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A THEORY OF MORTALITY CLASSES

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INTRODUCTION

THE course of mortality for any large group depends on three basic things: first, the inherent nature of the organism—horses, for example, seem to be constitutionally incapable of living as long as men; among human beings such differences may exist according to sex and race, although conclusive evidence that this is necessarily so has not been adduced; second, the environment in the broadest sense, including not only climate but all the geographic, economic, scientific and sociological forces influencing longevity—it is to these forces that differences in mortality by race and possibly sex, may be attributable; third, the stages in the course of life in which the respective individuals in the group are found at any particular point.

2. It is the basic thesis of this paper that all mortality tables constructed on the experience of lives having in common the first and second of the foregoing elements differ from one another only as the stages of deterioration, to which the persons represented in the respective tables have progressed, differ. Accordingly, it may be postulated that the mortality embodied in different tables developed within a given period from the experience of lives, constitutionally similar, having a common environment may be expressed as varying manifestations of the mortality experienced in the population as a whole. In the development of this proposition a "mortality class" focuses attention exclusively on the members' prospects for longevity, and not on the accident through which they enter the experience, such as is the case with the collections of lives called the "class of annuitants," or the "class of medically examined insured lives," etc.

3. The mortality classes commonly encountered in life insurance experience are not homogeneous even in the loosest meaning of that term. In fact, two classes exhibiting mortality rates as different as those shown in a select table of insured lives on one hand, and a disabled life table on the other, will include many of the same lives. Even the collections of lives being accepted for standard insurance, despite the great skill of underwriters, are not homogeneous. Some of the poorest risks are accepted involuntarily as they manage to slip through the selection screen. One has merely to examine the causes of death resulting in first year claims among such lives to realize that even individuals suffering from degenerative diseases often far advanced had been admitted to the select group.

4. Homogeneity in organisms as complex as humans is an unattainable ideal. However, it may be assumed that, by a suitable selection process, a practical kind of homogeneity with respect to risk of death could be obtained within relatively small collections of lives. This hypothesis may be extended to encompass the notion that all lives in a given environment may be collected in mutually exclusive subgroups of this kind.

5. In this paper, a "mortality class" is a collection of lives at a given age composed in a particular way of these homogeneous subgroups. A class subject to light mortality might be thought of as including a larger proportion of the subgroups enjoying better health, engaged in less hazardous occupations, etc., while a class exhibiting a high rate of death might be considered as including a larger proportion of lives belonging to subgroups experiencing heavier mortality. Every mortality class, therefore, would normally include subgroups displaying different levels of mortality. A mortality class is heterogeneous by definition, but the heterogeneity is definite and specific for any particular class.

6. Since it will be assumed that the various subgroups are subject to rates of deterioration in vitality appropriate to each of them, it may be sensed that the career of a mortality class will be select in form, indefinitely; it will never normally reach an "ultimate" state, in which experience becomes equal to that of another class. This feature appears to be confirmed by modern select tables which trace mortality for 15 years without reaching the state where the "effects of selection" have worn off.

MORTALITY STRATA

7. In developing the thesis outlined, definite criteria for the determination of the "subgroups" appear necessary. As a first step, it is proposed that each of the lives in the population, *regardless of age*, may at any point be thought of as being subject to a risk of death within a year according to the physical condition and the environmental influences peculiar to such life.

8. Accordingly, let us imagine that the entire population of the United States is "underwritten" at one time by the application of ideal methods (including diagnostic procedures which might indicate physical impairments not, under present conditions, normally discoverable) so as to appraise for each person in the country the risk of death within a year. It may then be assumed that if these lives are ranked according to these probabilities they may be collected in several groups, each subject, for practical purposes, to a uniform risk of dying within a year. Each of these groups may at the point of being so appraised be considered as within a definite *mortality stratum*. Lives exhibiting the lowest risk of death within a year will be said to be then in the lowest stratum.

