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**MORTALITY EXPERIENCE OF UNION  
CIVIL WAR VETERANS**

ROBERT J. MYERS AND LOUIS O. SHUDDE

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NORMAN BRODIE:

The authors have presented a set of graduated death rates for ages 96 and over that they developed from recent mortality experience data on Union Civil War veterans. We have compared their rates with insurance company experience at these ages on immediate annuities, displaying the results in a tabular summary below. For this purpose we have used the intercompany experience between 1948 and 1953 contract anniversaries on immediate annuities issued in 1931 or later and in force six or more years, the data by individual ages having been made available to us by the Society's Committee on Ordinary Mortality.

There were 318 deaths in this experience at attained ages 96 to 105 for both sexes and for refund and nonrefund annuities combined, or about 50% more deaths than the number reflected in the Union Civil War veterans graduated death rates. The comparison is in the form of mortality ratios, the actual intercompany results being related to tabular deaths computed on the basis of the authors' smoothed death rates.

Since the intercompany data consisted solely of individuals who were at least 74 years of age at the time of purchase of their annuities and include a higher proportion of females, an over-all mortality ratio substantially below 100% would not have been unexpected. Actually, though, the ratio is surprisingly close to 100%—specifically 91%. This might be interpreted as tending to confirm the level of the death rates produced by Messrs. Myers and Shudde.

As for the relative mortality by sex at these high ages, we do observe a higher ratio on males than on females—99% and 89% respectively—but the difference is not statistically significant, *i.e.*, it is less than three probable errors. Therefore, there is no indication in these figures that it is improper to use the same mortality rates at the very high ages in the male and female population tables. It might also be mentioned that we do not find any statistically significant difference between the mortality ratios

for refund and nonrefund annuities at the ages involved in this comparison.

At attained age 95, which is not reflected in the foregoing comparison, there were 217 deaths in the intercompany experience for both sexes and types of annuities combined. These deaths yielded a crude death rate of .316. This compares with a value of .314 for  $q_{95}$  produced by extending the authors' logistic curve to that age. The observed relative mortality by sex and type of annuity, though, is not the same at this single age as at the

RELATIVE MORTALITY OF ANNUITANTS AND  
UNION CIVIL WAR VETERANS  
ACTUAL INTERCOMPANY MORTALITY ON IMMEDIATE ANNUITIES, 1948-1953 CONTRACT ANNIVERSARIES, DURATION 6 YEARS AND OVER, RELATED TO TABULAR DEATHS BASED ON SMOOTHED DEATH RATES FOR UNION CIVIL WAR VETERANS IN TSA VII, 66, TABLE 2

Sex or Attained Age of Annuitant	Type of Annuity	Actual Deaths by Contracts	Mortality Ratio
Ages 96 and over combined			
Male .....	Nonrefund	29	93% ± 9
Female .....	Nonrefund	98	91 ± 5
Male .....	Refund	44	103 ± 8
Female .....	Refund	147	88 ± 4
Male .....	Refund and Nonrefund	73	99 ± 6
Female .....	Refund and Nonrefund	245	89 ± 3
Both .....	Refund and Nonrefund	318	91 ± 3
By attained age, both sexes and types of annuity combined			
96 .....		119	92% ± 5
97 .....		72	79 ± 5
98 .....		66	109 ± 7
99 .....		37	104 ± 9
100-105 .....		24	77 ± 9
All .....		318	91% ± 3

older ages. The table below shows the actual deaths and death rates at age 95 for each sex and type of annuity.

Sex of Annuitant	Type of Annuity	Actual Deaths by Contracts	Crude Rate of Mortality
Male . . . . .	Nonrefund	7	.121
Female . . . . .	Nonrefund	42	.228
Male . . . . .	Refund	32	.352
Female . . . . .	Refund	136	.384
Male . . . . .	Refund and Nonrefund	39	.262
Female . . . . .	Refund and Nonrefund	178	.331
Both . . . . .	Refund and Nonrefund	217	.316

HENRY S. HUNTINGTON, III:

In presenting the mortality experience at advanced ages among Union Civil War veterans, Messrs. Myers and Shudde have made a valuable contribution to our storehouse of information. This experience seems very well suited for use as the basis at these ages for the 1949-51 U.S. Life Tables.

