DISCUSSION OF PRECEDING PAPER

ALFRED N. GUERTIN:

It should not be forgotten that the Special Committee of the Society that constructed the new mortality table was appointed for the purpose of assisting a committee of the National Association of Insurance Commissioners. Its formal report, therefore, was made relatively brief and in as nontechnical language as possible. The panel discussion before the Society last November, by its very informality, could not reach all the details of the studies made by the Special Committee. It seemed reasonable, therefore, that something more than these reports was needed for the membership. It was for this reason that Mr. Sternhell was asked to prepare this paper.

There is precedent for a paper such as this. When the CSO Table was prepared by the Committee on Non-Forfeiture Benefits and Other Matters in 1941, Mr. John S. Thompson, the nominee of the Actuarial Society on that committee, prepared a paper entitled "The Commissioners 1941 Standard Ordinary Mortality Table" (*TASA* XLII, 314), which was, in effect, a supplement to the committee report, but prepared specifically for actuaries. Similarly, Mr. Harry W. Jones, who had worked with Mr. Thompson on the mortality table project, in his discussion of the Thompson paper (*TASA* XLIII, 81) produced a graduated underlying table to round out the work of the committee. The Sternhell paper does for Table X_{17} what the Thompson paper and the Jones discussion did for the CSO Table.

Mr. Sternhell is well qualified for this task. As assistant to Mr. James T. Phillips, a member of the committee, he had charge of the calculations which were needed by the committee and attended nearly all of its meetings. While the paper is not an official expression of the committee, it was given advance distribution to all the members for comment, and there is reflection of such comments in the paper. Not only the members of the committee, but also the members of the Society, are indebted to Mr. Sternhell for undertaking the task of preparing this paper and doing it so well.

I do not at this time propose to discuss the content of the paper. That I propose to leave to others. I merely wish to say that the paper reflects accurately the decisions of the committee and the work done on behalf of the committee. I do not think it is appropriate to discuss here, either, the pro-

cedures by which the new table was presented to the National Association of Insurance Commissioners nor the actions taken by it or by its committees. I will reserve my statement on these matters for the informal discussion on the general subject of new mortality tables and kindred items later in this meeting.

W. HAROLD BITTEL:

This paper constitutes another outstanding contribution by Mr. Sternhell, in addition to that previously made by him and his company through the tremendous volume of work described in the paper, in the solution of a problem in which neither he nor his company had any direct interest at the time. His concise description of the step-by-step construction of this table should dispel the fears expressed by some that adequate consideration might not have been given to every desired attribute of a statutory mortality table for the valuation of policy liabilities.

In this discussion, I shall confine my comments to a brief summary of the reasons for the recommendation by the Commissioner's Sub-Committee that the loadings added to the observed mortality rates should be modified to the extent necessary to keep the final rates from exceeding those developed in recent population tables. It should be noted that, while the ultimate effect of this recommendation was a table in which the mortality rates did not exceed those in the U.S. White Males 1949–51 Table except at the very old ages, the implied criticism was directed at a pattern of proposed loadings which was of an arbitrary nature and for which no essential purpose could be developed at the ages involved. We would not question the validity of the points raised by Mr. Fassel in this connection last October if we were dealing with actual rates of mortality developed from the experience of insured lives. However, these criticisms would hardly apply when the required modification involved loadings which were of both an artificial and arbitrary nature. In other words, we did not, and do not advocate population data as a proper measure of mortality on insured lives, but we did feel that such data provided a very convenient and practical vehicle for placing a ceiling on loadings which appeared to be unnecessary and of a very arbitrary nature.

The modifications which were made at ages 3 to 16 reduced flat arbitrary loadings to amounts in the final table which represent margins in excess of 50% of the rates of mortality on the basic table at those ages. It should also be noted that the mortality rate in Table X_{17} for age 0 was derived from population data and that the final rates in all of the proposed tables for ages 0, 1 and 2 were substantially lower than those on the U.S. White Males 1949–51 Table. This was also true for ages over 16.

Under these circumstances, it seemed to us that the higher rates for the ages in question could not be justified.

The situation at the older ages was also indefensible, in our opinion. The excesses of the proposed rates of mortality on insured lives in earlier tables over the rates on population tables were due to arbitrary increases in percentage loadings at the higher ages. The use of a flat percentage loading, such as the 15% in Table X_{17} from ages 52 to 93, produced margins which appeared to be adequate to cover all conceivable contingencies on the basis of the extensive studies which were made of this problem. The resulting margins are greater percentagewise than those in the 1941 CSO Table for ages over 60 and even provide greater dollar margins for ages over 66.

There is one final observation which I feel needs to be made on this subject entirely apart from the arguments for or against our recommendation. This falls in the area of public relations and should be of vital concern to life companies as well as to Insurance Commissioners. It is just not possible for a layman to understand why a mortality table based on selected standard lives should reflect higher mortality rates than those experienced by the general population which includes not only those classed as substandard but also all of those who would be rejected as uninsurable.

ALLEN L. MAYERSON:

Mr. Sternhell is to be commended for a very thorough and interesting compilation of the data underlying Mortality Table X_{17} and for a lucid presentation of the manner of constructing the table.

