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ANALYSIS OF APPROXIMATE VALUATION METHODS

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INTRODUCTION

SINCE Frank Shailer's paper "Approximate Methods of Valuation" appeared in 1924,[1] our actuarial literature has omitted any further development of this subject, except for occasional discussions. In the thirty-one year period new forms of insurance, modifications of old forms of insurance, and new valuation methods and bases have appeared.

During the same period unit expenses have generally increased, and the deadlines for statement work have not been extended. Some of the larger companies are successfully attacking the greater complexity of year-end valuation, inflated expenses and time limitations through the use of electronic data processing machines. Since these are not generally available to smaller companies, it seems that alternative means of expense reduction should be sought and used whenever possible. One such means is the use of approximate methods of valuation.

This paper will not attempt to cover the details of every possible approximate method, but will endeavor to illustrate the use of principles and general methods. It is hoped that the application of these principles and methods will help develop particular solutions to many of the varied valuation problems of today.

PRINCIPLES

Substantial accuracy, saving in expense and saving in time are the advantages that justify approximate valuation methods.

These are attained when the following principles are followed:

A. Principles applying to the method:
   1. At least one of the following advantages should be obtained:
      a) Improvement in timing in meeting a deadline.
      b) Saving in clerical time.
      c) Saving in time of technical personnel.
      d) Elimination of scheduling problems involving machines or personnel.
   2. Method should be simple.
   3. The result should be capable of being checked easily.

[1] *TASA* XXV, 80.

4. Periodic verification of the method should be possible, preferably in slack periods.
5. Method should involve minimum reasonable error.
6. Errors should be preferably on the safe side.
7. Method must be acceptable to state insurance examiners (substantially accurate, on the safe side, and reasonably easy to review).

B. Principles based on nature of item:

1. A detailed valuation of the item in question would be complicated, involving an expense incommensurate with its importance.
2. The amount of the item compared to the total liability should be small.
3. The statement item should generally be composed of a large number of small individual amounts.
4. No single amount should be large enough to influence the total of the item greatly. Ages, durations, or plans with disproportionately large reserves per unit should be valued separately, if necessary.
5. Some items, such as reserves for unreported claims and outstanding A & H claims, have no alternative exact means of evaluation.

METHODS

The methods used today for approximate valuation are quite numerous, being limited only by the ingenuity of the actuary. The methods used for any particular item will vary between companies because of variations in accounting systems, basic valuation methods and composition of business.

These approximate methods may be broken down into eight general categories, which may be applied singly or in combination to the valuation of a particular item.

I. *Inclusion Method*

One of the easiest approximate methods is that of including a few difficult items with the regular valuation of similar items. The substitution of regular reserves (on perhaps a modified basis) should be considered when the following factors are present:

1. The difficult items may not be handled simply, but require calculation, or too close supervision, by technically trained personnel.
2. A similar, larger group of items must be evaluated anyway.
3. The inclusion, with or without an initial modification, of the difficult items in the larger group would require little additional yearly supervision by technical personnel.
4. The aggregate value of the items is small, thus keeping to negligible proportions any error introduced.

When these factors suggest the possibility of the inclusion method, a calculation is required to test possible means of inclusion. The immediate

and ultimate reserves, calculated by the proposed method, need to be compared with the true reserves.

An example of such a calculation for the application of this method to a relatively few continuous income ordinary life policies follows (these policies provide income for life to the beneficiary, after the face amount has been paid in instalments).

A means of including these reserves with ordinary life reserves is sought. Going back to first principles, to the prospective formula for the ordinary life reserves (${}_tV_x = A_{x+t} - P_x\ddot{a}_{x+t}$), we have two possibilities suggested:

(1) a uniform percentage increase in reserves, obtained by increasing the face amount and net premium by the same percentage; and
(2) a reduction in the net premium.

The reserve basis used in this example is $3\frac{1}{2}\%$ American Experience net level; illustrated is the calculation for age 35 at issue for both the insured and the beneficiary.