With regard to the manner in which the data are graduated and the nature of the resulting smoothed rates there may be room for a difference of opinion. The authors undoubtedly had good reasons for processing the data as they did. The purpose of this discussion is to include in the record a statement of reasons for possibly preferring a set of smoothed rates differing from that in the paper by having a steeper age slope and probably being at least slightly concave upward (the authors' set is practically linear).

The reasoning behind this preference involves the following considerations:

#### 1. *Analysis of Union Civil War Veterans' Experience*

The total of the expected deaths for ages 96-107, the range of ages used by the authors in their graduation, is 199.85 on the basis of their smoothed death rates as compared with corresponding actual deaths of 205. A second set of smoothed death rates has been derived from the authors' set by the method described on pages 11-12 of Morton D. Miller's *Elements of Graduation*. The resulting rates, together with the actual number of deaths, the crude rates and the authors' smoothed rates are set forth below. This second set of smoothed death rates yields total

expected deaths for ages 96-107 of 205.11, and also reduces the sum of the squares of the deviations from 83.29 in the case of the authors' set to 72.72. (The authors' method of graduation seems to take no account of the variation in volume of exposure by age.)

Age	Actual Deaths	Crude Death Rate	Smoothed Death Rate*	Smoothed Death Rate†
96.....	6	.333	.329	.331
97.....	12	.308	.344	.348
98.....	22	.328	.360	.366
99.....	34	.382	.376	.384
100.....	41	.471	.392	.402
101.....	33	.465	.409	.421
102.....	17	.370	.426	.440
103.....	15	.429	.442	.458
104.....	11	.478	.460	.478
105.....	8	.500	.477	.497
106.....	2	.250	.494	.516
107.....	4	.800	.511	.535

\* By authors.

† Derived from authors' smoothed death rates as described above.

## 2. Implications with Respect to Oldest Ages Attained by Humans

The U.S. Life Table for White Females, 1949-51, which for the extremely old ages makes use of the authors' set of smoothed death rates, yields a stationary population for ages 85 and older of about 103,000, of whom 1 is 110 years old. Now the number of white females aged 85 and older in the United States in July 1954 is estimated by the Bureau of the Census to have been 407,000. If stationary population conditions applied to this actual population it would follow that there were then about four white females aged 110 years in the United States. Actually, the probability is greater that there were two or three than that there were four such females.

If the mortality rates of the above-mentioned mortality table are projected to age 115 it may be shown that the resulting probability of survival to age 115 of a white female aged 110 is about .01. Under current conditions this would indicate that we could expect a white female in the United States to survive to age 115 about two or three times in a century. This implication, particularly when translated into world-wide terms, appears inconsistent with the widely accepted conclusion cited by the authors that age 110 is probably about the highest that any human being has attained.

### 3. *Relationship of Results to Those of Other Studies*

The authors considered the work done by Vincent on mortality rates at the extremely old ages among the residents of four European countries, from which he obtained a single set of smoothed death rates which is concave upward. They concluded that their data were not compatible with an upward concavity and adopted smoothed death rates which are almost linear. It is true that the relatively high crude death rates associated with the relatively large exposures at ages 100 and 101 do not lend themselves to a graduation exhibiting upward concavity. On the other hand, it may be felt that Vincent's results, drawn from four experiences all of which show close agreement with each other, have established some degree of presumption as to the general pattern of human mortality rates for the extremely old ages, and that a larger volume of experience than that here presented would be needed to substantiate so material a departure from that pattern.

#### W. RULON WILLIAMSON:

Mr. Shudde, a Texan, open-mindedly followed the Union Veterans till the last man faltered but did not fall. Then he and Mr. Myers told us the story.