9. The probability of death within a year in the case of a life in the lowest stratum, 1, at the beginning of the year will be expressed in the symbol ${}_1q$; corresponding probabilities for the succeeding higher strata by ${}_2q$, ${}_3q$, etc., to ${}_zq$, the rate characteristic of the highest stratum z. Complementary probabilities of survival within the respective strata will be designated ${}_1p$, ${}_2p$, ${}_3p$, . . . ${}_zp$.

10. These rates being independent of age, some individuals of advanced age may be placed in the lowest stratum by reason of excellence of physical condition and the favorable nature of the other forces having an effect on survival, and, obviously, some young lives may, for quite different reasons, be placed in the highest stratum. Of course the vital strength of the older lives in a given stratum will, on the average, deteriorate sooner than that of the younger lives and we may expect that with the passage of time fewer of the older lives will be found in the stratum in which they are originally placed than will be the case with the younger lives.

11. While individuals do experience changes in health, residence, occupation, etc., which would place them in higher strata as they grow older, most individuals, except at the more advanced ages, tend to remain in the same stratum from one year to the next for many years. However, occasionally changes occur in the physical condition or in the general environment of some individuals which might warrant a move to a lower stratum. In view of the fact that the tide is in the direction of deterioration, if anything, it is assumed that any changes to lower strata will be more than compensated for by changes in the reverse direction. Any changes in strata, accordingly, will ignore the likelihood of movement from a higher stratum to a lower.

RATE OF DETERIORATION

12. An individual is classifiable at every point of his lifetime in one stratum or another. If his life continues one year, as he advances from one age to the next he may remain in the same stratum or, provided he is not already in the highest stratum, he may move to a higher stratum. The function measuring the probability of remaining in the same stratum or moving to a higher stratum is referred to as the *rate of deterioration* and will be designated by the symbol α, modified on the left by a superscript indicating the stratum at the beginning of the year of age and by a subscript for the stratum one year later; the age at the beginning of this interval is shown in the conventional position. Thus, ${}^{s}_{s+t}\alpha_x$ will represent the probability that a life aged x in stratum s surviving one year will be in a stratum $s + t$ $(t \geq 0)$ at age $x + 1$.

13. With reference to those surviving to age $x + 1$, it is obvious that

$$\sum_{t=0}^{z-s} {}_{s+t}^{\;\;s}a_x = 1\,. \tag{1}$$

With reference to those living at age x, ${}_sp \cdot {}_{s+t}^{\;\;s}a_x$ represents the probability that an individual in stratum s living at that age will, one year later, be in stratum $s + t$. From (1) it is evident that

$$\sum_{t=0}^{z-s} {}_sp \cdot {}_{s+t}^{\;\;s}a_x = {}_sp\,. \tag{2}$$

14. If rates of deterioration are known, all subsequent vital facts with regard to any collection of lives of the same age in a given stratum originally are fully determined: the proportions of lives surviving and dying at each age are told by the characteristic ${}_sp$'s and ${}_sq$'s of the several strata; and the number of lives in each stratum from year to year is given by the rates of deterioration. It will be observed, therefore, that, for lives, constitutionally similar, and subject to a common environment, the rate of deterioration is the basic function measuring the incidence of the breakdown of the living machine.

MORTALITY SETS

15. The lives of a given age within the mutually exclusive subgroups to which reference has been made will hereafter be called sets. A set, therefore, is a collection of lives of the same age in a given stratum. A set composed of lives aged x years who are in stratum s at formation of the set will be designated $l_{[x(s)]}$.

16. The survivors of $l_{[x(s)]}$ living at age $x + n$ will be indicated by the symbol $l_{[x(s)]+n}$. The individuals composing $l_{[x(s)]+n}$ will, except where s is the highest stratum, normally be distributed in more than one stratum, and the latter symbol will be modified by a subscript on the left to indicate these strata. Thus, the survivors of the original set $l_{[x(s)]}$ who are in stratum $s + t$ at the end of n years will be represented by ${}_{s+t}l_{[x(s)]+n}$. Contemporaries in all strata at that time, of course, represent the total number of survivors, *i.e.*,

