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Pension Forecasts, Part 2: The Model Has No Clothes

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Editor's Note: In the previous issue of this newsletter, Part 1 of this article described a simplified problem in pension plan financing and presented two questions about how that pension plan can be modeled. The questions are repeated here, together with answers.

Consider this simplified pension plan and funding system. The liabilities consist of a single known benefit payment to be made 20 years from today. That benefit payment can be matched in timing and amount by a portfolio of 20-year zero-coupon Treasury bonds with a market value of $1 million. The plan assets also equal $1 million.

The company will make no interim contributions to or withdrawals from the plan. At the end of year 20, the company will wind up the plan by withdrawing the surplus or contributing to cover the deficit. (We ignore taxes and assume that there is no risk of default by the company.[1])

The corporate sponsor of this plan asks for your help. The assets are currently invested in the matching Treasury portfolio, which will ensure full funding of the plan with a zero company cost. The sponsor believes that, over a 20-year horizon, equity investments would give rise to potential withdrawals that greatly outweigh the potential contributions, in both probability and magnitude. So he asks you *Question #1: Ignoring taxes, how would shifting the $1 million from Treasuries into equities affect shareholder value?*

You decide to use a pension forecasting model. You prepare a series of 20-year simulations that show a range of terminal company contributions or withdrawals. To provide a single answer to Question #1, you need to discount each of these terminal payments to a present value. This presents *Question #2: What discount rate should you use—the Treasury yield, the expected return on the plan assets, the company's borrowing rate, the company's weighted average cost of capital, or some other rate?*

Answers

At the end of year 20, the company will withdraw from the plan an amount equal to the cumulative change in the assets minus the cumulative change in the liability (or contribute the difference, if negative). Because the matching Treasury portfolio mimics the liability, we can think of the withdrawal as the total asset return minus the total return of the matching Treasury portfolio (the "liability return"). If the assets are in fact invested in that matching Treasury portfolio, the asset and liability returns are of course identical and the withdrawal is zero. If the assets are equities, readers familiar with swaps will recognize that the company is engaging in a simple debt-for-equity swap, paying the return on a specific Treasury portfolio and receiving the return on an equity portfolio of equal size. *The value of such a swap is zero. Therefore the proposed equity investment would leave shareholder value unchanged.*

This result may seem quite counterintuitive to those who have not studied swaps, and a simple swap illustration may be helpful. We ignore tax considerations, transaction costs, and other frictions, and assume that you and I both have flawless credit—we can borrow at Treasury rates.

Let's agree to engage in the following swap transaction:

I'll pay you the return of the S&P 500 on a $1 million investment for the next 20 years (or I'll collect from you if the return is negative). You'll pay me the return on $1 million of 20-year zero-coupon Treasury bonds. Although swaps are commonly for shorter periods with periodic interim settlement, we'll duplicate the pension problem by waiting and settling the entire difference at the end of 20 years.

Both history and common sense indicate that you're much more likely to collect than to pay, and your likely collections are much larger than your likely payments. It seems that you are receiving, and I am paying, something with a substantial positive present value. So, would it be fair for you to pay me a little extra to get this deal—say, 2% annually on top of the Treasury return?

The correct answer is that the swap is a fair deal and no additional payment is appropriate. We can show that the swap is fair by demonstrating that I can hedge my position:

1. I borrow $1 million at the Treasury rate, with all interest and principal due in 20 years.
2. I invest the loan proceeds in the S&P 500. During the next 20 years, I earn the S&P return on my $1 million investment.
3. I pay that S&P return to you in exchange for 20 years of Treasury bond interest.
4. I use that interest plus the original $1 million investment to repay my loan.

This hedge assures me of breaking even on the swap. If you're willing to give me any extra payment beyond the Treasury bond interest, I can pocket it as pure and certain profit, which I make without putting up any capital or taking any risk. Therefore my offer to pay you the equity return minus the Treasury bond return has a true present value of zero. (These results can easily be generalized to any pair of marketable portfolios and any length period, and the swap market reflects this zero present value.)

To put the matter in its starkest form, $1 *million worth of equity is not worth more than a $1 million worth of Treasury bonds. Current shareholder value is unaffected when the company replaces one with the other, or with any other marketable asset.* (A change in investment strategy can affect shareholder value if other factors, such as corporate taxation and PBGC premiums, are considered.) The corporation can hope, even expect, that the equity will be worth more in the future than the Treasuries, but *that higher expected return is only anticipated compensation for bearing risk, not additional present value.*

Question #2, the discount rate for company withdrawals (or contributions), becomes moot in our example, because we have determined from general principles that the true present value of the company's withdrawals must be zero. The expected equity return exceeds the Treasury return, so the company withdrawal, before discounting, has a positive expected value. The expected value remains positive after applying any single finite discount rate. We conclude that *any single finite discount rate gives a positive and therefore incorrect discounted present value of the company withdrawals, just as it would incorrectly attribute a non-zero value to a swap.*

Although the correct expected present value of the company cost is zero, we may still wish to discount the individually simulated paths to understand the risks inherent in the distribution of costs around their zero mean. Is there any discounting procedure that enables us to observe the distribution while preserving a zero mean? Corporate finance principles require that a discount rate reflect the risk of the cash-flow stream to which it is applied. For example, we would discount the scheduled flows from a noncallable bond at the market yield appropriate to the bond's quality, and the discounted value would be the fair market price. By discounting expected equity returns at the expected equity return rate, we similarly arrive at the market value of the equity.

In our pension fund example, the cash flow that we seek to discount is actually the difference between two flows—the asset return and the liability return—with different risks. We must recognize that these two components should have separate discount rates to reflect their different risk levels. We can then discount each simulated terminal value of assets and liability, as the market does, at its own appropriate discount rate—we discount the Treasury bond maturity value (liability value) at the Treasury rate, and the simulated terminal asset values at the expected asset return rate.[2]

On any particular simulation, the discounted terminal asset value may differ from the initial market value, but the expected discounted value will equal that initial market value. For each simulation, we can then net the separately discounted values of terminal assets and liabilities, with a correct expected net present value of zero.[3]

The standard pension modeling practice of using a single discount rate or yield curve gives the wrong answer: It fails to adjust for the different risks of the asset and liability components of cost, and would therefore show a net present value gain for any asset reallocation (or swap) that raises expected return.

The simple model presented in this article does not offer a unique or all-purpose solution to forecasting questions. It serves two lesser purposes: to illustrate some financial principles regarding the valuation of risky cash flows that any model must respect—arbitrage-free pricing in particular—and to provide a setting in which a model may be tested for compliance with these principles. In real-world pension funding, various deferrals mask the underlying exchange of liability returns for asset returns. But to the plan sponsor, the financial essence of funding remains a swap, which customary pension discount methodology clearly misvalues. So our final question: *If traditional actuarial models and techniques stumble over questions about pension cost and asset allocation for the simple case described here, is there any reason to think that they get it right for real-world pension plans and funding practices?*

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Footnotes

1) The assumption of no default risk was inadvertently omitted from Part 1 as published in the previous issue. The discussion following initially reflects this assumption, but a footnote explains how to adjust for default risk.

2) If the corporate sponsor has default risk, we use its own borrowing rate rather than the Treasury rate on the unfunded portion of the terminal liability. (An unfunded liability can arise only if the assets are *not* invested in the matching Treasury portfolio.) This higher discount rate lowers the liability. By investing in risky assets, the sponsor can then show an average *gain* on the plan, with a corresponding loss to the participants or guarantee agency.

3) Interpreting the results of individual simulations raises some interesting issues that are outside the scope of this brief article.