As to final dependable ages not going beyond age 110, I have two items to record. An adventurous friend of mine several years ago was in Borneo. There he visited an aged Chinaman. At the age of 100 that gentleman of veracity moved into the room with his coffin. At the time of the visit he had been there 32 years, and my friend took his picture. The procedure exhausted the 132-year-old, but he survived. His death has not yet been reported. The other case was in the Sudan. Retired from a disbanded harem, a Sudanese eunuch became a bank messenger at 100. After serving one bank and then a new branch opening up as a pioneering venture, a fall over some unguarded construction resulted in a disability pension at about 115. His death at 132 was recently reported to a pension representative at Ottawa.

The only other comment I am making is that the Civil War veterans pensions started out as disability benefits too—disablement being logical from the unwholesome conditions in the "dark South." Yet these recipients, letting down the bars to their less privileged fellow-veterans, have been able to show practically standard mortality. It seems a well-authenticated actuarial legend that nursing an incurable ailment is an excellent way to prolong life—provided the ailment is accompanied by an income.

T. N. E. GREVILLE:

This is a most interesting paper, since it is so rare that mortality rates for ages beyond 100 can be correctly computed from actual data, where the deaths and exposures are known to relate to the same group of persons and the ages have been determined with reasonable accuracy. These would appear to be the most trustworthy mortality rates for such advanced ages ever published.

The authors have graduated the crude rates by fitting a logistic curve in the form:

$$q_x = \frac{1}{1 + B c^x}.$$

Comparison with other possible mortality curves is facilitated by re-defining  $B$  and  $c$  as the reciprocals of the quantities so denominated by the authors of the paper, so that we shall have:

$$q_x = \frac{B c^x}{1 + B c^x}, \quad \frac{q_x}{p_x} = B c^x,$$

and the latter convention will be employed throughout the remainder of this discussion.

As these mortality rates have been used to close the U.S. Life Tables for 1949-51, and may well be similarly used in connection with other mortality tables in the future, I was interested in knowing whether they could have been graduated equally well by the more familiar Gompertz law. Under this assumption, of course, both  $\mu_x$  and  $\log p_x$  have the form  $Bc^x$  (with slightly different values of  $B$  in the two cases, for the same mortality table). I have also tried the assumption made by Vincent in the article cited by the authors—that  $q_x$  itself has the form  $Bc^x$ . Like the authors, I have made use of the crude data only between ages 96 and 107, inclusive, but I have used a different criterion of fit: exact reproduction of the actual number of deaths and the average age at death for all deaths between these ages. I have also made a regraduation by the logistic curve on the basis of this criterion.

Table 1 compares the rates of mortality obtained by the three new graduations with the crude rates and with the graduated rates given in the paper. The three new graduations scarcely differ at those ages which would ordinarily be tabulated, and it becomes largely a matter of personal taste which of the three is preferred. The marked similarity of the three curves seems to be the result of the criterion of fit employed: there would have been greater differences if they had been fitted by the method of least squares to the crude rates (without regard to the exposure at each age) as was done in the paper. They yield at age 99 a mortality rate

which is approximately equal to both the crude rate and that obtained in the original graduation, but they produce a noticeably steeper  $q_x$  curve than the authors' graduation gives. Merely to illustrate the behavior of the four curves at subsequent ages, values for ages 120 and 150 have been included.

TABLE 1  
COMPARISON OF DIFFERENT GRADUATIONS OF MORTALITY  
OF UNION CIVIL WAR VETERANS  
JULY 1, 1945 TO JUNE 30, 1954

AGE	CRUDE DEATH RATE	SMOOTHED DEATH RATES			
		Logistic		Gompertz	Vincent
		Original	Revised		
96.....	.333	.329	.331	.333	.335
97.....	.308	.344	.348	.349	.350
98.....	.328	.360	.366	.366	.366
99.....	.382	.376	.384	.383	.383
100.....	.471	.392	.402	.401	.400
101.....	.465	.409	.420	.420	.419
102.....	.370	.426	.439	.439	.438
103.....	.429	.442	.458	.458	.458
104.....	.478	.460	.478	.478	.478
105.....	.500	.477	.497	.498	.500
106.....	.250	.494	.516	.519	.522
107.....	.800	.511	.535	.540	.547
108.....		.528	.554	.562	.572
109.....		.545	.573	.583	.598
110.....		.562	.592	.605	.625
111.....		.579	.610	.627	.653
112.....		.596	.629	.648	.683
120.....		.719	.758	.814	.975
150.....		.953	.969	.99995	

