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## THE NEW STANDARD ORDINARY MORTALITY TABLE

# PROPOSED BY THE SUBCOMMITTEE ON DEFICIENCY RESERVES OF THE NATIONAL ASSOCIATION OF INSURANCE COMMISSIONERS

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#### I. INTRODUCTION

THE meeting of the Society of Actuaries on November 13, 1956, there was presented, in the form of a panel discussion, a report of the Special Committee on New Mortality Tables of the Society. The abstract of that discussion and the formal report of the Special Committee, appended thereto, form a basic record of the work of the Special Committee.

This paper is presented at the suggestion of members of the Special Committee as a means of explaining, in greater detail than was appropriate in its formal report, the methods used in the construction of the new standard ordinary mortality table, temporarily designated as Mortality Table  $X_{17}$ , and the corresponding experience mortality table, temporarily designated as Mortality Table  $X_{18}$ . The work described in this paper was carried out under the instructions of the Special Committee and all basic decisions were either made by, or approved by, the Special Committee.

In general, material on this subject published in the November 1956 *Transactions* of the Society will not be repeated here but will be referred to when pertinent.

## II. MORTALITY RATES REPRESENTING AVERAGE COMPANY EXPERIENCE Ages 20 and over

The first step was to develop mortality rates based on recent ultimate experience among lives insured under standard ordinary policies. In order to determine the most appropriate basis for these mortality rates, a number of experimental mortality tables were constructed on the basis of various combinations of the data compiled by the Committee on Mortali-

ty under Ordinary Insurances and Annuities and published in the annual reports on mortality under standard ordinary insurance issues.

Three different policy anniversary periods, 1946–1950, 1950–1954, and 1946–1954, were considered. The period between 1950 and 1954 anniversaries was selected as the most appropriate because it provided a sufficiently large volume of homogeneous data and was most representative of recent mortality experience. Some consideration was given to the desirability of giving equal weight to each year of experience. As the experience period covered only four years, it was decided that this refinement was not necessary. War deaths arising from the Korean War were shown separately in the basic data and were excluded in order to avoid any distortion of mortality rates at the young adult ages.

After exploring the effect of excluding data for various select periods, it was agreed that the ultimate experience should be based on the experience at durations six and over (i.e., excluding the first five policy years). While the published annual reports for the sixteenth and subsequent policy years show data covering the experience on medical and nonmedical issues combined, the data for the sixth to fifteenth policy years are shown separately for medical issues and for nonmedical issues. It was decided that the experience for the sixth to fifteenth policy years should be based on medical issues only, as tests indicated that including the experience on nonmedical issues would have only a negligible effect on the final mortality rates. The experience for the sixteenth and subsequent policy years includes only a relatively small proportion of nonmedical issues.

As one of the sixteen companies that contributed data for the first fifteen policy years did not contribute data for the sixteenth and subsequent policy years, it was decided to exclude that company's experience at durations six to fifteen and develop ultimate mortality rates on the basis of the combined experience of the remaining fifteen companies, which contributed data for all policy years. The experience of a particular company could easily be excluded because it had been decided, in connection with the problem of determining appropriate margins for individual company variations in mortality, to obtain separate basic data for each contributing company from the Committee on Mortality under Ordinary Insurances and Annuities.

The experience at durations six to fifteen for issue ages 65 and over was also excluded. As individual companies had different maximum issue ages, it was not feasible to determine an appropriate central issue age for the age group covering issue ages 65 and over.

The next problem was to determine an appropriate method for combining the data for the sixth to fifteenth policy years, which are submitted

by quinquennial issue age groups, with the data for the sixteenth and subsequent policy years, which are submitted by individual attained ages. In view of the fact that it was desirable to use the same method for each of the fifteen companies as that used for all fifteen companies combined, an approximate method, which would simplify the work and still produce sufficiently accurate results, was used to combine the data for the sixth to fifteenth policy years with the data for the sixteenth and subsequent policy years.

The method used is illustrated below by showing the particular durations and age groups for which data were combined in order to obtain the exposures and actual deaths that produce the unadjusted ultimate mortality rate at attained age 32. The approximate central attained age at the beginning of the policy year was assumed to be equal to the central issue age plus the average duration minus one.

Policy Year Durations	Average Duration	Age Gro	прв	Central Issue Age	Approximate Central Attained Age
6-8	7	Issue Ages	25-29	27	33
9-13	11	Issue Ages	20-24	22	32
14-15	$14\frac{1}{2}$	Issue Ages	15-19	17	30 <del>1</del>
16 and over		Attained Age	s 30–34		32

The basic data obtained for each company from the Committee on Mortality under Ordinary Insurances and Annuities are described below:

#### 1. Experience on medical issues during policy years 6 to 15

Exposures and actual claims (excluding war deaths) for the combined period between 1950 and 1954 policy anniversaries by quinquennial age groups at issue (10-14, 15-19, . . . 60-64) and policy year duration groups 6-8, 9-13, and 14-15.

2. Combined experience on medical and nonmedical issues during policy years 16 and over

Exposures and actual claims (excluding war deaths) for the combined period between 1950 and 1954 policy anniversaries by quinquennial attained age groups (25-29, 30-34, ... 85-89, 90-95).

Table 1 shows exposures and actual deaths for the combined experience of the fifteen large companies that contributed data for all policy years and indicates exactly how the data for various durations and age groups were combined. The unadjusted death rates per 1,000 obtained from the basic data in Table 1 are shown in Table 2.