$$\sum_{t=0}^{z-s} {}_{s+t}l_{[x(s)]+n} = l_{[x(s)]+n}\,. \tag{3}$$

17. The members of any set, it is assumed, form a homogeneous subgroup, as far as mortality is concerned, in the year following the establishment of the set. However, this homogeneity does not persist. In the second

year the survivors of the original set will be distributed in several strata by reason of the deterioration of vital force, adverse change in environment, etc., and this process will continue progressively from year to year as the age increases. The survivors of the original set in any stratum at any age themselves constitute a set at that age. That is to say, sets break down into sets. As far as the risk of death within a year is concerned, therefore, ${}_{s}l_{[x(s)]+n}$ is not different from $l_{[x+n(s)]}$, nor ${}_{s+t}l_{[x(s)]+n}$ from $l_{[x+n(s+t)]}$. In effect then, sets are fixed elements, varying in any given collection of lives which they constitute only in relative size.

FUNCTIONS EXPRESSIBLE IN TERMS OF, OR DERIVED FROM, $l_{[x(s)]+n}$

18. Deaths occurring among the survivors of any set may be denoted in the usual way. That is to say,

$$l_{[x(s)]+n} - l_{[x(s)]+n+1} = d_{[x(s)]+n} \,. \tag{4}$$

Similarly, the probabilities of surviving and of dying within a year may be designated by the customary ratios, *i.e.*,

$$\frac{l_{[x(s)]+n+1}}{l_{[x(s)]+n}} = p_{[x(s)]+n} \,; \tag{5}$$

$$\frac{d_{[x(s)]+n}}{l_{[x(s)+n]}} = q_{[x(s)]+n} \,. \tag{6}$$

Commutation functions may be developed in the conventional manner; thus

$$D_{[x(s)]+n} = v^{x+n} l_{[x(s)]+n}$$

$$C_{[x(s)]+n} = v^{x+n+1} d_{[x(s)]+n}$$

$$N_{[x(s)]+n} = \sum_{n}^{\omega} D_{[x(s)]+t}$$

$$M_{[x(s)]+n} = \sum_{n}^{\omega} C_{[x(s)]+t} \quad .$$

19. The symbol $q_{[x(s)]}$ represents the probability of death within a year of a life aged x who is then in stratum s. From what has been said it will be observed that this probability is the absolute rate ${}_{s}q$ to which all lives in stratum s are subject; that is, $q_{[x(s)]}$ has the same value as ${}_{s}q$. However, since, when $n > 0$, some lives in $l_{[x(s)]+n}$ are not in stratum s, $q_{[x(s)]+n}$ is not equal to ${}_{s}q$.

SET RADIXES

20. The references already made to "mortality class" have indicated that such a class is limited to one age and is characterized by the distribu-

tion peculiar to it of lives in the several strata. A mortality class will be designated by the conventional select notation $l_{[x]}$ and the survivors n years later by $l_{[x]+n}$. In all respects a mortality class is treated as a collection of lives entering the experience at a particular point of time which will be called the date of "formation" of "organization" or of "establishment" of the class.

21. Sets are components of a mortality class, so that

$$\sum_{s=1}^{z} l_{[x(s)]} = l_{[x]} . \qquad (7)$$

Sets are the common elements or "building blocks" of different classes, and the radixes of the sets in any class will depend on the proportions which the lives in any stratum bear to the total number of lives in the class. These proportions, obviously, are $l_{[x(1)]}/l_{[x]}$, $l_{[x(2)]}/l_{[x]}$, etc., and in view of their importance, a special symbol ρ, called the *Set Radix* will be employed to identify them. Hence

$$\frac{l_{[x(1)]}}{l_{[x]}} \text{ will be designated } {}_1\rho_{[x]}$$

$$\frac{l_{[x(2)]}}{l_{[x]}} \text{ will be designated } {}_2\rho_{[x]}, \text{ etc.}$$

Thus

$$\sum_{s=1}^{z} {}_s\rho_{[x]} = 1 \qquad (8)$$

and

$$\sum_{s=1}^{z} {}_s\rho_{[x]} l_{[x]} = l_{[x]} . \qquad (9)$$

If the set radixes characteristic of a mortality class are given, it will be noted that the sizes of the several sets composing the class may directly be determined.