Policy Year	Actual Mean Reserve	110% of Ord. Life Res.	Ratio to Actual	Att. Age Valuation Using $.95P_x$	Ratio to Actual
11.....	154.50	169.03	1.09	169.02	1.09
21.....	331.06	363.24	1.10	342.31	1.03
31.....	525.08	576.66	1.10	532.74	1.01

The level percentage increase appears more satisfactory, and would be an excellent approximation for the group or seriatim methods. For the attained age method, the reduction of net premium is more convenient.

This method, as applied in either fashion, is simple, provides savings in time in meeting deadlines and savings in clerical and technical time, and involves small errors on the conservative side. Periodic verifications, as recommended in the statement of principles, are not easy to make, but on the other hand may not be so important when the effect on future reserves is carefully tested in advance. It is necessary to be watchful for odd combinations of ages; not only must the typical combination of insured age and beneficiary age be considered, but also any extremes that may be present.

The inclusion method is appropriate for the valuation of joint life reserves on a single-life basis, with age at issue modified, and for the evaluation of cash refund annuities as instalment refunds, and vice versa. Cash refund annuities, when evaluated as instalment refunds, will require a slight modification to avoid an understatement of reserves at the time of

adoption of the method; in a few years, however, in a closed group, the modification will be reduced to zero when all the annuities involved have entered the life annuity stage.

II. *Method of Overstatement of Small Liabilities*

Another method which requires little attention after its adoption is that of evaluating small items on the basis of constant overstatement of liability for all future years. This method is particularly applicable for closed groups when the aggregate value of the items is an insignificant fraction of surplus and the inclusion method may not be applied.

A company with a few survivorship annuities might calculate the present exact reserve, and, noting that all future reserves are smaller than the current year's exact reserve, use the present reserve indefinitely. Adjustments would be made only for termination.

Another application of the overstatement method is to assign an average reserve which is known to be high for all plans, ages and durations. The waiver death benefit on junior policies (which is a decreasing term insurance benefit providing for waiver of premiums on death of the payor) is one which may be evaluated in this way. A relatively high level reserve per thousand of face amount is used, resulting in an aggregate reserve which is still a very small percentage of total reserves.

This method is in full accord with the statement of principles applying to the method, and in general accord with the principles based on the nature of the item.

III. *Substitution Method*

The next general method, that of the substitution of an average or averages for certain distributions, and the other methods following, require closer supervision after their establishment. Usually, in this method a single average is assumed to replace one of the following distributions:

a) Ages at issue
b) Durations
c) Terms of insurance
d) Years to maturity or expiry
e) Attained ages
f) Plans of insurance
g) Age at incurrence of claim
h) Due dates of premiums, interest or payments

The substitution method is applicable when there are large numbers of small amounts whose aggregate value is relatively small and for which an exact valuation would involve a large number of ages at issue, durations, plans of insurance, or other factors.

When an average is substituted for a distribution, proper care must be exercised to insure its being representative of the distribution. For example, a common application of this method is the use of the tables in *Disability Reserves* published by the Actuarial Society of America in 1930, in which the Class 3 disabled reserves averaged for ages 25, 35, 45 and 55 at incurrence of claim are substituted for reserves based on the true distribution of ages. For some companies, a more realistic valuation might give greater weight to the older ages; a recalculation of the average reserves based on proportions of actual ages at incurrence of claim centering on ages 25, 35, 45 and 55, might give somewhat greater liabilities at the earlier durations. A greater number of claims are likely to be incurred in the 40's and in the 50's than in the 20's and 30's. The resulting table of average reserves would be used in the same manner as the published table.