In Table 2, which shows the deviations of the actual from the expected number of deaths, and the accumulated deviations, by the old and new graduations, the Gompertz graduation is taken as the representative of the three new ones, since it gives mortality rates intermediate between the other two (in those cases where they differ at all). The sum of the deviations without regard to sign and the maximum accumulated deviation are slightly smaller for the Gompertz graduation. The latter

gives a mortality rate which is closer to the crude rate than is the graduated rate given in the paper in 7 cases out of the 12 in the age interval 96-107 used for fitting purposes. While the new graduations fail to show any marked superiority over that of the authors, they would perhaps be judged to give a slightly better fit on the basis of the comparison made here.

TABLE 2  
COMPARISON OF ACTUAL AND EXPECTED DEATHS UNDER ORIGINAL AND GOMPERTZ GRADUATIONS OF MORTALITY OF UNION CIVIL WAR VETERANS  
JULY 1, 1945 TO JUNE 30, 1954

AGE	ACTUAL DEATHS	ORIGINAL GRADUATION		GOMPERTZ GRADUATION	
		Deviation*	Accumulated Deviation	Deviation*	Accumulated Deviation
96.....	6	.1	.1	.0	.0
97.....	12	- 1.4	-1.3	- 1.6	-1.6
98.....	22	- 2.1	-3.4	- 2.5	-4.1
99.....	34	.5	-2.9	- .1	-4.2
100.....	41	6.9	4.0	6.1	1.9
101.....	33	4.0	8.0	3.2	5.1
102.....	17	- 2.6	5.4	- 3.2	1.9
103.....	15	- .5	4.9	- 1.0	.9
104.....	11	.4	5.3	.0	.9
105.....	8	.4	5.7	.0	.9
106.....	2	- 2.0	3.7	- 2.2	-1.3
107.....	4	1.4	5.1	1.3	.0
Total†.....	205	22.3	.....	21.2	.....

\* Actual minus expected deaths.

† Without regard to sign.

NOTE.—The average age at death for the deaths which occurred at ages 96-107 was 100.48. This value is, of course, reproduced by the Gompertz graduation. The corresponding value for the original graduation is 100.46.

The values of the numerical constants under the four graduations are as follows (using for the original graduation the reciprocals of those given by the authors):

	LOGISTIC		GOMPERTZ	VINCENT
	Original	Revised		
B.....	.0006699	.0003049	.0013601	.0046898
c.....	1.071128	1.080022	1.061122	1.045477

Some may object to the Vincent assumption on the ground that  $q_x$  becomes unity at a finite age (age 121 in the present case). Under both the other assumptions, this does not occur, but  $q_x$  approaches the limit 1 (and, in consequence,  $\mu_x$  tends to infinity) with increasing  $x$ . The two curves differ, however, in the rate of increase of  $\mu_x$  for very large values of  $x$ . This is most easily seen by comparing the expressions obtained for  $\Delta\mu_x$ . For the logistic curve,

$$\Delta\mu_x = q_x \log_e c ,$$

while the Gompertz curve gives

$$\Delta\mu_x = (c - 1) \mu_x = \text{colog}_e p_x \log_e c .$$

Thus, for the Gompertz curve  $\Delta\mu_x$  tends to infinity for increasing  $x$ , while, for the logistic curve, its limit is the constant value  $\log_e c$ . For example, under the graduation used in the paper the increase per year of age in the value of  $\mu_x$  can never reach .07, while, under the revised logistic graduation, it can never reach .08 (under the Gompertz graduation presented in this discussion, the former value would be reached at age 115 and the latter at age 117).

