	17	23	9	19	18	42	12	2	3	4	36	6	7
Pneumonia and Influenza	8	4	3	2	11	3	20	8	6	6	26	7	9	11	1	9	10	11
Accidents	3	1	7	6	10	3	7	3	2	2	9	3	3	4	0	0	3	3
All Other Causes	29	13	28	22	57	17	23	9	23	22	46	13	14	17	2	18	16	17
Total	218	100%	127	100%	345	100%	252	100%	104	100%	356	100%	81	100%	11	100%	92	100%

The foregoing quotations are naturally only statements of opinion; no one can know what the future will bring. Perhaps some reader, dealing mainly with life insurance and inclined toward a proper amount of actuarial conservatism, will be led by his experience to discount somewhat the spirit of optimism pervading the nonactuarial opinions stated above. But if this be the case, he should give serious consideration to two facts. First, that paradoxically it is conservative to assume radical decreases in mortality rates for annuities. Second, the men quoted, including the actuaries, have given more extensive and more careful thought to these matters than have many actuaries; they have at their disposal more information on this subject than is generally available, and they are outstanding men in their respective professions. The weight of such opinion is considerable and cannot be disregarded.

VIII. Long-Term Mortality Decreases Assumed by Others

A number of long-term forecasts of mortality rates[30] in the general population have been made by students of population and public health chiefly for the purpose of projecting future population. Projections have also been made of annuitant mortality by actuaries in Great Britain in connection with the British Offices annuitants investigation (1900–1920)[31] and the British Government life annuitants investigation (1900–1920)[32]. The various approaches to mortality forecasting have all been largely empirical, even though the problem has been attacked in many different ways.

It is also important to bear in mind that the population forecasts have usually been in the nature of "most probable" estimates of the future, as distinguished from prognostications conservative for annuity purposes. Students of population and public health have generally based their projections on the assumption of continued and more intensive application of existing knowledge, rather than on expectations of major advances comparable to those of recent years in the field of antibiotics. It is not surprising, therefore, that several of the forecasters have seen their long-range projections almost realized within a short period of years.

A recent series of projections made by P. K. Whelpton and his associates[33] deserve special attention. These forecasts supersede previous forecasts made in 1943 by Whelpton in collaboration with W. S. Thompson[34] and a still earlier series of forecasts published in 1937.[35] The forecasts prepared in 1943 were soon found to be at variance with the relatively light civilian mortality experienced during the war years. In his latest series of projections, Whelpton not only embodied the results of his experience with previous forecasts but also made elaborate studies of past mortality trends in the United States by age, sex, state, and cause of death, as well as of death rates in other countries with relatively low mortality. He reached the conclusion that past mortality trends should not be extrapolated into the future on the basis of any mathematical formulae. Accordingly, he based these latest forecasts on three alternative, empirical assumptions designated as "high," "medium," and "low." The annual rates of decrease in mortality implicit in the "medium" and "low" mortality assumptions are shown in Table 18. It should be noted that the "medium" assumptions provide for only a slight improvement in mortality at age 60 and none beyond age 70, while his "low" assumptions provide for larger improvement up to age 70 but none beyond age 80.

In the preparation of long-range cost estimates for Old-Age and Survivors Insurance, R. J. Myers assumed[36] for his "high-cost" estimates that up to age 65 mortality would follow the "low" assumptions previously made by Thompson and Whelpton in 1943, but he allowed (quoting Myers) "for a greater improvement in mortality beyond that age so as to take into account the possible great gains which may be made in the future through geriatric medical research." The annual rates of decrease in mortality implicit in Myers' assumptions are shown in Table 18. They provide for higher mortality rates than Whelpton's recent "low" mortality assumptions through age 60, but lower mortality in the age range from 70 through 90.

Another interesting method of mortality forecasting was that developed by A. J. Coale for population projections of European countries.[37] This method was more recently used in forecasting the population of Canada. It takes account not only of past mortality trends but also of the absolute level of mortality, and thus produces different rates of decrease in mortality for males and females. The annual rates of decrease in mortality implicit in Coale's long-range assumptions are also shown in Table 18. They provide for rather small improvement in mortality in the age range from 70 through 80 and none beyond age 80.

TABLE 18
AVERAGE RATES OF DECREASE PER YEAR
(GEOMETRICAL BASIS)
ASSUMED BY VARIOUS FORECASTERS IN
PROJECTING MORTALITY RATES

Age	P. K. Whelpton 1939-40 to 2000		R. J. Myers 1940 to 2000	A. J. Coale and F. W. Notestein Long Range	
	Medium Mortality	Low Mortality		Male	Female
20.....	1.5%	2.3%	2.0%	1.7%	1.6%
30.....	1.3	2.0	1.7	1.8	1.8
40.....	1.1	1.7	1.5	1.6	1.5
50.....	.8	1.5	1.5	1.0	.9
60.....	.4	1.2	1.1	.6	.8
70.....	0	.6	.7	.5	.5
80.....	.....	0	.5	.3	.3
90.....	.....	.....	.3	.....	.....

The actuarial forecasts of annuitant mortality made in connection with the British Offices annuitants investigation and the British Government life annuitants investigation were essentially extrapolations from the experience during the last three or four decades of the nineteenth century and the first two decades of the twentieth. As such, they visualized only gradual decreases in mortality. This is indicated by the assumption of asymptotic limits for q_x (equal to 63 percent of the ultimate 1900–1920 rates in the British Offices experience and about 80 percent of the ultimate 1900–1920 rates in the British Government life annuitants experience).

In the British Offices annuitants investigation forecast mortality rates were published for quinquennial age groups, applicable to calendar years 1925, 1935, and 1945.[38] The annual rates of decrease in mortality implicit in the mortality differential between 1925 and 1945 were in the range from 0.3% to 0.8% per year, the highest rates of decrease being assumed for the age group 65–69 and tapering off to very small figures in the 80's. However, for calculating annuity values a simplified method was adopted based on the theory that the mortality to be expected in the future would, apart from temporary selection, be a function of the year of entry.

In the British Government life annuitants investigation the forecasts were made on the theory that the mortality to be expected in the future would be a function of the calendar year passed through. Mortality tables based on forecast rates were not published, but from the data given it can be deduced that the annual rates of decrease implicit in the forecasting for the period from 1928 to 1948 did not apparently exceed 0.4% per year.[39]

Because the basic experiences upon which the British projections were based related to annuitants in a country passing through stages of economic, social, and medical development markedly different from those now unfolding in America, the applicability of these forecasts to American annuitants today is questionable.

IX. Projection Scales for Future Mortality Decreases

The purpose of this section is to present and explain the assumptions made by the authors in regard to possible or probable future decreases in mortality rates among annuitants. These assumptions are represented by "projection scale A" and "projection scale B," shown in Table 19 and Chart 5. Section X will show the effects of these assumptions on the values of life annuities of the more important types.

Both projection scales assume that future mortality rates among annuitants will vary with the year of exposure or the year passed through, rather than with the year of issue of the annuity or the year of birth as in the "generation" hypothesis, which has received the attention of a number of British and Scandinavian actuaries and others.[40] Each projection scale further assumes that mortality rates will continue decreasing from year to year indefinitely, at a rate which is constant at each attained age but decreases with advancing age. Projection scales A and B differ from each other only as to the size of the rates of decrease. In addition to rates of decrease per year, Table 19 shows the total reductions in mortality rates that will have taken place after twenty years, according to the respective scales.

TABLE 19

AVERAGE RATES OF DECREASE PER YEAR (GEOMETRICAL BASIS) ASSUMED IN PROJECTING ANNUITY TABLE FOR 1949

Age	Projection Scale A		Projection Scale B	
	Rate of Decrease per Year in Mortality Rate	Equivalent Reduction in Mortality Rate, End of 20 Years	Rate of Decrease per Year in Mortality Rate	Equivalent Reduction in Mortality Rate, End of 20 Years
20.....	2.8%	43.3%	1.25%	22.3%
30.....	2.4	38.5	1.25	22.3
40.....	2.0	33.3	1.25	22.3
50.....	1.6	27.6	1.25	22.3
60.....	1.2	21.5	1.20	21.5
65.....	1.0	18.2	1.10	19.8
70.....	.8	14.9	.95	17.4
75.....	.6	11.4	.75	14.0
80.....	.4	7.7	.50	9.5
85.....	.2	3.9	.25	4.8
90.....	.0	.0	.0	.0

The authors decided to follow the year of exposure hypothesis in their projection scales for the following reasons:

(1) In the authors' judgment, the major factors operating to produce lower mortality in the past have been: first (in chronological order), sanitation and better personal hygiene, followed later by higher living standards and improved conditions of work, then modern public health measures, and most recently advances in medical and surgical treatment. The effects of these factors on mortality rates have for the most part been quite direct, affecting all ages at much the same time though to differing degree. This has been particularly so in the case of the discovery of the "sulpha" drugs, penicillin, and other antibiotics, the wartime improvements in surgery, and the provision of better facilities for the early diagnosis and treatment of disease, all of which were so largely responsible for the recent marked decreases in mortality noted in Section VI. The likelihood of further progress along these lines argues strongly that mortality trends can be most readily interpreted on the year of exposure hypothesis, under which it is assumed that mortality rates will, apart from age, vary chiefly according to calendar year of exposure.

(2) An examination of the long-term decreases in mortality in Massachusetts since 1870 and in the expanding registration states since 1901 did not indicate that trends were more easily discernible on the "generation" hypothesis. Table 16 presents the evidence on this point.

(3) Although a number of British actuaries and others are adherents of the "generation" hypothesis and the British offices annuity tables were based on the year of issue theory, the weight of opinion does not seem to favor this hypothesis. It should be borne in mind that the hypothesis does not assert that mortality improvement depends solely on the year of birth and is independent of other factors, but rather that among many factors, other than age, the calendar year of birth is the most important in its influence on mortality rates. Considering that this conclusion was reached at a time when the impact of scientific and medical advances was very much less obvious than it has been during the past decade, some modification of the hypothesis might well be in order today. Moreover, the "generation" hypothesis did not even produce a decisively superior graduation when it was first advanced, as may be gauged from the following comments of Professor M. Greenwood, a foremost British statistician:[41]

Indeed, all I should feel justified in claiming is that, so far as concerns the generation method, the very simple plan of using it arithmetically, proposed by Kermack, McKendrick and McKinlay in their first paper and adopted by me, gives results not significantly different from those reached by Dr. Rhodes. His reexamination of the material does not, I think, modify the conclusions reached by Cramer and Wold and by me, *viz.*, that it is not possible to say decisively whether, from the point of view of graduation, a generation or a period method is the better.

(4) The year of exposure hypothesis appears to have a decided edge on the score of plausibility and ease of comprehension. Thus, R. A. Hohaus, Actuary of the Metropolitan Life Insurance Company, recently had the following to say on this subject:[42]

Like others who have had occasion to be concerned with the task of selecting future mortality assumptions for employee retirement plans, whether on a Group Annuity or Pension Fund basis, I have often struggled with the problem of trying to find a satisfactory basis for making allowance for future improvement in mortality. The year of issue theory was clearly not the answer. The year of exposure theory is by far the more appealing one, not only for the reasons suggested by the author but also for the very practical reason that it is probably the basis on which the employer, who puts up the money, is most likely to accept as an understandable, proper and sound explanation for the increased outlay required of him.

Projection scales A and B are perhaps best distinguished from each other by regarding the former as retrospective and the latter as prospective. In other words, scale A was designed to assume a continuation of the long-term mortality trends shown in Table 14, as well as they can be gauged. Scale B, however, looks to the future and assumes trends will be different from those in the past, notably smaller rates of decrease in mortality at the younger ages at which past reductions have already produced very low mortality rates, and somewhat higher rates of decrease at ages over 60, which seem to be the ages most susceptible to the many current efforts to reduce mortality from cardiovascular-renal diseases and cancer.

In forming projection scale A, the authors were also influenced by the long-term mortality decreases assumed for the future by others, particularly those discussed in Section VIII. It should be noted that all of the forecasts reviewed in that section, except that of R. J. Myers, were largely extrapolations from past experience.

In forming projection scale B, the authors were guided in part by the informed opinions quoted in Section VII and partly by the long-term projections of R. J. Myers, who made special allowance at ages beyond 65 for reductions in mortality greater in the future than in the past, so as to take into account the possible great changes proceeding from geriatric medical research. At the younger ages the authors took the view that improvements in public health, sanitation, and personal hygiene have possibly attained somewhere near their maximum effects in so far as mortality is concerned, so that it would appear unreasonable to expect future decreases in mortality at these ages to be as large as those experienced in the past. Moreover, as shown in Section XIII, the particular assumption made for ages under 60, and especially at ages under 40, is relatively unimportant for annuity purposes, because of the very low levels of mortality now prevailing at these ages.

The authors were much impressed with the fact that the age incidence of past mortality changes has not exhibited any definite pattern. This is indicated by the data in Table 16 for Massachusetts since 1870 and for the expanding registration states since 1901. Professor Greenwood reached a similar conclusion in his study of English death rates.[43] Because mortality rates are influenced by so many factors and the relative weight of each doubtless will change with the passage of time, as it appears to have done in the past, the authors concluded that history very likely will not repeat itself. Projection scale B is based largely on this conclusion.

The authors sought to obtain the opinions of several authorities regarding the particular mortality assumptions represented by projection scales A and B. In a communication to them, Dr. Louis I. Dublin stated:

Obviously, if we are to be guided by the trends of the last 50 years, there is every indication that mortality will continue to improve from age 45 onward, perhaps to a limiting age of 75, but on a decreasing scale from the earlier to the later age. I doubt very much whether we can predicate at this time any material improvement beyond 75.

The extent of the improvement at the older ages in the immediate future will depend largely upon the outcome of research in the fields of cancer, the cardiovascular-renal diseases, and the diseases of disturbed metabolism. The outlook is altogether promising. Furthermore, both clinical medicine and public health are giving increasing emphasis to preventive work and to enlarging the personnel and facilities for the care of those suffering from the diseases and disabilities of old age. There is also the possibility of very definite reduction in accident fatalities among older persons, providing communities take this problem seriously.... Gains in longevity after middle life are, therefore, indicated for future years.

For these reasons I concur in your general approach in assuming sizable decreases in the mortality of both sexes in the immediate future between ages 45 and 64, and a lesser improvement between 65 and 74.

The mortality decreases assumed for ages 60 and over in projection scale B would be attained if at the end of the next twenty years:

(1) Death rates from cardiovascular-renal diseases had been reduced on the average by about 10%, the reductions varying from about 15% at age 60 to 5% or less at age 80.
(2) Death rates from cancer among females had been reduced on the average by about 30%, and among males by 15%, ranging downward from a 35% reduction for females and a 17½% reduction for males at age 60.
(3) Death rates from influenza and pneumonia had been reduced by about 30%, from accidents by about 25% and from other causes by about 20%, on the average.

Of course, other combinations of decreases by cause of death would also yield the same over-all results.

Since cardiovascular-renal diseases and cancer cause about three-fourths of all deaths among annuitants at age 65 and over (see Table 17), the authors communicated with several authorities in the fields of heart disease and cancer to learn their views regarding the possibility of reductions in mortality from cardiovascular-renal diseases and cancer of the magnitude indicated above.

When asked by the authors regarding the possibility of death rates from heart disease being reduced within the next twenty years by as much as 15% at age 60 and 7½% at age 75, Dr. I. S. Wright stated:

I am inclined to think that it is possible that the figures as included in your letter may be approximately correct within the next twenty years. It is even conceivable that they may be on the conservative side. The improvement will, as you point out, rest on further basic progress in our understanding of these diseases, and on clinical studies but the application of all available knowledge would almost immediately bear fruit and this aspect of the problem should not be belittled in the consideration of where support should be placed.

Dr. Paul D. White, another eminent American heart specialist, said:

I would think that your estimate of a reduction in mortality from heart disease in the five-year age periods after [age] 60 is as reasonable as any. Ten percent might be an over-all figure for the whole lot. Certainly a reduction of rheumatic and hypertensive heart disease because of at least partial control of the factors is to be expected and perhaps we can do something even about early coronary disease. Reduction of these factors would certainly favor the 60–70–year old group especially.

The Committee on Federal Medical Services of the Commission on the Organization of the Executive Branch of the Government indicated in one of its reports[44] that it considered a 10% reduction in heart disease mortality to be a reasonable hypothesis.

In regard to cancer mortality, the authors approached Dr. C. P. Rhoads, whose opinion regarding the authors' assumptions as to future reductions in cancer mortality was as follows:

Under the circumstances stated [assuming progress which will provide us new controlling or curative procedures of the type not now available] I would not have the slightest doubt in predicting the rates of reduction in cancer mortality within the next twenty years which you specify as follows:

	Males	Females
At age 60	17½%	35%
At age 65	15	30
At age 70	12½	25

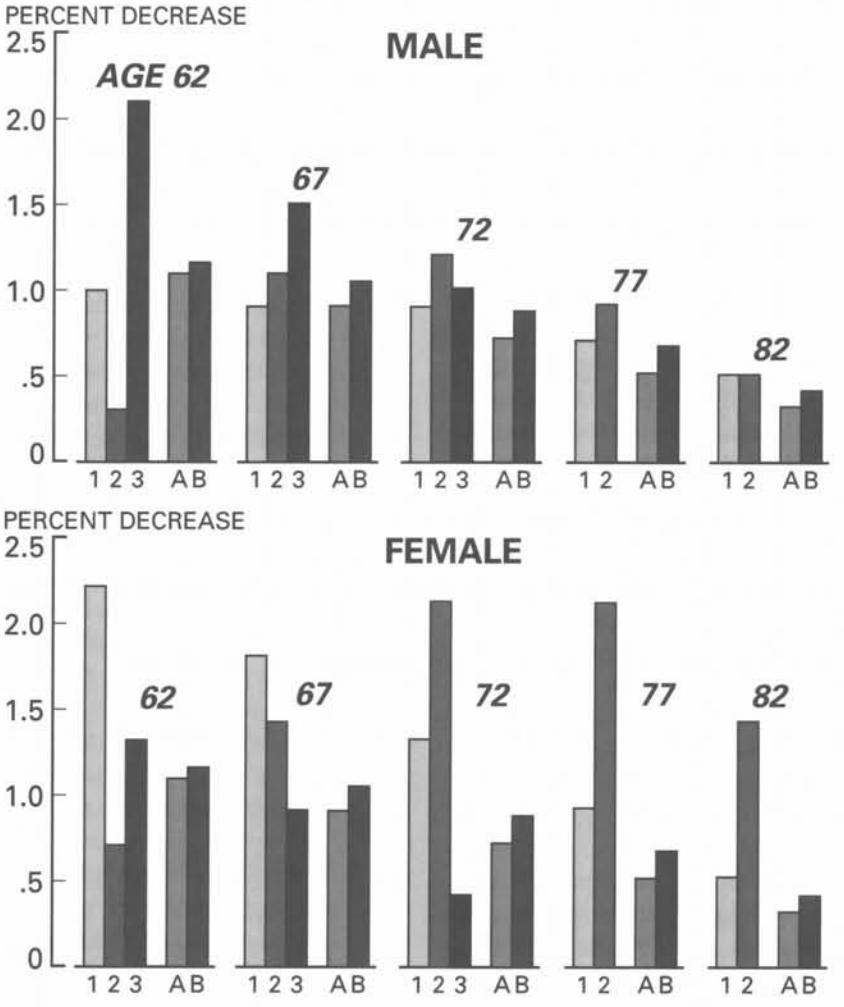
Your second question—"Could the improvement in mortality indicated above be obtained within the next twenty years merely assuming more intensive application of existing knowledge ...?"—can also be answered in the affirmative. I feel certain that this is the case because this rate of improvement is being attained today in institutions where adequate specialized attention can be given on a sufficient scale to the early diagnosis and the best possible therapeutic care of neoplastic disease.

Concerning the essential characteristics of projection scales A and B, the authors wish to emphasize particularly that their main objective was to present a choice of two scales of age incidence, rather than of general level. As to the general level of future mortality decreases, there are a large number of alternatives from which the actuary can choose, since he can easily modify upward or downward the general level of either scale so as to suit his judgment. Thus, irrespective of whether or not the individual actuary agrees with the authors in their choice of the general level of these scales, he can make use of them to arrive at his own solution; he need only select the age incidence scale he prefers and then adjust the projection factors for that scale, presented in Section X, to the level of mortality reduction he considers appropriate for the future. In view of the almost universal view that future mortality decreases are inevitable, the use of some kind of projection factor in connection with the Annuity Table for 1949 will probably be considered essential.

CHART 5

AVERAGE RATES OF DECREASE PER YEAR* IN ANNUITANTS' MORTALITY
PAST TRENDS COMPARED WITH ASSUMPTIONS USED IN
PROJECTING ANNUITY TABLE FOR 1949

(1) From American Annuitants Ult. To 1943 Experience Table Ult.
(2) From U.S. Annuitants Ult. to 1943 Experience Table Ult.
(3) Settlement Options Experience from 1934–40 to 1940–45
(A) Projection Scale A
(B) Projection Scale B



* Geometrical Basis

Both the general level and the age incidence of the projection scales were selected in the hope that the resulting scales would make appropriate and adequate provision for future mortality decreases arising not only from more intensive application of existing knowledge and measures, but also to a reasonable degree from new discoveries and techniques foreshadowed by current research endeavors. The scales do not, however, provide a sufficient margin for revolutionary discoveries, such as an infallible cancer cure, in addition to the mortality decreases allowed for.

In their choice of the general level of the projection scales, the authors aimed at a moderate degree of conservatism. This may be contrasted with the "most probable" estimates made by Whelpton, Coale, and others in developing their mortality projections for forecasting population. At the same time the authors attempted to avoid a degree of conservatism so great as would open their projections to criticism on the ground of implausibility. The authors believe that the projection scales here presented provide a reasonably satisfactory solution to the problem of premiums and reserves for annuity contracts to be issued over the next five or ten years, in that they allow conservatively for future mortality decreases without rendering the resulting annuity values impracticable.

X. Projection Factors for Future Mortality Decreases

Presented in this section are tables of projection factors calculated in accordance with projection scales A and B, described in Section IX. These factors were computed so that, when multiplied by appropriate annuity values based on the Annuity Table for 1949 (without projection), they produce annuity values which make provision for future mortality decreases in accordance with scale A or B.

Annuity and other values calculated on this basis are referred to in this paper as based on the "Annuity Table for 1949 (with projection)" or the "1949 table (with projection)"; and annuity values, mortality rates, etc., in or derived from the 1949 table without adjustment for future mortality decreases are referred to as based on the "Annuity Table for 1949, (without projection)," the "1949 table (without projection)," the "Annuity Table for 1949," or simply the "1949 table."

If a series of columns were prepared, each showing mortality rates assumed to prevail at all ages in a specified calendar year in accordance with projection scale A or B, and these columns were arranged chronologically from left to right, the projected annuity value would be calculated from mortality rates extending diagonally to the right and downward. In this way, each contract year the mortality rate advances one year of age and one calendar year. The projection factor is the ratio of the projected annuity value to the corresponding annuity value from the 1949 table (without projection).

Algebraically, if the annuity period begins in the calendar year y at age x, if q_x is the death rate at age x in the 1949 table (without projection), and if s_x represents the annual percentage decrease in the mortality rate at age x according to a particular projection scale, then the projected annuity value is based on a first policy year mortality rate of $q_x(1 - [s_x/100])^{y-1950}$, a second policy year rate of $q_{x+1}(1 - [s_{x+1}/100])^{y-1949}$, a third policy year rate of $q_{x+2}(1 - [s_{x+2}/100])^{y-1948}$, etc. Thus, it is assumed that annuity periods begin on January 1 in the year stated, and that the first annual mortality decrease from the 1949 table occurs at the end of 1950. Separate tables of projection factors are shown for each sex. While all of the projection factors were actually computed from ultimate annuity values, they may also be used to obtain select annuity values, because the resulting errors are negligible.

The tables presented in this section show projection factors which were calculated to apply only to contracts or policies to be issued within the next five or ten years. The authors assumed that the factors would be reviewed regularly, say every five or ten years, and revised in conformity with changing mortality trends.