The substitution method might also be used for the valuation of the nondeduction of fractional premium item. An exact valuation of this liability would require a complicated calculation involving small amounts of insurance (equal to the deferred net premiums) at each age of issue for each term of insurance (life, for ordinary life policies, or the number of years to expiry, maturity or paid-up date, for other plans), and for each year of issue. In addition, deferred premiums fluctuate from year to year as a result of changes in mode of premium payment. We have three factors to consider, as follows:

1. The average age at issue based on amount of deferred premiums may differ from the average age of all policies based on amount of insurance because of two factors: first, the tendency towards greater frequency of payment at younger ages; and second, the greater weight given the older ages because of larger premiums per thousand of insurance. Tests for various periods of issue are suggested to help determine how closely the average ages at issue based on deferred premiums and amounts of insurance agree. These will indicate modifications, if any, to be made to an average age derived from records based on amounts of insurance.
2. An investigation of the premium-paying plans of insurance in force for each year of issue would reveal the proportions of the business to be evaluated as ordinary life and term insurance, respectively, and would indicate the average term of insurance for the latter. This information, combined with the average age for each year of issue, would make possible a detailed and fairly accurate valuation. It would seem that such a calculation for smaller companies, at least, would be too involved, requiring too much time and effort for the importance of this item. Therefore, an approximation which used the ordinary life reserve for all plans might seem best, since it is both simple and conservative.

3. The amount of deferred premiums at each year of issue determines the amount of insurance by duration. If a punch card system is used, then year of issue may be available in the master premium card. If it is, a tabulation of the master premium cards which involve deferred premiums can be made showing the amount of net premiums for each year of issue (including net premiums for disability and accidental death on an approximate basis such as 85% of gross). In the absence of a punch card system, the tabulation would probably have to be made by hand. A calculation of the net deferreds for each year, based on this tabulation, may then be made; it is a rather long calculation which should be made perhaps only periodically. An approximation of the amounts in force at each year of issue can be made for each annual valuation from this periodic tabulation.

We may now multiply these amounts of insurance by the ordinary life reserve for the age at issue selected to determine the reserve. (Under Method IV, *Projection Method*, is another suggestion for handling this troublesome liability.)

The accepted basis for valuation of life insurance reserves, of course, is based on the substitution method, since a midyear premium due date is substituted for the actual distribution of due dates. In evaluating supplementary contracts and dividends accumulated at interest the assumption of midmonth due dates of payments and interest is a similar, but less approximate, method which results in considerable savings when such records are tabulated on punched cards or otherwise by due month. For each interest rate of dividends at interest, for example, a tabulation showing the balances in each effective month requires merely the addition of interest from the middle of the effective month to December 31st to give the liability.

In Mr. Shailer's paper a description of the application of this method in the valuation of paid-up extended term and reduced paid-up endowment insurance is given.

The substitution method is generally simple and conservative, resulting in valuable savings. Care is required in the determination of averages, but since the selection of averages is made during slack periods, sufficient time and effort can be expended to insure their being representative of the underlying distributions.

IV. *Projection Method*

The projection method involves the assumption that distributions will follow a pattern, as does the substitution method. Instead of adopting a set of assumptions each year and then applying factors to obtain the reserves, here a detailed calculation is made every three years or so. At this

time an average reserve is calculated for the current year and for three years later; interpolation gives the projected average values.

Whenever it may be assumed that a current distribution will continue to be representative for several years (with modification for new entries in open groups) then the projection method ought to be considered, if the item is otherwise suitable for approximate valuation.

The liability for nondeduction of fractional premiums could be calculated during a slack period, as of the previous year end, taking into consideration the factors described under the substitution method to the degree of refinement desired. An average reserve per thousand of deferred net premiums would be calculated as of the year end preceding the date of calculation and as of three years later. Present trends as to amounts of new business and average age and plans of new business should be taken into account in arriving at the latter reserve. Interpolation between these values would give the average reserve for each of the two intervening years. Then, for each of the next three year-end valuations, multiplication of the total deferred net premiums by the projected average reserve would give the liability for nondeduction of fractional premiums quickly and easily, with an excellent degree of accuracy.

Double indemnity and active disability reserves may also be evaluated by this method as described in Mr. Shailer's paper.

The projection method attains all the objectives of approximate methods, and is in conformity with the principles. The method is likely to introduce significant errors only when new business differs greatly in amount or composition from that allowed for in making the projections. The composition or amount of new business entering a valuation using the projection method should therefore be watched each year.