I was particularly interested in Tables 12 and 13 which show the ratio of actual to expected mortality on the CSO Table, Table X_{17}, and Table X_{18} for the fifteen large companies whose mortality data were used in constructing the table and for thirty-three smaller companies. It is interesting to note that the company experiencing the lowest over-all mortality among the fifteen large companies ranks behind twenty-six of the smaller companies on the basis of the CSO Table. This supplies further evidence, if any is still needed, of the obsolescence of the CSO Table. I presume that the extremely low mortality experienced by many of the smaller companies when measured by the CSO Table is the result of a concentration of business at younger ages and early durations.

This very low ratio of actual to expected mortality based upon the CSO Table changes considerably when based upon a modern table. For example, Company 9 in Table 13 ranks 9th on the basis of the CSO Table, ties for 28th on the basis of Table X_{17}, and shows the highest mortality of all based on Table X_{18}. The fact that most of the small companies show

better ratios of actual to expected mortality than the fifteen large companies, even on Table X_{17}, is probably due to the select period extending beyond five years, as well as to the rather heavy loading, percentagewise, on the latter table at the younger ages. Table X_{18} appears to present a much more realistic picture.

This may be carping about a rather small point, but I was rather confused by the mention at the last meeting of a "U-shaped pattern of margins" in Table X_{17}. This statement again appears on page 13 of Mr. Sternhell's paper. His Table 9, however, makes clear the fact that the margin is actually 15% of the unloaded mortality rates for ages 52 to 93, with substantially higher percentage margins at the younger ages. Although the over-all margin seems to be adequate and reasonable, it is unfortunate that the Committee felt it necessary to use such heavy percentage loadings at the younger ages. However, the effect in dollars and cents will be quite small and the data presented in Tables 6 and 7 seem to indicate that the variations among companies were higher percentagewise at the younger ages than at the older ages.

Mr. Sternhell did not mention what to me appears to be the most serious drawback to the new table: namely, the fact that it is a mixed table rather than one based solely on the experience of male lives. It is unfortunate that data on male and female mortality could not have been supplied separately. This disadvantage of the new table may become more important as the practice of offering lower rates to women (by charging them the premium for a male several years younger) becomes more prevalent.

As Mr. Fassel so ably pointed out in his discussion at the last meeting, there is no intrinsic reason why a mortality table intended for insurance use should bear any direct relationship to a population mortality table. There is, however, a public relations problem involved in this, and I am very glad that the Committee was cognizant of it. It is very hard to explain to a member of the general public why a table based upon carefully selected insured lives should show higher mortality than one based upon the general population. The fact that the Committee has taken this into consideration and that the new table shows mortality rates no higher than the U.S. White Males 1949–51 Table will certainly be of considerable help to those of us who are frequently in the position of having to explain such matters to laymen.

On the whole, Table X_{17} is a notable achievement, and I am especially impressed by the speed with which it was constructed. The Committee deserves a vote of thanks for its excellent work in constructing the table and Mr. Sternhell deserves special praise for explaining it to us so clearly.

NORMAN F. BUCK:

The purpose of this discussion is to examine intercompany select mortality experience between 1950 and 1954 policy anniversaries and to append select mortality rates to Table X_{18}. The development of this discussion owes much to the work of Mr. John H. Miller, who developed select rates for the material underlying the CSO Table (*TASA* XLIII, 12).

The mortality rates herein were derived entirely from two sources, Table X_{18} itself and the intercompany experience on standard business between 1950 and 1954 policy anniversaries as published in the 1952, 1953, 1954 and 1955 reports numbers of the *Transactions*. The experience used covers only medical issues in the first fifteen policy years but both medical and nonmedical in the sixteenth and subsequent years. It excludes war deaths. At the adult ages it differs from the experience underlying Tables X_{17} and X_{18} only in that it includes also the first five policy years and includes, for the first fifteen policy years, the contribution of the sixteenth company that was omitted in the development of X_{17} and X_{18}.

NATURE OF THE DATA

While the mortality table which is the end product of this project has the appearance of a select and ultimate table, it is not such in the usual sense of these terms since it does not follow one group of issues through successive policy years. The published intercompany experience between 1950 and 1954 policy anniversaries contains the same number of years of exposure, *i.e.* four, at each policy duration. The exposure at duration 1 comes from issues of 1950 through 1953, duration 2 from issues of 1949 through 1952, duration 20 from issues of 1931 through 1934. Thus the experience at each policy duration comes from a unique but overlapping group of years of issue. As Mr. Elston has put it (*TASA* XLVIII, 264), the results form a set of "associated mortality tables."

There are a number of other reasons why the final table needs to be used with care. Changes in home office underwriting techniques and skill over the years have probably affected the results by policy duration. Improvement in underwriting would tend to lower mortality on recent issues, hence make the rates artificially low in the early policy years compared with the later policy years. An increase over the years in the proportion of the total insurance issued on women would have a similar effect. On the other hand, liberalization in underwriting practices may have brought into the recent standard issues cases that some years ago would have been excluded as being substandard. Advances in medical knowledge may have

altered the nature of the group of persons surviving to secure standard insurance.