The experience between 1950 and 1954 anniversaries is based on an exposure of about \$170,000,000,000 and actual claims of nearly \$2,000,000,000. Although this experience covers only four years, it is larger than the experience between 1930 and 1940 anniversaries which was used in

#### TABLE 1

# MORTALITY EXPERIENCE BY AMOUNTS, EXCLUDING FIRST 5 POLICY YEARS COMBINED EXPERIENCE OF 15 LARGE COMPANIES BETWEEN 1950 AND 1954 POLICY ANNIVERSARIES

				**********	=		
		Exposu	RES AND ACTUAL	DEATHS			
CENTRAL ATTAINED AGE	Durations 6-8*	Durations 9-13*	Durations 14-15*	Durations 16 and over†	Durations 6 and over		
x	Issue Ages $x-7$ to $x-3$ (1)	Issue Ages $x-12$ to $x-8$ (2)	Issue Ages $x-17$ to $x-13$ (3)	Attained Ages $x-2$ to $x+2$ (4)	(1)+(2)+ (3)+(4) (5)		
	Exposures (Unit \$1,000)						
22	1,488,987 2,962,535 4,830,558 6,503,525 6,654,540	1,237,343 2,372,901 4,253,909 5,900,226 6,733,783	528,732 865,398 1,892,723 2,475,233	814,302 2,008,967 4,561,072 8,163,708	2,726,330 6,678,470 11,958,832 18,857,546 24,027,264		
47	5,290,420 3,172,269 1,700,613 635,517 162,842	6,297,852 4,829,019 3,183,331 1,688,383 643,780	2,565,549 2,183,646 1,631,592 970,561 463,613	11,927,337 13,969,847 14,613,129 12,386,938 8,408,595	26,081,158 24,154,781 21,128,665 15,681,399 9,678,830		
72	33,401,806	180,571 	186,060 52,819 	4,915,006 2,444,837 921,787 224,470 38,228 85,398,223	5,281,637 2,497,656 921,787 224,470 38,228 169,937,053		
	Actual Deaths (Unit \$1,000)‡						
22	1,317 2,604 5,166 11,068 19,065	1,382 2,408 4,405 9,399 19,647	546 967 2,683 6,096	927 2,675 8,090 26,841	2,699 6,485 13,213 31,240 71,649		
47	23,310 22,947 18,360 8,988 3,216	31,311 36,985 35,758 25,064 14,464	11,003 15,739 17,933 15,928 12,984	62,975 122,602 207,167 281,925 287,960	128,599 198,273 279,218 331,905 318,624		
72		6,235	6,939 2,505	258,284 182,367 103,828 36,135 8,576	271,458 184,872 103,828 36,135 8,576		
Total	116,041	187,058	93,323	1,590,352	1,986,774		

<sup>\*</sup> Medical Issues Only.

<sup>†</sup> Medical and Nonmedical Issues Combined.

<sup>‡</sup> Excluding War Deaths.

the construction of the 1941 CSO Table. The 1930-1940 experience was based on an exposure of about \$150,000,000,000 and actual claims of about \$1,800,000,000.

Table 3 compares the unadjusted ultimate mortality rates shown in column (5) of Table 2 with the unadjusted ultimate mortality rates that were used in the construction of the 1941 CSO Table. This comparison

TABLE 2

MORTALITY EXPERIENCE BY AMOUNTS, EXCLUDING FIRST 5 POLICY YEARS

COMBINED EXPERIENCE OF 15 LARGE COMPANIES BETWEEN

1950 AND 1954 POLICY ANNIVERSARIES

	Unadj	USTED DEATH RA	TE PER 1,000 D	ERIVED FROM TA	вце 1
CENTRAL ATTAINED AGE	Durations 6-8*	Durations 9-13*	Durations 14-15*	Durations 16 and over†	Durations 6 and over
x	Issue Ages $x-7$ to $x-3$ (1)	Issue Ages x-12 to x-8 (2)	Issue Ages $x-17$ to $x-13$ (3)	Attained Ages $x-2$ to $x+2$ (4)	Attained Age x (5)
22	.88 .88 1.07 1.70 2.86	1.12 1.01 1.04 1.59 2.92	1.03 1.12 1.42 2.46	1 .14 1 .33 1 .77 3 .29	.99 .97 1.10 1.66 2.98
17	4.41 7.23 10.80 14.14 19.75	4.97 7.66 11.23 14.84 22.47	4.29 7.21 10.99 16.41 28.01	. 5.28 8.78 14.18 22.76 34.25	4.93 8.21 13.22 21.17 32.92
72 77 32 37				52.55 74.59 112.64 160.98 224.34	51.40 74.02 112.64 160.98 224.34

<sup>\*</sup> Medical Issues Only.

indicates the extent of the improvement in mortality between the 1930–1940 period used for the 1941 CSO Table and the 1950–1954 period used for the new mortality table.

It was agreed that the dip in mortality rates at age 27, shown in column (5) of Table 2, should be eliminated in order to avoid anomalies in net premiums and reserves. This was accomplished by decreasing the mortality rate at age 22 from .99 to .89 and increasing the mortality rate at age 32 from 1.10 to 1.20. The mortality rates at ages 37 and over did not

<sup>†</sup> Medical and Nonmedical Issues Combined.

<sup>‡</sup> Excluding War Deaths.

require any preliminary adjustment or graduation because of the extremely large volume of data and the smooth progression of these mortality rates. The mortality rates shown in column (5) of Table 2, with the adjustments at ages 22 and 32 referred to above, were considered to represent recent average company ultimate experience among standard insured lives at ages 20 and over.

TABLE 3
UNADJUSTED DEATH RATE PEK 1,000

Central Attained Age	Experience be- tween 1930 and 1940 Anniversaries*	Experience be- tween 1950 and 1954 Anniversaries (2)	Ratio of (2) to (1) (3)
22		.99	49.5%
27		.97	48.3
32		1.10	47.4
37		1.66	<b>54.2</b>
42	4.93	2.98	60.4
47	7.25	4.93	68.0
52	. 12.00	8.21	68.4
57	. 18.30	13.22	72.2
62	. 28.26	21.17	74.9
67		32.92	77.0
72	63.46	51.40	81.0
77	95.16	74.02	77.8
32		112.64	79.7
37		160.98	76.4
92		224.34	79.3

<sup>\*</sup> TASA XLII, 323.

### Ages under 20

In order to develop mortality rates based on recent experience among standard insured lives under age 20, Mr. Bert A. Winter, a member of the Special Committee, obtained and analyzed available data from four large life insurance companies. These data consisted of exposures and actual deaths, by amounts of insurance, on standard ordinary policies issued at ages 0 to 9. The data covered the experience on both medical and non-medical issues. The periods of observation for which data were available, while close to the 1950–1954 period used for the adult ages, varied somewhat by company as follows:

Equitable: 1937 and later issues observed for the policy year beginning in 1950; 1940 and later issues for the policy years beginning in 1951 and 1952. War deaths were excluded.

Metropolitan: 1939 and later issues observed for the calendar years 1949 through 1951 and for the policy years beginning in 1951 and 1952. (One-half of calendar year 1951 figures were excluded.) War deaths were included.