V. *Accumulation Method*

An accumulation formula is a simple, easy-to-use method which, however, needs rather frequent checking because errors tend to be cumulative. The accumulation method is recommended whenever the factors in the formula are readily available or may be closely estimated.

One application of this method is the valuation of the reserve for dividend additions. All items of the regular formula, $MV_1 = MV_0 + P + I - TR - \frac{1}{2}(TC_0 + TC_1)$, are readily available except TC_0 and TC_1, the tabular costs of insurance. P, the net premium, is usually equal to the amount left to purchase dividend additions; TR, the terminal reserves released, is equal to the cash values allowed on surrender. To obtain TC_0 and TC_1, average costs of insurance per thousand of paid-up insurance are estimated by the projection method, based on periodic attained age

valuations which yield both the tabular cost and the mean reserve for establishing a new starting point.

Mr. Shailer's paper describes a modified accumulation formula for dividend additions, using records of face amount and changes in face amounts only.

This method is in accord with all the principles of approximate valuation; the principle of periodic verification during slack periods is relatively more important for this method, however.

VI. *Midyear Method*

A method which does not reduce the total time or effort in producing a valuation, but is justified in improved timing in meeting year-end deadlines, is that of making detailed calculation during the year, then adjusting the result for changes between the valuation date and December 31. The adjustment might be made in any one of the three following ways:

(1) Approximate valuation of entries and exits, the main reserve having been calculated as a year-end reserve, or
(2) average reserve projection factor to December 31, or
(3) addition of premiums and interest and deduction of estimated tabular claims (using the accumulation method for part of one year).

Supplementary contracts not involving life contingencies might be evaluated by this method. A seriatim valuation during a slack period, giving the liability as of, say, July 1st, would be the basis. Total amounts left under supplementary contracts not involving life contingencies would be added with $\frac{1}{4}$ year's interest, total payments (including interest payments) and withdrawals would be deducted with $\frac{1}{4}$ year's interest, and $\frac{1}{2}$ year's interest on the July 1st liability would be added. The error introduced would be small, involving the interest on amounts left and payments and withdrawals made; it depends on the variation between the actual distribution of dates when amounts are left and payments and withdrawals are made, and October 1st, the midpoint of the period. The year-end work would be simple, materially helping in meeting statement deadlines.

As mentioned above, the midyear method requires some additional calculating work. The additional work, however, is light and generally worth the gains coming from rescheduling of heavier, detailed work during slack periods. Substantial accuracy is obtained.

VII. *Percentage Method*

A method described in Mr. J. S. Thompson's discussion of Mr. Shailer's paper is that of making accurate calculations of the principal plans of insurance (which would have to make up perhaps 90% of the total liability

item), with a percentage increase, based on an earlier year's accurate valuation of all plans, to get the total reserve. Mr. Thompson describes the application of this percentage method to the evaluation of active disability benefits.

Since this method assumes the continuation of a certain distribution of plans, its use would be suggested when it is expected that the distribution of plans will not markedly change from year to year. After adoption, the method should be checked carefully whenever the proportion of principal plans to total in force undergoes a definite change. The amount of savings made would depend on the number of plans omitted from detailed calculation.

VIII. *Method of Relating to Similar Items*

A method which provides great savings in time and expense where applicable is that of relating an item to be evaluated to a similar item which is readily available, having been obtained for another purpose. An example of this method's use is the valuation of Y.R.T. reinsurance reserves. The renewal premiums may be related to the CSO cost of insurance, and the first year premiums to half the CSO cost of insurance, in a typical schedule of reinsurance premiums. A simple approximation of this reserve is, then:

$$MV = (P_1 + \tfrac{1}{2}P_r) \times K$$

where P_1 and P_r are, respectively, the first year and renewal incurred premiums, and K is twice the ratio of CSO reserves to renewal premiums. The additional liability under substandard reinsurance is automatically provided for under this method.