Much of the data underlying the CSO Table was derived from the experience in the sixth and later policy years on issues of 1925 through 1934 carried to policy anniversaries in 1940. In a triangular experience of this sort the results at the short durations are more heavily weighted with the exposures in the early years of the interval than are the results at the long

EXHIBIT 1

TRENDS OF MORTALITY RATES BY AMOUNTS
RATE DURING EACH ANALYSIS YEAR AS PERCENTAGE
OF RATE FOR ENTIRE PERIOD

Age Group*	Policy Years 16 and over				Policy Years 1–5			
	1950–51	1951–52	1952–53	1953–54	1950–51	1951–52	1952–53	1953–54
10–14	.....	.....	.....	.....	113%	104%	80%	100%
15–19	.....	.....	.....	.....	85	96	136	86
20–24	.....	.....	.....	.....	102	101	115	80
25–29	104%	113%	96%	89%	103	101	110	88
30–34	105	102	94	101	93	97	105	103
35–39	97	105	100	98	107	103	96	94
40–44	110	100	103	88	105	102	96	97
45–49	105	100	98	98	112	110	94	86
50–54	103	102	98	96	98	87	123	92
55–59	102	102	100	96	102	114	98	88
60–64	103	98	104	96	112	95	97	97
65–69	101	99	104	96	.....	.....	.....	.....
70–74	103	104	100	93	.....	.....	.....	.....
75–79	99	97	102	101	.....	.....	.....	.....
80–84	104	94	96	105	.....	.....	.....	.....
Simple Average	103%	101%	100%	96%	103%	101%	105%	92%

* Attained ages for durations 16 and over; issue ages for durations 1–5.

durations. For instance, in this example the experience in the sixth policy year was incurred in all ten analysis years from 1930 to 1940 anniversaries, but the experience in the fifteenth policy year was incurred only between 1939 and 1940 anniversaries. Accordingly any downward secular trend during this ten-year interval would cause an understatement of the mortality rates in the fifteenth policy year relative to the rates in the first policy year.

In the 1950–54 experience, which was trapezoidal in form, there was less chance for a secular trend to affect the results, not only because of the shorter interval of exposure but also because all policy durations had the

same number of years of exposure and essentially the same calendar period of exposure. Furthermore, an increase in the proportion of insurance issued on women and any improvement in underwriting would understate the mortality rates in the early policy years relative to those in the later policy years of this experience. This is just the opposite from the effect of a downward secular trend in a triangular experience of the CSO type. Hence the published mortality results for the first fifteen policy years were used without adjustment for any of these factors. This decision seemed particularly appropriate since the Committee made no adjustment for these factors in developing Tables X_{17} and X_{18}.

Also in conformity with the technique used for Tables X_{17} and X_{18}, the crude select mortality rates were obtained by dividing the deaths for the four years combined by the exposures for the four years combined. Tests revealed that giving equal weight to each year of experience would have produced virtually the same rates.

Exhibit 1 provides a measure of the trends operating within the four years between 1950 and 1954 policy anniversaries. Results are shown only for age groups where the total exposure exceeded $400,000,000. For the first five policy years combined and for the sixteenth and later years combined, the crude mortality rates for the individual years were compared with the rates for all four years combined. As might be expected, the percentages were more erratic in the first five years than in the ultimate years. As examples, the figure of 136% in 1952–53 was based on only $819,000 in claims and the 80% in 1953–54 on only $1,133,000.

EFFECTS OF SELECTION

Exhibit 2 presents crude mortality rates for each of the first fifteen policy years on standard Ordinary medically examined business. They were developed from the material published in the reports numbers of the *Transactions*.

Exhibit 3 gives mortality rates for the sixteenth and later policy years developed from the corresponding experience on medical and nonmedical business combined. The experience for each five-year attained age group was assumed to be concentrated at the middle age of the group, for instance at 42 for the 40–44 group. Since for the purpose at hand no elaborate graduation process was necessary, the intervening values were obtained by Gauss's forward formula

$$u_{x+t} = u_x + t\Delta u_x + \frac{t\,(t-1)}{2}\,\Delta^2 u_{x-1}.$$

In deriving the rates for ages 28 through 31 the second difference was assumed to be zero.

EXHIBIT 2

MORTALITY RATES PER 1,000, 1950–54 POLICY ANNIVERSARIES

POLICY YEARS 1–15

POLICY DURATION	ISSUE AGE GROUP										
	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64
1	0.20	1.03	0.87	0.55	0.62	0.85	1.34	2.37	3.38	3.72	6.50
2	0.32	0.68	0.80	0.64	0.88	1.29	1.79	3.12	5.20	6.61	8.14
3	0.53	0.92	0.68	0.75	0.98	1.58	2.34	4.37	5.58	8.92	9.06
4	0.55	1.13	0.77	0.87	1.25	1.64	3.06	4.78	6.40	10.71	15.64
5	0.59	0.91	0.92	0.95	1.15	2.08	3.22	5.00	9.29	11.73	17.56
6	0.80	0.82	1.01	1.16	1.48	2.66	3.80	6.35	9.72	13.71	17.86
7	1.23	0.90	0.87	1.00	1.75	2.70	4.56	7.09	11.17	14.48	18.02
8	0.83	1.09	0.94	1.11	1.94	3.30	4.89	8.80	11.72	14.69	23.66
9	1.12	1.01	1.00	1.38	2.32	3.69	6.59	8.94	12.12	17.41	25.68
10	1.17	1.07	1.08	1.54	2.53	4.42	6.74	10.12	13.61	23.31	31.34
11	1.21	0.94	1.01	1.51	2.82	5.00	7.21	11.93	15.39	22.34	36.15
12	1.21	1.02	1.08	1.60	3.08	6.03	8.24	13.57	17.15	25.31	44.58
13	0.88	0.86	1.12	1.93	3.81	6.07	10.15	13.62	19.44	27.85	37.45
14	1.15	1.01	1.30	2.20	3.91	6.93	10.29	14.99	25.79	30.77	43.54
15	0.90	1.20	1.48	2.67	4.72	7.54	11.51	17.33	30.45	42.92	50.14