In passing, it may be of interest to mention a certain mathematical relationship that exists between the different curves referred to. This is indicated in the following tabulation:

$$\text{Vincent: } B c^x = q_x = d_x / l_x$$

$$\text{Gompertz: } B c^x = \text{colog}_e p_x = d_x / l_{x+\theta}$$

$$\text{Logistic: } B c^x = q_x / p_x = d_x / l_{x+1}$$

Here  $l_{x+\theta}$  is an average value of the  $l$ -function over the year of age, obtained by weighting each part of the year according to the intensity of mortality then prevailing. In other words,

$$l_{x+\theta} = \int_0^1 l_{x+t} \mu_{x+t} dt / \int_0^1 \mu_{x+t} dt .$$

Thus,  $l_{x+\theta}$  would reduce to  $l_x$  or to  $l_{x+1}$  if all deaths in the year of age were assumed to occur at the beginning or at the end of the year, respectively.

The 1949-51 U.S. Life Tables, employing at the very old ages the mortality rates developed in this study, show, out of a total stationary population of 6.8 million, roughly 400 at ages 100 and over and 1 at ages 109 and above. As the population of the world is roughly 350 times as great, we might set as extreme upper limits populations of 140,000 at ages 100 and over and 350 at ages 109 and over for the entire world. These

figures would be drastically reduced because of the fact that mortality rates in most of the world are substantially higher than in the United States, and further because the world population is not stationary but has steadily increased over the past 100 years.

The 1950 census reported 4,475 persons aged 100 and over in the United States. The ratio  $T_{109}/T_{100}$  on the basis of the graduated Civil War veteran mortality rates obtained in this discussion works out to about 1/600. (This is somewhat less than the value of about 1/400 based on the original graduation and reflected in the published values cited above.) Applying the smaller ratio to the reported population at ages 100 and over would give about 7 for the estimated population at ages 109 and over. Bearing in mind that the census figure is probably exaggerated, we should not expect to find more than 3 or 4 persons in this country at ages 109 and above. There is now one surviving Civil War veteran stated to be at age 109, and, as brought out in the paper, one is reported to have died at age 110. In the tabulation of the 1950 census, a single code was used for all ages above 99, so that the number reporting themselves at age 109 or over cannot be ascertained from the punched cards.

(AUTHORS' REVIEW OF DISCUSSION)

ROBERT J. MYERS AND LOUIS O. SHUDE:

We are very glad to have the four discussions of our paper, which add considerably to its value. Two of the discussants, Mr. Huntington and Dr. Greville, have suggested more refined and elegant methods of graduation of the crude data.

We agree that it probably would have been desirable to do a somewhat more elaborate job, especially so as to more nearly equate actual and tabular deaths. The paucity of the data and the resulting random fluctuations led us not to lay too much stress on the elaborateness of the graduation method. Actually, a graphic graduation would have been sufficient, but we believed that an elementary mathematical formula basis would be better so that a definite set of results would be uniformly available. Actually, there is relatively little difference between the several sets of smooth death rates. It is interesting to note that there is almost exact agreement between Mr. Huntington's rates and Dr. Greville's revised logistic rates.

Both of the preceding discussants have also given some interesting projections as to the population at the very advanced ages in this country and throughout the world. Our personal opinion is that age 110 appears to all intents and purposes to be the effective limiting age for a radix of

100,000 at birth. Since  $q_x$  probably never becomes unity, some individuals no doubt have survived beyond that age. Naturally, if the radix of the mortality table is larger, the limiting age is increased. For example, for a radix of 100 million—the approximate annual number of births in the world currently—the limiting age is 116 for the U.S. White Males Life Table for 1949–51 and 117 for the corresponding female table. These ages approach, but do not nearly attain, those in Mr. Williamson's two cases.

Mr. Brodie presents some interesting data based on the experience at these ages under immediate annuities issued by insurance companies. The accuracy of age reporting may be considered fairly good, although it is to be noted that all the individuals covered were at least 74 years old at the time of purchase of the annuity and none of them had been under observation for more than 22 years. Although Mr. Brodie's data include 50% more deaths than in our study, the number of deaths at ages 100 and over were 132 in our data against 24 in his data. It seems to us that the basic value of our study was the obtaining of rates as reliable as possible at age 100 and thereafter. Unfortunately, very little of the considerable amount of detailed actual experience data of the veterans at age 95 and thereabouts can be obtained, since most of it occurred before 1945. In any event, it is reassuring that Mr. Brodie's results confirm, to a considerable extent, the level of death rates that we found.