The following calculation, using typical reinsurance premiums, with $K = 1$, illustrates the closeness with which this method approximates the reserve, at the more important attained ages:

Y.R.T. RESERVES CSO 2½%

Age	Reserve	1st Year Premium	Ratio to Reserve	Renewal Premium	½ Renewal Premium	Ratio to Reserve
10	.96	1.33	139%	2.66	1.33	139%
25	1.41	1.73	123	3.45	1.725	123
40	3.02	3.09	102	6.18	3.09	102
55	8.79	8.99	102	17.98	8.99	102
70	26.75	29.65	111	59.30	29.65	111

As shown, this method using this particular premium schedule and $K = 1$ is conservative for accepted reinsurance and slightly deficient for

ceded reinsurance, since there is a small overstatement of the reserves. For a company whose accepted reinsurance is greater than or approximately equal to its ceded reinsurance, the method for this scale of premiums might be adequate. The conservatism in the valuation of accepted reinsurance would tend to counterbalance the deficiency in the ceded reinsurance valuation. For other companies, K might be set at .90 or .95, depending on the distribution of its business by attained age; or K might vary between types of reinsurance within the same company. Whenever reinsurance premiums are changed, the effect on reserves calculated by this method must be checked.

Another simple application of this method would be to relate group life term reserves to the corresponding premiums.

The method of relating to similar items is certainly time-saving, is simple in its application, and may be made as conservative as desired by adjustment of the constant, K. Its use is recommended wherever a close, parallel relationship exists between an item required for the statement and one available in the ledger or similar record.

CONCLUSION

One might say that nearly all of the actuary's work consists of dealing with approximations. Furthermore, our legal reserves themselves are based on approximation—that is, conservative projections into the future of mortality and interest rates derived from the past. It is clear, then, that ample precedent exists for approximations involving relatively minor liabilities and assets and negligible effects on surplus. In addition, modern business conditions virtually require that the actuary be continually alert to the opportunities for the extension and improvement of approximate methods of valuation.

DISCUSSION OF PRECEDING PAPER

RALPH E. EDWARDS:

This paper is an excellent summary of various valuation methods. It will be a fine source of reference for those of us who encounter valuation problems in our everyday work, and we are indebted to Mr. Arnold for this contribution to actuarial literature.

It has been said that a chrysanthemum by any other name would be a lot easier to spell. It occurs to me that approximations by any other name would be a lot easier to sell. The word "approximation" suggests something that is second rate or that needs to be apologized for. To the contrary, the methods of this paper are often superior even from the viewpoint of regulatory officials.

Pursuing this thought further brings to light the fact that our nomenclature in this field is not only poor semantics but also far from precise. When we speak of approximate methods we insinuate that the customary method is precise, when the truth is that it is only approximate and sometimes not a very accurate approximation. It is merely that the customary method has become acceptable on this continent so universally that it has become an unquestioned standard. I suggest that we change our terminology and refer to this as the "standard" method and the other methods as "special" methods.

One of the least used of Mr. Arnold's methods is number VIII. An application of it is in the valuation of dismemberment benefits in Industrial policies. The benefit decreases with duration (compared to the life benefit which is level) and the rate of dismemberment increases more rapidly with age than does the rate of mortality. By assuming that these factors are offsetting, the reserve can be taken as a percentage of the life reserve. The name given to Method VIII is "Method of Relating to Similar Items," but someone has suggested that in this particular application we are dealing with dissimilarities rather than similarities.

I think it may be worth pointing out that special methods are often applied to benefits which do not provide a value in the event of voluntary termination. As a result the reserve will be considerably higher than the actual liability. As a consequence it may be possible to justify the use of the special method even though it is not a very close estimate of the reserve obtained by the standard method.

GEORGE C. CAMPBELL:

The author, in his introduction, more or less restricts the application of his paper to small companies by his reference to the use of electronic machinery in large companies. Without detracting from the great speed and capacity of the electronic machines, it looks as if approximate methods of valuation will continue to be very useful for many purposes even in large companies.