EXHIBIT 3

MORTALITY RATES PER 1,000, 1950–54 POLICY ANNIVERSARIES

POLICY YEARS 16 AND OVER

Attained Age	Mortality Rate	Attained Age	Mortality Rate	Attained Age	Mortality Rate
27	1.14	47	5.28	67	34.25
28	1.18	48	5.86	68	37.37
29	1.22	49	6.50	69	40.75
30	1.25	50	7.20	70	44.41
31	1.29	51	7.96	71	48.35
32	1.33	52	8.78	72	52.55
33	1.40	53	9.71	73	56.66
34	1.48	54	10.71	74	60.92
35	1.56	55	11.79	75	65.33
36	1.66	56	12.95	76	69.88
37	1.77	57	14.18	77	74.59
38	1.99	58	15.64	78	80.92
39	2.25	59	17.23	79	87.89
40	2.55	60	18.95	80	95.50
41	2.90	61	20.79	81	103.75
42	3.29	62	22.76	82	112.64
43	3.65	63	24.83		
44	4.03	64	27.01		
45	4.43	65	29.30		
46	4.84	66	31.72		

Exhibit 4 shows the coefficients of selection for these mortality rates, based on Exhibits 2 and 3, where

$$CS_{[x]+n} = 1 - \frac{q_{[x]+n}}{q_{x+n}}.$$

This array indicates that the coefficients diminish with advancing policy duration, increase with age, and remain substantial even in the fifteenth policy year. Why do the effects of selection appear to persist for so long?

EXHIBIT 4

COEFFICIENTS OF SELECTION

1950–54 SELECT COMPARED WITH 1950–54 ULTIMATE

POLICY DURATION	ISSUE AGE GROUP								SIMPLE AVERAGE
	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	
1.....	.518	.534	.520	.593	.551	.615	.738	.714	.598
2.....	.458	.371	.352	.510	.468	.464	.577	.672	.484
3.....	.385	.338	.298	.419	.328	.479	.482	.665	.424
4.....	.304	.199	.357	.309	.336	.457	.435	.466	.358
5.....	.264	.307	.283	.335	.372	.283	.436	.446	.341
6.....	.128	.164	.191	.280	.277	.315	.398	.479	.279
7.....	.286	.121	.260	.222	.270	.286	.417	.518	.297
8.....	.250	.138	.181	.248	.178	.320	.456	.419	.274
9.....	.115	.090	.167	.085	.242	.360	.406	.422	.236
10.....	.072	.128	.087	.153	.219	.345	.265	.352	.203
11.....	.147	.143	.053	.179	.159	.324	.348	.312	.208
12.....	.196	.156	−.029	.151	.132	.309	.323	.213	.181
13.....	.142	.055	.066	.052	.210	.280	.317	.385	.188
14.....	.137	.117	.037	.127	.209	.120	.307	.334	.173
15.....	.079	.025	.053	.111	.166	.040	.112	.282	.108
	Simple Averages of Above Coefficients								
1– 5...	.386	.350	.362	.433	.411	.460	.534	.593	.441
6–10...	.170	.128	.177	.198	.237	.325	.388	.438	.258
11–15...	.140	.099	.036	.124	.175	.215	.281	.305	.172
1–15...	.232	.192	.192	.252	.274	.333	.401	.445	.290

Perhaps they really do. Perhaps adverse selection by withdrawals from the insured group causes the mortality rates to increase indefinitely by policy duration. On the other hand, such factors as continuing improvements in underwriting and relative increases in issues on female lives may create this appearance.

Exhibit 5 shows the ratios of the mortality rates of Table X_{18} to the corresponding rates in Exhibit 3. These percentages give a measure of the

decrease in the rates in X_{18} as a result of including the experience in the sixth through fifteenth policy years.

Chart I displays mortality rates by policy duration for a fixed attained age where the effects of selection persist for more than fifteen policy years. The line *BL* represents the mortality rates actually experienced during the years when selection has some effect, the mortality rate theoretically re-

EXHIBIT 5

RATIOS OF MORTALITY RATES

RATES OF TABLE X_{18} AS PERCENTAGES OF 1950–54 ULTIMATE RATES

Attained Age	Ratio	Attained Age	Ratio	Attained Age	Ratio
27	86.0%	47	93.2%	67	96.6%
28	84.7	48	93.2	68	97.0
29	85.2	49	93.2	69	97.3
30	86.4	50	93.3	70	97.5
31	87.6	51	93.6	71	97.4
32	88.7	52	93.5	72	97.1
33	88.6	53	92.9	73	97.1
34	89.2	54	92.6	74	97.2
35	90.4	55	92.5	75	97.7
36	92.2	56	92.7	76	98.5
37	94.9	57	93.2	77	99.9
38	94.0	58	93.0	78	100.0
39	93.3	59	92.8	79	100.1
40	92.5	60	92.7	80	100.1
41	91.0	61	92.7	81	100.0
42	89.7	62	92.8	82	99.7
43	89.9	63	93.0	83	99.8
44	90.1	64	93.5	84	99.8
45	90.7	65	94.2	85	99.8
46	91.9	66	95.2	86	99.9

maining level at *L* for all policy durations thereafter. Of course for purposes of illustration this chart has been simplified; for instance, *BL* is not likely in fact to be a straight line.