Table 20 shows projection factors for immediate life annuities—nonrefund, 10 years guaranteed, and 20 years guaranteed—issued in 1950, 1955, and 1960 with first annual annuity payments one year after issue. Perhaps the most interesting fact which can be perceived from this table is that the factors based on projection scale A differ very little from the factors based on projection scale B, in spite of the substantially different age incidences represented by these scales. The reasons will be evident as to why the factors (a) are uniformly lower for females than for males, (b) increase with advancing age in the upper portion of this table but diminish with advancing age in the lower portion, (c) decrease with increasing guaranteed period, and (d) increase with increasing period of deferment. In view of the completely different bases, it is perhaps surprising that the factors in Table 20 are as close as they are to the male 1.03 and female 1.04 factors recommended in the British Government (1900–20) Life Annuitants investigation.[45]

TABLE 20
PROJECTION FACTORS FOR IMMEDIATE NONREFUND ANNUITIES
APPLICABLE TO SINGLE LIFE ANNUITY VALUES BASED ON
ANNUITY TABLE FOR 1949 AT 2½% INTEREST

Age of Annuitant at Issue	Projection Scale A						Projection Scale B					
	Males			Females			Males			Females		
	Annuity Period Commences in			Annuity Period Commences in			Annuity Period Commences in			Annuity Period Commences in		
	1950	1955	1960	1950	1955	1960	1950	1955	1960	1950	1955	1960
15.....	1.034	1.039		1.023	1.026		1.033	1.037		1.023	1.025	
20.....	1.035	1.040	1.045	1.024	1.027	1.030	1.035	1.039	1.043	1.024	1.027	1.029
25.....	1.036	1.042	1.047	1.024	1.028	1.031	1.036	1.041	1.046	1.025	1.028	1.031
30.....	1.037	1.043	1.050	1.025	1.029	1.033	1.037	1.043	1.049	1.026	1.030	1.034
35.....	1.037	1.044	1.052	1.025	1.030	1.034	1.038	1.045	1.052	1.026	1.031	1.035
40.....	1.036	1.045	1.054	1.024	1.030	1.035	1.038	1.046	1.055	1.027	1.032	1.037
45.....	1.034	1.044	1.055	1.023	1.029	1.036	1.037	1.047	1.057	1.026	1.032	1.039
50.....	1.031	1.043	1.054	1.022	1.029	1.036	1.034	1.047	1.059	1.025	1.032	1.040
55.....	1.027	1.040	1.052	1.020	1.027	1.035	1.031	1.045	1.058	1.023	1.032	1.040
60.....	1.022	1.035	1.048	1.017	1.026	1.034	1.028	1.042	1.056	1.021	1.030	1.040
65.....	1.018	1.031	1.043	1.014	1.023	1.031	1.022	1.039	1.052	1.017	1.028	1.038
70.....	1.013	1.025	1.037	1.010	1.019	1.028	1.016	1.031	1.045	1.013	1.024	1.034
75.....	1.008	1.018	1.029	1.007	1.014	1.022	1.010	1.023	1.036	1.008	1.018	1.028
80.....	1.004	1.012	1.019	1.003	1.009	1.016	1.005	1.015	1.024	1.004	1.012	1.019
85.....	1.001	1.005	1.009	1.001	1.004	1.007	1.001	1.006	1.011	1.001	1.005	1.009

TABLE 20—*Continued*
PROJECTION FACTORS FOR IMMEDIATE ANNUITIES WITH 10-YEAR CERTAIN
PERIOD APPLICABLE TO SINGLE LIFE ANNUITY VALUES BASED ON
ANNUITY TABLE FOR 1949 AT 2½% INTEREST

Age of Annuitant at Issue	Projection Scale A						Projection Scale B					
	Males			Females			Males			Females		
	Annuity Period Commences in			Annuity Period Commences in			Annuity Period Commences in			Annuity Period Commences in		
	1950	1955	1960	1950	1955	1960	1950	1955	1960	1950	1955	1960
15.....	1.034	1.038		1.023	1.026		1.033	1.037		1.023	1.025	
20.....	1.035	1.040	1.045	1.024	1.027	1.030	1.034	1.039	1.043	1.024	1.027	1.029
25.....	1.036	1.042	1.047	1.024	1.028	1.031	1.036	1.041	1.046	1.025	1.028	1.031
30.....	1.036	1.043	1.049	1.024	1.029	1.033	1.037	1.043	1.049	1.026	1.030	1.033
35.....	1.036	1.044	1.051	1.024	1.029	1.034	1.037	1.044	1.051	1.026	1.031	1.035
40.....	1.035	1.044	1.052	1.024	1.029	1.034	1.037	1.045	1.054	1.026	1.031	1.037
45.....	1.033	1.042	1.052	1.023	1.029	1.034	1.036	1.045	1.055	1.026	1.032	1.038
50.....	1.029	1.039	1.049	1.021	1.028	1.034	1.033	1.044	1.054	1.025	1.031	1.038
55.....	1.024	1.035	1.045	1.019	1.026	1.032	1.029	1.040	1.051	1.022	1.030	1.037
60.....	1.019	1.029	1.038	1.016	1.022	1.029	1.024	1.034	1.045	1.019	1.027	1.035
65.....	1.013	1.021	1.029	1.012	1.018	1.024	1.017	1.027	1.035	1.015	1.022	1.029
70.....	1.008	1.013	1.019	1.007	1.012	1.017	1.010	1.016	1.023	1.009	1.015	1.021
75.....	1.003	1.006	1.009	1.003	1.006	1.009	1.004	1.007	1.011	1.004	1.008	1.011
80.....	1.001	1.001	1.002	1.001	1.001	1.002	1.001	1.002	1.003	1.001	1.002	1.003
85.....	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 20—*Continued*
PROJECTION FACTORS FOR IMMEDIATE ANNUITIES WITH 20-YEAR CERTAIN PERIOD APPLICABLE TO SINGLE LIFE ANNUITY VALUES BASED ON ANNUITY TABLE FOR 1949 AT 2½% INTEREST

Age of Annuitant at Issue	Projection Scale A						Projection Scale B					
	Males			Females			Males			Females		
	Annuity Period Commences in			Annuity Period Commences in			Annuity Period Commences in			Annuity Period Commences in		
	1950	1955	1960	1950	1955	1960	1950	1955	1960	1950	1955	1960
15.....	1.034	1.037		1.023	1.025		1.033	1.036		1.023	1.025	
20.....	1.035	1.039	1.043	1.023	1.026	1.029	1.034	1.038	1.042	1.024	1.026	1.029
25.....	1.035	1.040	1.045	1.024	1.027	1.030	1.035	1.040	1.044	1.025	1.028	1.031
30.....	1.035	1.041	1.046	1.024	1.027	1.031	1.036	1.041	1.047	1.025	1.029	1.032
35.....	1.034	1.040	1.046	1.023	1.027	1.031	1.035	1.042	1.048	1.025	1.029	1.033
40.....	1.031	1.038	1.045	1.022	1.027	1.031	1.034	1.041	1.047	1.025	1.029	1.034
45.....	1.027	1.034	1.040	1.020	1.025	1.029	1.030	1.037	1.044	1.023	1.028	1.033
50.....	1.021	1.028	1.034	1.018	1.022	1.026	1.025	1.032	1.038	1.021	1.026	1.031
55.....	1.015	1.020	1.025	1.014	1.018	1.022	1.018	1.024	1.030	1.016	1.021	1.026
60.....	1.009	1.012	1.015	1.009	1.012	1.015	1.011	1.015	1.019	1.011	1.015	1.018
65.....	1.004	1.005	1.007	1.004	1.006	1.008	1.004	1.007	1.008	1.005	1.007	1.009
70.....	1.001	1.001	1.002	1.001	1.002	1.002	1.001	1.002	1.002	1.001	1.002	1.003
75.....	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 21 shows the projection factors for life income settlement options 10 years and 20 years guaranteed, with the annual annuity payments beginning in 1965, 1970, and 1975, in which years, on the average, it might be assumed that life income settlement options in life insurance policies issued within the next several years will mature by death. In comparing these factors with those shown in Table 20 for immediate annuities, it should be kept in mind that under immediate annuities the first annual annuity payment is due one year after issue of the contract, which date of issue is assumed to be in 1950, 1955, or 1960, whereas under life income settlement options, the first annual annuity payment is due in advance in 1965, 1970, or 1975, as the case might be. A comparison of the factors in Tables 21 and 20 brings out that the factors for life income settlement options are greater than those for immediate annuities, because the annuity payments under the former extend much further into the future than those under the latter. It might also be noted that the factors for life income settlement options based on projection scale A do not differ much from the factors based on projection scale B, just as in the case of the factors for immediate annuities.

Table 22 was prepared to aid the actuary in gauging the values of the annuities available at maturity of the contracts in the case of retirement income life insurance policies and deferred annuities issued within the next several years. It should be noted that the projection factors in this table apply to the values of the immediate annuities, with the first annual annuity payment in advance, at the time the contracts mature (that is, at the time the annuity payments commence) and not to the values of the deferred annuities at time of issue of the original contracts. The factors in Table 22 are shown for life annuities 10 years and 20 years guaranteed, with the annuity payments beginning at maturity ages 55, 60, and 65, applicable to retirement income life insurance policies and deferred annuities issued in 1950 and 1955 at the issue ages of the original contracts indicated. Because Table 22 is different in form from that of the two preceding tables, the pattern of the factors is changed. At the older ages of issue, the factors for retirement income life insurance policies and deferred annuities could, of course, also be taken from Table 21 for the same form of annuity, same age when annuity payments commence, and same period of deferment; at the younger ages, the periods of deferment under retirement income insurances and deferred annuities are larger than those shown in Table 21 and the projection factors in Table 22 are, therefore, correspondingly increased.

Tables of projection factors are not shown for group annuities because of the considerable variety of forms of annuity involved. On request, the authors' basic tables will be made available to any actuary.

Table 23 may be useful for "pure" deferred annuities, that is, deferred annuities of any form in the case of which account is taken of the probabilities of survival and the rate of discount during the period of deferment. Table 23 shows the values of ${}_nP_x$, based on the Annuity Table for 1949 Ultimate (with projection), for the same issue years and maturity ages as the annuity values presented in Table 22, so that these two tables can be used together.

Table 24 is of interest in connection with immediate annuities; it compares ultimate annuity values based on the 1949 table (with projection B) at 2½% interest with those based on the 1937 Standard Annuity Table together with the 2% interest rate now widely used.

Chart 6 portrays the factors from Tables 20 to 22 for the more important types of annuity here considered, the original contracts being issued in 1950 and the settlement option annuities beginning after 20 years.

To recapitulate, the mechanism for obtaining a single life annuity value at date of issue of a policy contract, based on the Annuity Table for 1949 (with projection), can be summarized as follows:

(1) The annuity value is computed in the usual manner from the Annuity Table for 1949 (without projection), select or ultimate.

(2) The value from (1) is multiplied by the appropriate projection factor from Table 20, 21, or 22. The actuary may use these factors unchanged, or he may increase or decrease them, in his judgment. Factors for refund annuities can be determined by interpolation or extrapolation from the factors for annuities with 10-year and 20-year guarantees.

(3) If the annuity is other than an immediate nonrefund annuity, further adjustment for the difference in mortality level by type of annuity can be made as explained in Section XI.

(4) If a "pure" deferred annuity, the annuity value thus obtained is multiplied by the appropriate probability from Table 23 and the appropriate discount factor.

As to the reserve after an annuity has begun, the mechanism consists of these same steps (1), (2), and (3) indicated above, entering Table 20 with the current attained age and year of valuation (instead of issue age and year of issue) and making appropriate adjustment for the reduced guaranteed period, if any. Obviously, these mechanisms can be simplified by approximate methods, determined by the actuary to suit his requirements.

TABLE 21

PROJECTION FACTORS FOR LIFE INCOME SETTLEMENT OPTIONS WITH 10-YEAR CERTAIN PERIOD, APPLICABLE TO SINGLE LIFE ANNUITY VALUES BASED ON ANNUITY TABLE FOR 1949 AT 2½% INTEREST

Age of Payee When the Income Commences	Projection Scale A						Projection Scale B					
	Males			Females			Males			Females		
	Life Income Commences			Life Income Commences			Life Income Commences			Life Income Commences		
	1965	1970	1975	1965	1970	1975	1965	1970	1975	1965	1970	1975
30.....	1.053	1.058		1.035	1.038		1.052	1.057		1.035	1.039	
35.....	1.056	1.062	1.068	1.037	1.040	1.044	1.056	1.062	1.068	1.038	1.042	1.045
40.....	1.058	1.065	1.072	1.038	1.042	1.046	1.059	1.066	1.073	1.040	1.044	1.049
45.....	1.059	1.067	1.075	1.038	1.043	1.048	1.061	1.070	1.078	1.042	1.047	1.052
50.....	1.057	1.066	1.075	1.038	1.044	1.049	1.062	1.072	1.081	1.043	1.049	1.054
55.....	1.053	1.062	1.071	1.037	1.043	1.049	1.060	1.070	1.080	1.043	1.049	1.056
60.....	1.046	1.055	1.063	1.035	1.041	1.047	1.054	1.064	1.074	1.041	1.048	1.055
65.....	1.037	1.044	1.052	1.030	1.035	1.041	1.045	1.054	1.063	1.036	1.043	1.050
70.....	1.025	1.031	1.037	1.022	1.027	1.032	1.031	1.038	1.045	1.027	1.033	1.039
75.....	1.013	1.016	1.019	1.013	1.016	1.019	1.016	1.020	1.024	1.016	1.020	1.023
80.....	1.004	1.005	1.006	1.004	1.005	1.006	1.005	1.006	1.007	1.005	1.007	1.008
85.....	1.000	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001

TABLE 21—*Continued*

PROJECTION FACTORS FOR LIFE INCOME SETTLEMENT OPTIONS WITH 20-YEAR CERTAIN PERIOD, APPLICABLE TO SINGLE LIFE ANNUITY VALUES BASED ON ANNUITY TABLE FOR 1949 AT 2½% INTEREST

Age of Payee When the Income Commences	Projection Scale A						Projection Scale B					
	Males			Females			Males			Females		
	Life Income Commences			Life Income Commences			Life Income Commences			Life Income Commences		
	1965	1970	1975	1965	1970	1975	1965	1970	1975	1965	1970	1975
30.....	1.050	1.055		1.033	1.036		1.050	1.055		1.034	1.037	
35.....	1.051	1.056	1.062	1.034	1.037	1.041	1.052	1.058	1.063	1.036	1.039	1.043
40.....	1.050	1.056	1.062	1.034	1.038	1.042	1.053	1.059	1.065	1.037	1.041	1.045
45.....	1.047	1.053	1.059	1.033	1.037	1.041	1.051	1.057	1.064	1.037	1.041	1.046
50.....	1.040	1.046	1.052	1.031	1.035	1.039	1.045	1.052	1.058	1.035	1.040	1.044
55.....	1.031	1.036	1.041	1.026	1.030	1.034	1.036	1.042	1.048	1.031	1.035	1.040
60.....	1.020	1.024	1.027	1.019	1.022	1.025	1.024	1.029	1.033	1.023	1.027	1.030
65.....	1.010	1.012	1.014	1.010	1.012	1.014	1.012	1.014	1.017	1.013	1.015	1.018
70.....	1.003	1.003	1.004	1.004	1.004	1.005	1.004	1.004	1.005	1.004	1.005	1.006
75.....	1.000	1.000	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001

TABLE 22

PROJECTION FACTORS FOR RETIREMENT INCOME INSURANCES AND DEFERRED ANNUITIES WITH 10-YEAR CERTAIN PERIOD, APPLICABLE TO SINGLE LIFE ANNUITY VALUES BASED ON ANNUITY TABLE FOR 1949 AT 2½% INTEREST

Age of Annuitant at Issue of Original Contract	Projection Scale A						Projection Scale B					
	Males			Females			Males			Females		
	Life Income Commences			Life Income Commences			Life Income Commences			Life Income Commences		
	Age 55	Age 60	Age 65	Age 55	Age 60	Age 65	Age 55	Age 60	Age 65	Age 55	Age 60	Age 65
	(a) Original Contracts Issued in 1950											
15.....	1.097	1.096	1.088	1.065	1.069	1.068	1.108	1.112	1.106	1.074	1.081	1.082
20.....	1.089	1.089	1.081	1.060	1.064	1.063	1.099	1.103	1.098	1.068	1.075	1.076
25.....	1.080	1.080	1.074	1.055	1.058	1.058	1.090	1.094	1.089	1.062	1.068	1.070
30.....	1.071	1.072	1.067	1.049	1.053	1.052	1.080	1.084	1.081	1.056	1.062	1.063
35.....	1.062	1.063	1.060	1.043	1.047	1.047	1.070	1.074	1.072	1.049	1.055	1.057
40.....	1.053	1.055	1.052	1.037	1.041	1.041	1.060	1.064	1.063	1.043	1.048	1.050
45.....	1.043	1.046	1.044	1.031	1.035	1.035	1.049	1.054	1.054	1.036	1.041	1.043
	(b) Original Contracts Issued in 1955											
15.....	1.105	1.104	1.095	1.071	1.075	1.074	1.117	1.121	1.114	1.080	1.087	1.088
20.....	1.097	1.096	1.088	1.065	1.069	1.068	1.108	1.112	1.106	1.074	1.081	1.082
25.....	1.089	1.089	1.081	1.060	1.064	1.063	1.099	1.103	1.098	1.068	1.075	1.076
30.....	1.080	1.080	1.074	1.055	1.058	1.058	1.090	1.094	1.089	1.062	1.068	1.070
35.....	1.071	1.072	1.067	1.049	1.053	1.052	1.080	1.084	1.081	1.056	1.062	1.063
40.....	1.062	1.063	1.060	1.043	1.047	1.047	1.070	1.074	1.072	1.049	1.055	1.057
45.....	1.053	1.055	1.052	1.037	1.041	1.041	1.060	1.064	1.063	1.043	1.048	1.050
50.....	1.043	1.046	1.044	1.031	1.035	1.035	1.049	1.054	1.054	1.036	1.041	1.043

TABLE 22—*Continued*

PROJECTION FACTORS FOR RETIREMENT INCOME INSURANCES AND DEFERRED ANNUITIES WITH 20-YEAR CERTAIN PERIOD, APPLICABLE TO SINGLE LIFE ANNUITY VALUES BASED ON ANNUITY TABLE FOR 1949 AT 2½% INTEREST

Age of Annuitant at Issue of Original Contract	Projection Scale A						Projection Scale B					
	Males			Females			Males			Females		
	Life Income Commences			Life Income Commences			Life Income Commences			Life Income Commences		
	Age 55	Age 60	Age 65	Age 55	Age 60	Age 65	Age 55	Age 60	Age 65	Age 55	Age 60	Age 65
	(a) Original Contracts Issued in 1950											
15.....	1.055	1.041	1.023	1.044	1.037	1.023	1.065	1.050	1.029	1.052	1.045	1.029
20.....	1.051	1.038	1.021	1.041	1.034	1.022	1.059	1.046	1.026	1.048	1.041	1.027
25.....	1.046	1.034	1.019	1.037	1.031	1.020	1.054	1.041	1.024	1.044	1.038	1.025
30.....	1.041	1.031	1.017	1.034	1.028	1.018	1.048	1.037	1.022	1.040	1.034	1.022
35.....	1.036	1.027	1.015	1.030	1.025	1.016	1.042	1.033	1.019	1.035	1.030	1.020
40.....	1.031	1.024	1.014	1.026	1.022	1.014	1.036	1.029	1.017	1.031	1.027	1.018
45.....	1.026	1.020	1.012	1.022	1.019	1.012	1.030	1.024	1.014	1.026	1.023	1.015
	(b) Original Contracts Issued in 1955											
15.....	1.060	1.045	1.025	1.048	1.040	1.026	1.070	1.054	1.031	1.056	1.048	1.031
20.....	1.055	1.041	1.023	1.044	1.037	1.023	1.065	1.050	1.029	1.052	1.045	1.029
25.....	1.051	1.038	1.021	1.041	1.034	1.022	1.059	1.046	1.026	1.048	1.041	1.027
30.....	1.046	1.034	1.019	1.037	1.031	1.020	1.054	1.041	1.024	1.044	1.038	1.025
35.....	1.041	1.031	1.017	1.034	1.028	1.018	1.048	1.037	1.022	1.040	1.034	1.022
40.....	1.036	1.027	1.015	1.030	1.025	1.016	1.042	1.033	1.019	1.035	1.030	1.020
45.....	1.031	1.024	1.014	1.026	1.022	1.014	1.036	1.029	1.017	1.031	1.027	1.018
50.....	1.026	1.020	1.012	1.022	1.019	1.012	1.030	1.024	1.014	1.026	1.023	1.015

XI. Adjustments for Various Kinds of Annuities

Tables 25, 26, and 27 are offered as an aid to the actuary in deciding whether or not the mortality basis he selects for immediate nonrefund annuities is suitable without adjustment for other kinds of annuities. And, if an adjustment is to be made, these tables can aid him in determining its size.

Table 25 compares the ultimate mortality ratios experienced under the various types of annuities, during recent five-year periods, as indicated. The ratios shown in this table are based on the 1943 Experience Table (ultimate) and the data correspondingly relate to durations 2 and over except that the group annuity data cover all durations. The probable error of the mortality ratio for each kind of annuity is shown for all ages combined; thus, the significance of the differences between the experience under immediate nonrefund annuities and those under various other types of annuity can be gauged. If the actuary decides that the mortality level for any kind of annuity differs sufficiently from the immediate nonrefund annuity experience and he makes the not unreasonable assumption that the relative mortality levels shown in Table 25 will also prevail in the future, he is justified in assuming that the Annuity Table for 1949 (with projection) needs a corresponding adjustment. In the case of life income settlement options, the actuary desiring to do so can make a distinction between payee-elected and non-payee-elected options on the basis of the pertinent data shown in Table 25.