We have valued our Industrial and Ordinary business for many years by a system of preliminary valuations based on the in-force at one or two dates during the year, such as January 1, June 30 or September 30 for different branches. Averages are obtained from these preliminary valuations by year of issue and, in some cases, by plans or plan groups. The averages from the preliminary valuations are used at the year-end, generally after applying some method of projection to allow for changing distributions.

The method of projection used for Ordinary business in the larger branches depends on valuing the business in year-plan-age detail at two preliminary dates and projecting averages by year of issue and plan groups (Life, Endowment and Term) to the end of the year. This method of projection approximately corrects the averages both for changing plan distributions and for changing age distributions.

The Industrial averages by year of issue are projected to the end of the year from a single year-plan-age valuation in June. June averages by plan and year of issue are applied against the September in-force to produce another set of averages by year of issue only. The June and September averages by year of issue are extrapolated to the end of the year. This method of projection corrects for changing plan distributions but not for changing age distributions.

Sometimes we are asked about the accuracy of these methods. Consider the Industrial Weekly Premium-Paying valuation, for example, where the reserve is over $1.6 billion. The projection corrects almost completely for the changing plan distribution. A test later showed that the reserve was only $39,000 below what it would have been if factors by plan and year of issue had been used at the year-end.

The principal error that must be tested in this approximation arises from the changing age distributions for which our projection does not correct. Our valuation of the June in-force for Industrial business is made with reserve factors as of December of the current year and, at the same time, with another set of factors as of December of the following year. The averages as of December of the following year are adjusted for poli-

cies maturing and becoming paid-up, which would not be valued as of December of the following year. Averages by plan and year of issue derived from the December 1955 reserve factors and the June 1954 in-force can be tested by comparison with the June 1955 valuation. These plan and year of issue averages derived from the June 1954 in-force gave a reserve $700,000 too high when applied against the June 1955 in-force. This is the error from changing age distributions in a whole year. Assuming that the changing age distribution was reasonably uniform over the year, it would have given a reserve about $350,000 too high at the end of 1954. Combining this result with the $39,000 in the opposite direction from the plan distribution leaves a net overstatement in the reserve at the end of 1954 of about $300,000 compared with the reserve of $1,600,000,000, or something of the order of 0.02%.

These preliminary valuations, together with the necessary test-checking, take more actual work than a single valuation at the end of the year, but they shift the load from the year-end peak to more convenient times.

When we came to transfer much of our valuation to Univac this year, we considered going to a single valuation either at the end of the year or at some date very late in the year. We felt it unwise at this stage, however, to schedule a valuation involving several billion dollars of reserve so late in the year that we would not have time to make any reasonable review before signing the valuation affidavit. Consequently, we are using Univac to perform the heavy arithmetical work using the same principles of approximation we had used before with punch card machinery. This is a transition stage, of course, and it is not unlikely that extensive modifications will be made in our valuation methods to make better use of electronic methods.

On the other hand, it did not seem economical this year to transfer to Univac many small items that must be included in the valuation and it may not be economical to do so for some time. Approximate methods where appropriate will be continued in the meantime.

An approximate method must be suitable for the purpose, and it must be tailor-made to the operating records of the company. When approximating large items one must make sure that the error is controlled sufficiently so that the difference in the errors between one year and the next, even if the errors are in opposite directions, will not distort the surplus earnings. Sometimes the test-checking can be built into the projection so that the end of one projection can be compared against the starting point of the next projection. If a projection stays within satisfactory limits for five years, for example, there is little reason to test shorter periods.

Approximations to small items should not be unreasonable, but it is

practical to permit much larger percentage errors here because they have so little effect on the resulting surplus position. There is little or no justification for any extensive test-checking on many very small items after a method has been established. A competent actuary can tell in many cases by general reasoning that a method must give reasonable results satisfactory for the purpose.

One should not lose sight of the point the author makes in his conclusion that the reserves are approximations themselves, based on assumed mortality and assumed interest. A moderate change in either of these factors would make substantial changes in our reserves. Consequently, it is not in the service of our policyholders to spend money refining the accuracy of the arithmetical calculations, beyond the point of practical usefulness. Money spent on calculations is an actual expense to our policyholders, but any reasonable error of approximation in the aggregate reserve has no ultimate effect on the finances of our policyholders.