New York Life: 1942 and later issues observed between policy year anniversaries in 1948 and 1953. War deaths were included.

Prudential: 1938 and later issues observed for the calendar years 1949 through 1953. War deaths were included.

The above data were supplemented by intercompany experience during the first five policy years on standard ordinary policies issued at ages 10 to 14. These intercompany data were compiled by the Committee on Mortality under Ordinary Insurances and Annuities and were obtained from the published annual reports on mortality under standard ordinary insurance issues. The data covered the experience on medical issues between 1948 and 1950 anniversaries and the experience on both medical and nonmedical issues between 1950 and 1953 anniversaries. War deaths were included because they were relatively insignificant at these issue ages and durations.

After comparing the mortality rates obtained from these five sources, it was agreed that the five sets of data could be combined even though they were not exactly homogeneous. It should be noted, however, that the Metropolitan and Prudential calendar year data required special treatment so that they would reflect the experience in policy years following integral ages. The combined experience at ages 0 to 19 is based on over \$8,000,000 of actual claims. The unadjusted mortality rates based on this combined experience are shown in Table 4.

After some preliminary graduations of these rates were examined, it was agreed that the unadjusted mortality rates at ages 2, 7, 12, and 17 could be used to represent recent average company experience among standard insured lives at ages 2 to 19.

At ages under 2, because of the rapidly changing slope of the mortality curve, it seemed advisable to take more refined account of the exact age distribution of policies issued at age 0 nearest birthday. As the Prudential contributed more than half the total data at age 0, a sample was taken of 500 Prudential policies recently issued at age 0 nearest birthday. The issue age was expressed in completed weeks for the first two months of life and in completed months for the remaining four months involved. The age distribution produced by this sample is shown in Table 5.

This sample indicated that the average issue age for policies issued at age 0 nearest birthday was about 55 days. Smaller samples for the Metropolitan in 1947 and 1953, and for the Prudential in 1947, gave similar distributions. It was, therefore, assumed that this distribution by exact issue age applies to the combined data for the four large companies.

Using this age distribution, an average first policy year mortality rate for age at issue 0 was calculated on the basis of U.S. Total White population mortality experience in 1949–1951 (as set forth in Numbers 1 and 3 of Volume 41 of the Special Reports of the National Office of Vital Statistics, U.S. Department of Health, Education, and Welfare). This experience shows separate mortality rates by days in the first week of life, by weeks in the first month of life, and by months in the first year of life. The

TABLE 4

MORTALITY EXPERIENCE BY AMOUNTS AT AGES 0 TO 19—EXPERIENCE OF 4 LARGE COMPANIES FOR ISSUE AGES 0 TO 9 COMBINED WITH INTERCOMPANY EXPERIENCE FOR ISSUE AGES 10 TO 14 (FIRST 5 POLICY YEARS)—EXPERIENCE GENERALLY COVERS PERIOD BETWEEN 1948 AND 1953 POLICY ANNIVERSARIES

Attained Age	Unadjusted Death Rate per 1,000	Attained Age	Unadjusted Death Rate per 1,000
0	2.74 1.00 .75 .67	10	.38 .33 .37 .39 .54
5	.53 .46 .44 .40 .43	15	.50 .74 .72 .93 1.10

population mortality rate derived from the above data was 7.65 per 1,000, corresponding to the first policy year mortality rate on lives distributed in accordance with the Prudential sample which had an average effective age at issue of about 55 days. The corresponding population mortality rate for a year beginning at age one day is 17.69 per 1,000 and for a year beginning at birth is 27.22 per 1,000.

As the age 0 mortality rate for insured lives would also tend to increase as the true average age at issue decreases, the unadjusted mortality rate of 2.74 per 1,000 shown for age 0 in Table 4 was adjusted to reflect an assumed issue age of one day. An issue age of one day was selected in order to eliminate the relatively large number of deaths occurring during the first day of life. It was felt that very few lives would be insured during the first day of life.

This adjustment was made by assuming that the ratio of the insured

life mortality rate to the population mortality rate for a group of lives distributed in accordance with the Prudential sample for age 0 at issue (i.e., 35.8% = 2.74/7.65) would also be applicable for an issue age of one day. The adjusted age 0 mortality rate for insured lives, based on an assumed issue age of one day, was calculated to be 6.33 per 1,000 by taking 35.8% of the corresponding population mortality rate of 17.69 per 1,000. A similar test indicated that the effect of a more refined age distribution on the mortality rate for age 1 would be negligible.

TABLE 5
AGE DISTRIBUTION OF 500 PRUDENTIAL POLICIES
ISSUED AT AGE 0 NEAREST BIRTHDAY IN 1956

Completed Period of Life	Average Number of Days (1)	% of Sample (2)	(1)×(2) (3)
Less than 1 week	3	3%	.09
1 week	10	11	1.10
2 weeks	17	13	2.21
3 weeks	24	10	2.40
4 weeks	31	9	2.79
5 weeks	38	8 1	3.04
6 weeks	45	6	2.70
7 weeks	54	7	3.78
2 months	75	12	9.00
3 months	105	8	8.40
4 months	135	7	9.45
5 months	165	6	9.90
Total	55	100%	54.86

It was agreed to use the adjusted mortality rate of 6.33 per 1,000 for age 0 and the unadjusted mortality rate of 1.00 per 1,000 shown in Table 4 for age 1 to represent recent average company experience among standard insured lives at attained ages 0 and 1, respectively.

Although the data used to develop mortality rates based on recent experience among standard insured lives under age 20 are not exactly comparable to the data used at ages 20 and over, it was felt that the two sets of mortality rates could be joined to form a basis for the construction of a new mortality table. It was decided to use readily available data for lives under age 20 rather than to delay the entire job by attempting to obtain special data that were exactly comparable to the data used at ages 20 and over and that would probably produce mortality rates that did not differ significantly from those produced by the available data at ages under 20.

#### III. MORTALITY RATES COVERING INDIVIDUAL COMPANY VARIATIONS

It was agreed that mortality rates representing average company experience, such as those presented in Section II, would not provide an appropriate basis for the construction of a mortality table that all companies operating in the United States could safely use for the valuation of policies issued at standard premium rates. The various reasons for using an adjusted mortality table with appropriate margins over average experience were clearly described by Mr. Henry F. Rood (TSA VIII, 506) and will not be repeated here.