TABLE 23

VALUES OF ${}_np_x$

DERIVED FROM ANNUITY TABLE FOR 1949 ULTIMATE (PROJECTED)

Age x	Projection Scale A						Projection Scale B					
	Males $x+n$			Females $x+n$			Males $x+n$			Females $x+n$		
	55	60	65	55	60	65	55	60	65	55	60	65
	(a) Age x Attained in 1950											
15.....	.940275	.907021	.858938	.967123	.951410	.925686	.931908	.897555	.850995	.962264	.945970	.920964
20.....	.937449	.902067	.851563	.965486	.948739	.921633	.930402	.893983	.844891	.961442	.944148	.917713
25.....	.935016	.897360	.844301	.964310	.946453	.917880	.929320	.890697	.838924	.961098	.942731	.914775
30.....	.933379	.893267	.837492	.963548	.944498	.914370	.929024	.888030	.833398	.961450	.941932	.912347
35.....	.933160	.890372	.831669	.964479	.944121	.912302	.930105	.886535	.828813	.962862	.942099	.910761
40.....	.935424	.889662	.827729	.966901	.945112	.911459	.933565	.887141	.826017	.965966	.943839	.910591
45.....	.942616	.893419	.827756	.972226	.948837	.913141	.941758	.892046	.827010	.971811	.948165	.912797
	(b) Age x Attained in 1955											
15.....	.945498	.914165	.868281	.970114	.955352	.930907	.935914	.903425	.859157	.964516	.949144	.925457
20.....	.942801	.909458	.861246	.968563	.952827	.927065	.934496	.900048	.853358	.963741	.947421	.922377
25.....	.940452	.904958	.854291	.967416	.950636	.923475	.933474	.896935	.847680	.963425	.946096	.919607
30.....	.938814	.901004	.847729	.966900	.948994	.920345	.933197	.894412	.842424	.963751	.945334	.917301
35.....	.938478	.898146	.842067	.967403	.948276	.918028	.934218	.892995	.838058	.965082	.945490	.915795
40.....	.940421	.897299	.838139	.969533	.949069	.917083	.937471	.893556	.835377	.967998	.947123	.915617
45.....	.946923	.900598	.837902	.974344	.952389	.918476	.945196	.898194	.836308	.973500	.951201	.917695
50.....	.964273	.913946	.846776	.983611	.959948	.923834	.963597	.912732	.846188	.983297	.959372	.923585

TABLE 24

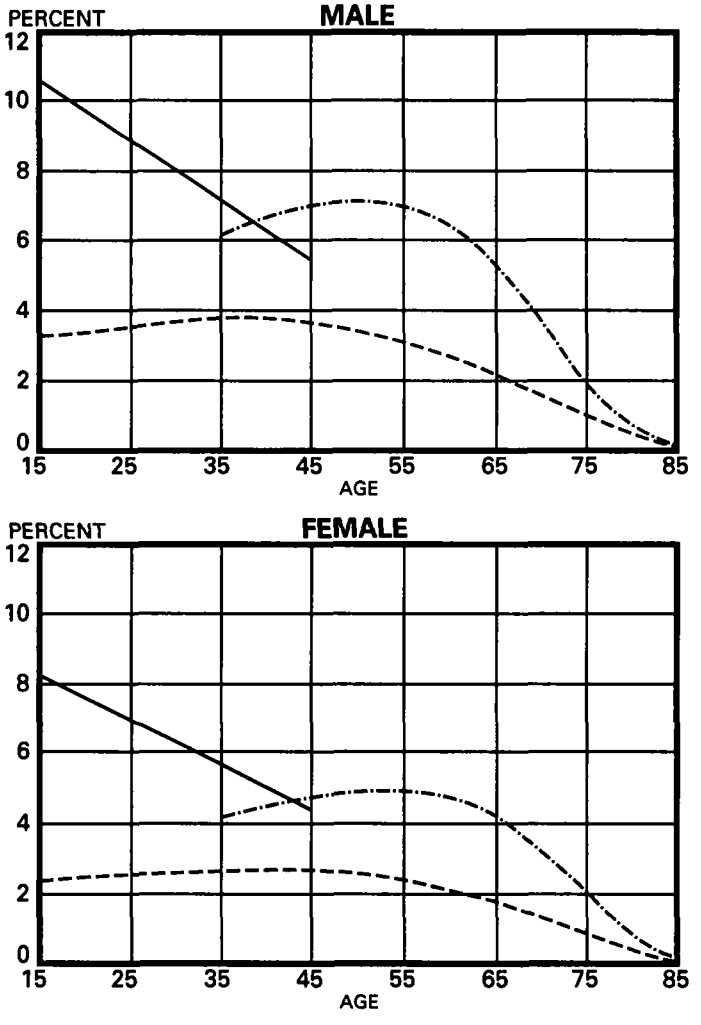
COMPARISON OF a_x ON ANNUITY TABLE FOR 1949 (WITH PROJECTION B) AT 2½% INTEREST WITH a_x ON 1937 STANDARD ANNUITY TABLE AT 2% INTEREST CONTRACTS ISSUED IN 1950

Age x	Male			Female		
	1949 Table Ultimate Projected 2½% Interest (1)	1937 Standard Annuity Table 2% Interest (2)	Ratio (1) ÷ (2) (3)	1949 Table Ultimate Projected 2½% Interest (4)	1937 Standard Annuity Table 2% Interest (5)	Ratio (4) ÷ (5) (6)
15.....	30.917	32.344	.956	31.935	33.807	.945
25.....	28.296	28.940	.978	29.611	30.722	.964
35.....	24.962	24.946	1.001	26.672	27.007	.988
45.....	20.849	20.544	1.015	23.018	22.784	1.010
55.....	16.330	15.956	1.023	18.640	18.255	1.021
65.....	11.744	11.508	1.021	13.686	13.691	1.000
75.....	7.396	7.581	.976	8.714	9.457	.921
85.....	3.927	4.483	.876	4.564	5.916	.771

CHART 6

PERCENTAGE BY WHICH ANNUITY VALUES ON ANNUITY TABLE FOR 1949 (WITH PROJECTION B) EXCEED THOSE ON ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION) AT 2½% INTEREST

— — — Immediate Nonrefund Annuity Issued in 1950
—·—·— Life Income Settlement Option (10 Years Certain) Beginning in 1970
——— Annuity Deferred to Age 65 (10 Years Certain) Issued in 1950



The retired lives group annuity experience shown in Table 25 departs significantly from the immediate nonrefund annuity experience, and adjustment of the 1949 table (with projection) on this account is indicated. It should be noted that the experience among persons retiring before normal retirement age is not included in the retired lives group annuity experience presented in Table 25. This experience covers all occupational groups and differs in this respect from the active lives experience on predominantly clerical groups that was used in the construction of the 1943 Experience Table at the younger ages. The group annuity mortality committee has not published any experience data for retired lives in occupational subdivisions.

The significance of the departure in the case of male immediate refund annuities is lessened by the fact that a considerable portion of the excess mortality indicated occurred on issues of 1931–35, which may have been abnormal.

Table 26 shows for individual annuities the first year mortality ratios corresponding to the ultimate ratios shown in Table 25. From this table the actuary can judge for himself the extent to which first year death rates in the Annuity Table for 1949 (with projection) need modification for the various kinds of annuities. This may reduce to a decision to use either ultimate or select annuity values.

Table 27 corresponds to and is based on the same data as Table 25, but deals with ultimate annuity values instead of mortality ratios and omits deferred annuities because of the immaturity of the data for such contracts. The annuity values shown in Table 27 are based on the experience at durations 2 and over in the case of individual annuities and on the experience at all durations for group annuities. Annuity values, calculated from the ungraduated death rates for each kind of annuity, are compared with the graduated annuity values calculated on the 1943 Experience Table (ultimate). Differences brought out by these comparisons can be used as measures of the adjustment to be applied to annuity values based on the 1949 table (with projection). The statistical significance of these differences may be gauged by the standard deviations of the annuity values, next shown in the table, calculated[46] on the basis of the actual exposures for each type of annuity and the 1943 Experience Table Ultimate mortality rates. The following column shows these differences expressed as multiples of the standard deviations. The last three columns show in each instance the age setback (–) or set forward (+), in years, which when applied to the annuity value based on the 1943 Experience Table would produce a value equal to the annuity value calculated from the ungraduated experience. For convenience, each of these figures is placed in a column headed by the indicated probabilities that differences as large as these would be due to chance fluctuations from the 1943 Experience Table Ultimate mortality rates.

The reader is referred to the Joint Mortality Committee's 1948 report for other comparisons between the experience under immediate nonrefund annuities, under refund annuities, under deferred annuities, and under life income settlement options.

Attention is also directed to the fact that the mortality level for a given kind of annuity tends to vary from company to company. Variations of this kind were discussed in the Joint Mortality Committee's 1947 report on life income settlement options and again in its 1948 report on immediate annuities. Mr. Elston has shown[47] that variations of this kind in ordinary insurance are of considerable size in some instances. These analyses emphasize the importance of an examination of a company's own experience, if of sufficient size, to test the applicability of the data and tables in this paper.

XII. Annuity Tables for 1959 and 1979

The authors deemed it advisable to present two mortality tables in the usual form which may represent conservative estimates of annuity mortality rates prevailing ten and thirty years hence, *i.e.*, in 1959 and 1979. These tables are ultimate in form, correspond to the Annuity Table for 1949 (without projection), and were obtained from that table by (1) applying to the ultimate death rates in the 1949 table the annual rates of decrease in mortality rates shown in Table 19 for projection scale B, and (2) graduating the rates so obtained, as described below. Because the differences between the results of applying projection scales A and B (shown in Section X) are relatively unimportant for most kinds of annuity, tables for 1959 and 1979 in accordance with projection scale A were not prepared.

TABLE 25
ULTIMATE* EXPERIENCE ON VARIOUS TYPES OF ANNUITY
ON 1943 EXPERIENCE TABLE (ULTIMATE)—MALES

Age Group	Immediate Annuities		Settlement Options			Deferred Annuities†		Group Annuities
	Joint Committee Experience 1941 to 1946 Anniversaries (By Number of Contracts)		Joint Committee Experience 1940 to 1945 Anniversaries (By Number of Contracts)			Joint Committee Experience 1940 to 1945 Anniversaries (By Number of Contracts)		Intercompany 1941-45 Exper. on Lives Retired on or after Normal Retirement Date (By Number of Lives)
	Nonrefund	Refund	Payee	Unknown	Payee and Unknown	Nonrefund	Refund	
	Ratios of Actual to Expected Mortality							
Under 60.....	111%	120%	75%	102%	90%	95%	115%	261%
60-69.....	102	117	93	89	92	98	110%	116
70-79.....	99	107	91	96	93	96	111	123
80-89.....	100	109	111	130	118	51	68	131
90 and over.....	98	83	176	75	132	.....	.....	107
Total.....	100%± 1.0	109%±0.8	94%±2.6	98%±3.6	95%±2.1	96%±6.3	110%±2.5	121%±1.0
	Actual Deaths							
Under 60.....	142	474	26	44	70	5	40	40
60-69.....	770	1,826	259	111	370	58	394	2,348
70-79.....	1,971	3,595	262	123	385	45	360	2,684
80-89.....	1,382	2,366	78	53	131	1	5	710
90 and over.....	195	217	6	2	8	.....	.....	48
Total.....	4,460	8,478	631	333	964	109	799	5,830

* Duration 2 and over in the case of Immediate Annuities, Settlement Options and Deferred Annuities; all durations combined in the case of Group Annuities.
† Matured contracts.

Mortality rates and life annuity values at 2½% interest for the Annuity Tables for 1959 and 1979 (ultimate) are shown in Tables 28 and 29. To aid in the calculation of joint annuity values, these tables were graduated on the same general plan as the 1949 table, *i.e.*, by Makeham curves with the equivalent of modifications of the constant A at the younger ages. The constants at ages 60 to 87 for $\text{colog}_e(p_x) = A + Bc^x$ are as follows:

	Annuity Table for 1959		Annuity Table for 1979	
	Male Table	Female Table	Male Table	Female Table
1,000 A.....	3.70	0.98	3.10	0.94
1,000 B.....	.0204	.00493	.00885	.00213
$\text{Log}_{10} c$	.045	.051	.049	.055

TABLE 25—*CONTINUED*
ULTIMATE* EXPERIENCE ON VARIOUS TYPES OF ANNUITY ON 1943 EXPERIENCE TABLE (ULTIMATE)—FEMALES

Age Group	Immediate Annuities		Settlement Options				Deferred Annuities‡		Group Annuities
	Joint Committee Experience 1941 to 1946 Anniversaries (By Number of Contracts)		Joint Committee Experience 1940 to 1945 Anniversaries (By Number of Contracts)				Joint Committee Experience 1940 to 1945 Anniversaries (By Number of Contracts)		Intercompany 1941-45 Exper. on Lives Retired on or after Normal Retirement Date (By Number of Lives)
	Non-refund	Refund	Payee	Non-payee	Unknown	Total	Non-refund	Refund	
	Ratios of Actual to Expected Mortality								
Under 60...	96%	130%	109%	138%	118%	119%	75%	162%	118%
60-69.....	100	103	92	103	110	102	97	100	129
70-79.....	99	104	94	108	111	105	106	99	141
80-89.....	103	107	97	110	120	112	184	80	158
90 and over	94	89	121	63	65	68		0	72
Total.....	100%±0.7	105%±0.5	95%±2.0	110%± 2.5	112%± 1.7	106±1.1	101% ± 3.1	101%±1.9	136%±3.5
	Actual Deaths								
Under 60...	164	683	159	139	288	586	6	60	11
60-69.....	1,426	2,693	397	201	613	1,211	255	675	226
70-79.....	4,174	6,601	358	280	633	1,271	195	472	188
80-89.....	2,938	4,913	91	139	273	503	11	12	53
90 and over	301	534	5	11	20	36		0	2
Total.....	9,003	15,424	1,010	770	1,827	3,607	467	1,219	480

* Duration 2 and over in the case of Immediate Annuities, Settlement Option and Deferred Annuities; all durations combined in the case of Group Annuities.
† Matured contracts.

Below age 60 the mortality rates in the 1959 and 1979 tables were actually calculated to equal 88.1802% and 68.5667%, respectively, of the rates appearing in the 1949 table. These percentages are a consequence of projection scale B at age 50 and under and bridge satisfactorily the gap between ages 50 and 60. The resulting death rates at ages below 60 can be represented by Makeham curves with the same values of the constants B and c as at ages 60 and over but with modified values of the constant A. In order to avoid higher death rates than those in the 1949 table at some of the very advanced ages, the mortality rates in the 1959 and 1979 tables were graded into those of the 1949 table over the age range from 88 to 94 and made equal to those of the 1949 table at ages 95 and over. Complete tests of these graduations are not presented; it can be stated, however, that the death rates were changed but little by graduation, the largest percentage changes being 0.7% for the male and female 1959 tables, and 2.0% for the male and female 1979 tables.

A comparison of the graduations of the 1949, 1959, and 1979 tables reveals an even progression of the Makeham constants. Interpolation between or extrapolation from these constants thus provides a simple method for obtaining corresponding mortality tables for other years.

First year (select) mortality rates for the 1959 and 1979 tables may be calculated in the same manner as for the 1949 table, that is, as 75% of the ultimate rates in the case of males and as 50% of the ultimate rates in the case of females.

The 1959 and 1979 tables were prepared for two purposes. First, some actuaries may desire to observe the mortality rates and annuity values on the Annuity Table for 1949 projected ten and thirty years by scale B.

The other reason is that, in spite of the relative simplicity of the projection factor method presented in Section X and even greater simplicity obtainable by the use of approximations, some actuaries may prefer to use the customary form of mortality table without projection. If this were done, the actuary will, of course, appreciate that the 1959 and 1979 tables would not be appropriate for annuities beginning in the years appearing in the names of these tables; in this respect, the 1959 and 1979 tables correspond strictly to the Annuity Table for 1949 (without projection).

TABLE 26
FIRST POLICY YEAR EXPERIENCE ON VARIOUS TYPES OF ANNUITY ON 1943 EXPERIENCE TABLE (ULTIMATE)—MALES

Age Group	Immediate Annuities		Settlement Options			Deferred Annuities*	
	Joint Committee Experience 1941 to 1946 Anniversaries (By Number of Contracts)		Joint Committee Experience 1940 to 1945 Anniversaries (By Number of Contracts)			Joint Committee Experience 1940 to 1945 Anniversaries (By Number of Contracts)	
	Non-refund	Refund	Payee	Unknown	Payee and Unknown	Non-refund	Refund
	Ratios of Actual to Expected Mortality						
Under 60.....	115%	144%	136%	152%	142%	0%	216%
60-69.....	94	101	90	75	87	93	118
70-79.....	77	104	111	113	112	66	67
80-89.....	57	57	79	0	64	.....	0
90 and over.....	0	49	.....	.....	.....	.....	.....
Total.....	80%±4.8	97%±3.2	99%±5.8	95%±9.7	98%±5.0	81%±13.1	110%±4.9
	Actual Deaths						
Under 60.....	12	56	18	12	30	0	28
60-69.....	50	134	78	21	99	17	138
70-79.....	72	171	31	12	43	4	37
80-89.....	21	49	3	0	3	.....	0
90 and over.....	0	2	.....	.....	.....	.....	.....
Total.....	155	412	130	45	175	21	203

* Matured contracts.

TABLE 26—*CONTINUED*
FIRST POLICY YEAR EXPERIENCE ON VARIOUS TYPES OF ANNUITY ON 1943 EXPERIENCE TABLE (ULTIMATE)—FEMALES

Age Group	Immediate Annuities		Settlement Options				Deferred Annuities*	
	Joint Committee Experience 1941 to 1946 Anniversaries (By Number of Contracts)		Joint Committee Experience 1940 to 1945 Anniversaries (By Number of Contracts)				Joint Committee Experience 1940 to 1945 Anniversaries (By Number of Contracts)	
	Nonrefund	Refund	Payee	Nonpayee	Unknown	Total	Nonrefund	Refund
	Ratios of Actual to Expected Mortality							
Under 60.....	74%	98%	134%	94%	69%	102%	29%	116%
60-69.....	52	77	60	100	87	76	57	111
70-79.....	50	70	55	108	25	62	50	111
80-89.....	45	68	86	86	82	85	.....	0
90 and over.....	0	109	0	0	0	0	.....	
Total.....	51%±3.9	74%± 2.7	78%±5.4	100%±8.2	70%±6.3	80%±3.7	54%±7.4	111%±4.5
	Actual Deaths							
Under 60.....	13	48	53	17	23	93	1	15
60-69.....	49	139	46	23	49	118	36	179
70-79.....	69	188	18	22	5	45	7	50
80-89.....	22	84	3	4	2	9	.....	0
90 and over.....	0	6	0	0	0	0	.....	
Total.....	153	465	120	66	79	265	44	244

* Matured contracts.

Use of the 1959 and 1979 tables (without projection) would obviously involve rougher and less equitable approximations to annuity values than are produced by the method of projection factors. In general, younger annuitants would be favored in comparison with older annuitants, and annuites under which the first payment is deferred for a relatively long period of time would be favored in comparison with annuities under which payments begin at the same attained age but are deferred for a shorter period of time. However, if the table chosen (not necessarily the 1959 or 1979 table) were sufficiently conservative, there might be less objection to its use without projection for participating annuities; this is because in the case of participating annuities inequities could probably be minimized or eliminated through dividends. It should be noted that if approximations introduced into premium rates are to be equitably offset through dividends, it is necessary that the approximations err in the direction of conservatism in all cases, not merely on the average.

Even though in some circumstances it may be satisfactory to use tables such as the 1959 or 1979 tables without projection for participating premium rates on immediate annuities, these tables would not produce adequate annuity reserves after some time had elapsed. This is because when an average mortality level that remains constant with the passage of time is substituted for a mortality level that decreases with the passage of time, the annuity reserves on the former basis will tend to be lower than those on the latter basis. Adjustments to offset such reserve deficiencies might involve processes no simpler than the method of projection factors presented in Section X.

Related to this matter of reserves is the incidence of annual statement gains and losses from mortality under annuity contracts.

TABLE 27

TEST OF VARIABILITY OF MORTALITY UNDER VARIOUS TYPES OF ANNUITY VALUES OF a_x BASED ON UNGRADUATED ULTIMATE EXPERIENCE COMPARED WITH a_x BASED ON 1943 EXPERIENCE TABLE ULTIMATE

Age x	Males						
	a_x on Ungraduated Experience (1)	(1) − a_x on 1943 Experience Table (2)	$\sigma\{a_x\}$ on 1943 Experience Table (3)	Difference As Multiple of $\sigma\{a_x\}$ (2) ÷ (3) (4)	Set Forward (+) or Set Back (−) in Years of Age		
					Probability Range More Than 5%	Probability Range 5% to 1%	Probability Range Less Than 1%
	Immediate Annuity—Nonrefund						
50	16.893	−0.354	0.196	−1.806	+0.8		
55	15.127	+0.003	0.158	+0.019	−0.0		
60	13.059	+0.016	0.119	+0.134	0.0		
65	10.925	−0.039	0.100	−0.390	+0.1		
70	8.927	+0.011	0.092	+0.120	0.0		
75	6.932	−0.048	0.097	−0.495	+0.1		
80	5.242	+0.009	0.112	+0.080	0.0		
	Immediate Annuity—Refund						
50	16.534	−0.713	0.117	−6.094			+1.7
55	14.489	−0.635	0.099	−6.414			+1.5
60	12.459	−0.584	0.084	−6.952			+1.4
65	10.590	−0.374	0.074	−5.054			+0.9
70	8.629	−0.287	0.072	−3.986			+0.7
75	6.796	−0.184	0.077	−2.390		+0.5	
80	5.042	−0.191	0.091	−2.099		+0.6	
	Settlement Options—Payee Elections						
50	18.192	+0.945	0.426	+2.218		−2.2	
55	15.715	+0.591	0.318	+1.858	−1.4		
60	13.319	+0.276	0.272	+1.015	−0.7		
65	11.253	+0.289	0.262	+1.103	−0.7		
70	9.095	+0.179	0.310	+0.577	−0.4		
75	7.109	+0.129	0.397	+0.325	−0.3		
80	4.863	−0.370	0.543	−0.681	+1.2		

XIII. Effect of Certain Mortality Changes on Annuity Values

The relatively low mortality rates exhibited by the Annuity Table for 1949 and the strong probability that death rates will continue to decrease (for which projection factors are developed in Section X) suggest that some analysis of the effect on annuity values of certain other reductions in mortality may be informative.

One obvious characteristic of the 1949 table (without projection) is that ultimate mortality rates do not reach 2 per 1,000 until age 40 in the case of males and age 45 in the case of females. Therefore, until middle age the annual probability of survival exceeds .998, and further mortality decreases at younger ages cannot have a marked effect on immediate annuity values no matter how large they may be percentagewise. This is demonstrated in Part A of Table 30, which shows that the complete and immediate elimination of all deaths (zero mortality) at ages under 40 would increase immediate life annuity values (according to the 1949 ultimate table and 2½% interest) by 1.5% or less for males and by 1.0% or less for females. These percentages bring out that the particular scale of decreases in mortality at ages under 40 is relatively unimportant, and show why the markedly different assumptions at these ages under projection scales A and B do not produce markedly different immediate annuity values. Part A of Table 30 also demonstrates that, when an immediate annuity involves mortality rates as low as those in the 1949 table at ages under 40, its yield cannot be significantly larger than that which would be produced by the substitution of an annuity certain throughout the low-mortality period.

TABLE 27—*Continued*

Age x	Males						
	a_x on Ungraduated Experience (1)	(1) − a_x on 1943 Experience Table (2)	$\sigma\{a_x\}$ on 1943 Experience Table (3)	Difference As Multiple of $\sigma\{a_x\}$ (2) ÷ (3) (4)	Set Forward (+) or Set Back (−) in Years of Age		
					Probability Range More Than 5%	Probability Range 5% to 1%	Probability Range Less Than 1%
	Settlement Options—Payee and Unknown Elections						
50	17.916	+0.669	0.279	+2.398		−1.6	
55	15.681	+0.557	0.240	+2.321		−1.3	
60	13.282	+0.239	0.217	+1.101	−0.6		
65	11.089	+0.125	0.216	+0.579	−0.3		
70	8.882	−0.034	0.255	−0.133	+0.1		
75	6.779	−0.201	0.325	−0.618	+0.6		
80	4.562	−0.671	0.445	−1.508	+2.2		
	Group Annuity—Lives Retired on or after Normal Retirement Date						
50	11.543	−5.704	0.797	−7.157			+13.6
55	12.939	−2.185	0.351	−6.225			+5.3
60	11.916	−1.127	0.162	−6.957			+2.7
65	9.972	−0.992	0.091	−10.901			+2.4
70	7.894	−1.022	0.110	−9.291			+2.6
75	5.961	−1.019	0.140	−7.279			+2.9
80	4.305	−0.928	0.191	−4.859			+3.1

TABLE 27—*Continued*

Age x	Females						
	a_x on Ungraduated Experience (1)	(1) − a_x on 1943 Experience Table (2)	$\sigma\{a_x\}$ on 1943 Experience Table (3)	Difference As Multiple of $\sigma\{a_x\}$ (2)÷(3) (4)	Set Forward (+) or Set Back (−) in Years of Age		
					Probability Range More Than 5%	Probability Range 5% to 1%	Probability Range Less Than 1%
	Immediate Annuity—Nonrefund						
50	19.701	−0.031	.104	−0.298	+0.1		
55	17.600	+0.052	.079	+0.658	−0.1		
60	15.210	−0.022	.066	−0.333	0.0		
65	12.880	+0.037	.061	+0.607	−0.1		
70	10.464	0.000	.063	0.000	0.0		
75	8.174	−0.015	.070	−0.214	0.0		
80	6.051	−0.064	.086	−0.744	+0.2		
	Immediate Annuity—Refund						
50	19.399	−0.333	.061	−5.459			+0.8
55	17.277	−0.271	.053	−5.113			+0.6
60	15.052	−0.180	.049	−3.673			+0.4
65	12.697	−0.146	.048	−3.042			+0.3
70	10.302	−0.162	.050	−3.240			+0.4
75	8.036	−0.153	.055	−2.782			+0.4
80	5.979	−0.136	.066	−2.061		+0.4	
	Settlement Options—Payee Elections						
50	19.854	+0.122	.192	+0.635	−0.3		
55	17.750	+0.202	.206	+0.981	−0.5		
60	15.507	+0.275	.229	+1.201	−0.6		
65	13.142	+0.299	.265	+1.128	−0.6		
70	10.693	+0.229	.318	+0.720	−0.5		
75	8.404	+0.215	.401	+0.536	−0.5		
80	6.205	+0.090	.544	+0.165	−0.2		

The two projection scales, suggested in this paper to anticipate future decreases in mortality, assume decreases in mortality at ages 40 to 60 which will average about 25% by the end of the next twenty years. (More precisely, these decreases are shown in Table 19 to vary under scale A from 33.3% at the age 40 down to 21.5% at age 60, and under scale B from 22.3% at age 40 down to 21.5% at age 60.) A natural question is: supposing that these assumed decreases prove to be too small and that mortality rates at ages 40 to 60 actually reduce by 50%, instead of by approximately 25%, by the end of the next twenty years, what would be the effect on immediate annuity values? Part B of Table 30, based on the 1949 ultimate table and 2½% interest, shows that a reduction in mortality rates of this magnitude occurring immediately would increase immediate life annuity values by 4.1% or less in the case of males and by 2.0% or less in the case of females. If this reduction occurred gradually over the 20-year period, instead of immediately, the percentage changes would, of course, be less.