22. At the end of one year from the date of organization of the class, the survivors

$$\sum_{s=1}^{z} {}_s\rho_{[x]} l_{[x]} \cdot {}_s p = l_{[x]} p_{[x]} . \qquad (10)$$

On subtracting equation (10) from (9) and dividing by $l_{[x]}$, we obtain

$$\sum_{s=1}^{z} {}_s\rho_{[x]} \cdot {}_s q = q_{[x]} ; \qquad (11)$$

which is to say that, for the class, the average probability of dying within a year after the formation of a class is equivalent to the weighting by the set radixes of the absolute rates of mortality respectively operative in the several sets comprising the class.

23. On the basis of the particular radixes of the sets composing a class

$$\sum_{s=1}^{z} d_{[x(s)]+n} = d_{[x]+n} \tag{12}$$

and

$$\frac{\sum_{s=1}^{z} d_{[x(s)]+n}}{\sum_{s=1}^{z} l_{[x(s)]+n}} = q_{[x]+n}\,. \tag{13}$$

24. Commutation functions developed for sets may also be added to equal the corresponding functions of the class; thus

$$\sum_{s=1}^{z} \mathrm{D}_{[x(s)]+n} = \mathrm{D}_{[x]+n}$$

$$\sum_{s=1}^{z} \mathrm{C}_{[x(s)]+n} = \mathrm{C}_{[x]+n}$$

$$\sum_{s=1}^{z} \mathrm{N}_{[x(s)]+n} = \mathrm{N}_{[x]+n}$$

$$\sum_{s=1}^{z} \mathrm{M}_{[x(s)]+n} = \mathrm{M}_{[x]+n}$$

25. In carrying out the foregoing summations, it should be remembered that the elements being added vary in amount from one class to another according to the values of ρ employed in determining the sets. General commutation functions for mortality classes cannot be developed, therefore, since each class will depend on its peculiar composition.

POPULATION TABLES

26. A mortality class implies some process by means of which the proportions of the lives in the several strata come to be included in, or excluded from, the class. However, despite the fact that in the population all lives are included and none omitted, the lives at any age in the population may be thought of as a class since they may be considered as falling

into regular sets. At the same time the persons living at any age may be considered to be the survivors of classes established at younger ages. The population table, accordingly, may be regarded either as a series of classes organized at the several ages (the prospective view, as it were) or as the survivors of a class formed at some prior age (the retrospective view).

27. Considering individuals in the population as classes organized at the several ages, we have for the lives in the general population at age x:

$$\sum_{s=1}^{z} {}_s\rho_{[x]} l_x = l_x \,, \tag{14}$$

where the x in the symbol for this peculiar class is, in accordance with conventional usage, written without brackets. However, the age subscripts in the set radixes are placed in brackets to indicate the age at which the class is formed. The survivors of this class living at age $x + 1$ are distributed in the several strata as follows:

Stratum	Survivors	
1	${}_1\rho_{[x]} l_x \cdot {}_1p \cdot {}_1^1a_x$	(15)
2	$\begin{cases} {}_1\rho_{[x]} l_x \cdot {}_1p \cdot {}_2^1a_x \\ + {}_2\rho_{[x]} l_x \cdot {}_2p \cdot {}_2^2a_x \end{cases}$	(16)
3	$\begin{cases} {}_1\rho_{[x]} l_x \cdot {}_1p \cdot {}_3^1a_x \\ + {}_2\rho_{[x]} l_x \cdot {}_2p \cdot {}_3^2a_x \\ + {}_3\rho_{[x]} l_x \cdot {}_3p \cdot {}_3^3a_x, \text{ etc.} \end{cases}$	(17)