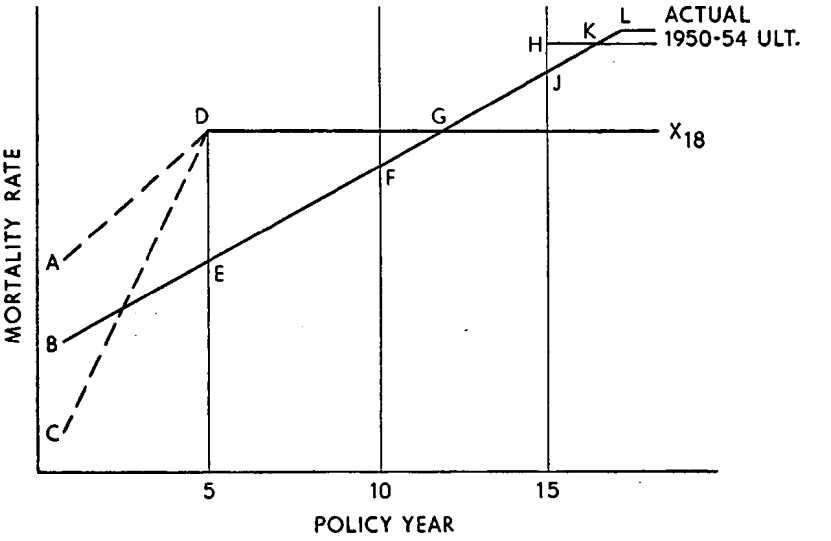
The intercompany experience, when arranged as in Exhibits 2 and 3, gives a result corresponding to the line *BJ* for the first fifteen policy years and a constant rate at the level of *H* in the sixteenth and later policy years, with some discontinuity between *J* and *H* at the end of the fifteenth year.

Table X_{18} gives a constant mortality rate for durations six and later at the level of *D*. The percentages in Exhibit 5 therefore express the ratios of level *D* to level *H* for the various attained ages.

SELECT TABLES

One way to produce a smoothed 1950–54 experience table with a fifteen year select period is to graduate the coefficients of selection in Exhibit 4 and multiply the resulting complements by the ultimate mortality rate in Exhibit 3 for the same attained age. The degree of discontinuity at the end of the fifteenth policy year (*HJ*) will depend upon the gradation of the coefficients.

CHART I



However, the immediate problem is to append to Table X_{18} a set of select mortality rates for the first five or fewer policy years. In solving this problem there are at least three choices.

The first way is to develop a set of values that will grade smoothly into Table X_{18} and reproduce the aggregate experience of the first five policy years. If the total tabular mortality for the first five policy years is to equal the total experience mortality, then in order to get a smooth gradation into X_{18} it will be necessary at least at some ages to understate the experience mortality rates in the first two policy years or so and overstate them in the fourth and fifth years. This will give a set of select rates of the form *CD* on the chart to represent the actual mortality *BE*.

The second choice is to relate the experience mortality rate in each of the first five policy years to the experience mortality rate at the same attained age five policy years later (giving ratios of each of the five points

on segment BE to the corresponding points on EF). Subtracting these ratios from 1 gives coefficients of selection of the form

$$CS'_{[x]+n} = 1 - \frac{q_{[x]+n}}{q_{[x-s]+n+s}}.$$

Graduating these coefficients and applying the complements to X_{18} at the same attained age will give select values running from A to D where q_A/q_D equals q_B/q_E. The form of line AD will depend on how rapidly the coefficients are brought to zero. This method has the advantage of producing select rates which merge smoothly into the ultimate section of the table but overstates the select rates for any age where D exceeds E.

EXHIBIT 6

COEFFICIENTS OF SELECTION

1950–54 DURATIONS 1–5 COMPARED WITH DURATIONS 6–10

POLICY DURATION	ISSUE AGE GROUP										SIMPLE AVERAGE
	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	
1.....	−.288	−.061	.455	.466	.426	.496	.376	.468	.617	.526	.348
2.....	.447	.111	.264	.120	.263	.337	.316	.267	.408	.438	.297
3.....	−.108	.376	.202	.117	.186	.291	.106	.366	.239	.383	.216
4.....	−.009	.238	.130	.094	.293	.171	.275	.284	.116	.102	.169
5.....	.222	.140	.120	.253	.178	.271	.258	.082	.138	.247	.191
Simple Average..	.053	.161	.234	.210	.269	.313	.266	.293	.304	.339	.244

The ungraduated coefficients of selection on this basis for the 1950–54 experience are shown in Exhibit 6. As with the data underlying the CSO Table (Mr. Miller's Table V), these new coefficients increase with age; however, they are generally larger and persist to longer durations.

The third choice in deriving select rates is to adhere closely to the actual experience in the first five policy years, in this instance, even though this produces a sharp discontinuity (DE) between the rates in the fifth policy year and those in the ultimate portion of the table. In the development of select rates for the British Table for Assured Lives 1949–52 (*TFA* 23, 169), the coefficients of selection were found to remain at a level just below twenty percent in the third, fourth and fifth policy years. Accordingly a select period of only two years was adopted and the discontinuity between the second year rates and the ultimate rates was tolerated.

Exhibit 7 shows both crude and graduated coefficients of selection at quinquennial ages on this third basis. The crude coefficients are the com-

plements of the ratios of the mortality rates for the first five policy years in Exhibit 2 to the rate at the same attained age in Table X_{18}.