TABLE 27—*Continued*

Age x	Females						
	a_x on Ungraduated Experience (1)	(1) − a_x on 1943 Experience Table (2)	$\sigma\{a_x\}$ on 1943 Experience Table (3)	Difference As Multiple of $\sigma\{a_x\}$ (2) ÷ (3) (4)	Set Forward (+) or Set Back (−) in Years of Age		
					Probability Range More Than 5%	Probability Range 5% to 1%	Probability Range Less Than 1%
	Settlement Options—Nonpayee Elections						
50	19.289	−0.443	.197	−2.249		+1.0	
55	17.188	−0.360	.206	−1.748	+0.8		
60	14.991	−0.241	.221	−1.090	+0.5		
65	12.593	−0.250	.242	−1.033	+0.5		
70	10.178	−0.286	.273	−1.048	+0.6		
75	7.739	−0.450	.320	−1.406	+1.1		
80	5.978	−0.137	.400	−0.343	+0.4		
	Settlement Options—Payee, Nonpayee and Unknown Elections						
50	19.479	−0.253	.092	−2.750			+0.6
55	17.337	−0.211	.097	−2.175		+0.5	
60	15.023	−0.209	.106	−1.972		+0.4	
65	12.655	−0.188	.119	−1.580	+0.4		
70	10.220	−0.244	.138	−1.768	+0.5		
75	7.842	−0.347	.167	−2.078		+0.8	
80	5.909	−0.206	.214	−0.963	+0.6		
	Group Annuity—Lives Retired on or after Normal Retirement Date						
50	20.077	+0.345	.964	+0.358	−0.8		
55	16.191	−1.357	.366	−3.708			+2.9
60	13.688	−1.544	.359	−4.301			+3.2
65	11.277	−1.566	.412	−3.801			+3.3
70	8.789	−1.675	.495	−3.384			+3.7
75	6.684	−1.505	.621	−2.424		+3.6	
80	4.586	−1.529	.816	−1.874	+4.3		

TABLE 28
ANNUITY TABLE FOR 1959
ULTIMATE MORTALITY RATES AND ANNUITY VALUES—MALE

Age x	$1000q_x$	a_x at 2½% Interest	Age x	$1000q_x$	a_x at 2½% Interest
10	.426	31.278	53	7.795	17.146
11	.434	31.074	54	8.538	16.713
12	.443	30.865	55	9.316	16.278
13	.451	30.651	56	10.133	15.842
14	.462	30.431	57	10.987	15.404
15	.474	30.206	58	11.883	14.965
16	.486	29.976	59	12.823	14.524
17	.500	29.740	60	13.828	14.080
18	.515	29.499	61	14.928	13.634
19	.532	29.252	62	16.147	13.187
20	.550	28.949	63	17.497	12.739
21	.571	28.740	64	18.992	12.290
22	.594	28.475	65	20.648	11.841
23	.619	28.204	66	22.482	11.393
24	.646	27.927	67	24.511	10.946
25	.677	27.644	68	26.758	10.502
26	.711	27.354	69	29.243	10.060
27	.749	27.058	70	31.993	9.622
28	.790	26.755	71	35.033	9.188
29	.835	26.446	72	38.395	8.760
30	.885	26.130	73	42.109	8.338
31	.941	25.807	74	46.213	7.922
32	1.002	25.477	75	50.743	7.513
33	1.070	25.140	76	55.744	7.112
34	1.144	24.796	77	61.259	6.720
35	1.227	24.445	78	67.339	6.338
36	1.317	24.087	79	74.036	5.965
37	1.417	23.722	80	81.409	5.603
38	1.528	23.350	81	89.517	5.252
39	1.651	22.970	82	98.428	4.913
40	1.786	22.583	83	108.209	4.586
41	1.958	22.189	84	118.934	4.271
42	2.188	21.788	85	130.678	3.969
43	2.473	21.382	86	143.523	3.680
44	2.810	20.971	87	157.547	3.404
45	3.197	20.556	88	172.815	3.142
46	3.629	20.137	89	189.362	2.893
47	4.107	19.716	90	207.211	2.658
48	4.626	19.292	91	226.382	2.436
49	5.185	18.866	92	246.895	2.228
50	5.782	18.438	93	268.775	2.032
51	6.417	18.009	94	292.070	1.849
52	7.088	17.578	95*	316.834	1.677

* The values at ages 95 and over are the same as those shown for the Annuity Table for 1949.

TABLE 28—*Continued*
ANNUITY TABLE FOR 1959
ULTIMATE MORTALITY RATES AND ANNUITY VALUES—FEMALE

Age x	$1000q_x$	a_x at 2½% Interest	Age x	$1000q_x$	a_x at 2½% Interest
10	.168	32.380	53	3.489	19.431
11	.183	32.195	54	3.801	18.987
12	.198	32.006	55	4.149	18.536
13	.213	31.813	56	4.538	18.079
14	.229	31.615	57	4.973	17.615
15	.245	31.413	58	5.461	17.146
16	.261	31.206	59	6.007	16.671
17	.278	30.995	60	6.618	16.191
18	.295	30.779	61	7.319	15.706
19	.312	30.558	62	8.106	15.217
20	.332	30.332	63	8.990	14.725
21	.351	30.101	64	9.984	14.230
22	.371	29.864	65	11.100	13.733
23	.393	29.622	66	12.354	13.234
24	.417	29.375	67	13.762	12.735
25	.442	29.122	68	15.344	12.236
26	.468	28.863	69	17.119	11.737
27	.496	28.598	70	19.112	11.240
28	.527	28.328	71	21.348	10.746
29	.561	28.051	72	23.857	10.255
30	.597	27.768	73	26.670	9.768
31	.636	27.479	74	29.825	9.287
32	.679	27.184	75	33.360	8.812
33	.725	26.883	76	37.321	8.344
34	.775	26.575	77	41.756	7.884
35	.831	26.260	78	46.718	7.433
36	.891	25.939	79	52.269	6.992
37	.957	25.611	80	58.472	6.562
38	1.029	25.276	81	65.400	6.144
39	1.108	24.935	82	73.131	5.738
40	1.195	24.587	83	81.748	5.346
41	1.291	24.232	84	91.344	4.968
42	1.396	23.870	85	102.015	4.604
43	1.512	23.501	86	113.867	4.255
44	1.639	23.125	87	127.008	3.922
45	1.780	22.742	88	141.528	3.605
46	1.936	22.352	89	157.507	3.304
47	2.108	21.955	90	175.022	3.020
48	2.298	21.551	91	194.147	2.752
49	2.509	21.141	92	214.952	2.500
50	2.742	20.724	93	237.507	2.264
51	2.964	20.300	94	261.883	2.044
52	3.212	19.869	95*	288.153	1.838

*The values at ages 95 and over are the same as those shown for the Annuity Table for 1949.

TABLE 29
ANNUITY TABLE FOR 1979
ULTIMATE MORTALITY RATES AND ANNUITY VALUES—MALE

Age x	$1000q_x$	a_x at 2½% Interest	Age x	$1000q_x$	a_x at 2½% Interest
10	.331	31.727	53	6.061	17.975
11	.337	31.531	54	6.639	17.537
12	.344	31.330	55	7.244	17.096
13	.351	31.124	56	7.879	16.651
14	.359	30.913	57	8.543	16.203
15	.368	30.697	58	9.240	15.751
16	.378	30.476	59	9.971	15.295
17	.389	30.250	60	10.750	14.835
18	.400	30.018	61	11.660	14.371
19	.413	29.781	62	12.678	13.904
20	.428	29.538	63	13.817	13.435
21	.444	29.289	64	15.089	12.964
22	.462	29.035	65	16.512	12.492
23	.481	28.775	66	18.103	12.019
24	.503	28.509	67	19.880	11.547
25	.527	28.236	68	21.866	11.076
26	.553	27.957	69	24.084	10.607
27	.582	27.672	70	26.561	10.140
28	.614	27.380	71	29.326	9.677
29	.649	27.082	72	32.413	9.219
30	.688	26.777	73	35.856	8.766
31	.732	26.465	74	39.697	8.319
32	.779	26.146	75	43.978	7.879
33	.832	25.821	76	48.747	7.447
34	.889	25.489	77	54.058	7.024
35	.954	25.149	78	59.968	6.611
36	1.024	24.802	79	66.540	6.209
37	1.102	24.448	80	73.843	5.818
38	1.188	24.087	81	81.950	5.439
39	1.284	23.719	82	90.941	5.073
40	1.388	23.343	83	100.901	4.720
41	1.522	22.960	84	111.922	4.381
42	1.701	22.570	85	124.099	4.056
43	1.923	22.174	86	137.533	3.746
44	2.185	21.772	87	152.327	3.452
45	2.486	21.365	88	168.546	3.174
46	2.822	20.954	89	186.170	2.913
47	3.193	20.539	90	205.094	2.669
48	3.597	20.120	91	225.168	2.442
49	4.032	19.697	92	246.328	2.230
50	4.496	19.271	93	268.584	2.033
51	4.990	18.842	94	292.036	1.849
52	5.511	18.410	95*	316.834	1.677

* The values at ages 95 and over are the same as those shown for the Annuity Table for 1949.

TABLE 29—*Continued*
ANNUITY TABLE FOR 1979
ULTIMATE MORTALITY RATES AND ANNUITY VALUES—FEMALE

Age x	$1000q_x$	a_x at 2½% Interest	Age x	$1000q_x$	a_x at 2½% Interest
10	.131	32.686	53	2.713	20.017
11	.143	32.508	54	2.955	19.573
12	.154	32.325	55	3.226	19.122
13	.166	32.138	56	3.528	18.664
14	.178	31.947	57	3.867	18.198
15	.191	31.752	58	4.246	17.725
16	.203	31.552	59	4.671	17.246
17	.216	31.347	60	5.176	16.760
18	.229	31.138	61	5.747	16.268
19	.243	30.924	62	6.394	15.771
20	.258	30.705	63	7.129	15.269
21	.273	30.481	64	7.961	14.763
22	.289	30.252	65	8.905	14.254
23	.306	30.017	66	9.976	13.742
24	.324	29.777	67	11.190	13.227
25	.344	29.531	68	12.566	12.711
26	.364	29.280	69	14.125	12.195
27	.386	29.023	70	15.892	11.679
28	.410	28.760	71	17.893	11.164
29	.436	28.491	72	20.160	10.652
30	.464	28.216	73	22.726	10.143
31	.494	27.935	74	25.631	9.638
32	.528	27.648	75	28.918	9.139
33	.564	27.354	76	32.634	8.646
34	.603	27.054	77	36.836	8.161
35	.646	26.747	78	41.582	7.685
36	.693	26.433	79	46.941	7.219
37	.744	26.113	80	52.988	6.764
38	.800	25.786	81	59.804	6.321
39	.861	25.452	82	67.481	5.891
40	.929	25.111	83	76.118	5.475
41	1.004	24.763	84	85.825	5.074
42	1.085	24.408	85	96.719	4.689
43	1.176	24.045	86	108.926	4.321
44	1.275	23.675	87	122.582	3.970
45	1.384	23.298	88	137.771	3.638
46	1.506	22.914	89	154.554	3.325
47	1.639	22.522	90	172.961	3.031
48	1.787	22.123	91	192.937	2.757
49	1.951	21.717	92	214.397	2.502
50	2.132	21.303	93	237.343	2.265
51	2.305	20.882	94	261.867	2.044
52	2.497	20.453	95*	288.153	1.838

* The values at ages 95 and over are the same as those shown for the Annuity Table for 1949.

TABLE 30

EFFECT OF CHANGES IN MORTALITY ON IMMEDIATE ANNUITY VALUES AT AGES UNDER 60

A. EFFECT OF ZERO MORTALITY AT AGES UNDER 40 ON IMMEDIATE ANNUITY VALUES BASED ON ANNUITY TABLE FOR 1949 (ULTIMATE, WITHOUT PROJECTION) AT 2½% INTEREST

Age x	a_x 1949 Annuity Table Ult. 2½% Interest		a_x with Zero Mortality to Age 40* 2½% Interest		Ratio of (i) Annuity Values With Zero Mortality to Age 40* to (ii) Annuity Values on 1949 Annuity Table Ult.	
	Male	Female	Male	Female	Male	Female
15.....	29.932	31.222	30.380	31.530	101.5%	101.0%
20.....	28.700	30.121	29.115	30.417	101.4	101.0
25.....	27.317	28.890	27.685	29.157	101.3	100.9
30.....	25.773	27.516	26.066	27.732	101.1	100.8
35.....	24.057	25.988	24.236	26.120	100.7	100.5

B. EFFECT OF A 50% REDUCTION IN MORTALITY AT AGES 40–59 ON IMMEDIATE ANNUITY VALUES BASED ON ANNUITY TABLE FOR 1949 (ULTIMATE, WITHOUT PROJECTION) AT 2½% INTEREST

Age	a_x 1949 Annuity Table Ult. 2½% Interest		a_x with Mortality of 50% of 1949 Annuity Table Ult. to Age 60† 2½% Interest		Ratio of (i) Annuity Values with Mortality of 50% of 1949 Annuity Table Ult. to Age 60† to (ii) Annuity Values on 1949 Annuity Table Ult.	
	Male	Female	Male	Female	Male	Female
40.....	22.165	24.295	23.026	24.781	103.9%	102.0%
45.....	20.112	22.433	20.946	22.885	104.1	102.0
50.....	17.984	20.404	18.695	20.778	104.0	101.8
55.....	15.837	18.215	16.279	18.448	102.8	101.3

* qx by 1949 Annuity Table at ages 40 and over.
† qx by 1949 Annuity Table at ages 60 and over.

Thus, even very radical reductions in mortality at ages under 60 would not result in extremely large increases of immediate life annuity values, unless accompanied by similar changes at ages over 60. It should be noted, however, that the applicability of this statement to life income settlement options, maturity settlements of retirement income insurances, and other types of deferred annuities, becomes less as the period of deferment lengthens.

It can be concluded from the foregoing paragraphs, and from the fact that most individual annuities are issued at the older ages, that the important future changes in annuitant mortality are those at ages 60 and over; changes at these ages will have the controlling effect on most annuity values in practice. The projection scales presented in Section IX were designed to make appropriate and adequate provision for mortality decreases arising from more intensive application of existing knowledge and, to a reasonable degree, from new discoveries foreshadowed by current research but not from revolutionary discoveries, such as an infallible cancer cure, in addition to the mortality decreases provided for. Therefore, in relation to the important ages over 60, the effect on annuity values of complete or

partial elimination of each major cause of death deserves special attention.

Table 31 considers immediate life annuity values at ages 60 and over, calculated on the Annuity Table for 1949 (ultimate, without projection) and 2½% interest, and indicates how these annuity values would be increased by the immediate elimination of all deaths arising from each of the following causes:

(1) Cardiovascular-renal diseases.
(2) Cancer.
(3) Influenza and pneumonia.
(4) Accidents.
(5) All other causes.

In preparing each portion of this table, the 1949 table central death rates were reduced in the proportions that deaths resulting from the cause in question bear to the total deaths. These proportions, graded by age, were derived largely from Table 17. It was thus assumed that each such change would be accompanied by no change in the death rates from causes other than that eliminated.

While the hypothesis of immediate elimination of all deaths from any of the major causes may be regarded as a theoretical supposition, the figures in Table 31 also serve to indicate what effect a partial reduction in deaths from any of the major causes would have on immediate life annuity values. A given percentage reduction in deaths from any major cause would result in an increase in annuity value slightly smaller than that percentage of the increase in annuity value shown in Table 31 for complete elimination of the cause of death.

Even though the complete elimination of all deaths from cardiovascular-renal diseases seems quite unthinkable, if not impossible, the data in Table 31 indicate that such an eventuality would call for an entirely new perspective on longevity at ages 60 and over, because immediate annuity values would thereby be increased, according to the calculations, by over 25% at age 60 and would be about doubled at age 85. Of more immediate concern are the substantial increases in annuity values that Table 31 indicates would result from a partial, say 10% or 20%, reduction in death rates from the cardiovascular-renal diseases.

It is also noteworthy that, according to Table 31, the eradication of all deaths from cancer would increase immediate annuity values by from 4.1% to 5.7%—a substantial amount, but perhaps less than might have been expected by some. The eradication of deaths from influenza and pneumonia would increase immediate annuity values by from 1.9% to 6.2%—again a substantial amount, possibly larger than expected by some.

Because it is altogether unlikely that mortality from any major cause at ages 60 and over would be completely or even largely eliminated in a relatively short period of time, the authors believe that it should be possible for actuaries to keep annuity values at these ages in reasonable conformity with actual decreases in mortality. This will require careful study of mortality trends and periodic adjustment of mortality tables and projection factors so as to provide conservative allowance for impending changes.

The possibility, however unlikely, of a very rapid elimination of some major cause of death underlines the need for an adequate safety margin in annuity mortality tables used for premium and reserve purposes.

XIV. Joint Life Annuities

Although in constructing the Annuity Table for 1949 (ultimate, without projection) it was not possible to use a Makeham curve over the whole age range, a relatively simple method for computing values of joint life annuities for two lives has been developed for this table. The general procedure is the usual one associated with Makehamized tables, *i.e.*, the substitution of a joint annuity at equal ages for the joint annuity at the two different ages involved. It is suggested that for two lives of the same sex the usual procedure for obtaining equal ages be used without modification. For one male and one female, a special procedure for obtaining equal ages, described below, is suggested. In the latter part of this section, the projection of joint annuity values is discussed. While only ultimate annuity values are considered in this section, the methods described here can readily be extended to obtain select values.

Values of joint life annuities at equal ages on the Annuity Table for 1949 (without projection) at 2½% interest are presented in Table 32 for the combinations of two males, two females, and one male and one female. The equivalent equal ages in the case of two males or two females may be determined from the table of uniform seniority presented in Table 33. These tables were all calculated by the standard formulae applicable where the Makeham constants hold throughout the entire age range.

TABLE 31
EFFECT OF ELIMINATING MAJOR CAUSES OF DEATH ON IMMEDIATE ANNUITY VALUES AT AGES 60 AND OVER

Age x	a_x 1949 Annuity Table (Ultimate) at 2½% Interest	a_x 1949 Annuity Table (Ultimate) at 2½% Interest, Modified by Eliminating All Deaths from:					Ratios of Modified Annuities to a_x 1949 Annuity Table (Ultimate) at 2½% Interest				
		Cardiovascular-renal Diseases*	Cancer*	Influenza and Pneumonia*	Accidents*	All Other Causes*	Cardiovascular-renal Diseases Eliminated	Cancer Eliminated	Influenza and Pneumonia Eliminated	Accidents Eliminated	All Other Causes Eliminated
	Males										
60.....	13.676	18.671	14.240	13.997	13.827	14.609	1.365	1.041	1.023	1.011	1.068
65.....	11.495	16.571	12.016	11.821	11.643	12.388	1.442	1.045	1.028	1.013	1.078
70.....	9.351	14.397	9.808	9.674	9.486	10.179	1.540	1.049	1.035	1.014	1.089
75.....	7.323	12.211	7.696	7.630	7.448	8.071	1.667	1.051	1.042	1.017	1.102
80.....	5.492	10.084	5.779	5.773	5.602	6.128	1.836	1.052	1.051	1.020	1.116
85.....	3.923	8.088	4.127	4.167	4.013	4.423	2.062	1.052	1.062	1.023	1.127
	Females										
60.....	15.881	20.318	16.537	16.175	16.016	16.487	1.279	1.041	1.019	1.009	1.038
65.....	13.454	18.113	14.086	13.766	13.593	14.075	1.346	1.047	1.023	1.010	1.046
70.....	11.009	15.793	11.578	11.327	11.145	11.626	1.435	1.052	1.029	1.012	1.056
75.....	8.642	13.409	9.118	8.955	8.770	9.247	1.552	1.055	1.036	1.015	1.070
80.....	6.458	11.040	6.824	6.751	6.572	7.021	1.710	1.057	1.045	1.018	1.087
85.....	4.560	8.772	4.813	4.817	4.655	5.031	1.924	1.055	1.056	1.021	1.103

* Distribution of annuitant deaths by cause of death based mainly on data in Table 17.

The 1949 table was constructed by using a Makeham curve at ages 60 and over in the male table and at ages 50 and over in the female table, retaining the same values of B and c at the younger ages, but gradually reducing the value of A with decreasing age. Consequently, for two lives of the same sex the annuity values produced by the customary procedure are exact in all cases where both males are aged 60 or over and where both females are aged 50 or over, but such annuity values will be slightly lower than the true values in cases where either male life is below age 60 and where either female life is below age 50. It is suggested, nevertheless, that Tables 32 and 33 be used without adjustment.

Table 34 compares the annuity values produced by this suggested method with the exact values, for two lives of the same sex at various combinations of ages. It indicates that for two females, the maximum error is less than 1%. For two males, the maximum error is about 3%, but errors of this magnitude occur only for unusual combinations of ages such as 25 and 65. The maximum error is less than 1% in the more common cases where the ages of the two male lives differ by less than 10 years and also where both male lives are aged 50 or over. Because the joint life annuity values on the 1949 table should in practice be adjusted by projection factors or some other equivalent method, the authors thought that the simple procedure suggested above for two lives of the same sex is preferable to an elaborate procedure producing more accurate values.

The problem of obtaining equal ages for a joint life annuity on one male life and one female life is complicated by the fact that the Makeham constant c in the male Annuity Table for 1949 (without projection) differs from that in the corresponding female table. However, a relatively simple method of obtaining the equal age for one male and one female is presented in Table 35. This table is based on the principle that a joint life annuity for a male aged x and a female aged y is equal to a joint life annuity on a male aged w and a female aged w for some value of t, if w is determined from the following equation:

$$bc^{x+t}+BC^{y+t} = bc^{w+t}+BC^{w+t}$$

the small letters representing the Makeham constants on the male table and the capital letters the Makeham constants on the female table. For any combination of ages x and y, there always is some value of t which will produce a value of w such that $a_{ww} = a_{xy}$. The problem was to determine a single value of t which produces reasonably close estimates of the joint life annuity values over the whole range of ages.

It was found that if the value of w is obtained from the above equation by using $t = 5$, the values of a_{ww} shown in Table 32 for one male and one female generally come very close to the exact values from the Annuity Table for 1949 (without projection) at 2½% interest. A comparison of the life annuity values produced by this method with the exact values at various ages is shown in Table 36. This table indicates that the maximum error produced by the suggested method is just over 2%, while the maximum error for the more important age combinations is much less than 1%.