28. The individuals aged $x + 1$ in the population regarded as a class may be indicated in the same manner as in (14):

$$\sum_{s=1}^{z} {}_s\rho_{[x+1]} l_{x+1} = l_{x+1} \,. \tag{18}$$

The number of lives in (18), of course, is the same as the survivors of (14) at age $x + 1$; and the expressions in (15), (16), (17), etc., are respectively equal to the terms for the corresponding strata in (18) since they each represent the number of lives at age $x + 1$ in the indicated strata. That is to say, for stratum 1,

$${}_1\rho_{[x]} l_x \cdot {}_1p \cdot {}_1^1a_x = {}_1\rho_{[x+1]} l_{x+1} \,, \tag{19}$$

whence

$${}_1\rho_{[x]} \cdot {}_1p \cdot {}_1^1a_x = {}_1\rho_{[x+1]} p_x \,; \tag{20}$$

for stratum 2,

$$ {}_1\rho_{[x]} \cdot {}_1p \cdot {}^1_2a_x + {}_2\rho_{[x]} \cdot {}_2p \cdot {}^2_2a_x = {}_2\rho_{[x+1]} \cdot p_x \; ; \tag{21} $$

for stratum 3,

$$ \sum_{s=1}^{3} {}_s\rho_{[x]} \cdot {}_sp \cdot {}^s_3a_x = {}_3\rho_{[x+1]} p_x \; ; \tag{22} $$

for stratum 4,

$$ \sum_{s=1}^{4} {}_s\rho_{[x]} \cdot {}_sp \cdot {}^s_4a_x = {}_4\rho_{[x+1]} p_x \; ; \tag{23} $$

etc .

29. In deriving rates of deterioration it will be assumed (in paragraphs 34 *et seq.*) that these rates, already subject to the condition that

$$ \sum_{t=0}^{z-s} {}^s_{s+t}a_x = 1 \, , $$

are expressible as a frequency distribution such that

$$ {}^s_{s+t}a_x = f_t\left({}^s_sa_x\right) . \tag{24} $$

30. On solving (20) for ${}^1_1a_x$ we obtain

$$ {}^1_1a_x = \frac{{}_1\rho_{[x+1]} p_x}{{}_1\rho_{[x]} \cdot {}_1p} . \tag{25} $$

The solution of (21) yields

$$ {}^2_2a_x = \frac{{}_2\rho_{[x+1]} p_x - {}_1\rho_{[x]} \cdot {}_1p \cdot {}^1_2a_x}{{}_2\rho_{[x]} \cdot {}_2p} \tag{26} $$

$$ = \frac{{}_2\rho_{[x+1]} p_x - {}_1\rho_{[x]} \cdot {}_1p \cdot f_1\left({}^1_1a_x\right)}{{}_2\rho_{[x]} \cdot {}_2p} . \tag{27} $$

In similar fashion the solutions of (22) and (23) give, respectively,

$$ {}^3_3a_x = \frac{{}_3\rho_{[x+1]} p_x - {}_1\rho_{[x]} \cdot {}_1p \cdot f_2\left({}^1_1a_x\right) - {}_2\rho_{[x]} \cdot {}_2p \cdot f_1\left({}^2_2a_x\right)}{{}_3\rho_{[x]} \cdot {}_3p} , \tag{28} $$

$$ {}^4_4a_x = $$

$$ \frac{{}_4\rho_{[x+1]} p_x - {}_1\rho_{[x]} \cdot {}_1p \cdot f_3\left({}^1_1a_x\right) - {}_2\rho_{[x]} \cdot {}_2p \cdot f_2\left({}^2_2a_x\right) - {}_3\rho_{[x]} \cdot {}_3p \cdot f_1\left({}^3_3a_x\right)}{{}_4\rho_{[x]} \cdot {}_4p} , \text{ etc.} \tag{29} $$

These equalities are useful in computing rates of deterioration and will be referred to later.

31. Rates of deterioration are basic to all the variety of death rates experienced among lives having in common certain general constitutional features as well as the broad environment in which they exist. Unfortunately, as a practical matter these rates cannot be obtained by direct observation. For one thing, criteria for measuring deterioration in vitality are lacking in quantitative refinement; for another, it is virtually impossible to trace sufficiently large groups over a long enough period to determine rates of deterioration having statistical validity.