The graduation was accomplished by the following steps: (*a*) The experience for each issue age group was assumed to be concentrated at the middle age of that group, *e.g.* 12 for 10–14. (*b*) The graduated coefficients were set at zero for issue ages 12 and younger. This decision was based on the observed coefficients for issue ages 12 and 17 and on the fact that the

EXHIBIT 7

COEFFICIENTS OF SELECTION

1950–54 SELECT COMPARED WITH TABLE X_{18}

POLICY DURATION	ISSUE AGE GROUP										
	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64
	Crude Observed Coefficients										
1	.535	−.537	.022	.439	.475	.494	.546	.518	.588	.719	.692
2	.319	.093	.111	.360	.290	.310	.454	.429	.424	.546	.648
3	−.039	−.136	.261	.279	.258	.248	.355	.279	.438	.442	.641
4	.000	−.329	.172	.194	.113	.305	.239	.289	.413	.390	.434
5	.033	−.046	.032	.159	.248	.212	.276	.329	.226	.392	.419
	Graduated Coefficients										
1	.000	.199	.323	.372	.422	.472	.521	.571	.620	.670	.720
2	.000	.151	.246	.283	.321	.359	.396	.435	.472	.510	.548
3	.000	.136	.220	.254	.288	.322	.355	.389	.423	.457	.491
4	.000	.108	.175	.202	.229	.256	.283	.310	.337	.364	.391
5	.000	.097	.158	.182	.206	.231	.255	.279	.303	.328	.352

data for Tables X_{17} and X_{18} included the first five policy years of experience on issue ages 14 and younger. (*c*) The crude value for issue age 22 in the first policy year was raised from .022 to .220 to bring it more into line with the surrounding data. The new figure lies midway between .439 at age 27 and 0.000. (*d*) The straight line of least squares was fitted to the first policy year values from ages 22 through 62 and extended to age 70. Values for intervening ages were obtained by linear interpolation. (*e*) Adjusted coefficients for ages 13 through 21 were obtained by passing a quadratic equation through the graduated values at ages 12, 22 and 27.

(*f*) For each of the first five policy years the total of the crude coefficients was obtained for quinquennial issue ages 22 through 62. The ratios of these totals to the totals for duration one were found to be: .761 for duration two, .682 for duration three, .543 for duration four, .489 for duration five. (*g*) These ratios were applied at all issue ages to the graduated coefficients for the first policy year to get the graduated coefficients for the next four policy years.

The complements of these graduated coefficients were multiplied by the mortality rate at the same attained age in Table X_{18}. Four of the resulting

EXHIBIT 8

RATIOS OF ACTUAL TO TABULAR SELECT MORTALITY

Policy Duration	Ratio
1	101.1%
2	99.6
3	104.0
4	100.5
5	99.4
Issue Age Group	
10–19	124.5%
20–29	101.5
30–39	101.9
40–49	101.2
50–59	99.2
60–64	87.0
10–64	100.8%

mortality rates at issue ages 20 and 21 were adjusted by 0.00001, two up and two down, to prevent a decrease in rate from one age to the next. It will be recalled that in the development of Tables X_{17} and X_{18} a larger adjustment was necessary at attained age 22. In addition a few minor changes, none under age 44 and none amounting to more than 0.00004, were made in the rates in order to smooth the series of third differences. The final table of select and ultimate mortality rates is presented in Exhibit 9.

As a final test, the select mortality rates of Exhibit 9 at quinquennial issue ages were then multiplied by the 1950–54 exposures for the corresponding age groups at issue and policy durations. Exhibit 8 compares the resulting tabular mortality with the actual deaths.

EXHIBIT 9

ORDINARY SELECT 1950–54 MORTALITY TABLE

ANNUAL RATES OF MORTALITY PER 1,000

Issue Age [x]	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	$q_{[x]+3}$	$q_{[x]+4}$	q_{x+5}	Attained Age x+5
						6.33	0
						1.00	1
						.78	2
						.66	3
						.58	4
0	6.33	1.00	.78	.66	.58	.52	5
1	1.00	.78	.66	.58	.52	.47	6
2	.78	.66	.58	.52	.47	.43	7
3	.66	.58	.52	.47	.43	.40	8
4	.58	.52	.47	.43	.40	.38	9
5	.52	.47	.43	.40	.38	.37	10
6	.47	.43	.40	.38	.37	.39	11
7	.43	.40	.38	.37	.39	.43	12
8	.40	.38	.37	.39	.43	.47	13
9	.38	.37	.39	.43	.47	.51	14
10	.37	.39	.43	.47	.51	.55	15
11	.39	.43	.47	.51	.55	.61	16
12	.43	.47	.51	.55	.61	.67	17
13	.45	.49	.53	.59	.66	.75	18
14	.46	.51	.57	.64	.72	.81	19
15	.48	.55	.61	.70	.76	.85	20
16	.51	.59	.67	.74	.78	.87	21
17	.54	.64	.70	.76	.79	.89	22
18	.58	.67	.72	.76	.79	.90	23
19	.60	.68	.72	.77	.79	.92	24
20	.60	.68	.72	.77	.79	.93	25
21	.60	.68	.72	.77	.79	.95	26
22	.60	.68	.72	.77	.80	.98	27
23	.60	.69	.72	.78	.82	1.00	28
24	.60	.69	.73	.80	.83	1.04	29
25	.60	.69	.74	.81	.86	1.08	30
26	.61	.71	.75	.84	.89	1.13	31
27	.62	.72	.78	.86	.92	1.18	32
28	.62	.74	.80	.90	.96	1.24	33
29	.63	.76	.83	.93	1.00	1.32	34
30	.65	.78	.86	.97	1.06	1.41	35
31	.66	.81	.89	1.02	1.13	1.53	36
32	.68	.84	.94	1.09	1.21	1.68	37
33	.70	.89	.99	1.17	1.33	1.87	38
34	.74	.94	1.07	1.28	1.47	2.10	39
35	.77	1.00	1.16	1.41	1.64	2.36	40
36	.82	1.09	1.28	1.57	1.83	2.64	41
37	.89	1.20	1.42	1.76	2.03	2.95	42
38	.97	1.33	1.58	1.95	2.25	3.28	43
39	1.07	1.48	1.75	2.16	2.49	3.63	44
40	1.18	1.63	1.94	2.38	2.74	4.02	45
41	1.29	1.80	2.14	2.62	3.02	4.45	46
42	1.41	1.98	2.34	2.88	3.32	4.92	47
43	1.54	2.16	2.56	3.17	3.64	5.46	48
44	1.67	2.36	2.81	3.49	4.01	6.06	49