The methods suggested above for obtaining joint life annuity values on the Annuity Table for 1949 (without projection) are, in general, applicable also to the Annuity Tables for 1959 and 1979. This is so because these tables were constructed on the same pattern as the 1949 table, *i.e.*, by using Makeham curves with the equivalent of modifications of the constant A at the younger ages and with different values of the constant c for the male and female tables. The auxiliary tables for obtaining joint life annuities on the 1949 table are, of course, not applicable to the 1959 and 1979 tables because the Makeham constants differ and also because of the adjustments introduced into these latter tables over age 87.

One method of adjusting joint life annuities based on the Annuity Table for 1949 (without projection) so as to project mortality in accordance with scale A or B is to increase the unprojected joint annuity value by the projection factor shown for a single life annuity at the equal age, using the male factor for one male and one female. The errors introduced by this method, which is referred to as method A, are shown in Table 37 for projection scale B; they do not exceed 1.0%.

Another method, referred to as method B, is to convert the projection factor for each single life annuity into an equivalent age setback on the basis of the 1949 table (without projection). Thus, the age setbacks, h and k, are fixed by the equation:

$$a'_x \text{ (with projection)} = a_{x-h} \text{ (without projection)}$$

$$a'_y \text{ (with projection)} = a_{y-k} \text{ (without projection)}$$

Then, the joint life annuity adjusted for projection can be obtained by computing $a_{x-h:y-k}$ on the Annuity Table for 1949 (without projection). As shown in Table 37 for projection scale B the maximum errors introduced by this method are 0.4% for two females, 0.2% for two males, and 0.2% for one male and one female.

In addition to the errors cited in Table 37, account must also be taken of the errors introduced by the approximations tested in Tables 34 and 36. The total maximum errors, including all types of error, were calculated to be 2.5% for two males under method A mentioned above and 3.1% under method B mentioned above, 1.0% for two females under method A and 0.8% under method B, and 1.8% for one male and one female under both methods, using projection scale B in all instances. For the more important cases where both lives are over age 45, the maximum total error is less than 1.0% for all sex combinations and both methods.

TABLE 32
JOINT LIFE ANNUITIES FOR TWO LIVES AT EQUAL AGES ON THE ANNUITY TABLE FOR 1949 (WITHOUT PROJECTION) AT 2½% INTEREST ULTIMATE

Age x	Two Males a_{xx}	Two Females a_{xx}	Male and Female a_{xx}	Age x	Two Males a_{xx}	Two Females a_{xx}	Male and Female a_{xx}
10.....	29.150	30.711	29.866	60.....	10.284	12.772	11.398
11.....	28.908	30.491	29.633	61.....	9.879	12.290	10.959
12.....	28.660	30.266	29.395	62.....	9.476	11.808	10.521
13.....	28.406	30.037	29.152	63.....	9.076	11.328	10.085
14.....	28.146	29.803	28.903	64.....	8.680	10.850	9.652
15.....	27.880	29.564	28.649	65.....	8.288	10.374	9.223
16.....	27.608	29.320	28.389	66.....	7.901	9.902	8.798
17.....	27.329	29.071	28.123	67.....	7.520	9.435	8.378
18.....	27.044	28.817	27.851	68.....	7.145	8.973	7.964
19.....	26.752	28.557	27.574	69.....	6.777	8.517	7.557
20.....	26.454	28.292	27.290	70.....	6.416	8.069	7.157
21.....	26.149	28.021	27.000	71.....	6.063	7.629	6.766
22.....	25.838	27.744	26.704	72.....	5.719	7.198	6.383
23.....	25.520	27.462	26.402	73.....	5.384	6.776	6.010
24.....	25.195	27.174	26.093	74.....	5.059	6.365	5.647
25.....	24.863	26.880	25.778	75.....	4.744	5.965	5.294
26.....	24.524	26.580	25.456	76.....	4.439	5.577	4.952
27.....	24.178	26.273	25.127	77.....	4.145	5.202	4.622
28.....	23.825	25.960	24.792	78.....	3.862	4.840	4.304
29.....	23.464	25.641	24.450	79.....	3.590	4.491	3.998
30.....	23.096	25.316	24.101	80.....	3.330	4.156	3.705
31.....	22.721	24.984	23.745	81.....	3.081	3.836	3.425
32.....	22.339	24.646	23.382	82.....	2.844	3.530	3.158
33.....	21.950	24.301	23.012	83.....	2.619	3.239	2.903
34.....	21.553	23.950	22.635	84.....	2.405	2.963	2.662
35.....	21.149	23.592	22.251	85.....	2.202	2.702	2.433
36.....	20.738	23.227	21.861	86.....	2.011	2.456	2.217
37.....	20.320	22.856	21.464	87.....	1.831	2.225	2.014
38.....	19.895	22.478	21.060	88.....	1.662	2.008	1.823
39.....	19.463	22.094	20.649	89.....	1.503	1.805	1.644
40.....	19.024	21.703	20.232	90.....	1.355	1.616	1.477
41.....	18.579	21.306	19.808	91.....	1.217	1.441	1.322
42.....	18.128	20.903	19.378	92.....	1.088	1.279	1.178
43.....	17.674	20.494	18.943	93.....	.969	1.129	1.045
44.....	17.218	20.079	18.505	94.....	.859	.991	.922
45.....	16.761	19.658	18.064	95.....	.758	.865	.809
46.....	16.305	19.231	17.621	96.....	.665	.750	.706
47.....	15.851	18.799	17.176	97.....	.580	.646	.612
48.....	15.400	18.361	16.730	98.....	.502	.552	.526
49.....	14.952	17.918	16.284	99.....	.432	.468	.449
50.....	14.508	17.471	15.838	100.....	.368	.392	.380
51.....	14.068	17.020	15.392	101.....	.311	.325	.318
52.....	13.632	16.563	14.946	102.....	.260	.266	.263
53.....	13.200	16.101	14.500	103.....	.215	.214	.215
54.....	12.772	15.635	14.055	104.....	.176	.170	.173
55.....	12.349	15.165	13.610	105.....	.141	.132	.137
56.....	11.929	14.691	13.166	106.....	.112	.100	.106
57.....	11.513	14.214	12.723	107.....	.087	.074	.080
58.....	11.101	13.735	12.281	108.....	.063	.052	.057
59.....	10.691	13.254	11.839	109.....	.000	.000	.000

TABLE 33*

ADDITIONS TO PRODUCE EQUAL AGES FOR TWO MALES OR TWO FEMALES FOR USE IN COMPUTING JOINT LIFE ANNUITIES ON THE ANNUITY TABLE FOR 1949

ULTIMATE

$$a_{x:x+h} = a_{x+t:x+t}$$

Difference of Age in Years h	Addition to Younger Age in Years		Difference of Age in Years h	Addition to Younger Age in Years	
	Two Males t	Two Females t		Two Males t	Two Females t
1.....	.512	.514	31.....	24.458	25.121
2.....	1.049	1.057	32.....	25.415	26.093
3.....	1.611	1.626	33.....	26.377	27.068
4.....	2.197	2.224	34.....	27.342	28.046
5.....	2.806	2.848	35.....	28.310	29.026
6.....	3.439	3.499	36.....	29.281	30.008
7.....	4.094	4.174	37.....	30.255	30.992
8.....	4.772	4.873	38.....	31.231	31.977
9.....	5.471	5.596	39.....	32.210	32.965
10.....	6.190	6.342	40.....	33.190	33.953
11.....	6.929	7.107	41.....	34.172	34.943
12.....	7.686	7.892	42.....	35.156	35.934
13.....	8.461	8.696	43.....	36.141	36.926
14.....	9.253	9.517	44.....	37.128	37.918
15.....	10.061	10.354	45.....	38.116	38.912
16.....	10.884	11.206	46.....	39.105	39.906
17.....	11.720	12.071	47.....	40.095	40.900
18.....	12.570	12.949	48.....	41.086	41.896
19.....	13.432	13.839	49.....	42.078	42.891
20.....	14.305	14.739	50.....	43.071	43.888
21.....	15.189	15.649	51.....	44.064	44.885
22.....	16.083	16.568	52.....	45.058	45.882
23.....	16.986	17.495	53.....	46.052	46.879
24.....	17.896	18.429	54.....	47.048	47.876
25.....	18.815	19.369	55.....	48.043	48.875
26.....	19.741	20.316	56.....	49.039	49.872
27.....	20.673	21.268	57.....	50.035	50.871
28.....	21.612	22.225	58.....	51.031	51.869
29.....	22.556	23.187	59.....	52.028	52.868
30.....	23.504	24.152	60.....	53.026	53.867

* The values of $a_{\overline{x}}$ for two males and two females are shown in Table 32 The additions shown above will produce exact values of the joint life annuity in all cases where both males are age 60 or over or both females are age 50 or over. In cases where one of the male lives is under age 60 or one of the female lives is under age 50, an approximate value of the joint life annuity will be produced The error introduced by this method may be estimated from Table 34.

TABLE 34
TEST OF PROPOSED METHOD* FOR OBTAINING JOINT LIFE ANNUITIES FOR TWO MALES OR TWO FEMALES ON ANNUITY TABLE FOR 1949 AT 2½% INTEREST ULTIMATE

Younger Life Age x	Older Life Age y	Two Males				Two Females			
		Exact Value a_{xy}	Approximate Value a_{ww}	Error		Exact Value a_{xy}	Approximate Value a_{ww}	Error	
				Value of $a_{xy}-a_{ww}$	% of a_{xy}			Value of $a_{xy}-a_{ww}$	% of a_{xy}
25.....	35	22.669	22.648	.021	0.1	24.893	24.868	.025	0.1
	45	19.419	19.329	.090	0.5	21.878	21.805	.073	0.3
	55	15.528	15.174	.354	2.3	17.951	17.850	.101	0.6
	65	11.368	11.023	.345	3.0	13.342	13.277	.065	0.5
	75	7.278	7.119	.159	2.2	8.601	8.568	.033	0.4
	85	3.910	3.855	.055	1.4	4.548	4.537	.011	0.2
35.....	45	18.536	18.493	.043	0.2	21.191	21.168	.023	0.1
	55	15.094	14.817	.277	1.8	17.637	17.588	.049	0.3
	65	11.201	10.894	.307	2.7	13.216	13.181	.035	0.3
	75	7.225	7.075	.150	2.1	8.558	8.538	.020	0.2
	85	3.895	3.843	.052	1.3	4.536	4.530	.006	0.1
45.....	55	14.083	13.985	.098	0.7	16.871	16.864	.007	0.0
	65	10.719	10.567	.152	1.4	12.904	12.898	.006	0.0
	75	7.046	6.960	.086	1.2	8.453	8.449	.004	0.0
	85	3.846	3.810	.036	0.9	4.510	4.507	.003	0.1
55.....	65	9.811	9.802	.009	0.1	12.125	12.125	.000	0.0
	75	6.673	6.667	.006	0.1	8.185	8.186	−.001	0.0
	85	3.728	3.725	.003	0.1	4.439	4.440	−.001	0.0

* Proposed method produces exact values if both male lives are over age 60, or if both female lives are over age 50.

TABLE 35

AUXILIARY TABLE FOR OBTAINING EQUAL AGES FOR ONE MALE AND ONE FEMALE FOR USE IN COMPUTING JOINT LIFE ANNUITIES ON THE ANNUITY TABLE FOR 1949

ULTIMATE

$$a_{xy} = a_{ww}^{*}$$

Age in Years x, y, w	(1) Male Life 1000 bc^{x+5}	(2) Female Life 1000 BC^{y+5}	(3) Male and Female 1000 $(bc^{w+5}+BC^{w+5})$	Age in Years x, y, w	(1) Male Life 1000 bc^{x+5}	(2) Female Life 1000 BC^{y+5}	(3) Male and Female 1000 $(bc^{w+5}+BC^{w+5})$
10.....	.13689	.04074	.17763	56.....	13.01253	7.31242	20.32495
11.....	.15113	.04561	.19674	57.....	14.36685	8.18580	22.55265
12.....	.16686	.05106	.21792	58.....	15.86214	9.16350	25.02564
13.....	.18423	.05716	.24139	59.....	17.51305	10.25797	27.77102
14.....	.20340	.06398	.26738	60.....	19.33578	11.48316	30.81894
15.....	.22458	.07162	.29620	61.....	21.34822	12.85468	34.20290
16.....	.24795	.08018	.32813	62.....	23.57011	14.39002	37.96013
17.....	.27375	.08976	.36351	63.....	26.02326	16.10873	42.13199
18.....	.30225	.10048	.40273	64.....	28.73172	18.03272	46.76444
19.....	.33370	.11248	.44618	65.....	31.72208	20.18651	51.90859
20.....	.36844	.12591	.49435	66.....	35.02367	22.59755	57.62122
21.....	.40678	.14095	.54773	67.....	38.66889	25.29655	63.96544
22.....	.44912	.15778	.60690	68.....	42.69349	28.31791	71.01140
23.....	.49586	.17663	.67249	69.....	47.13697	31.70015	78.83712
24.....	.54747	.19772	.74519	70.....	52.04292	35.48634	87.52926
25.....	.60445	.22134	.82579	71.....	57.45948	39.72476	97.18424
26.....	.66736	.24778	.91514	72.....	63.43978	44.46940	107.90918
27.....	.73682	.27737	1.01419	73.....	70.04251	49.78073	119.82324
28.....	.81351	.31050	1.12401	74.....	77.33244	55.72644	133.05888
29.....	.89818	.34759	1.24577	75.....	85.38109	62.38228	147.76337
30.....	.99166	.38910	1.38076	76.....	94.26744	69.83309	164.10053
31.....	1.09487	.43557	1.53044	77.....	104.07866	78.17381	182.25247
32.....	1.20882	.48760	1.69642	78.....	114.91102	87.51072	202.42174
33.....	1.33463	.54584	1.88047	79.....	126.87080	97.96282	224.83362
34.....	1.47354	.61103	2.08457	80.....	140.07534	109.66329	249.73863
35.....	1.62690	.68401	2.31091	81.....	154.65419	122.76124	277.41543
36.....	1.79623	.76571	2.56194	82.....	170.75039	137.42358	308.17397
37.....	1.98318	.85716	2.84034	83.....	188.52185	153.83716	342.35901
38.....	2.18958	.95954	3.14912	94.....	208.14294	172.21115	380.35409
39.....	2.41747	1.07414	3.49161	85.....	229.80617	192.77968	422.58585
40.....	2.66908	1.20243	3.87151	86.....	253.72408	215.80488	469.52896
41.....	2.94687	1.34605	4.29292	87.....	280.13134	241.58016	521.71150
42.....	3.25358	1.50682	4.76040	88.....	309.28702	270.43398	579.72100
43.....	3.59221	1.68679	5.27900	89.....	341.47719	302.73404	644.21123
44.....	3.96608	1.88826	5.85434	90.....	377.01766	338.89196	715.90962
45.....	4.37887	2.11379	6.49266	91.....	416.25714	379.36850	795.62564
46.....	4.83461	2.36625	7.20086	92.....	459.58061	424.67947	884.26008
47.....	5.33779	2.64887	7.98666	93.....	507.41312	475.40228	982.81540
48.....	5.89334	2.96525	8.85859	94.....	560.22398	532.18333	1092.40731
49.....	6.50671	3.31941	9.82612	95.....	618.53132	595.74618	1214.27750
50.....	7.18392	3.71588	10.89980	96.....	682.90720	666.90084	1349.80804

TABLE 35—*Continued*

Age in Years x, y, w	(1) Male Life 1000 bc^{x+5}	(2) Female Life 1000 BC^{y+5}	(3) Male and Female 1000 (bc^{w+5}+ BC^{w+5})	Age in Years x, y, w	(1) Male Life 1000 bc^{x+5}	(2) Female Life 1000 BC^{y+5}	(3) Male and Female 1000 (bc^{w+5}+ BC^{w+5})
51.....	7.93162	4.15969	12.09131	97.....	753.98324	746.55406	1500.53730
52.....	8.75713	4.65652	13.41365	98.....	832.45678	835.72090	1668.17768
53.....	9.66856	5.21268	14.88124	99.....	919.09773	935.53764	1854.63537
54.....	10.67485	5.83527	16.51012	100.....	1014.75615	1047.27627	2062.03242
55.....	11.78587	6.53223	18.31810				

* Procedure: To compute a_{xy} on a male life age x and female life age y, add $1000bc^{x+5}$ from column (1) and $1000BC^{y+5}$ from column (2) and use the sum to enter column (3) and determine the age w that corresponds to the sum, so that $bc^{x+5} + BC^{y+5} = bc^{w+5} + BC^{w+5}$. Values of a_{ww} for one male and one female are shown in Table 32. Only approximate values of the joint life annuities are produced by this method and the extent of the errors may be estimated from Table 36.

TABLE 36
TEST OF PROPOSED METHOD FOR OBTAINING JOINT LIFE ANNUITIES FOR ONE MALE AND ONE FEMALE ON ANNUITY TABLE FOR 1949 AT 2½% INTEREST
ULTIMATE

Male Age x	Female Age y	Exact Value a_{xy}	Approximate Value a_{ww}	Error: Value of $a_{xy} - a_{ww}$	Error: % of a_{xy}	Male Age x	Female Age y	Exact Value a_{xy}	Approximate Value a_{ww}	Error: Value of $a_{xy} - a_{ww}$	Error: % of a_{xy}
25...	35	24.147	24.340	–.193	–0.8	65...	25	11.412	11.252	.160	1.4
	45	21.474	21.638	–.164	–0.8		35	11.324	11.192	.132	1.2
	55	17.776	17.661	.115	0.6		45	11.101	11.006	.095	0.9
	65	13.279	13.073	.206	1.6		55	10.540	10.490	.050	0.5
	75	8.581	8.443	.138	1.6		75	6.876	6.891	–.015	–0.2
	85	4.542	4.446	.096	2.1		85	4.031	4.003	.028	0.7
35...	25	23.242	23.077	.165	0.7	75...	25	7.293	7.246	.047	0.6
	45	20.300	20.375	–.075	–0.4		35	7.261	7.224	.037	0.5
	55	17.174	17.088	.086	0.5		45	7.184	7.159	.025	0.3
	65	13.034	12.870	.164	1.3		55	6.989	6.979	.010	0.1
	75	8.507	8.376	.131	1.5		65	6.467	6.466	.001	0.0
	85	4.525	4.430	.095	2.1		85	3.437	3.419	.018	0.5
45...	25	19.707	19.525	.182	0.9	85...	25	3.915	3.937	–.022	–0.6
	35	19.210	19.124	.086	0.4		35	3.906	3.931	–.025	–0.6
	55	15.833	15.834	–.001	–0.0		45	3.884	3.914	–.030	–0.8
	65	12.382	12.363	.019	0.2		55	3.830	3.863	–.033	–0.9
	75	8.261	8.214	.047	0.6		65	3.677	3.702	–.025	–0.7
	85	4.459	4.387	.072	1.6		75	3.273	3.297	–.024	–0.7
55...	25	15.645	15.424	.221	1.4						
	35	15.424	15.264	.160	1.0						
	45	14.876	14.798	.078	0.5						
	65	11.183	11.248	–.065	–0.6						
	75	7.767	7.800	–.033	–0.4						
	85	4.308	4.275	.033	0.8						

TABLE 37

TEST OF PROPOSED METHODS FOR OBTAINING JOINT LIFE ANNUITIES ON ANNUITY TABLE FOR 1949 (WITH PROJECTION) AT 2½% INTEREST ULTIMATE—PROJECTION B

Age of Younger Life	Age of Older Life	Ratio (%) of (1) Exact Increase in Annuity Value Due to Projection Less Approx. Increase by Proposed Method to (2) Exact Value of Projected Annuity							
		Two Males		Two Females		One Male and One Female			
						Male Younger		Female Younger	
		Method A*	Method B†	Method A*	Method B†	Method A*	Method B†	Method A*	Method B†
25.....	25	1.0%	.1%	1.0%	.4%	.6%	.2%		
35.....	35	1.0	.0	1.0	.4	.5	.2		
	45	.8	−.2	.7	.2	.1	−.1	.4%	.2%
	55	.3	−.2	.3	.1	−.5	−.2	.1	.1
	65	−.3	.0	−.2	.0	−.8	−.1	−.3	.1
	75	−.7	−.1	−.4	.0	−1.0	−.2	−.5	.0
45.....	45	.9	−.2	.9	.3	.5	.1		
55.....	55	.7	−.1	.8	.1	.4	.0		
	65	.4	−.1	.4	.0	.1	−.1	.1	.0
	75	−.3	−.2	−.1	−.1	.4	−.1	−.3	.0
65.....	65	.4	−.1	.5	.1	.2	.0		
	75	.0	−.1	.2	.0	.0	−.1	.0	.0
75.....	75	.2	−.1	.2	.0	.1	.0		

* Method A: Joint life annuity on Annuity Table for 1949 (without projection) increased by projection factor for a single life of same sex at equivalent equal age (male factor used for one male and one female)

† Method B: Joint life annuity on Annuity Table for 1949 (without projection) with age setbacks corresponding to the projection factors for single lives at the given ages.

TABLE A
1943 EXPERIENCE TABLE—1000 q_x

Age x	Males		Females	
	First Policy Year	Ultimate	First Policy Year	Ultimate
15	.600	.800	.216	.432
16	.611	.815	.228	.455
17	.623	.831	.240	.479
18	.636	.848	.252	.504
19	.650	.867	.265	.530
20	.666	.888	.278	.557
21	.683	.911	.292	.585
22	.703	.937	.307	.614
23	.725	.966	.323	.646
24	.749	.998	.340	.681
25	.776	1.034	.360	.719
26	.806	1.075	.380	.759
27	.842	1.123	.400	.800
28	.884	1.179	.422	.843
29	.933	1.244	.445	.890
30	.989	1.318	.470	.939
31	1.049	1.398	.496	.992
32	1.113	1.484	.526	1.052
33	1.181	1.575	.560	1.119
34	1.254	1.672	.595	1.190
35	1.334	1.779	.633	1.266
36	1.421	1.894	.676	1.352
37	1.519	2.025	.724	1.449
38	1.635	2.180	.778	1.557
39	1.775	2.367	.838	1.677
40	1.940	2.587	.904	1.807
41	2.134	2.845	.974	1.947
42	2.364	3.152	1.052	2.104
43	2.642	3.522	1.140	2.281
44	2.975	3.966	1.238	2.476
45	3.367	4.489	1.344	2.689
46	3.810	5.080	1.462	2.923
47	4.298	5.730	1.590	3.179
48	4.826	6.435	1.724	3.448
49	5.393	7.190	1.868	3.737
50	5.995	7.993	2.021	4.042
51	6.630	8.840	2.179	4.358
52	7.307	9.743	2.345	4.690
53	8.036	10.714	2.526	5.053
54	8.817	11.756	2.730	5.461
55	9.657	12.876	2.960	5.920
56	10.546	14.061	3.217	6.434
57	11.462	15.282	3.506	7.011
58	12.386	16.515	3.826	7.652
59	13.319	17.759	4.182	8.365

TABLE A—*Continued*

Age x	Males		Females	
	First Policy Year	Ultimate	First Policy Year	Ultimate
60.....	14.260	19.013	4.586	9.172
61.....	15.234	20.312	5.047	10.094
62.....	16.307	21.743	5.560	11.120
63.....	17.489	23.318	6.130	12.260
64.....	18.789	25.052	6.764	13.529
65.....	20.219	26.959	7.470	14.940
66.....	21.794	29.058	8.254	16.508
67.....	23.524	31.365	9.126	18.251
68.....	25.427	33.903	10.094	20.189
69.....	27.521	36.694	11.170	22.341
70.....	29.820	39.760	12.366	24.733
71.....	32.348	43.130	13.694	27.388
72.....	35.123	46.831	15.168	30.337
73.....	38.171	50.895	16.804	33.609
74.....	41.516	55.355	18.620	37.240
75.....	45.186	60.248	20.634	41.267
76.....	49.211	65.614	22.866	45.731
77.....	53.621	71.494	25.339	50.678
78.....	58.452	77.936	28.078	56.156
79.....	63.740	84.987	31.110	62.220
80.....	69.525	92.700	34.464	68.927
81.....	75.848	101.131	38.170	76.340
82.....	82.754	110.339	42.264	84.527
83.....	90.290	120.387	46.780	93.561
84.....	98.505	131.340	51.760	103.520
85.....	107.451	143.268	57.244	114.487
86.....		156.241		126.549
87.....		170.334		139.799
88.....		185.623		154.333
89.....		202.183		170.251
90.....		220.091		187.654
91.....		239.423		206.645
92.....		260.254		227.327
93.....		282.653		249.801
94.....		306.687		274.162
95.....		332.413		300.501
96.....		359.881		328.896
97.....		389.132		359.417
98.....		420.191		392.113
99.....		453.071		427.017
100.....		487.766		464.139
101.....		524.253		503.463
102.....		562.488		544.943
103.....		602.404		588.503
104.....		643.915		634.033
105.....		686.908		681.392
106.....		731.250		730.401
107.....		776.785		780.853
108.....		823.338		832.510
109.....		1000.000		1000.000

TABLE B

TEST OF GRADUATION OF 1943 EXPERIENCE TABLE (ULTIMATE)—MALES COMPARISON OF ACTUAL AND EXPECTED DEATHS ON BASIS INDICATED

At ages 55 and under actual deaths and exposures are those for the group annuities 1939–40 and 1946–47 experience among active lives on contracts covering predominantly clerical employees, the actual deaths including an adjustment for deaths among ill-health terminations on basis of group conversions mortality.