32. One possible method of deriving rates of deterioration not dependent on direct methods is to estimate such rates from the internal evidence supplied by large exposures in which possible distortions due to withdrawals of lives are not present. A census study would seem to provide a suitable medium for this purpose, and analyses have been made of the data given in the *United States Life Tables 1949–51* with a view to extracting these rates.

33. This table appears to be a particularly suitable one for this purpose, inasmuch as it was constructed in a period when migration in both directions was quite small and since it is reasonably contemporary with several intercompany studies of insurance experience on a select basis, such as the *1946–1949 Select Basic Table* and the mortality experienced on disabled lives given in *TSA* 1952 Reports, each of which provides opportunities to test the application of this theory in practical instances.

DERIVATION OF PRACTICAL RATES OF DETERIORATION

34. Rates of deterioration derived from the *United States Life Tables 1949–51* were based on the following assumptions:

(1) that rates of deterioration applicable to lives in a given stratum at a given age might be expressed as a frequency distribution represented by the terms of a binomial having as many terms as the number of strata to which a life might move in one year;

(2) that the entire range of mortality might be embraced within six strata, the q's of which are in the following geometric progression with ratio 4.5,

Rate	Stratum	Rate of Death
${}_1q$	1	.00040
${}_2q$	2	.00180
${}_3q$	3	.00810
${}_4q$	4	.03645
${}_5q$	5	.164025
${}_6q$	6	.7381125

35. The relative ease of operating on the binomial contributed to its employment in this paper. Choosing it was an empirical matter and does not indicate a conviction that deterioration in vitality actually follows such a distribution. Other frequency distributions may very well be superior for the purpose. Furthermore, it is realized that the use of strata as broad as those indicated introduces some statistical complications. However, the results flowing from these assumptions exemplified in the applications illustrated later seem to be reasonable by the usual pragmatic tests.

36. A life in stratum 1, accordingly, could, after one year, either continue in that stratum or move to one of the other five strata. In accordance with the first assumption, therefore, rates of deterioration might be equated respectively to the six terms of the binomial expansion $[{}_1b_x + (1 - {}_1b_x)]^5$ as indicated in the following tabulation:

To Stratum	1	2	3	4	5	6
Term	${}^1_1a_x$	${}^1_2a_x$	${}^1_3a_x$	${}^1_4a_x$	${}^1_5a_x$	${}^1_6a_x$
Value	${}_1b_x^5$	$5\,{}_1b_x^4(1-{}_1b_x)$	$10\,{}_1b_x^3(1-{}_1b_x)^2$	$10\,{}_1b_x^2(1-{}_1b_x)^3$	$5\,{}_1b_x(1-{}_1b_x)^4$	$(1-{}_1b_x)^5$

Since

$$ {}^1_1a_x = \frac{{}_1p_{[x+1]}\,p_x}{{}_1p_{[x]}\cdot {}_1p} $$

by equation (25),

$$ {}_1b_x = \sqrt[5]{\frac{{}_1p_{[x+1]}\,p_x}{{}_1p_{[x]}\cdot {}_1p}}\,, $$

from which all the terms in the expansion may be evaluated once the values of p are derived (see paragraphs 39 *et seq.*). The rates of deterioration from stratum 1, accordingly, are completely determinable.

37. Correspondingly, a life in stratum 2 might, on surviving one year, be in any one of five strata. The a's would be represented as follows:

To Stratum	2	3	4	5	6
Term	${}^2_2a_x$	${}^2_3a_x$	${}^2_4a_x$	${}^2_5a_x$	${}^2_6a_x$
Value	${}_2b_x^4$	$4\,{}_2b_x^3(1-{}_2b_x)$	$6\,{}_2b_x^2(1-{}_2b_x)^2$	$4\,{}_2b_x(1-{}_2b_x)^3$	$(1-{}_2b_x)^4$

By equation (27),

$$ {}_2^2a_x = \frac{{}_2\rho_{[x+1]}\,p_x - {}_1\rho_{[x]}\cdot{}_1p\cdot 5\,{}_1b_x^4\,(1-{}_1b_x)}{{}_2\rho_{[x]}\cdot{}_2p} . $$

The fourth root of the expression on the right, accordingly, yields ${}_2b_x$, from which, after obtaining the indicated ρ functions, the five rates of deterioration from stratum 2 may be computed.