In order to obtain some guide as to the appropriate level for these margins, it was decided to obtain more specific information about variations in ultimate mortality rates by individual company. Unadjusted death rates, corresponding to those shown in column (5) of Table 2 for the combined experience of fifteen large companies, were obtained for each of these companies. Table 6 shows, for the quinquennial age group corresponding to each central age, the ratios of the five highest individual company death rates for that age group to the corresponding death rate based on the combined experience of the fifteen large companies. Comparable values of the differences between the individual company death rates per 1,000 and the corresponding combined experience death rate per 1,000 are also shown in Table 6.

In order to take account of the mortality experience of companies that do not contribute to the annual mortality studies of the Society, a special questionnaire was sent on August 24, 1956 to the senior actuary of each of these companies that had an actuary who was a member of the Society. This questionnaire was sent to companies of all sizes, all locations and of various types and asked them to contribute available data indicative of their current mortality levels. The companies were requested to submit data corresponding to those available for the companies that contribute to the annual mortality studies of the Society and to note appropriate exceptions where the data submitted did not exactly correspond.

Appropriate mortality data were submitted by 33 companies that do not contribute to the annual mortality studies of the Society. While these data were generally not exactly homogeneous with the data available for the fifteen large companies, it was felt that the differences in mortality rates arising from lack of homogeneity were not of sufficient magnitude for these 33 companies to prevent the use of their mortality rates as an auxiliary guide for the determination of appropriate mortality margins to cover individual company variations. The death rates of these 33 companies are compared in Table 7 with the unadjusted death rates representing the combined experience of the fifteen large companies.

TABLE 6

MORTALITY EXPERIENCE BY AMOUNTS, EXCLUDING FIRST 5 POLICY YEARS

EXPERIENCE OF EACH OF 15 LARGE COMPANIES BETWEEN

1950 AND 1954 POLICY ANNIVERSARIES

	DATA FOR COMPANIES THAT HAVE THE 5 HIGHEST							
CENTRAL		DEATH RA	TES AT EACH CE	NTRAL AGE				
Age	5th Highest	4th Highest	2-3 77:-1	2nd Highest	Highest			
	Rate	Rate	3rd Highest Rate	Rate	Rate			
	Ratio of Individual Company Death Rate to Combined  Experience Death Rate (15 Large Companies)							
		T			<u> </u>			
22	124.2%	128.3%	134.3%	145.5%	151.5%			
27	108.2	108.2	114.4	160.8	199.0			
32	106.4	107.3	108.2	116.4	138.2			
37	106.0	107.8	110.8	116.9	127.7			
42	106.0	106.4	110.7	117.4	128.5			
47,	103.7%	103.9%	106.3%	107.7%	110.3%			
52	103.8	103.9	104.6	105.0	111.4			
57	101.6	103.0	104.1	105.8	112.4			
52	103.1	103.6	103.8	110.1	114.7			
57	101.9	102.2	102.2	103,0	109.6			
72	100.0%	100.1%	102.5%	102.6%	106.4%			
77	107.5	109.5	109.7	111.5	113.8			
82	102.3	102.4	108.1	111.3	116.8			
87	108.3	110.4	111.0	117.4	157.4			
92	118.7	122.8	127.8	128.0	161.8			
,								
	Excess of Individual Company Death Rate per 1,000 over Com-							
	b	ined Experience	Death Rate (15	Large Companie	es)			
22	.24	.28	.34	.45	.51			
27	.08	.08	.14	.59	.96			
	.07	.08	.09	.18	.42			
32								
37	.10	.13	.18	.28	.46			
12	.18	.19	.32	.52	.85			
1								
	.18	.19	.31	.38	.51			
52	.31	.32	.38	.41	.94			
52	.31 .21	.32 .40						
52 57 52	.31	.32 .40 .77	.38 .54 .80	.41 .77 2.13	.94 1.64 3.11			
52 57 52	.31 .21	.32 .40	.38 .54	.41 .77	.94 1.64			
52 57 62 67	.31 .21 .65 .62	.32 .40 .77 .71	.38 .54 .80 .74	.41 .77 2.13 .98	.94 1.64 3.11 3.15			
52 57 62 67	.31 .21 .65 .62	.32 .40 .77 .71	.38 .54 .80 .74 1.26 7.17	.41 .77 2.13 .98	.94 1.64 3.11 3.15			
52	.31 .21 .65 .62	.32 .40 .77 .71	.38 .54 .80 .74	.41 .77 2.13 .98	.94 1.64 3.11 3.15			
47 52	.31 .21 .65 .62 .00 5.54	.32 .40 .77 .71 .03 7.03	.38 .54 .80 .74 1.26 7.17	.41 .77 2.13 .98 1.35 8.51	.94 1.64 3.11 3.15 3.27 10.24			
52	.31 .21 .65 .62 .00 5.54 2.60	.32 .40 .77 .71 .03 7.03 2.65	.38 .54 .80 .74 1.26 7.17 9.10	.41 .77 2.13 .98 1.35 8.51 12.70	.94 1.64 3.11 3.15 3.27 10.24 18.91			