EXHIBIT 9—*Continued*

Issue Age [x]	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	$q_{[x]+3}$	$q_{[x]+4}$	q_{x+5}	Attained Age $x+5$
45	1.80	2.59	3.07	3.83	4.43	6.72	50
46	1.95	2.82	3.37	4.21	4.88	7.45	51
47	2.11	3.08	3.70	4.64	5.37	8.21	52
48	2.29	3.38	4.06	5.10	5.88	9.02	53
49	2.48	3.70	4.45	5.57	6.41	9.92	54
50	2.68	4.05	4.84	6.08	7.00	10.91	55
51	2.90	4.40	5.26	6.63	7.65	12.01	56
52	3.12	4.77	5.72	7.23	8.37	13.22	57
53	3.34	5.17	6.22	7.90	9.15	14.55	58
54	3.57	5.60	6.77	8.62	10.00	15.99	59
55	3.82	6.07	7.36	9.41	10.91	17.57	60
56	4.08	6.58	8.00	10.27	11.89	19.28	61
57	4.36	7.13	8.68	11.18	12.96	21.12	62
58	4.66	7.72	9.42	12.17	14.10	23.10	63
59	4.96	8.35	10.20	13.22	15.32	25.25	64
60	5.27	8.99	11.05	14.33	16.63	27.61	65
61	5.59	9.69	11.92	15.52	18.04	30.21	66
62	5.91	10.43	12.85	16.82	19.58	33.08	67
63	6.24	11.21	13.86	18.25	21.27	36.24	68
64	6.56	12.07	14.95	19.80	23.12	39.66	69
65	6.90	12.98	16.14	21.49	25.10	43.30	70
66	7.25	13.96	17.45	23.28	27.18	47.09	71
67	7.61	15.00	18.84	25.17	29.34	51.00	72
68	7.97	16.10	20.26	27.12	31.55	55.01	73
69	8.33	17.26	21.71	29.12	33.80	59.23	74
70	8.66	18.43	23.15	31.14	36.07	63.80	75
						68.85	76
						74.52	77
						80.92	78
						87.99	79
						95.64	80
						103.78	81
						112.32	82
						121.20	83
						130.45	84
						140.12	85
						150.27	86
						160.98	87
						172.39	88
						184.75	89
						198.38	90
						213.71	91
						231.24	92
						251.47	93
						274.90	94
						303.03	95
						343.36	96
						409.79	97
						522.62	98
						708.55	99
						1000.00	100

HARRY F. GUNDY:

The Canadian Association of Actuaries has accumulated the mortality experience since 1949 of a number of the life insurance companies operating in Canada in connection with policies issued at standard rates to persons resident in Canada. A comparison of this experience with Table X_{18} has been made and will be of interest.

The comparison has been made by calculating the expected deaths by Table X_{18}, using the exposure between the policy anniversaries in 1950 and 1954 for policy years 6 and over. The data are therefore on a comparable basis with the data used in the preparation of Table X_{18}. They include medical and nonmedical issues and, in general, exclude substandard issues except for a small proportion of such business which could not be excluded by a few of the contributing companies. Twenty-one companies contributed to the study and the results are given in the following table:

STANDARD ORDINARY BUSINESS IN CANADA
EXPERIENCE BETWEEN THE 1950 AND 1954
POLICY ANNIVERSARIES
EXCLUDING THE FIRST FIVE POLICY YEARS

Attained Ages	Actual Deaths (Dollars)*	Ratio of Actual to Expected Deaths by Table X_{18}
5- 9	$ 257,785	133.6%
10-19	387,563	123.1
20-29	2,270,812	109.2
30-39	6,226,735	95.6
40-49	16,464,893	95.1
50-59	30,286,633	97.2
60-69	42,059,575	100.1
70-79	24,316,797	94.3
80 and over	5,806,455	96.9
Total	$128,077,248	97.5%
20 and over	$127,431,900	97.4%

* Excluding war deaths.

The following points of difference between the basic data for Table X_{18} and the Canadian Association of Actuaries data should be noted:

(1) The Canadian data include all nonmedical issues whereas the data for Table X_{18} include nonmedical issues for durations 16 and over only.
(2) The Canadian data include the data for issue ages 65 and over at durations 6 to 15 inclusive whereas such data were omitted in the preparation of Table X_{18}.