At ages 55 and over actual deaths and exposures are those from the Joint Committee 1941–46 experience under immediate nonrefund annuities at duration 2 and over (by number of contracts).

Age Group	Actual Deaths	Expected Deaths	Actual/Expected Ratio	Probable Error of Actual/Expected Ratio	Deviation in Terms of Probable Error
21-25.....	12.32	10.35	119.0%	20.8%	.9
26-30.....	38.54	34.65	111.2	11.3	1.0
31-35.....	69.42	77.16	90.0	7.6	–1.3
36-40.....	119.37	119.65	99.8	6.1	.0
41-45.....	193.40	179.90	107.5	5.0	1.5
46-50.....	276.08	267.55	103.2	4.1	.8
51-55.....	312.48	350.56	89.1	3.6	–3.0
Total Ages 21-55...	1021.61	1039.82	98.2	2.1	– .9
55-59.....	81.	81.14	99.8	7.4	.0
60-64.....	234.	236.92	98.8	4.3	– .3
65-69.....	536.	518.80	103.3	2.9	1.1
70-74.....	906.	945.39	95.8	2.2	–1.9
75-79.....	1065.	1038.56	102.5	2.1	1.2
80-84.....	868.	881.38	98.5	2.2	– .7
85-89.....	514.	497.67	103.3	3.0	1.1
90-94.....	172.	173.55	99.1	5.1	– .2
95-99.....	23.	25.62	89.8	13.2	– .8
Total Ages 55-99...	4399.	4399.03	100.0	1.0	.0
60-69.....	770.	755.72	101.9	2.4	.8
70-84.....	2839.	2865.33	99.1	1.2	– .8
85-99.....	709.	696.84	101.7	2.5	.7
Total Ages 60-99...	4318.	4317.89	100.0	1.0	.0

TABLE B—*Continued*

TEST OF GRADUATION OF 1943 EXPERIENCE TABLE (ULTIMATE)—FEMALES

COMPARISON OF ACTUAL AND EXPECTED DEATHS ON BASIS INDICATED

At ages 50 and under actual deaths and exposures are those for the group annuities 1939–40 and 1946–47 experience among active lives on contracts covering predominantly clerical employees, the actual deaths including an adjustment for deaths among ill-health terminations on basis of group conversions mortality.

At ages 50 and over actual deaths and exposures are those from the Joint Committee 1941–46 experience under immediate nonrefund annuities at duration 2 and over (by number of contracts).

Age Group	Actual Deaths	Expected Deaths	Actual/Expected Ratio	Probable Error of Actual/Expected Ratio	Deviation in Terms of Probable Error
21-25.....	22.55	21.94	102.8%	14.2%	.2
26-30.....	21.60	22.80	94.7	14.0	- .4
31-35.....	25.06	24.61	101.8	13.4	.1
36-40.....	29.19	31.69	92.1	11.9	- .7
41-45.....	31.20	35.54	87.8	11.2	-1.1
46-50.....	49.22	39.63	124.2	10.6	2.3
Total Ages 21-50...	178.82	176.21	101.5	5.0	.3
50-54.....	39.	34.63	112.6	11.3	1.1
55-59.....	115.	123.12	93.4	6.0	-1.1
60-64.....	419.	386.91	108.3	3.4	2.4
65-69.....	1007.	1043.64	96.5	2.1	-1.7
70-74.....	1910.	1932.67	98.8	1.5	- .8
75-79.....	2264.	2302.20	98.3	1.4	-1.2
80-84.....	1925.	1842.07	104.5	1.6	2.8
85-89.....	1013.	1012.55	100.0	2.1	.0
90-94.....	272.	289.64	93.9	3.9	-1.6
95-99.....	29.	28.90	100.3	12.4	.0
Total Ages 50-99...	8993.	8996.33	100.0	0.7	.0
60-69.....	1426.	1430.55	99.7	1.8	- .2
70-84.....	6099.	6076.94	100.4	0.9	.4
85-99.....	1314.	1331.09	98.7	1.8	- .7
Total Ages 60-99...	8839.	8838.58	100.0	0.7	.0

TABLE B—*Continued*

TEST OF GRADUATION OF 1943 EXPERIENCE TABLE (ULTIMATE)
CRITERIA FOR CLOSENESS OF FIT TO THE JOINT COMMITTEE 1941–46 EXPERIENCE*
UNDER IMMEDIATE NONREFUND ANNUITIES

By Number of Contracts	Ages 60 to 99	
	Males	Females
(1) Sum of Expected Deaths	4,317.89	8,838.58
(2) Sum of Actual Deaths	4,318	8,839
(3) Deviations = Actual Deaths (A)—Expected Deaths (E)		
(a) Sum	.11	.42
(b) Sum of Positive Values	196.04	348.42
(c) Sum of Negative Values	–195.93	–348.00
(d) Changes in Sign	24	21
(4) Accumulated Deviations = $\sum_{60}^{x}(A-E)$		
(a) Sum	- 4.84	- 4.61
(b) Sum of Positive Values	171.04	357.47
(c) Sum of Negative Values	-175.88	-362.08
(d) Changes in Sign	11	12

* Durations 2 and over.

Discussion of Preceding Paper

William M. Rae

I would term the paper of Mr. Jenkins and Mr. Lew "a monumental masterpiece." Even these words are inadequate to describe the magnificent job which they have done for us.

Perhaps the most important parts of the paper are the "projection factors" by which they take into account the improvement in mortality that is bound to come in the future. In the last few years we have, in our company, endeavored to take this future improvement directly into account in determining some of our premium rates, particularly group annuity and group permanent. Our method, although differing in detail, is fundamentally the same as theirs. First, we estimated what we thought annuitant mortality would be in 1950. Then, by assuming rates of improvement varying with attained age, we estimated it for each subsequent year. We did all this on two different bases—a "medium cost," which we regarded as "most probable"; and a "high cost," above which we thought it very unlikely the "cost" would go. Next, we calculated two sets of experience premiums. The high cost experience premiums indicated the general level for gross premiums (participating). The medium cost indicated the margins available for dividends and surplus improvement. If these margins were thought too thin, the medium cost also influenced gross premiums. We did not regard "high cost" as equivalent to "maximum cost." Miraculous improvement in geriatrics could upset our high cost estimates.

Interestingly enough, Mr. Jenkins' and Mr. Lew's factors for future mortality improvement confirmed our assumptions. Their factors fall comfortably in between our medium and high cost assumptions. Naturally, in setting a single mortality improvement rate (either Projection A or B), they are being more conservative than the "most probable," and justifiably less conservative than a "high cost" assumption. Also, as they pointed out, they made no allowance whatsoever for participation features. For participating premiums (and a settlement option that pays excess interest is a participating benefit, incidentally) I feel that some reasonable "high cost" assumption must be used.

To give numerical examples, certain key factors from the authors' Tables 20 and 22 are reproduced in my Illustrations 1 and 2 together with our comparable "projection factors." Illustration 1 (and their Table 20) pertain to the allowance for future mortality improvement on immediate annuities commencing in 1950, with a first annual payment due in 1951. Illustration 2 (and their Table 22) pertain to annuities (or settlement options to insured or beneficiary) commencing in the future.

In their Table 22 the authors illustrate only 10cc (10 years certain and continuous for life thereafter) and 20cc annuities. Yet many Retirement Annuity contracts are written on a life annuity basis, without certain period. Also a nonrefund annuity or a 5cc annuity is now frequently included as a settlement option in insurance policies. Consequently, Illustration 3 shows our projection factors for nonrefund annuities commencing in the future. These are materially greater than those for 10cc annuities shown in Illustration 2.

Naturally the projection factors for pure deferred life annuities, such as are frequently used in Group Annuity contracts, are larger still.

ILLUSTRATION 1
PROJECTION FACTORS FOR NONREFUND ANNUITIES—MALE
ANNUITY PERIOD COMMENCES IN 1950

Age at Issue	Rae Medium Cost	Jenkins and Lew Projection		Rae High Cost
		A	B	
25.....	1.029	1.036	1.036	1.057
35.....	1.028	1.037	1.038	1.056
45.....	1.025	1.034	1.037	1.051
55.....	1.022	1.027	1.031	1.045
65.....	1.018	1.018	1.022	1.037

ILLUSTRATION 2
PROJECTION FACTORS FOR 10CC* ANNUITIES—MALE
ANNUITY PAYMENTS (ANNUAL) COMMENCE AT AGE 65

Years from 1950 When Annuity Commences	Rae Medium Cost	Jenkins and Lew Projection		Rae High Cost
		A	B	
10.....	1.024	.....	.....	1.051
20.....	1.036	1.044	1.054	1.076
30.....	1.048	1.060	1.072	1.101
40.....	1.061	1.074	1.089	1.128

*See text.

The authors have not attempted to show precisely how projection factors can be best utilized in the actual calculation of rates and reserves for all types of coverage, such as Income Endowment contracts and settlement options. I suspect that we will find this to be a long and involved subject in itself. Among the numerous possibilities are:

(1) Direct application of projection factors (on an exact basis) for both rates and reserves. This seems close to impractical because of the complexities involved.

(2) Direct application of projection factors (on a very approximate basis, with broad groupings) for both rates and reserves. This, at least, seems feasible. One serious question is the extent to which settlement option rates would vary according to the year in which the income commenced. Another is that some actuaries may object to making projection factors an integral part of the annual valuation system. Are there any legal obstacles to this approach—I do not know. Perhaps the major advantage of this approach is that it faces reality and forces us, and others, to constantly consider future mortality improvement as a definite and substantial item.

ILLUSTRATION 3
PROJECTION FACTORS FOR NONREFUND ANNUITIES—MALE
ANNUITY PAYMENTS (ANNUAL) COMMENCE AT AGE 65

Years from 1950 When Annuity Commences	Rae Medium Cost	Rae High Cost
10	1.034	1.069
20	1.051	1.104
30	1.068	1.140
40	1.086	1.176

(3) Use of the projection factors as guideposts only. The actual rate and reserve calculations would be made by conventional methods, using actuarial assumptions which indirectly made approximate allowance for future mortality improvement. These assumptions would include age ratings (up or down), or a forecast table such as the Annuity Table for 1959, or interest ratings (up or down), or loading adjustments, or a combination of these. Some actuaries, at least in the near future, may prefer this approach. While it may produce satisfactory results for rates (except possibly *nonparticipating* settlement options) its weakness is that reserves will become progressively inadequate unless adjustments are made. These adjustments might even take the form of special projection factors computed from the *conventional basis used* rather than from, say, the Annuity Table for 1949 at 2½%. There is some advantage, however, in that the adjustments would be considerably smaller than those required from, say, the Annuity Table for 1949 at 2½%. They would probably be nominal for a few years after issue, and might even be safely ignored until a change was made in the *conventional basis* for new issues and reserves, if not too long delayed. They would not necessarily need to be an integral part of the annual valuation system. The authors have doubtless given this matter considerable thought, and I am sure we should all appreciate getting their views in their discussion.

Without in any way detracting from the excellence of the paper, the fact is that it is but one leg of a three legged stool, the other two legs being interest and expense. Time does not permit me to elaborate, but in view of past history and today's conditions, there is at least a fair possibility that the long-term trend of interest will continue to be downward and the long-term trend of expense will continue to be upward. If we

allow for future improvement in mortality, but fail to allow for a continually declining interest rate and a continually increasing expense rate, we may find ourselves in the position of the housewife who carefully locks the windows and then goes away on vacation leaving the back door open.

Walter G. Bowerman

It seems evident that the literature of annuity mortality has now received a milestone of monumental size. In future years that literature will have been divided into two parts, one before and the other after this paper. It is a good omen for the Society of Actuaries to receive such a scholarly and encyclopedic paper at its first meeting.

In recent months the papers and magazines have contained a number of articles regarding new methods of attack upon the degenerative diseases, including diabetes, cancer and the cardiovascular-renal system. To life insurance these news items are a heartening guarantee of future success and prosperity. But as to the annuity section of the business, the picture is more that of a man out on a limb at which he is sawing vigorously somewhere between his own position and that of the supporting tree trunk. The problem is of vital import to all of us. The lowering interest rates and the lowering death rates form the Scylla and Charybdis between which the Actuary has to pass as a modern Ulysses.

In Table I, I show typical annuity values, giving effect to decreased rates of both mortality and interest. The 1949 Basis at 2½% would give one-third less yield at age 20 as compared to the American Annuitants at 4%. At age 80 the yield is one-eighth less than by the former standard. In view of the increase in the cost of living during the intervening years, there is evidently a field for purchase of additional annuities on behalf of present annuitants.

TABLE 1
IMMEDIATE ANNUITY VALUES (NET)

Age	Amer. Annuitants Select: 1900–1920 at 4%	1949 (Not Projected) Select at 2½%	1959 Ultimate at 2½%	1979 Ultimate: at 2½%
	Male Lives			
20.....	19.6	28.7	28.9	29.5
35.....	17.1	24.1	24.4	25.1
50.....	13.4	18.0	18.4	19.3
65.....	9.0	11.6	11.8	12.5
80.....	4.9	5.6	5.6	5.8
	Female Lives			
20.....	20.1	30.1	30.3	30.7
35.....	17.8	26.0	26.3	26.7
50.....	14.4	20.4	20.7	21.3
65.....	10.1	13.5	13.7	14.3
80.....	5.8	6.7	6.6	6.8

TABLE 1—*Continued*
FOR EVERY $1,000 OF ANNUITY PAYMENTS ON THE AMERICAN ANNUITANTS BASIS THE 1949 BASIS WOULD YIELD A PAYMENT OF

Age	Male	Female
20.....	$683	$668
35.....	710	684
50.....	744	706
65.....	776	748
80.....	875	866

The other matter which I would mention deals with graduation of the annuity tables in this paper. In 1920 Mr. Valentine Howell produced a regraduation of the American Men table of mortality and stated that more accurate results may be obtained by *not* forcing the data into the Makeham mould. He recommended finding a value of log c as a basis of joint life calculations, but otherwise merely smoothing the table so as to retain its native characteristics. In May 1927 Mr. C. D. Rutherford followed this advice in preparing the Canadian annuity table. He used the Henderson-Whittaker formula B. Since then the British actuaries have experienced many difficulties in studying whole families of curves, such as those named after Mr. Perks. They could have saved themselves much trouble if they had availed themselves of Mr. Howell's suggestion. The procedure seems analogous to Vaihinger's famous doctrine of "as if." The table does not follow Makeham's law, but let us use it just as though it did follow that law.

This line of thought focuses attention upon the values of log c. In the present paper these values are:

	1949	1959	1979
Male	.043	.045	.049
Female	.049	.051	.055

These are fairly close to the values used in the English Government Annuities 1900–1920, namely, .052 for males and .046 for females. They are definitey higher than the .038 of the O^{am} (1893) table and the .035 of American Annuitants (1920) and Combined Annuity table (1928). Turning to insurance tables we may note .046 for American Experience, .034 for American Men and .039 for the CSO table.

Henry E. Blagden

Last week I sat next to one of the authors of this paper at a meeting of the Actuaries' Club in New York and I congratulated him on his monumental paper. He did not like the use of the term "monumental." He associated it with something that is dying.

Upon reflection, it occurred to me that perhaps it is the 1947 Standard Annuity Table that is dying.

Those of us who are making guarantees on the basis of present-day statistics are naturally very much interested in a paper of this nature. As with individual annuities, so with group annuities—we must face the problem of future reductions in mortality; I wonder, however, if we need to solve the problem by adoption of the projection factor method for the calculation of gross rates in a deferred annuity type of group annuity contract. The use of annuity rates increasing with time elapsed to date of entry upon annuity does, however, appeal to me for use in deposit administration group annuity contracts.

If we are (and we probably are) going to finance Bethlehem-type programs in which the benefits will be reduced as social security benefits are increased, deposit administration guarantees-which currently are tied to the money at the time it is paid—will prove to be more extensive than they look today. To explain a little further, the money we take in during the first five years will on the basis of today's conditions probably emerge in the form of annuities in the first ten years, so we are guaranteeing annuity rates for people retiring within the next ten years.

On the other hand, if the same amount of money is paid during the first five years and insured benefits are cut back, we may be taking care of retirements during the next 20 years, and if mortality is improving, as many

of us think it is, our guarantees may mean more than we thought they did when we put them in the contract.

Earlier I said I do not think that mortality rates reducing on a year-to-year basis need be used to calculate gross rates for deferred annuity group annuity contracts. This is because the need for individual equity as between individual lives is not there. The equity that we need to maintain in the group annuity business is between individual employers and not between individual employees. If we adopt appropriate and efficient rating and dividend policies and make sure that our over-all reserves per contract are adequate, we should be able to accomplish the result which for individual policies can only be accomplished by the use of progressively increasing annuity rates. I do think, however, that one application that we can make of the principles developed in the paper is in the handling of our dividend formula.

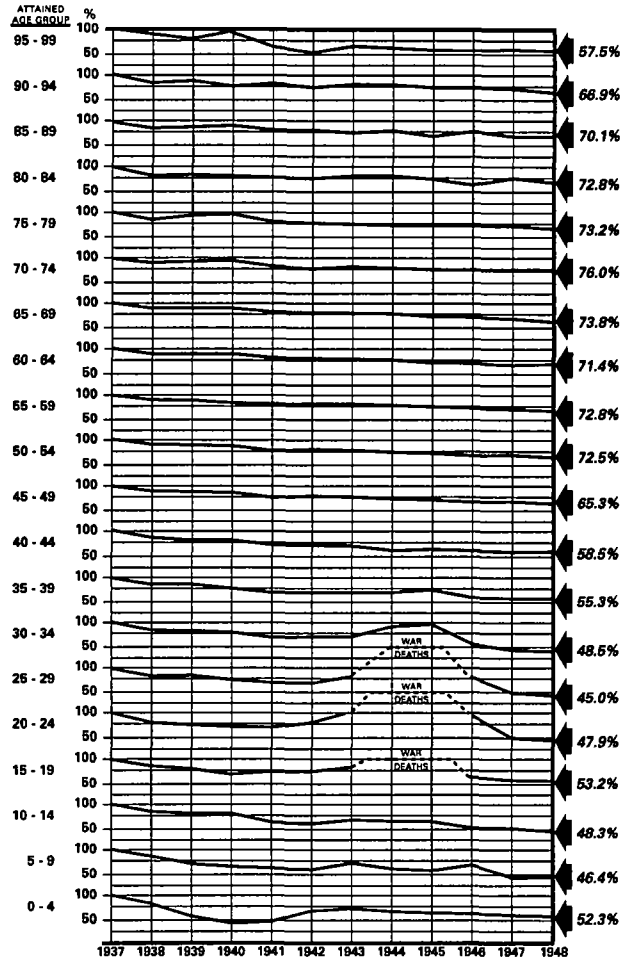
Turning to some particular points in the development of the paper, I am not quite sure that I understand how the terminations in poor health were handled.

It looks to me as if the effect of them is at least to distort the mortality. It seems that at the younger ages we have overstated the mortality and at the higher ages we have understated it. Maybe in the discussion the authors will go into that further.

There is something which has puzzled me for a long time. That is: Why the most recently published group annuity mortality on retired lives apparently shows no improvement over earlier results. Since this mortality rate furnishes the basis for one of the comparisons in Table 25, it is of some importance. I just do not think that there is no improvement, because statistics of the general population show improvement, and I might say that, in our company, we recently took out some mortality statistics on our weekly premium policyholders. We plotted the mortality for each of the calendar years 1937 to 1948 and then expressed 1948 mortality as a percentage of the 1937 mortality. A graphic presentation of this experience shows that above age 55 there was no age at which the 1948 mortality was higher than 76 percent of the 1937 mortality. Even though we realize the limitations of such an experience, that is quite a drop.

With that kind of experience before us, I find it hard to understand why the retired life mortality of group annuitants does not show a similar trend or certainly that type of trend. It occurs to me that there might be some factors in explanation of this. For one, group annuities have been expanding rapidly. Possibly in the early stages they covered mainly types of groups which have a relatively low mortality. With passage of time, we have had a broadening of the base of coverage, bringing in a higher mortality group, which has masked the improvement in the original group.

WEEKLY PREMIUM INDUSTRIAL MORTALITY BY AMOUNTS EXPRESSED AS PERCENTAGES OF 1937 RATES



We must remember also that the 1941–1945 experience covered the wartime period. During the war there were a great many men over the normal retirement age who kept on working. A lot of people write articles about "retire and die" and that sort of thing, implying that retirement shortens a man's life. I have heard of individual cases where that could have been true but the

chances are that a group of men who continue working under wartime pressure is subject at least to some accident rate and for that and other reasons generally will experience a higher mortality than they would if retired.

Then again, there is the tendency, when an employer puts in a pension plan, to include as retired lives a block of superannuated employees. These are people who should have been retired but were retired on the payroll, and when they get into the retired life experience you have a high mortality group. A great many group annuity contracts were written in the early 40's.

I do not know that these are the elements responsible for the apparent lack of improvement in mortality. I think they might have something to do with it and I am going to be interested to see what the statistics show for the years 1947, 1948, and 1949, when we are getting beyond the influence of the war. These statistics at least in part will be available shortly and if they confirm the impressions I have, based upon our own experience, a definite decrease in mortality rates can be expected to be shown.

I may say also that we are taking out a mortality experience on our own retired lives and, if I get it done in time, I shall incorporate it in this discussion.

The data shown below are taken from two studies of the mortality of Prudential employees retired other than for disability. The two studies cover the following experience:

A. Experience from date of retirement to retirement anniversary in 1945 of lives retired during the years 1931–1944, inclusive. The central year of exposure is, roughly, 1940.
B. Experience from retirement anniversary in 1945 to anniversary in 1948 of lives retired during the years 1931–1947, inclusive. The central year of exposure is, roughly, 1947.

CLERICAL MALES

	Study	Age at Death under 75		Age at Death 75 and over	
		A	B	A	B
	Actual Deaths	91	41	25	14
Combined Annuity Table.....	Expected Deaths	69.75	38.88	20.57	17.36
	Ratio $\frac{\text{Actual}}{\text{Expected}}$	130.5%	105.5%	121.5%	80.6%
Standard Annuity Table.....	Expected Deaths	57.51	32.11	16.44	13.82
	Ratio $\frac{\text{Actual}}{\text{Expected}}$	158.2%	127.7%	152.1%	101.3%
Annuity Table, 1949.....	Expected Deaths	48.22	26.82	15.37	13.22
	Ratio $\frac{\text{Actual}}{\text{Expected}}$	188.7%	152.9%	162.7%	105.9%

CLERICAL FEMALES

	Study	Age at Death under 75	
		A	B
Male Combined Annuity Table rated down 4 years	Actual Deaths	49	24
	Expected Deaths	46.28	33.67
	Ratio $\frac{\text{Actual}}{\text{Expected}}$	105.9%	71.3%
Male Standard Annuity Table rated down 5 years	Expected Deaths	36.36	26.52
	Ratio $\frac{\text{Actual}}{\text{Expected}}$	134.8%	90.5%
Annuity Table, 1949	Expected Deaths	24.40	17.68
	Ratio $\frac{\text{Actual}}{\text{Expected}}$	200.8%	135.7%

* There was not enough experience over age 75 to be significant.