#### TABLE 7

## MORTALITY EXPERIENCE BY AMOUNTS, GENERALLY EXCLUDING FIRST 5 POLICY YEARS

## EXPERIENCE OF EACH OF 33 COMPANIES, GENERALLY BETWEEN 1950 AND 1954 POLICY ANNIVERSARIES

Central	DATA FOR COMPANIES THAT HAVE THE 5 HIGHEST DEATH RATES AT EACH CENTRAL AGE									
Age	5th Highest	4th Highest	3rd Highest	2nd Highest	Highest					
	Rate	Rate	Rate	Rate	Rate					
	Rat	Ratio of Individual Company Death Rate to Combined Experience Death Rate (15 Large Companies)								
22	160.6%	189.9%	191.9%	218.2%	535.4%					
	145.4	146.4	173.2	178.4	253.6					
	138.2	142.7	143.6	144.5	165.5					
	126.5	127.7	138.6	162.0	174.7					
	124.8	126.8	144.3	151.7	156.0					
47	114.2%	116.6%	121.3%	125.8%	175.5%					
	115.0	116.6	117.2	119.6	128.5					
	116.9	118.4	119.3	120.0	141.4					
	105.9	106.6	115.8	118.8	130.3					
	106.4	110.7	111.5	114.9	122.2					
72	112.5%	114.5%	116.6%	128.9%	141.0%					
	105.9	111.1	115.4	120.0	131.7					
	126.7	138.9	144.2	166.5	368.2					
	129.3	152.7	199.7	227.6	231.1					
	115.8	177.0	179.3	196.1	202.5					
		of Individual Co ned Experience								
22	.60 .44 .42 .44	.89 .45 .47 .46	.91 .71 .48 .64 1.32	1.17 .76 .49 1.03 1.54	4.31 1.49 .72 1.24 1.67					
47	.70	.82	1.05	1.27	3.72					
	1.23	1.36	1.41	1.61	2.34					
	2.23	2.43	2.55	2.64	5.47					
	1.25	1.39	3.34	3.99	6.42					
	2.12	3.51	3.79	4.89	7.32					
72	6.43	7.46	8.52	14.87	21.07					
	4.35	8.25	11.41	14.79	23.45					
	30.12	43.83	49.75	74.94	302.05					
	47.16	84.77	160.45	205.43	211.11					
	35.55	172.76	177.81	215.58	229.87					

This analysis of variations in individual company mortality rates indicated that a U-shaped pattern of margins might be appropriate with a minimum margin of from 10% to 15% in the central range of ages between 47 and 77 and with higher percentage margins at younger and older ages. These preliminary margins were not determined by attempting to clear the highest individual company mortality rate at each quinquennial age, but rather by attempting to draw a smooth curve that would clear, on the average, about 90% of the individual company mortality rates. In other words, the preliminary margins were determined so that, on the average, only about 4 or 5 of the 48 companies would have higher mortality rates at a particular central age than those produced by the curve. Naturally, the companies with higher mortality rates at one central age were not the same as those with higher mortality rates at another central age.

The preliminary margins that were selected and the various reasons for modifying these preliminary margins were described in the discussion prepared by Mr. James T. Phillips (TSA VIII, 509). In modifying the preliminary margins, account was taken of the suggestion of the NAIC Subcommittee on Deficiency Reserves that the mortality rates in the final table should not exceed those shown in the U.S. White Males 1949–51 Table except at the very old ages. At the November 1956 meeting of the Society, arguments for adopting this criterion were presented by Mr. W. Harold Bittel (TSA VIII, 519) and arguments against adopting this criterion were presented by Mr. Elgin G. Fassel (TSA VIII, 520).

The margins finally adopted were as follows:

- 1. A flat 15% margin was used at ages 52 to 92.
- 2. The 15% margin at age 52 is equivalent to a margin of 1.23 deaths per 1,000 and this constant margin was graded down to a value of .24 per 1,000 at age 12 and then up to a value of .65 per 1,000 at ages 1 and 0.

Table 8 shows mortality rates representing average company experience, the final margins represented both as a constant and as a percentage, mortality rates covering individual company variations, and U.S. White Males 1949-51 mortality rates.

While a flat 15% margin was used at ages 52 to 92, it was not possible to obtain a simple mathematical formula that would produce satisfactory margins below age 52. The margins over  $1,000q_x$  below age 52 were determined so that they would grade fairly smoothly into the margin over  $1,000q_x$  at age 52 and so that they would produce graduated mortality rates that would not exceed those shown in the U.S. White Males 1949–51 Table (see Table 9).

#### IV. CONSTRUCTION OF MORTALITY TABLES $X_{17}$ AND $X_{18}$

The next problem was to determine which set of mortality rates should be graduated first, the mortality rates representing average company experience or the mortality rates covering individual company variations. While the 1941 CSO Table was graduated both before and after margins were added, the major concern of the Special Committee was the construction of an adjusted mortality table with appropriate margins over average experience. It was decided, therefore, to proceed directly with the graduation of the mortality rates covering individual company variations.

It was felt that the primary emphasis in selecting a method of graduation should be on closeness of fit. Preliminary tests quickly demonstrated the fact that a Makeham formula would not produce a satisfactory fit over a wide range of ages. In view of the importance of reproducing closely the agreed upon level of mortality rates, it was felt that the best results

TABLE 8

MARGINS REQUIRED TO COVER
INDIVIDUAL COMPANY VARIATIONS IN MORTALITY
(Mortality rate per 1,000)

	MORTALITY RATE REPRE-	Ma	RGIN	MORTALITY RATE COVERING	U.S. White Males
Age	senting Aver- age Company Experience (1)	Over 1,000q <sub>x</sub> (2)	% of (1) (3)	Individual Company Variations (4) = (1) + (2)	1949-51 Mortality Rate (5)
0	6.33	.65	10.3%	6.98	30.69
1	1.00	.65	65.0	1.65	2,12
2	.75	.49	65.3	1.24	1.38
7	.44	.29	65.9	.73	.68
12	.37	.24	64.9	.61	.67
17	.72	.58	80.6	1.30	1.33
22	.89 .97 1.20 1.66 2.98 4.93 8.21	.63 .68 .78 .88 .98 1.08	70.8 70.1 65.0 53.0 32.9 21.9 15.0	1.52 1.65 1.98 2.54 3.96 6.01 9.44	1.74 1.69 2.01 2.94 4.77 7.71 12.12
57	13.22	1.98	15.0	15.20	18.82
	21.17	3.18	15.0	24.35	27.72
	32.92	4.94	15.0	37.86	40.00
	51.40	7.71	15.0	59.11	58.87
	74.02	11.10	15.0	85.12	87.89
	112.64	16.90	15.0	129.54	128.02
	160.98	24.15	15.0	185.13	188.98
	224.34	33.65	15.0	257.99	257.66

would be obtained by using a modified osculatory interpolation formula. The particular formula selected was the fifth difference formula developed by Mr. W. A. Jenkins (TASA XXVIII, 202). While this formula does not exactly reproduce the pivotal values at quinquennial ages, the variations from the original data are small and tend to smooth out any irregularities in the original data. Both first and second derivatives are continuous in the interpolated series.