(3) The Canadian data include issue ages down to age 0 whereas the data for Table X_{18} at ages 20 and over include issue ages down to age 10 only. This means that policies issued at ages under 10 with long durations have been omitted from the basic data for Table X_{18}. It has not been possible to separate policies of long durations issued at ages under 10 from the Canadian data to ascertain the effect which the mortality experience on these policies may have on the mortality ratios in the age group 15 to 29.

MORTON D. MILLER:

Mr. Sternhell is much to be complimented on the masterly exposition of the basis and construction of the new Standard Ordinary Mortality Table, Table X_{17}. Speaking as Chairman of the Committee on Group Insurance Mortality, I should like to present a few thoughts on the implications of the new table for group insurance.

While we realize that the place of the new table is in the process of being determined, and that its intended purpose was to provide an alternate minimum mortality basis for Ordinary insurance reserves, should the new table be put into statute it is likely that pressures will develop for its use as a mortality standard for group life insurance premiums. Although Table X_{17} represents more closely the pattern of group life mortality than the Commissioners 1941 Standard Ordinary Table, it would appear to be an inappropriate mortality standard for group insurance, at least unless substantial additional loadings are provided for mortality fluctuations.

The following Table A compares Table X_{17} with the group life insurance

TABLE A

RATIOS OF ACTUAL CLAIMS TO EXPECTED CLAIMS ACCORDING TO TABLE X_{17}

INTERCOMPANY GROUP EXPERIENCE				ORDINARY POLICY EXPERIENCE	
Ages	Waiver of Premium	Extended Death Benefit	Total and Permanent Disability	Ratios	Ages
21–30	69.0%	62.8%	67.3%	60.3%	20–29
31–40	72.9	65.3	73.4	62.0	30–39
41–50	87.4	80.0	90.2	79.8	40–49
51–60	89.6	83.6	103.2	86.9	50–59
61–70	72.6	78.9	78.8	86.9	60–69
71–80	69.1	77.3	82.0	87.1	70–79
81 and over	72.9	87.8	86.7	87.0	80 and over
Total	78.7%	78.6%	86.2%	85.2%	Total

experience for nonrated (standard) industries for the calendar years 1950–1954, which appears in the 1955 Committee Report and is the last published group mortality experience. Table A shows separately for each of the three disability provisions commonly found in group life insurance the ratio of actual death and disability claims to the claims expected according to Table X_{17}. Under the waiver clause, only 75% of the disability claims are included, which allows for subsequent recoveries. The column in Mr. Sternhell's Table 11 which brings out the extent of the margins provided by Table X_{17} over the ordinary experience data is also shown. Thus, the difference in pattern between the group and ordinary experiences and the size of the margins in each instance may be observed. The age groupings in the group and ordinary experiences differ slightly, but not enough to affect the comparisons.

Under the waiver of premium clause, the premium for a person insured is waived upon receipt of proof of total disability of at least nine months duration occurring by age 60. Under the extended benefit clause, the face amount is payable in the event of death occurring within one year after termination of insurance and prior to age 60 or 65, if the employee was disabled at the time of termination and remained continuously disabled until death. Under the total and permanent disability clause, the face amount is settled in installments in the event of disability prior to age 60.

It should be noted that the group experience covers about the same period of observation as the data underlying Table X_{17}, but is based on lives and not amounts. The total group experience exceeds 37 million life years, which makes it probably about half as extensive as the experience on which Table X_{17} is based. The exposure is about equally distributed among the three disability clauses.

The difference in experience between the group policies with total and permanent disability benefits and the others is commonly recognized in the premium structure by an additional premium of $1.00 per annum per thousand, so that one table would not be expected to apply without modification to all three disability clauses. However, the pattern of Table X_{17} gives cause for concern. The ratios of actual to expected are highest at ages 40 to 60; in other words, the margins are thinnest at these most important group insurance ages.

The table also shows that even allowing for the effect of disability claims under the total and permanent disability clause experience, the ratios of actual to expected for group are generally higher under age 60 than for ordinary. This means, of course, that at those ages Table X_{17} provides less margin for group than for ordinary.

Since the group insurance experience is based on lives, undoubtedly the

actual experience, particularly under the present expansion of group life amounts and maximums, is considerably worse than appears in Table A.

While there is a considerable variation in group mortality by industry, at present all except a handful of industries are generally accepted without additional premium rating. Out of a total of 133 industry classifications used in our experience analysis, about 50 show extra claims per thousand of one or more, as compared with the average experience of all nonrated industries, and 30 have extra claims per thousand of 1.5 or more. In Table B the margins provided by Table X_{17} over current group experience have

TABLE B

EXCESS OF EXPECTED OVER ACTUAL DEATHS PER THOUSAND

AGES	INTERCOMPANY GROUP EXPERIENCE		
	Waiver of Premium	Extended Death Benefit	Total and Permanent Disability
Under 65.....	1.02	1.27	.66
65 and over.....	17.33	12.84	11.41
All.....	1.66	1.78	1.14

been expressed in terms of the excess of expected claims over actual claims per thousand. It is clear that on the basis of Table X_{17} the proportion of the business that requires rating on account of industry would be increased materially.

The answer lies in the development of a separate group insurance mortality table based on group insurance experience. The group annuity business developed its own tables a few years ago and it is probably time for group insurance to do so. A separate table would have the clear advantage of being meaningful as a standard for group insurance premiums, which could be applied to group permanent and group employee paid-up forms as well as group term life insurance. It would also provide an extremely useful tool for the many kinds of group insurance experience analysis, including projections of individual group policyholder costs for active and retired employees.

(*Discussion continued in Western Spring Meeting Number*)