38. To illustrate further, the survivor of a life in stratum 5 would be subject at the next year of age to the following rates of deterioration:

$$ {}_5^5a_x = {}_5b_x $$

$$ {}_6^5a_x = (1 - {}_5b_x) . $$

By the methods indicated above, these a's may also be calculated. A life in stratum 6, of course, has open to him only the same stratum, so that ${}_6^6a_x = 1$.

DERIVATION OF SET RADIXES FROM CENSUS TABLE

39. Set radixes derived from general population life tables, by making appropriate assumptions as to the distribution of lives by stratum (see par. 41 *et seq.*), may be used for mortality classes selected from that population.

40. In the computation of set radixes for the general population (census) table, the equality demonstrated in equation (11) is used, namely, that

$$ \sum_{s=1}^{6} {}_s\rho_{[x]}\cdot{}_sq = q_x , \qquad (30) $$

where the sum of the ρ's is 1. Also assumed is a distribution of the ρ's in accordance with the frequencies indicated in the binomial expansion. (See comments on the use of the binomial in paragraph 35.) Values of ρ have been determined from r's which satisfy the equation[1]

$$ .00040\,[r + (1-r)\,4.5]^5 = q_x \qquad (31) $$

by setting

${}_1\rho_{[x]}$ equal to r^5

${}_2\rho_{[x]}$ " " $5r^4(1-r)$

${}_3\rho_{[x]}$ " " $10r^3(1-r)^2$

${}_4\rho_{[x]}$ " " $10r^2(1-r)^3$

${}_5\rho_{[x]}$ " " $5r(1-r)^4$

${}_6\rho_{[x]}$ " " $(1-r)^5$

[1] Based on equation (11), and assumption (2) in paragraph 34.

41. The character of the rates of mortality experienced in any class from year to year is dependent on the relative magnitudes of the set radixes employed to determine the class. Set radixes for the census table may be arrived at empirically as described above. However, there is no systematic method of determining the set radixes for any class, so that resort must be taken to devices of various kinds to arrive at set radixes for each class which reproduce in satisfactory fashion the experience to be expressed in terms of classes.

42. One device found reasonably successful is the following procedure: an array of set radixes for a standard table (the census table, for example) computed on the basis of the binomial distribution is entered to determine the set radixes for such age which when applied to unit sets at the age being worked on reproduce some salient features of the class. To illustrate, suppose the experience data indicate that, under a class of lives, c deaths develop in n years. An age y in the census table may be found to describe the class so that the sum of the sets comprising the class

$$\left(\sum_{s=1}^{s} {}_{s}\rho_{[y]} l_{[x]}\right)$$

would all together produce the number of deaths satisfying the equation:

$$\sum_{t=0}^{n-1} \{ d_{[x(1)]+t} + d_{[x(2)]+t} + \ldots + d_{[x(s)]+t} \} = c\,. \qquad (32)$$

43. The device of utilizing the distribution of set radixes at age y of the census table in the characterization of a mortality class organized at age x will be employed in the applications described later. Such an age y will be identified by a symbol using angle brackets and written as the numerator of a fraction having the actual age of the lives involved as a denominator. Thus, $\langle y \rangle / x$ will indicate the distribution by stratum of a mortality class formed at age x having set radixes equal to those of the general population at age y. The denominator of the foregoing symbol may be omitted if the age being dealt with is clearly understood.

44. If the age y is taken from a census table and the class is a collection of medically examined insured lives, y will ordinarily be smaller than x. If, on the other hand, the class $l_{[x]}$ is composed of highly impaired lives, y may very well be greater than x.