The mortality rates shown in column (4) of Table 8 for quinquennial ages 2, 7, ..., 92 were used as pivotal values and graduated by the formula described in the preceding paragraph to produce graduated mortality rates at individual ages 2 to 92, inclusive. The mortality rates shown in column (4) of Table 8 for ages 0 and 1 were used without any modification as the graduated mortality rates at these ages.

The selection of an appropriate terminal age for the new mortality table was considered, and it was decided that the practical advantages of making the mortality rate at age 99 equal to 1, following the practice used in the construction of the 1941 CSO Table, outweighed any theoretical reasons for using a higher terminal age. Graduated mortality rates at ages 93 to 99, inclusive, were obtained by setting the mortality rate at age 99 equal to 1 and extending third differences of the mortality rates at ages 92 and under so as to make a smooth progression to the end of the mortality table.

The graduated mortality rates were tested for smoothness by inspection of the third differences. Only a few minor adjustments in the last decimal place were required to produce a satisfactory series of third differences.

This completed the construction of the final series of graduated mortality rates covering individual company variations that was designated as Mortality Table  $X_{17}$ . Values of  $1,000q_x$  on Mortality Table  $X_{17}$  are compared in Table 9 with corresponding values on the U.S. White Males 1949–51 Table. The Mortality Table  $X_{17}$  mortality rates do not exceed the U.S. White Males 1949–51 mortality rates at any age below 80, are slightly higher at ages 80 to 83, are lower at ages 84 to 90, and are higher at ages 91 and over.

As it was felt that there would be some interest in a graduated mortality table representing average company experience, this table was constructed by subtracting interpolated values of the margins shown in column (2) of Table 8 from the Mortality Table  $X_{17}$  mortality rates at ages 92 and under. Mortality rates at ages 93 to 100, inclusive, were obtained by setting the mortality rate at age 100 equal to 1 and extending third differences of the mortality rates at ages 92 and under so as to make

TABLE 9 Comparison of Values of 1,000 $q_z$  on Mortality Tables  $X_{17}$  and  $X_{18}$  with Values on U.S. White Males 1949–51 Table

		MAR	GIN	<del></del>	
Age	MORTALITY TABLE X <sub>18</sub> (1)	Over $1,000q_x$ $(4)-(1)$ $(2)$	% (2)÷(1) (3)	Mortality Table X <sub>17</sub> (4)	U.S. WHITE MALES 1949~51 (5)
0	6.33 1.00 .78 .66	.65 .65 .51 .40	10.3% 65.0 65.4 60.6 55.2	6.98 1.65 1.29 1.06	30.69 2.12 1.38 1.06 .90
5	.52 .47 .43 .40	.28 .26 .24 .23 .23	53.8 55.3 55.8 57.5 60.5	.80 .73 .67 .63	.82 .74 .68 .63
10	.37 .39 .43 .47	.23 .23 .24 .28 .36	62.2 59.0 55.8 59.6 70.6	.60 .62 .67 .75	.60 .62 .67 .76 .90
15 16 17 18 19	.55 .61 .67 .75 .81	.45 .52 .58 .59	81.8 85.2 86.6 78.7 74.1	1.00 1.13 1.25 1.34 1.41	1.05 1.20 1.33 1.43 1.53
20	.85 .87 .89 .90	.61 .62 .63 .64 .65	71.8 71.3 70.8 71.1 70.7	1.46 1.49 1.52 1.54 1.57	1.62 1.69 1.74 1.76 1.74
25	.93 .95 .98 1.00 1.04	.66 .67 .68 .70	71.0 70.5 69.4 70.0 69.2	1.59 1.62 1.66 1.70 1.76	1.71 1.68 1.69 1.72 1.76
30	1.08 1.13 1.18 1.24 1.32	.74 .76 .78 .80 .82	68.5 67.3 66.1 64.5 62.1	1.82 1.89 1.96 2.04 2.14	1.82 1.90 2.01 2.14 2.30
35	1.41 1.53 1.68 1.87 2.10	.84 .86 .88 .90	59.6 56.2 52.4 48.1 43.8	2.25 2.39 2.56 2.77 3.02	2.48 2.69 2.94 3.23 3.55

TABLE 9-Continued

			<del></del>		
	Margin				
Age	MORTALITY TABLE X18	Over $1,000q_x$ (4)-(1)	% (2)÷(1)	MORTALITY TABLE X17	U.S. White Males 1949-51
	(1)	(2)	(3)	(4)	(5)
40	2.36	.94	39.8%	3.30	3.91
	2.64	.96	36.4	3.60	4.31
	2.95	.98	33.2	3.93	4.77
	3.28	1.00	30.5	4.28	5.26
	3.63	1.02	28.1	4.65	5.79
45	4.02	1.04	25.9	5.06	6.37
	4.45	1.06	23.8	5.51	7.01
	4.92	1.08	22.0	6.00	7.71
	5.46	1.09	20.0	6.55	8.46
	6.06	1.10	18.2	7.16	9.26
50	6.72	1.12	16.7	7.84	10.12
	7.45	1.15	15.4	8.60	11.06
	8.21	1.23	15.0	9.44	12.12
	9.02	1.35	15.0	10.37	13.28
	9.92	1.49	15.0	11.41	14.53
55	10.91	1.64	15.0	12.55	15.87
	12.01	1.80	15.0	13.81	17.31
	13.22	1.98	15.0	15.20	18.82
	14.55	2.18	15.0	16.73	20.41
	15.99	2.40	15.0	18.39	22.06
60	17.57	2.63	15.0	20.20	23.81
	19.28	2.89	15.0	22.17	25.69
	21.12	3.16	15.0	24.28	27.72
	23.10	3.46	15.0	26.56	29.85
	25.25	3.79	15.0	29.04	32.07
65	27.61	4.14	15.0	31.75	34.45
	30.21	4.53	15.0	34.74	37.07
	33.08	4.96	15.0	38.04	40.00
	36.24	5.44	15.0	41.68	43.19
	39.66	5.95	15.0	45.61	46.64
70	43.30	6.49	15.0	49.79	50.27
	47.09	7.06	15.0	54.15	54:34
	51.00	7.65	15.0	58.65	58.87
	55.01	8.25	15.0	63.26	63.84
	59.23	8.89	15.0	68.12	69.21
75	63.80	9.57	15.0	73.37	74.99
	68.85	10.33	15.0	79.18	81.21
	74.52	11.18	15.0	85.70	87.89
	80.92	12.14	15.0	93.06	94.87
	87.99	13.20	15.0	101.19	102.14
80.	95.64	14.34	15.0	109.98	109.93
81.	103.78	15.57	15.0	119.35	118.48
82.	112.32	16.85	15.0	129.17	128.02
83.	121.20	18.18	15.0	139.38	138.75
84.	130.45	19.56	15.0	150.01	150.53