FIELD AGENCY MALES

	Study	Age at Death under 75		Age at Death 75 and over	
		A	B	A	B
	Actual Deaths	211	88	59	50
Combined Annuity Table	Expected Deaths	228.52	104.91	56.10	59.59
	Ratio $\frac{\text{Actual}}{\text{Expected}}$	92.3%	83.9%	105.2%	83.9%
Standard Annuity Table	Expected Deaths	188.35	86.48	44.86	47.44
	Ratio $\frac{\text{Actual}}{\text{Expected}}$	112.0%	101.8%	131.5%	105.4%
Annuity Table, 1949	Expected Deaths	157.79	72.52	41.85	45.24
	Ratio $\frac{\text{Actual}}{\text{Expected}}$	133.7%	121.3%	141.0%	110.5%

The results of the studies are shown separately for three groups of lives and for two groups by age at death. The experience is by number of lives, with expected deaths shown on the Combined Annuity Table, 1937 Standard Annuity Table, and the new Annuity Table for 1949 (Ultimate). The clerical experience groups are small and the comparisons may have been affected by some changes in retirement practice so that the remarkable reduction in mortality shown should be accepted with some reservations. Furthermore, it should be added that an earlier study which includes part of the experience entering into Experience A, strangely enough, for clerical employees shows mortality ratios lower than Experience A although significantly higher than Experience B. This earlier experience is suspected to contain statistical inaccuracies but time was not available to delve into it and for that reason the experience has not been included for either clerical or field agency employees even though for the latter the results are more in line with expectations.

Assuming that Experience A represents the year 1940 and Experience B the year 1947, the decreases in mortality ratios have taken place over a seven year period. The compound rate of decrease per year for the various groups is shown below, based on the ratios of actual mortality to that expected according to the Annuity Table for 1949 (Ultimate).

Age at Death	Clerical Males	Clerical Females	Field Agency Males
Under 75...	3.0%	5.4%	1.4%
75 and over.....	6.0%	.....	3.4%

Ray D. Murphy

Mr. Blagden suggested that this paper is burying the 1937 Standard Annuity Table. If that is the effect, there are some of us who will not regret it. The paper has been described as monumental. Certainly it displays an amount of study, thought, and effort which puts us all very much in debt to the authors. My only regret is that there has not been time since the paper became available to give adequate consideration to the theoretical and practical problems which are involved in any application of the principles and tables brought out in it. It is such an important paper that I hope some way will be found to extend the discussion upon a later occasion.

The Committee on Mortality under Ordinary Insurances and Annuities is presenting at this meeting the results of the experience from 1946 to 1948 anniversaries under immediate annuities, and there may be much interest in knowing how that experience compares with the authors' two tables, for 1943 and for 1949. The following tables exhibit this comparison, using the new data for the second and later contract years. It will be recalled that the authors used the corresponding data from the 1941 to 1946 anniversaries under nonrefund annuities to construct their 1943 table, and in Tables I and 2 there are also reproduced from the paper the age-group comparisons between such data and the resulting table.

TABLE 1
COMMITTEE ON MORTALITY—NONREFUND ANNUITY DATA
2D AND LATER CONTRACT YEARS
Number of Contracts

Attained Ages	Actual Deaths	Ratios of Actual to Expected Deaths		
		By 1943 Table		By 1949 Table
		1941-1946 Experience	1946-1948 Experience	1946-1948 Experience
	Male Lives			
to 59	52	111%	118%	144%
60 to 69	283	102	110	128
70 to 79	896	99	99	110
80 to 89	690	100	94	101
90 and over.....	121	98	116	122
All	2,042	100%	100%	110%
	Female Lives			
to 59	71	96%	134%	168%
60 to 69	459	100	92	110
70 to 79	1,891	99	94	108
80 to 89	1,711	103	100	110
90 and over	217	94	86	91
All	4,349	100%	96%	108%

It might be expected that the new experience would exhibit slightly lower mortality than the 1943 table, although the time interval has been very short. In general that has been the result in the aggregate figures. The one exception is under nonrefund annuities for male lives, for which the aggregate ratio is unchanged.

When we compare the 1946 to 1948 experience with the authors' 1949 table we should expect such experience to show ratios in excess of 100%. The short time interval might lead us to expect such a result to a minor degree, but, more importantly, we know that the authors used some degree of conservatism in projecting from the 1943 table to the 1949 table. This expectation is borne out and I think we may conclude that their 1949 table has a reasonable degree of conservatism in it for nonrefund annuities for which it was designed.

I wish to touch on only one other point. We naturally have a great interest in the effect of valuing reserves for outstanding annuities by the projected annuity values shown in the paper instead of employing the 1937 Standard Annuity Table. I adopted one model office distribution of immediate annuities, and found that, based on 2½% interest in both cases, the aggregate reserves by the Standard Annuity Table were about 1% higher than by the authors' projected annuity values. However, for the younger attained ages the projected annuity values gave considerably higher reserves, and it was only at the higher ages, where the probabilities of death by the Standard Annuity Table have proven quite low compared with current experience, that the reserves based on the Standard Annuity Table exceed those on the projected annuity values. This illustrates the danger of continued use of the Standard Annuity Table without taking age distribution carefully into account. This is true not only in valuing reserves but also in interpreting mortality data when the Standard Annuity Table is used to calculate the expected deaths.

TABLE 2
COMMITTEE ON MORTALITY—REFUND ANNUITY DATA
2D AND LATER CONTRACT YEARS
Number of Contracts

Attained Ages	Actual Deaths	Ratios of Actual to Expected Deaths		
		By 1943 Table		By 1949 Table
		1941-1946 Experience	1946-1948 Experience	1946-1948 Experience
	Male Lives			
to 59	163	120%	104%	128%
60 to 69	655	117	108	126
70 to 79	1,749	107	112	124
80 to 89	1,189	109	97	104
90 and over	175	83	111	116
All	3,931	109%	106%	117%
	Female Lives			
to 59	209	130%	111%	140%
60 to 69	1,144	103	108	130
70 to 79	3,266	104	102	118
80 to 89	2,732	107	101	112
90 and over	413	89	93	98
All	7,764	105%	103%	116%

Reuben I. Jacobson

This paper contains a wealth of material, all pertinent to the important problem of determining the mortality basis for annuities. The phase of the problem that strikes me as being the most important is the projection of the mortality scale. I am convinced we can expect an improvement fully as great as that shown by the projections developed by the authors.

The authors list four factors which they consider chiefly responsible for the year-by-year improvement in mortality, one of them being advances in medical and surgical treatment. Because these advances are just beginning to show their effect upon mortality they must be given the most serious consideration. It is the opinion of our Medical Department that we are now seeing only the beginning of the great improvement in mortality above age 40 which is now under way. We have every reason to expect that many drugs as revolutionary as the sulfonamides, penicillin and other antibiotics will be brought out in the near future. Our Medical Director pointed out a few instances of recent medical advances not referred to in the paper under discussion. Doctors Hench, Kendall and co-workers of the Mayo Clinic have shown the dramatic effect of Compound E, which is the hormone of adrenal cortex and the pituitary hormone, on the arthritides. It is true this compound is not available for general use as yet but you can be assured that the organic chemist will overcome that difficulty in the very near future. The results of the use of this Compound E are almost as dramatic as the use of the wonder drugs in infectious diseases—insulin in diabetes, liver extract in pernicious anemia. It is true that the arthritides, except acute rheumatic fever, are infrequent causes of death, but they do predispose their victims to other ailments, which often are fatal. This compound and others like it will have the effect of prolonging the lives of our annuitants even further than the projections developed by the authors.

Recently, the use of hormones in certain types of cancer has shown marvelous promise. In the control of prostatic cancer the use of the female hormone has been equally dramatic in not only controlling pain but now apparently prolonging life. Hormone therapy has also been used recently in cancer of the breast. These are steps forward in cancer therapy that have a most important significance. If these two cancers can be affected by hormone therapy, we can believe it opens even a greater field in saving and prolonging life.

There are two other points that I think should be recognized in trying to project the effect of recent advances in medical science on any annuity table. First, the fact that Cancer Detection Centers are developing all over the country. Secondly, one would have the right to believe that in the near future we can anticipate a biological test for cancer that might even revolutionize the whole field of cancer and its therapy.

Although I believe the authors have made an excellent contribution in evaluating the effects of medical science up to 1949, 1 believe we are going to see in the very near future greater accomplishments in the medical and allied sciences that will have a phenomenal effect on the longevity of annuitants—much greater than that of the accomplishments up to 1949.

Edward W. Marshall

Our gratitude and praise are certainly due the authors for this able and thorough study of a new mortality basis for annuities. Seldom have we had a paper which reflects so much constructive imagination, good judgment and sheer hard work.

The paper recognizes the material lengthening of annuitant longevity which has been and still is taking place. It relates the mortality basis for annuity net premiums and reserves to estimated probable future mortality, rather than to past heavier mortality rates. This of course is the only sound and realistic approach to the subject.

Doubtless most of us agree heartily with the main conclusions of the authors. We might differ in certain details or arrive at slightly different scales for projecting mortality into the future. But, by and large, their projections appear to be reasonable for the present.

It seems to me that the authors were justified in choosing the year of exposure hypothesis, rather than the generation hypothesis, for their approach. I prefer their projection scale B for estimating future mortality, as more realistic than scale A. Their study of the probable future effects of progress in medical science analyzed by cause of death is most interesting and suggestive. Recently this progress has been accelerated and there seems every reason to expect it to go much further in prolonging human life.

It is interesting that the projections beyond 1949 assume no future improvement in mortality for ages 90 and over, and only a moderate relative improvement for ages above about 75. Also, the authors used the same limiting age for all their mortality tables, whether for 1943, 1949, 1959 or 1979. These assumptions doubtless conform to past experience including the 1946–1948 results just reported by the Committee on Mortality.

Whether or not they are adequate for the future, remains to be seen. However, the authors' 1949 tables include a margin which would take care of a certain amount of future improvement in longevity. Thus, from a practical viewpoint, the projected tables are probably reasonable for present use even at the advanced ages.

The matter is not too important, as relatively few annuities are issued at advanced ages and many companies charge new annuitants over age 85 the same premium as for age 85 because of the uncertainties regarding longevity involved. However, I believe that progress in medical science is likely to result in increased longevity at the very advanced ages, and that the projections might well have taken this possibility into account. That an increase in longevity at these ages is possible is also suggested by the results of animal experiments conducted at Cornell University to determine the effects of diet and other factors on longevity.

The authors' excellent paper is most timely and should be of great value to the actuarial profession in the years immediately ahead.

J. Gordon Fletcher

The following comment on the mortality under Canadian government annuities is added to the discussion for the benefit of those concerned with Canadian mortality. Some statistics and 3% annuity values are given in the minutes of the Canadian Association of Actuaries for April 1949.

The experience covers the five fiscal years April 1, 1943, to March 31, 1948, by amount of annuity being paid, excluding group annuities. As a general idea of current mortality was required quickly, it was not feasible to separate the various types of annuity nor to look for selection. The results are reasonably comparable with the 1943 table of Jenkins and Lew, because the influence of guaranteed annuities toward raising mortality rates is offset by the fact that in this office mortality by amounts is lower than mortality by lives.

Experience annuity values at 2½% interest are compared below with 1943 values from the paper.

The mortality basis adopted in 1938 and continued till 1948, was the *a(m)* and *a(f)* tables rated down one year. In his report in 1937, the late Prof. M. A. Mackenzie stated that this basis "had it been used in the past would have anticipated just a few less deaths than actually did occur." It has not, however, been adequate to cover mortality improvement since 1938. In fact, it was probably inadequate for male lives in 1938. The annuity fund has incurred mortality loss which is heavier, in proportion to volume of business, for males than for females.

COMPARISON OF ANNUITY VALUES AT 2½% INTEREST

Age x	MALES			FEMALES		
	(1) 1943 Table Ultimate a_x	(2) Canadian Govt. Annuities 1943-48 Aggregate a_x	(3) (2) as Percentage of (1)	(4) 1943 Table Ultimate a_x	(5) Canadian Govt. Annuities 1943-48 Aggregate a_x	(6) (5) as Percentage of (4)
55	15.12	15.54	102.8%	17.55	17.34	98.8%
60	.....	13.34	.....	.....	15.18	.....
65	10.96	11.16	101.8	12.84	12.75	99.3
70	.....	8.97	.....	.....	10.23	.....
75	6.98	6.78	97.1	8.19	7.91	96.6
80	.....	4.85	.....	.....	5.74	.....
85	3.73	3.34	89.5	4.32	3.99	92.4

Although a precise measure of the decrease in mortality is not yet available, it is clear that the relative decrease is less at the high ages than at lower ages. Canadian experience seems to run parallel to American. Thus the curves of existing tables are not the curves of current mortality, and rating down no longer solves the premium problem equitably.

The authors have performed a monumental work and deserve our thanks for such a useful contribution to the art of trying to outguess annuitants.

Elgin G. Fassel

I wish to pay tribute to the very great contribution made by Mr. Jenkins and Mr. Lew in this excellent paper.

Life insurance is properly based on a static mortality table representing past experience and used for a period until, in turn, superseded by a later table. This has proven safe for insurance because with improving mortality rates the overstatement of mortality results in operating profit. Annuities are the inverse of insurance and for operating profit it is necessary to understate the mortality. The lesson of history has been one of continually improving annuitant mortality rates. Therefore, the most reasonable course is to base annuities on a table with projection that allows for reducing mortality rates.

The authors have furnished a static table representative of current annuitant mortality, the annuity table for 1949, without projection, and have furnished projection factors on various assumptions for typical forms of annuity contracts. Thus they have provided suitable means for use by actuaries in judging the propriety of existing premium rates and for the determination of new premiums if desired.

The authors do not develop the question of valuation of annuities, a different and important phase of the subject. Here the concern is an overall test of solvency and individual equities are not affected.

A desirable valuation table for annuities is one that permits joint lives and sex distinctions to be handled with maximum ease. The valuation table may differ appreciably in detail from the meticulous table upon which policy equities are based so long as the valuation total found by both tables is substantially the same. An appropriate valuation table may be found and demonstrated to be acceptable by referring to a typical distribution of business with regard to age, duration and sex.

The most desirable annuity valuation table would be one with a Gompertz graduation, with sex represented by an age rating; in other words a table with the properties of the 1937 Standard Annuity Table.

The desirable valuation table, as I see it, would be a counterpart of the authors' annuity table for 1949, without projection. It is important to note that in expanded detail this, in effect, constitutes a family of tables, one for each year of birth, but with a device reducing them to a single master table after all.

The expanded tables would constitute, for example, a mortality table for persons born in 1900, a table for those born in 1901, etc. The successive tables would be parallel (in the special sense applicable to geometrical progressions), differing slightly and consistently to represent improving mortality from table to table. Thus, the forecast principle regards all persons born in 1901 as having slightly lower mortality rates throughout life than persons born in 1900: and, in turn, those born in 1902 have correspondingly lower rates than those born in 1901, etc.

Continuing for the moment to regard these as a family of mortality tables, it is important to note that they bear no direct relation to the annuity table for 1949, without projection, in somewhat the way that in analytical geometry the successive intersections of a family of curves trace an envelope which is itself a different curve.

The annuity table for 1949, without projection, is the locus of a point representing the mortality in 1949 in the successive mortality curves of the family—thus, at age 47 in the curve for those born in 1902, at age 48 in the curve for those born in 1901, etc. The successive curves would at all ages be flatter than the annuity table for 1949, without projection, as is implied by their nature in allowing for improving mortality.

I referred to a device for reducing the family of curves to a simple master mortality table. This might, for example, be the particular curve of the family representing persons born in 1900. It would appear possible for the entire family of curves to be expressed by such a simple master curve, the distinction being through fractional rating of the ages up or down according to birth before or after 1900.

To recapitulate these remarks, my proposal is that the excellent means provided by the authors for use in determination of contract equities ought to be supplemented by a companion table, designed for maximum effectiveness in the mechanics of valuation. The acceptability of this table, as a proper test of solvency, would be established by approximate equivalence to the premium basis for a typical distribution of business. It is probable that the conditions imposed by the valuable properties of a Gompertz graduation, and of age rating up or down, as outlined, may demand a progression of mortality improvement differing in detail from the assumptions of the authors while agreeing with them in general.

In any case, the valuation basis will come up for examination periodically in the future and for correction as may be considered advisable from time to time.

Such a program offers the hope of obtaining annuity mortality gains instead of incurring losses and it is only then that actuaries can consider that the annuity problem has been conquered.

Ralph H. Maglathlin

Messrs. Jenkins & Lew's paper is a most timely one. The problem of determining adequate annuity rates, in view of the current and long-term trends of improving mortality, faces all companies today. Their most excellent and thorough treatment of this subject should aid considerably in solving this problem.

After reading this paper I was most interested in determining the adequacy, by the standards presented, of the single premiums being charged for immediate annuities by insurance companies today. In order to make such a comparison I have chosen, as representative of current rates, premiums calculated at 2% interest on the Standard Annuity Mortality Table, set back one year for males and six years for females, the net rate basis in use today by several of the larger companies selling nonparticipating immediate annuities. The table below compares these rates with premiums computed in accordance with the more realistic standards set forth in this paper, Projection B mortality and 2½% interest. (In computing some of the rates on this latter basis, certain minor approximations were used due to the unavailability of sufficient data.) The premiums are shown on a net basis in order that any loading complications may be eliminated.

Column (5) of the table shows the approximate year of issue when the two rates are equivalent, and may be used as a measure of the relative adequacy of the various premiums. It is seen that the current rates in Column (4) are more than adequate by this test at the present time for all ages for both refund and nonrefund annuities. As expected, the current rates for refund annuities contain more margin than nonrefund annuities, due to the lesser importance of the mortality element. However, it is perhaps surprising to note that the current female rates are more conservative than the male rates.

The table clearly shows the U-shaped pattern developed by the current arbitrary rate scale which produces decidedly more than adequate rates at the very young and very old ages but produces fairly realistic rates at the ages where the bulk of annuities are sold. This portrays one of the dangers involved in trying to duplicate current mortality by applying age setbacks and interest differentials to an outmoded mortality table, such expedients producing a distortion of equities by age, sex, and form of annuity.

Another interesting point which can be observed from the above table is a comparison of the reserves which emerge under the two assumptions. Reading Columns (1), (2) and (3) along the diagonal will give the realistic reserves for a single premium immediate annuity issued in 1950. A comparison of these with the reserves in Column (4) shows the distortion which would be involved if reserves are continuously maintained on the basis used in the premium calculations; and Column (5) indicates the year when such reserves will become inadequate for a given attained age.

NET SINGLE PREMIUMS PER $100 IMMEDIATE ANNUAL ANNUITY

Age and Sex	Jenkins-Lew Projection B Mortality and 2½% Interest, Issued in Year			Standard Annuity (Set Back 1 Year) and 2% Interest	Approximate Year When Equivalent
	1950 (1)	1960 (2)	1970 (3)	(4)	(5)
	Nonrefund Life Annuity				
M35	$2,498.10	$2,531.70	$2,565.40	$2,536.70	1961
45	2,087.50	2,127.70	2,168.00	2,099.70	1953
55	1,637.10	1,680.00	1,722.90	1,641.50	1951
65	1,181.80	1,216.50	1,252.40	1,193.60	1953
75	750.20	769.50	789.60	794.00	1972
F35	$2,667.50	$2,690.90	$2,714.30	$2,740.50	1981
45	2,303.90	2,333.10	2,362.30	2,322.40	1956
55	1,867.70	1,898.70	1,929.80	1,871.50	1951
65	1,377.00	1,405.50	1,433.90	1,413.90	1962
75	887.20	904.80	922.40	985.40	Beyond 2000
	Ten-Years-Certain-and-Life Annuity				
M35	$2,503.80	$2,537.60	$2,569.00	$2,553.40	1965
45	2,109.80	2,148.50	2,183.10	2,134.80	1956
55	1,693.70	1,729.90	1,762.90	1,714.20	1955
65	1,300.70	1,323.80	1,348.10	1,339.40	1966
75	1,007.40	1,014.40	1,023.50	1,067.60	Beyond 2000
F35	$2,672.60	$2,696.10	$2,716.90	$2,752.00	1986
45	2,315.50	2,342.60	2,365.10	2,346.60	1961
55	1,893.40	1,921.10	1,945.20	1,922.20	1960
65	1,446.70	1,466.60	1,486.60	1,517.50	1985
75	1,072.50	1,080.00	1,089.60	1,187.40	Beyond 2000

Charles M. Sternhell

The extensive tables of projection factors presented in this paper represent the ratios of various annuity values calculated on the Annuity Table for 1949 (with either Projection Scale A or Projection Scale B) to the corresponding annuity values computed on the Annuity Table for 1949 (without projection). A review of these projection factors indicates that they depend on a number of variables, as follows:

(1) The type of annuity issued—*e.g.*, nonrefund, 10 year certain, etc.
(2) The age at issue
(3) The year in which the annuity is issued
(4) The basic mortality level without projection—*e.g.*, male or female
(5) The projection scale—*e.g.*, Projection Scale A or Projection Scale B
(6) The interest rate.

As each projection factor depends on all of these variables, and is the end result of a lengthy calculation, the effect of a change in one of these variables on the projection factor is generally not readily apparent. This is particularly true because of the novelty involved in thinking of life contingencies in terms of steadily improving mortality. Some of the familiar relationships which were considered to be almost axiomatic up to now will be found to epend on the assumption of a stationary mortality table. We will also find that the assumption of steadily improving mortality will produce some new relationships which may startle us at first and may seem to contradict some of the basic notions we have held in the past.

It might be helpful, therefore, to indicate how the projection factors presented in this paper could be interpreted in the familiar terms of a stationary mortality table, namely the Annuity Table for 1949 (without projection). It will be shown that an annuity of $1.00 a year computed on the Annuity Table for 1949 (with projection) may be considered equivalent to an annuity with a systematically increasing amount payable each year computed on the Annuity Table for 1949 (without projection). Viewed in this way, the projection factor merely represents the average level payment that is equivalent to the systematically increasing payments. This interpretation may help us understand the underlying nature of the projection factors and will make it easier to estimate the effect on the projection factors of changes in the variables on which they depend.

In considering the relation between a particular projection scale and the resulting projection factors, the first step is to express the effect of the projection scale on p_x instead of q_x. Thus, while Table 19 in the paper indicates that Projection Scales A and B involve substantial percentage reductions in the mortality rates at the end of 20 years, the corresponding percentage increases in the values of p_x are quite small as indicated by the following table.

The table clearly indicates why the male projection factors are larger than the female projection factors and why the projection factors on Projection Scales A and B differ by as little as they do. It will also help us understand the variations in the projection factors by age at issue.

The next step may best be illustrated by breaking up a particular projection factor into its component parts. For example, let us take the case of an immediate nonrefund annuity issued in 1950 to a male aged 65. Table 20 in the paper indicates that the projection factor for this case on Projection Scale B is 1.022. This factor represents the ratio of an annuity issued at age 65 in 1950 (which we may designate by a_{65}^{1950}) computed on the Annuity Table for 1949 (with Projection Scale B) to a_{65} computed on the Annuity Table for 1949 (without projection), or

$$1.022 = \frac{a_{65}^{1950}}{a_{65}} = \frac{11.744}{11.496}.$$

Now

$$a_{65}^{1950} = v({}_1p_{65}^{1950}) + v^2({}_2p_{65}^{1950}) + \ldots + v^t({}_tp_{65}^{1950}) + \ldots,$$

where ${}_tp_{65}^{1950}$ represents the probability of a male aged 65 in 1950 surviving to age $65 + t$ in the year $1950 + t$ on the basis of Projection Scale B, and

$$a_{65} = v({}_1p_{65}) + v^2({}_2p_{65}) + \ldots + v^t({}_tp_{65}) + \ldots,$$

where ${}_tp_{65}$ represents the probability of a male aged 65 surviving to age $65 + t$ on the basis of the Annuity Table for 1949 (without projection).