ELI A. GROSSMAN:

These remarks are directed at an analysis of approximate valuation methods but have application to other accounting and actuarial reports.

The figures one reads in the Annual Statement of a life insurance company might have little meaning except to those closely associated with its preparation. A reason for this is that approximations are made without conveying the basic objective to the reader. For example, assuming no legal restrictions, one company might try to establish the most probable value of each asset and liability item, whereas another company might attempt to establish a very conservative value for each item. If the reader knew the method of calculation of the Annual Statement figures, then they would be much more meaningful.

The confidence limit could be a useful tool in this regard. Instead of just saying a certain statement item is conservative, we could give some quantitative measure of the word "conservative." To illustrate we might attempt to establish the value of every asset and liability so that it is conservative with a confidence limit of 99%. This would mean that we have a 99% confidence that each asset is not larger than stated and that each liability is not smaller than stated. Of course, legal requirements restrict the values assigned to certain items for the Statement that is filed with the Insurance Departments. However, even for these statements a description is known in regard to the basic pattern by which the items are computed. The pattern need not be the same for each item and could vary in many ways depending on the purpose of the Statement. Naturally there are problems connected with determining confidence limits, mo-

ments, or other statistical entities in connection with assets and liabilities, but a satisfactory solution should be available. This discussion does not explore these phases of the subject. However, there should be a definition stating what measure of the items is being used. In this way the Annual Statement will have much more meaning and would lend itself to various statistical measures.

BERTRAM N. PIKE:

I was pleased to see Mr. Arnold's paper appear in the *Transactions*. It is a good summary of various types of approximations that have been used to estimate reserve liabilities, and should be helpful also to students for examination preparation for Part 6.

It is mainly with regard to the author's statement of principles that I would like to offer some comments. The savings in expense and time, while preserving substantial accuracy, could be appreciable items, and it is desirable to take the necessary time to analyze the more troublesome items in order to find a good substitute method. Of more than academic interest are the saving of wear and tear on those charged with performing the many detailed calculations at a time when time seems to be so short, as well as the possible use of these methods for internal company statements. The author has commented on the desirability of approximate valuations when electronic machinery is not available. Perhaps it would be well to keep in mind that, even if such machinery has been installed, there may be so many other tasks for it to perform that its mere presence will not assure its use for relatively small scale problems, especially if approximate work might turn out cheaper.

I think we would all agree that if a known error is to be introduced into a calculation of one particular reserve, it should preferably be an error of conservatism. However, too much or too frequent conservatism is not necessarily satisfactory either, and I wonder if it might not be desirable to use approximate valuation methods which, rather than erring consistently on the high side, erred randomly both positively and negatively. For a number of such valuations, we might come closer to the correct value in the aggregate.

I wonder if we should not add two additional concepts to the author's statement of principles. The first would require that consideration be given to any loss of valuable by-product information usually obtainable from the full-scale calculation. Thus, if we make some saving from a short-cut valuation which also eliminates valuable intermediate information, we have to balance the disadvantages against the advantages to be sure that

the net result is sufficiently favorable to justify the use of the approximation.

The second additional concept would require that the reserve item being approximated should neither be obtained from, nor be used for, other approximate valuations; that is, we should avoid any snowballing of approximation errors.

I am not sure I fully appreciate the significance of the table shown under the Inclusion Method. It appears that for certain durations a 10% overstatement in ordinary life reserve will overstate the true combination reserve (the ordinary life policy with the continuous income feature) by 10%. Perhaps in his review of the discussion Mr. Arnold will indicate the corresponding ratios for other durations to illustrate the advantages of this method.

The only other comment I should like to make is with regard to the method of Overstatement of Small Liabilities. I wonder if a word of caution is not appropriate when the increase (or decrease) in overstated reserves is part of the determination of the divisible surplus for a closed group of policies on a fund accounting basis. Unless the company pays terminal dividends of some sort, a consistent conservatism in this area might become a penalty to the insured.