	Manager	Margin		35	U.S.
Асе	MORTALITY TABLE X <sub>18</sub> (1)	Over $1,000q_x$ (4) – (1) (2)	% (2)÷(1) (3)	MORTALITY TABLE X <sub>17</sub> (4)	WHITE MALES 1949-51 (5)
85	140.12	21.02	15.0%	161.14	163.04
	150.27	22.55	15.0	172.82	175.96
	160.98	24.15	15.0	185.13	188.98
	172.39	25.86	15.0	198.25	202.02
	184.75	27.71	15.0	212.46	215.28
90	198.38	29.76	15.0	228.14	228.90
	213.71	32.06	15.0	245.77	242.99
	231.24	34.69	15.0	265.93	257.66
	251.47	37.83	15.0	289.30	273.05
	274.90	41.76	15.2	316.66	289.06
95	303.03	48.21	15.9	351.24	305.53
	343.36	57.20	16.7	400.56	322.27
	409.79	78.63	19.2	488.42	339.11
	522.62	145.53	27.8	668.15	356.17
	708.55	291.45	41.1	1000.00	373.57
100	1000.00				391.12

TABLE 9-Continued

a smooth progression to the end of the mortality table. The decision to make the mortality rate at age 100 equal to 1 conforms with the practice followed for the unloaded mortality table used in the construction of the 1941 CSO Table and for the CSO Basic Table obtained by deloading the 1941 CSO Table.

The mortality rates obtained in this manner were tested for smoothness by inspection of the third differences. This test indicated that a special graduation of these mortality rates was not needed. Only a few minor adjustments in the last decimal place were required to produce a satisfactory series of third differences.

The final series of mortality rates representing average company experience was designated as Mortality Table  $X_{18}$ . Values of  $1,000q_x$  on Mortality Table  $X_{18}$  are compared in Table 9 with corresponding values of  $1,000q_x$  on Mortality Table  $X_{17}$ .

The margins provided by Mortality Table  $X_{17}$  over Mortality Table  $X_{18}$  are compared in Table 10 with the margins provided by the 1941 CSO Table over the CSO Basic Table.

Table 10 indicates that the percentage margins provided by Mortality Table  $X_{17}$  over Mortality Table  $X_{18}$  are generally similar to the percent-

age margins provided by the 1941 CSO Table over the CSO Basic Table at ages 62 and under and are higher at ages 67 and over. The higher margins at the older ages reflect some allowance for the lower mortality level at the older ages on fully paid-up policies than on premium-paying policies (TSA 1955 Reports, 10). It was felt that the recent trend towards lower premium forms of insurance would undoubtedly tend to reduce the proportion of paid-up business at the older ages. These margins were discussed in greater detail by Mr. James E. Hoskins (TSA VIII, 513).

#### V. MORTALITY EXPERIENCE ON BASIS OF NEW MORTALITY TABLES

In order to indicate how Mortality Tables  $X_{17}$  and  $X_{18}$  compare with recent average company ultimate mortality experience among standard insured lives at ages 20 and over, ratios of actual to expected deaths were computed for the combined experience of the fifteen large companies between 1950 and 1954 policy anniversaries. The exposures shown in column

TABLE 10 COMPARISON OF MARGINS PROVIDED BY MORTALITY TABLE  $X_{17}$  OVER MORTALITY TABLE  $X_{18}$  WITH MARGINS PROVIDED BY 1941 CSO TABLE OVER CSO BASIC TABLE

Age	MARGINS PROVIDED BY MORTALITY TABLE X <sub>17</sub> OVER MORTALITY TABLE X <sub>18</sub>		MARCINS PROVIDED BY 1941 CSO TABLE OVER CSO BASIC TABLE	
	Margin in 1,000q <sub>x</sub>	% Margin	Margin in 1,000q <sub>x</sub>	% Margin
2	.51	65.4%	.77	22.8%
	.24	55.8	.83	50.6
	.24	55.8	.89	86.4
	.58	86.6	.98	77.2
22	.63	70.8	1.07	70.4
	.68	69.4	1.19	62.0
	.78	66.1	1.34	51.9
	.88	52.4	1.52	41.9
	.98	33.2	1.75	33.1
47	1.08	22.0	2.05	26.1
	1.23	15.0	2.44	20.6
	1.98	15.0	2.97	16.5
	3.16	15.0	3.70	13.5
	4.96	15.0	4.73	11.3
72	7.65	15.0	6,21	9.8
	11.18	15.0	8,40	8.8
	16.85	15.0	11,73	8.2
	24.15	15.0	17,00	8.1
	34.69	15.0	26,09	8.8

(5) of Table 1 were used to compute expected deaths on the basis of the 1941 CSO Table, CSO Basic Table, Mortality Table  $X_{17}$  and Mortality Table  $X_{18}$ . Actual deaths shown in column (5) of Table 1 are compared in Table 11 with expected deaths computed on the basis of each of these four mortality tables.

TABLE 11

MORTALITY EXPERIENCE BY AMOUNTS, EXCLUDING FIRST 5 POLICY YEARS

COMBINED EXPERIENCE OF 15 LARGE COMPANIES BETWEEN

1950 AND 1954 POLICY ANNIVERSARIES

1	ACTUAL DEATHS (UNIT \$1,000)	RATIOS OF ACTUAL TO EXPECTED DEATHS ON			
AGES		1941 CSO Table	CSO Basic Table	Table X17	Table X18
20–29 30–39	\$ 9,184 44,453	33.0%	54.1% 44.8	60.3%	102.4% 97.1
40–49	200,248	46.9	60.3	79.8	100.5
50-59	477,491	60.5	71.5	86.9	100.0
60–69	650,529	69.2	77.8	86.9	99.9
70-79	456,330	72.7	79.5	87.1	100.2
80 and over	148,539	72.4	78.4	87.0	100.0
20 and over	\$1,986,774	62.9%	73.2%	85.2%	100.0%