Age	Projection Scale A			Projection Scale B		
	Equivalent Reduction in Mortality Rate at End of 20 Years	Equivalent Increase in Value of p_x at End of 20 Years		Equivalent Reduction in Mortality Rate at End of 20 Years	Equivalent Increase in Value of p_x at End of 20 Years	
	Males and Females	Males	Females	Males and Females	Males	Females
20.....	43.3%	.03%	.02%	22.3%	.01%	.01%
30.....	38.5	.04	.03	22.3	.02	.02
40.....	33.3	.07	.05	22.3	.05	.03
50.....	27.6	.18	.09	22.3	.15	.07
60.....	21.5	.34	.16	21.5	.34	.16
70.....	14.9	.54	.32	17.4	.63	.37
80.....	7.7	.72	.51	9.5	.89	.62
90.....	.0	.00	.00	.0	.00	.00

We may therefore write

$$a_{65}^{1950} = v({}_1p_{65})\left(\frac{{}_1p_{65}^{1950}}{{}_1p_{65}}\right) + v^2({}_2p_{65})\left(\frac{{}_2p_{65}^{1950}}{{}_2p_{65}}\right) + \ldots + v^t({}_tp_{65})\left(\frac{{}_tp_{65}^{1950}}{{}_tp_{65}}\right) + \ldots$$

or

$$a_{65}^{1950} = \frac{D_{66}\left(\frac{{}_1p_{65}^{1950}}{{}_1p_{65}}\right) + D_{67}\left(\frac{{}_2p_{65}^{1950}}{{}_2p_{65}}\right) + \ldots + D_{65+t}\left(\frac{{}_tp_{65}^{1950}}{{}_tp_{65}}\right) + \ldots}{D_{65}},$$

where the values of D_{65+t} are based on the Annuity Table for 1949 (without projection). From this expression, it is readily apparent that a level annuity on the basis of the Annuity Table for 1949 (with projection) is equivalent to an annuity providing for variable payments computed on the basis of the Annuity Table for 1949 (without projection). The variable payment at the end of the tth year merely represents the ratio of the probability of surviving to the end of the tth year on the basis of the Annuity Table for 1949 (with projection) to the corresponding probability on the basis of the Annuity Table for 1949 (without projection). As would be expected from general reasoning, the payment at the end of the tth year is simply increased to provide for the additional number of survivors due to the particular projection scale we have adopted to allow for improving mortality.

Now as the projection factor

$$1.022 = \frac{a_{65}^{1950}}{a_{65}}$$

it may be written as

$$1.022 = \frac{D_{66}\left(\frac{{}_1p_{65}^{1950}}{{}_1p_{65}}\right) + D_{67}\left(\frac{{}_2p_{65}^{1950}}{{}_2p_{65}}\right) + \ldots + D_{65+t}\left(\frac{{}_tp_{65}^{1950}}{{}_tp_{65}}\right) + \ldots}{D_{66} + D_{67} + \ldots + D_{65+t} + \ldots}.$$

This expression indicates that the projection factor represents the weighted arithmetic mean of the variable payments or, in other words, the equivalent level annual payment on the basis of the Annuity Table for 1949 (without projection).

The third step is to consider the variable payment made at the end of the tth year, namely ${}_tp_{65}^{1950}/{}_tp_{65}$. This ratio may be broken up into its component parts as follows:

$$\frac{{}_tp_{65}^{1950}}{{}_tp_{65}} = \left(\frac{p_{65}^{1950}}{p_{65}}\right)\left(\frac{p_{66}^{1951}}{p_{66}}\right)\cdots\left(\frac{p_{65+t-1}^{1950+t-1}}{p_{65+t-1}}\right),$$

where p_{65+t}^{1950+t} represents the probability of a male aged $65 + t$ in $1950 + t$ surviving one year on the basis of Projection Scale B and p_{65+t} represents the probability of a male aged $65 + t$ surviving one year on the basis of the Annuity Table for 1949 (without projection).

It follows, therefore, that the variable payment made at the end of the tth year represents the continued product of the ratios of p_x on the Annuity Table for 1949 (with projection) to the corresponding values of p_x on the Annuity Table for 1949 (without projection) from date of issue to the beginning of the tth year. These ratios were discussed above under the first step. The various steps involved in analyzing a projection factor are indicated in the following illustrative calculation based on the particular case we have used here.

It should be emphasized that the following table does not illustrate the method by which the projection factors were actually calculated but merely presents a different way of interpreting them in order to make it easier to estimate the effect on the projection factors of changes in the basic variables on which they depend. For example, it is obvious that a reduction in the interest rate would give greater weight relatively to the payments at the longer durations and would thereby increase the projection factors. Similarly a change from a nonrefund annuity to a refund annuity gives greater weight to the payments at the early durations and thereby reduces the projection factors. Further development along these lines has indicated the possibility of computing approximate values of annuities on the 1949 Annuity Table (with projection) directly from some supplementary tables by a relatively simple procedure.

ILLUSTRATIVE CALCULATION OF PROJECTION FACTOR FOR a_{65}^{1950} ON MALE LIFE BASED ON ANNUITY TABLE FOR 1949 (PROJECTION SCALE B)—2½% INTEREST

Duration t	Attained Age $65+t$	Year $1950+t$	p_{65+t}^{1950}/p_{65+t} (1)	tp_{65}^{1950}/tp_{65} = $(2)_{-1}X(1)_{-1}$ (2)	D_{65+t}/D_{65} on 1949 Ann. Table (without projection (3)	$\frac{D_{65+t}}{D_{65}}\left(\frac{tp_{65}^{1950}}{tp_{65}}\right)$ = (2)X(3) (4)
1.....	66	1951	1.00028	1.00000	.95311	.95311
2.....	67	1952	1.00058	1.00028	.90658	.90683
3.....	68	1953	1.00091	1.00085	.86042	.86116
4.....	69	1954	1.00128	1.00177	.81461	.81605
5.....	70	1955	1.00169	1.00306	.76915	.77150
10.....	75	1960	1.00418	1.01627	.54816	.55708
15.....	80	1965	1.00677	1.04320	.34474	.35963
20.....	85	1970	1.00757	1.08233	.17659	.19113
25.....	90	1975	1.00000	1.11352	.06460	.07193
30.....	95	1980	1.00000	1.11352	.01360	.01515
43.....	108	1993	1.00000	1.11352	.000001	.000001
Total...	.....	.....	.....	a_{65} = 11.49597	a_{65}^{1950} = 11.74417	

$$\text{The projection factor} = \frac{a_{65}^{1950}}{a_{65}} = \frac{11.74417}{11.49597} = 1.022.$$

In considering the effect of changes in the basic variables on the projection factors, there is one other point which might be mentioned. In section IX of this paper, the authors discuss the possibility of a general upward or downward revision of either projection scale without specifying the method by which the corresponding adjustment of the projection factors should be made. The above analysis suggests that straight line interpolation might be sufficiently accurate, using 1.000 as the projection factor for 0% of the projection scale and the published projection factors for 100% of the projection scale. Some actual calculations confirmed this suggestion and indicated that this method would give reliable results provided the basic projection scale was not increased too greatly. The results of these calculations are indicated in the following table.

PROJECTION FACTORS FOR IMMEDIATE NONREFUND ANNUITIES ISSUED IN 1950 ON ANNUITY TABLE FOR 1949 AT 2½% INTEREST

Age of Annuitant at Issue	0% of Projection Scale B	50% of Projection Scale B	100% of Projection Scale B	200% of Projection Scale B
	Males			
15	1.000	1.017	1.033	1.059
35	1.000	1.019	1.038	1.071
55	1.000	1.015	1.031	1.062
75	1.000	1.005	1.010	1.020
	Females			
15	1.000	1.012	1.023	1.040
35	1.000	1.014	1.026	1.049
55	1.000	1.012	1.023	1.046
75	1.000	1.004	1.008	1.016

Authors' Reviews of Discussion

Wilmer A. Jenkins

Even though the galley proofs of this long and involved paper were distributed only a month before this meeting, the discussions are a most valuable addition to it. It is very gratifying that the actuaries who have spoken have approved of the approach to the problem of annuity mortality which Mr. Lew and I adopted, particularly the main thesis of the paper that, in computing annuity premiums and reserves, the actuary should not fail to assume that lower levels of mortality will prevail in the future and should be sure to make, by one method or another, adequate allowance for this probability. Opinions expressed were unanimous that this thesis is correct.

There may be, in some minds, an impression that Mr. Lew and I intended to suggest that calculations of annuity premiums and reserves in actual practice must always be made—even in routine valuations, for example—by applying projection factors to values derived from the 1949 Table. If so, I should deny this intention. For some calculations use of the projection factors raises no practical difficulty, and in such circumstances we think that calculations should be made in this way. However, for other calculations in practice, more or less serious practical difficulties will probably lead the actuary to using the projection factors in approximate form or as only "guide posts" of the kind described by Mr. Rae. Our intention, stated perhaps not clearly enough in the last paragraph of Section II, was to provide the actuary with tools which, it was hoped, would assist him in deciding how his calculations should be made, whether he uses the 1949 Table or other table, a loading or interest rate margin, or whatever other method makes adequate and equitable allowance for future mortality decreases. But it is highly important that the particular method he decides to use be tested and proved by comparison with values derived by a projection of the 1949 Table or other table conservatively representative of current experience.

Mr. Lew and I fully realized that the determination of precisely how premium and reserve calculations should be made in practice presents a substantial question to each actuary now. This question, which relates to what may be termed "practical applications," was deemed to be outside the scope of the paper and intentionally was not discussed in it. I am delighted with Mr. Murphy's suggestion, seconded by our President, that the theoretical background of our paper and especially these practical applications be a topic of informal discussion at next spring's meetings when actuaries will have had time to study the paper fully.

I would like to comment on only a few of the many very informative and valuable discussions, in relation to practical applications. Mr. Rae's three alternatives summarize correctly, I think, the alternatives now open to the actuary. It seems obvious that the actuary may very

well decide to use different methods of computation for premiums and for valuation, and they may differ also as between immediate annuities, deferred annuities, settlement options, and group annuities. Mr. Fassel's interesting suggestion as to valuation is of the type included in Mr. Rae's alternative (3) and its mechanics are similar to those described by this author in *TASA* XLVII, 265–285. Mr. Blagden likewise thinks that alternative (3) is the most promising for group annuities of the usual type, although the use of projection factors may be feasible for deposit administration contracts.

Another interesting possibility for the practical applications is Mr. Sternhell's demonstration that the assumption of an increasing annuity amount can be equivalent to the assumption of decreasing death rates. It may be that, in practice, the assumption of an annuity amount increasing by a constant amount each year would produce reasonable results and at the same time be adaptable to a punch-card attained age valuation. In this connection the authors will, on request, be glad to furnish any actuary with a copy of a table of projected values of p_x which has been prepared at certain ages.

With these suggestions already made and a variety of others which doubtless can be devised, it is reasonable to expect that the question of practical applications can be met without difficulty.

Answering Mr. Blagden's question, the method of allowing for group ill-health terminations was to add to the actual deaths graded percentages (shown in Section III) of the actual ill-health terminations, thus determining the adjusted actual deaths shown in Table B in Section XV. These percentages approximately measure the present value at termination date of the extra mortality that would have prevailed thereafter if the terminations had not occurred and had yielded the mortality of group conversions, according to the experience of the Metropolitan Life Insurance Company.

While this analogy between ill-health terminations and group conversions is, of course, an imperfect one, it yielded reasonable results for a mortality table designed for use for all kinds of annuities and made it possible for us to utilize the active lives group annuity experience. Tests indicated that any reasonable method of allowing for ill-health terminations would result in only a relatively small change in the mortality rates. Thus the approximations introduced by the method we used are quite small in comparison with the allowance for decreases in mortality between 1943 and 1949 as well as the future decreases upon which the projection factors were based.

I would like to extend sincere thanks to the actuaries who have discussed our paper, and also to the Metropolitan Life Insurance Company for the very large amount of skilled assistance which that company made available to us and which made this paper possible.

Edward A. Lew

Mr. Jenkins and I are much indebted to all who discussed our paper for focusing attention so ably on the more important questions raised in the paper.

Foremost among these questions is the need for and the practicability of the device of projection factors. In so far as practical convenience for annuity calculations is concerned, a single mortality table in standard form would, of course, have been greatly preferred to the combination of a mortality table and projection factors. Early in our studies, however, it became apparent that the standard form of mortality table could not be simply adapted to include reasonably accurate provisions for future mortality decreases on most kinds of annuities, if such decreases were assumed to be a function of attained age and the calendar year passed through. This hypothesis produces allowances for future mortality decreases that vary by age and year of issue, as well as according to the period of time over which annuity payments extend. While annuity values could be calculated on the same assumptions by using a series of tables of commutation columns for different ages and years of issue, such a procedure would be much more laborious than recourse to projection factors.

The unsuitability of the usual form of mortality table for calculating annuity values under the assumption of decreasing mortality rates has long been recognized. For instance, in discussing the present concept of the static mortality table, Mr. Spoerl expressed* the situation admirably as follows: "The mortality table is a mythical beast but within the limits of error which concern me it is able to perform as much useful work as a real beast. Moreover, as long as its mortality rates are sufficient, I can use it just as I have always done and the company is not likely to lose money on its life insurance contracts."

But, Mr. Spoerl went on to say: "For annuity contracts, of course, I will use a different mythical beast."

* "Life Insurance and the Theory of Probability," by C. A. Spoerl, Institute of Actuaries Centenary Assembly, 1948.

The device of projection factors may, therefore, be regarded as a useful beast which should be put to work so that a company would not be likely to lose money on its annuity contracts, even though mortality rates continued to decline at a rate depending primarily on attained age and the calendar year of exposure. The degree of credibility given to this hypothesis might well govern the actuary's decision to use projection factors exactly or on an approximate basis or merely as a guidepost. The individual actuary's judgment as to the probable rate of decline in mortality from year to year by attained age would determine the magnitude of the projection factors he chose to consider.

Granting that the direct use of projection factors presents some practical difficulties, as in valuation and for settlement options, I nevertheless would stress, as Mr. Rae did, that their great advantage lies in their confronting us mechanically with a measure of the consequences of decreasing mortality rates. I believe there would be serious objection to any short-cut method which produced allowances for future mortality decreases that were materially smaller for important segments of the business than those calculated through the application of projection factors developed on assumptions satisfactory to the individual actuary.

Some of the difficulties inherent in practical solutions by short-cut methods can perhaps be illustrated with reference to the problem of appropriate guarantees for life income settlement options. One possible solution of this problem might be to base the guarantees on rates calculated to be adequate for settlements beginning in some future year, beyond which only a small proportion of the life income options arising from the policies issued within the next few years was likely to mature. For life income options maturing in the intervening years more liberal payments could, of course, be declared. To meet the extra cost of the life income options maturing beyond the year as of which the guarantees were based, additional surplus funds would have to be accumulated.

In this connection a word of caution appears necessary regarding the suitability for some companies of the projection factors given in Table 21. This table presents the factors applicable to life income settlement options with annual payments commencing in 1965, 1970, and 1975. In the paper it was indicated that these years might be regarded as the years in which life income options arising from the policies issued within the next few years could be expected, on the average, to mature by death. Later studies of the probable distribution in time of life income settlements arising from policies issued in 1950 suggest, however, that in some companies settlement options arising from policies issued in 1950 might on the average mature about 1980 or even 1985, rather than about 1970 or 1975, as is implied in Table 21.

Specifically, model office calculations based on the Metropolitan's recent experience with life income settlements suggest that payments under life income options may under some circumstances extend much further into the future than first surmised. Judging by these calculations, perhaps only about 40 percent of all life income settlements arising from policies issued in 1950 will have begun by 1975, and by the year 2000 perhaps only 75 percent of all life income settlements arising from policies issued in 1950 will have begun. These figures reflect in part the Metropolitan's relatively young age distribution of the issue. It might also be noted that the model office included only a small proportion of endowments maturing at age 65.

It is probable that in companies with an older average age at issue, the payments under life income settlement options would not extend as far into the future. In any event, however, the suitability for any company of the projection factors shown in Table 21 should be tested against that company's probable distribution in time of payments under life income options.

The table following sets forth the projection factors, based on projection scale B, applicable to life income options with annual payments commencing in 1980.

Inasmuch as the magnitude and incidence of future mortality decreases among annuitants lie at the heart of the annuity problem, there is clearly a need for more information as to the mortality being experienced under different types of annuity contracts and as to the mortality trends among annuitants. Here again the actuaries who discussed our paper have made valuable contributions to our knowledge.

Especially pertinent is the intercompany immediate annuity experience from 1946 to 1948 anniversaries, which Mr. Murphy presented in summary form. This study brings us up to date with the mortality facts on immediate annuities; its successors should help us to develop the facts on long time trends. The experience among retired employees of the Prudential, presented by Mr. Blagden, adduces some further evidence of the downward mortality trend among annuitants. The data for Canadian government annuities, which Mr. Fletcher

assembled, indicate that the experience under Canadian annuities has been broadly similar to that under United States annuities and that annuitant mortality in Canada has also decreased more at the younger than at the older ages.

Mr. Marshall has raised an intriguing question regarding the possibility of larger reductions in mortality at the very advanced ages. There appear to be two distinctly different views on this question. One view expresses skepticism of any material reductions in mortality at the very advanced ages on the ground that the persons surviving to these ages represent to an increasing degree a less selected group physically as compared with the more selected character of older people in the past. The other view rests partly on a contrary proposition, namely, that the persons surviving to the more advanced ages will as the years go by tend to include a larger proportion of physically superior individuals, because this generation will at the younger ages have been subjected to lesser damage from disease and infections than the preceding generation; it also relies strongly on the increasing effectiveness of both preventive and therapeutic medicine in reducing mortality at the older ages.

The Annuity Table for 1949 with projection B may be regarded as exemplifying a reasonably optimistic outlook on mortality at the advanced ages, since the provisions for future mortality decreases at the older ages included in the 1949 table when used together with projection B are materially greater than would have been warranted from a consideration of past mortality trends alone.

PROJECTION FACTORS BASED ON PROJECTION SCALE B FOR LIFE INCOME SETTLEMENT OPTIONS WITH ANNUAL PAYMENTS COMMENCING IN 1980 APPLICABLE TO SINGLE LIFE ANNUITY VALUES BASED ON ANNUITY TABLE FOR 1949 AT 2½% INTEREST

Age of Payee in 1980	10 Year Certain Period		20 Year Certain Period	
	Males	Females	Males	Females
35	1.074	1.049	1.068	1.046
40	1.080	1.053	1.071	1.049
45	1.086	1.057	1.070	1.050
50	1.090	1.060	1.065	1.049
55	1.090	1.062	1.054	1.044
60	1.084	1.062	1.037	1.034
65	1.072	1.057	1.019	1.020
70	1.052	1.045	1.008	1.007
75	1.028	1.027	1.001	1.001
80	1.009	1.009	1.000	1.000

End Notes

1. *TASA*, XLIX, 112.
2. *TASA*, XLVIII, 133.
3. *TASA*, XXXIX, 8.
4. References to the basic data are given in the later sections of the paper.
5. *TASA*, XLVIII, 133, and XLIX, 112.
6. For 1946 experience, see *TASA*, XLIX, 203. Experience for other years furnished the authors by Committee to Prepare Mortality Studies on Group Annuities.
7. See, for example, *TASA*, XXI, 171.
8. *TASA*, XXI, 157.
9. *TASA*, VI, 13, 137.
10. *TASA*, XXIX, 297; XXX, 237.
11. Latest report *TASA*, XLIX, 112.
12. *TASA*, XXXVII, 207.
13. *TASA*, XLII, 172.
14. *TASA*, XLVIII, 133.
15. For discussions of these hypotheses, see note on D. 413.
16. *Forecasts of the Population of the United States 1945-1975* by P. K. Whelpton, H. T. Eldridge, and J. S. Siegel, Bureau of the Census, Washington, 1947, p. 10.
17. "Mortality, Past and Future" by S. Peller, *Population Studies*, March, 1948, p. 453.
18. *JIA*, LXXIV, 67.
19. "Perspectives in Cancer Research" by C. P. Rhoads, *Perspectives in Medicine*, Columbia University Press, 1948, p. 96.
20. "Public Health and the Diseases of Old Age" by L. I. Dublin, *Public Health in the World Today*, Harvard University Press, 1949, p. 241.
21. "Perspectives in Cancer Research" by C. P. Rhoads, *Perspectives in Medicine*, Columbia University Press, 1948, p. 97.
22. Press release December 1947, Metropolitan Life Insurance Company.
23. The reference here is to the Final Report on the Evaluation of Anticoagulants in the Treatment of Coronary Thrombosis with Myocardial Infraction.
24. In a personal communication to the authors, Dr. H. F. Dorn of the National Cancer Institute has indicated (on the basis of independent estimates of his own) that in his judgment an average cure rate for all forms of cancer of approximately 60% might be obtained, assuming cases are discovered at an early stage permitting maximum benefits from treatment and assuming further that the best known skills and techniques are made generally available.
25. "Public Health and the Diseases of Old Age" by L. I. Dublin, *Public Health in the World Today*, Harvard University Press, 1949, p. 238.
26. "A Centennial of Public Health" by L. I. Dublin, *American Journal of Public Health*, December 1948, p. 1641.
27. *RAIA*, XXV, 361.
28. *TASA*, XLVII, 499.
29. "Mortality Among Annuitants" by R. D. Murphy, *Journal of the American Society of Chartered Life Underwriters*, II, p. 356–7.
30. In addition to references elsewhere in this paper, several important discussions of mortality forecasts, some of which debate the "generation" hypothesis, are: Davidson and Reid, *TFA*, 11, 183; Derrick, *JIA*, LVIII, 117; Elderton, *Skandinavisk Aktuarietidskrift*, 1932, p. 45; Kermack, McKendrick, and McKinlay, *The Lancet*, Vol. 226, p. 698, and the *Journal of Hygiene*, Vol. 34, p. 433; Cramer and Wold, *Skandinavisk Aktuarietidskrift*, 1935, p. 161; Rhodes, *Journal of the Royal Statistical Society*, Vol. 104, p. 15. Pertinent in this connection are: Jenkins, *TASA*, XLVII, 265, and *Institute of Actuaries Centenary*, 1948.
31. *JIA*, LIV, 43.
32. *JIA*, LV, 144.
33. *Forecasts of the Population of the United States 1945–1975* by P. K. Whelpton, H. T. Eldridge, and J. S. Siegel, Bureau of the Census, Washington, 1947.
34. *Estimates of Future Population of the United States 1940–2000* by W. S. Thompson and P. K. Whelpton, National Resources Planning Board, 1943.
35. *Population Statistics—National Data* by W. S. Thompson and P. K. Whelpton, National Resources Committee, 1937.
36. *Actuarial Study No. 23*, Federal Security Agency.
37. *The Future Population of Europe and the Soviet Union* by F. W. Notestein, A. J. Coale, and others, League of Nations, Geneva, 1944, pp. 183–189.
38. *JIA*, LIV, 43.
39. *JIA*, LV, 144.
40. For discussions of this hypothesis, see note on p. 413.
41. *Journal of the Royal Statistical Society*, Vol. 104, 1941, p. 15.

42. *TASA*, XLVII, 504.
43. *Journal of the Royal Statistical Society*, Vol. 99, p. 672.
44. *Task Force Report on Public Welfare (Appendix P), Prepared for the Commission on Organization of the Executive Branch of the Government*, Washington, Jan. 1949, p. 186.
45. *JIA*, LV, 144.
46. Calculated by J. F. Steffensen's formula for the standard deviation of an immediate annuity value; see p. 281 of *Fundamental Principles of Mathematical Statistics* by H. H. Wolfenden.
47. *TASA*, XLVIII, 239.