Table 11 indicates that the ratios of actual to expected deaths are 62.9% on the basis of the 1941 CSO Table, 73.2% on the basis of the CSO Basic Table, 85.2% on the basis of Mortality Table  $X_{17}$ , and 100.0% on the basis of Mortality Table  $X_{18}$ . The average percentage margin of Mortality Table  $X_{17}$  over Mortality Table  $X_{18}$  is

$$17\% \left(i.e., \frac{100.0\% - 85.2\%}{85.2\%}\right)$$

or about the same as the average percentage margin of the 1941 CSO Table over the CSO Basic Table on the basis of current exposures, namely,

$$16\% \left(i.e., \frac{73.2\% - 62.9\%}{62.9\%}\right).$$

Table 11 also indicates that Mortality Table  $X_{18}$  provides an excellent fit to recent average company experience. The minor variations from 100% at ages 20-29 and 30-39 reflect the decision to eliminate the dip in the unadjusted mortality rates in the 20-29 age range.

In order to indicate the range of variation in mortality ratios at ages

20 and over by individual company, the aggregate experience in Table 11 is shown separately in Table 12 for each of the fifteen large companies that contribute data for all policy years to the annual studies of mortality under standard ordinary insurance issues. The companies shown in Table 12 are listed in ascending order in accordance with the mortality ratio computed on the basis of the 1941 CSO Table. Individual company ratios were not computed on the basis of the CSO Basic Table.

TABLE 12

MORTALITY EXPERIENCE BY AMOUNTS, EXCLUDING
FIRST 5 POLICY YEARS

INDIVIDUAL EXPERIENCE OF 15 LARGE COMPANIES AT AGES 20 AND OVER BETWEEN 1950 AND 1954 POLICY ANNIVERSARIES

	RATIO OF ACTUAL TO EXPECTED DEATHS ON			
Company Letter	1941 CSO Table	Table X17	Table X18	
A	59.9%	80.4%	93.7%	
B	60.1	81.0	94.7	
C	60.9	86.3	102.7	
D	61.1	84.4	99.0	
E	61.4	81.0	94.4	
F	61.5	85.7	101.1	
G	62.2	82.6	96.3	
Н	63.8	85.2	99.4	
[ <i></i>	64.0	85.2	99.3	
J	64.0	87.0	102.0	
к	64.3	85.4	99.7	
L	64.3	86.3	100.6	
М	64.9	85.1	99.0	
N	67.2	89.2	103.8	
O	67.7	89.3	104.0	
Total	62.9%	85.2%	100.0%	

Table 12 indicates that, among the fifteen large companies, the highest individual company mortality ratio on the basis of Mortality Table  $X_{17}$  is 89.3%.

Table 13 shows comparable data for the 33 companies that did not contribute to the annual mortality studies of the Society. Among these 33 companies, the highest mortality ratio on the basis of Mortality Table  $X_{17}$  is 95.0%.

TABLE 13

## MORTALITY EXPERIENCE BY AMOUNTS, GENERALLY EXCLUDING FIRST 5 POLICY YEARS

## INDIVIDUAL EXPERIENCE OF 33 COMPANIES AT AGES 20 AND OVER

#### GENERALLY BETWEEN 1950 AND 1954 POLICY ANNIVERSARIES

	7.			
Company	RATIO OF ACTUAL TO EXPECTED DEATHS ON			
Number Number	1941 CSO Table	Table X17	Table X <sub>18</sub>	
1	32.0%	52.6%	69.1%	
	34.2	58.6	77.9	
	35.0	53.5	65.3	
	36.7	57.1	72.4	
	39.0	62.1	79.1	
6 7 8 9	44.6 45.4 49.0 50.0 51.1	62.2 66.8 66.2 87.6 69.3	74.0 80.4 78.7 115.7 81.3	
11	51.7	63.8	75.8	
	52.3	78.3	98.0	
	52.5	76.7	92.8	
	52.7	76.8	93.2	
	52.8	72.5	85.6	
16	53.2	82.2	100.4	
	53.5	81.1	100.7	
	54.5	87.1	110.5	
	55.6	75.2	88.4	
	56.5	85.0	103.7	
21	57.6	79.1	93.1	
	57.9	85.3	103.2	
	59.0	78.9	92.3	
	59.2	79.0	92.3	
	59.5	79.9	93.6	
26	59.7	83.9	99.4	
	60.4	87.0	104.1	
	61.1	82.8	97.2	
	62.9	87.7	104.2	
	63.1	89.4	107.0	
31	64.8	90.3	106.7	
32	66.0	87.6	102.0	
33	69.0	95.0	112.1	

#### VI. CONCLUSION

The "Report of the Special Committee on New Mortality Tables of the Society of Actuaries" (TSA VIII, 504) includes the following tables:

- (1) Values of  $1,000q_x$  on Mortality Table  $X_{17}$
- (2) Values of  $l_x$  and  $d_x$  on Mortality Table  $X_{17}$
- (3) Commutation Columns on Mortality Table  $X_{17}$ — $2\frac{1}{2}\%$  Interest
- (4) Comparison of Mortality Table X<sub>17</sub> with 1941 CSO Table
  - a) Values of  $1,000q_x$
  - b) Periods of Extended Term Insurance—2½% Interest
  - c) Net Premiums and Terminal Reserves -2\frac{1}{2}\% Interest
  - d) Mean Reserves—2½% Interest
  - e) Aggregate Mean Reserves—2½% Interest
- (5) Values of  $q_x$  on Mortality Table  $X_{18}$

This paper describes the final results of the work of the Special Committee. It does not provide any indication of the volume of work that was completed in the three-month period between the first meeting on August 3, 1956 and the meeting on November 5, 1956 at which the Special Committee agreed that Mortality Table  $X_{17}$  is suitable for use as a permissive basis for the valuation of standard ordinary insurance. The discussion prepared by Mr. James T. Phillips (TSA, VIII, 509) provides some indication of the scope of the preliminary work involved in the calculation of commutation columns, net premiums, terminal reserves and mean reserves for about twelve different experimental mortality tables. The completion of such a large volume of work in so short a period was accomplished through the use of the IBM 705 System of the New York Life. It has been estimated that the completion of this volume of work by the use of desk calculators would have required about two years.