HYBRID MORTALITY CLASSES

45. In his classical paper on select mortality tables presented to the British Institute in 1881 (*JIA* XXII) Sprague wrote:

. . . when we bear in mind that the rate of mortality among insured lives depends upon the rate of lapse, in such a way that, the greater the rate of lapse, the greater will be the rate of mortality, we see that, in order to determine the true value of the policy, we ought to know the values of assurances and annuities upon select lives among which there are no withdrawals. At present, however, I believe there are no tables in existence that will give us the values of these quantities. The fact is that, although we have ample materials for determining the exact rate of mortality among insured lives, and are able to determine with great accuracy the rate of mortality during the first insurance year for entrants of any age, and to trace the gradual increase in the rate of mortality among them caused by the combined operation of advancing age and the withdrawals of healthy lives, yet we have no means at present of determining how the rate of mortality would increase if there were no such withdrawals. All we can say is, that there is good reason for believing that the rate of mortality would be less than is now found to prevail among insured lives. . . .

(The spelling has been changed from the simplified form actually used by Sprague.)

46. The mortality classes so far described in this paper represent collections of lives all of whom are assumed to remain under observation from the formation of the class until death. Sprague's conclusions, which seem supported by general reasoning, are here adopted, and in this section a class is considered as hybrid when the mortality of those lives withdrawing is different[2] from that of the lives under observation. A mortality class, no members of which withdraw during the experience otherwise than by death, will be referred to as a "simple class."

47. Let it be assumed that such a hybrid class is composed of two subclasses experiencing mortality appropriate respectively to withdrawing lives (this subclass will be distinguished by a prime) and to lives not withdrawing (this subclass will be marked by a double prime). At the formation of the class, the number of lives in the former subclass represents $100a\%$ of the total number, and, of course, the number in the latter is 100 $(1 - a)\%$. If the radixes of the l's in these three classes each be equal to 1,

$$al'_{[x]} + (1 - a)\, l''_{[x]} = l_{[x]}\ . \qquad (33)$$

48. In the hybrid class $l_{[x]}$, the number withdrawing from observation in year n, $l_{[x]+n-1}(wq)_{[x]+n-1}$, will be indicated by $w_{[x]+n-1}$. The number of deaths in that year among those who had previously withdrawn (with-

[2] The principle would apply if the mortality of those withdrawing were heavier, as it might be, say, among those released from military service for reasons of health.

drawal being assumed to take place at end of year) will be designated $(wd)_{[x]+n-1}$, where

$$(wd)_{[x]+n-1} = q'_{[x]+n-1} \sum_{t=1}^{n-1} (w_{[x]+t-1} \cdot {}_{n-t-1}p'_{[x]+t}) \qquad (34)$$

and where the *total* number of deaths in that year among all the survivors of the original entrants, as in a double decrement table, is equal to

$$d_{[x]+n-1} + (wd)_{[x]+n-1} \, . \qquad (35)$$

49. In year n, on the other hand, the deaths arising among the two subclasses are equal, respectively, to $ad'_{[x]+n-1}$ and $(1-a)d''_{[x]+n-1}$. Equating the *total* number of deaths, therefore,

$$ad'_{[x]+n-1} + (1-a)\,d''_{[x]+n-1} = d_{[x]+n-1} + (wd)_{[x]+n-1} \, . \qquad (36)$$

50. The total number of deaths developing in the first g years is the sum for the g values of the several respective terms entering into equation (36); that is,

$$a\sum_{n=1}^{g} d'_{[x]+n-1} + (1-a)\sum_{n=1}^{g} d''_{[x]+n-1} = \sum_{n=1}^{g} d_{[x]+n-1} + \sum_{n=1}^{g} (wd)_{[x]+n-1} \, . \qquad (37)$$

51. Equation (36) may be solved for a, the proportion of the hybrid class composed of lives exhibiting mortality characteristic of "withdrawal prone" lives, to give

$$a = \frac{d_{[x]+n-1} - d''_{[x]+n-1} + (wd)_{[x]+n-1}}{d'_{[x]+n-1} - d''_{[x]+n-1}} \, . \qquad (38)$$

It should be remembered, however, that unless the hybrid table for $l_{[x]}$ is constructed on the basis of the elementary functions underlying the theory described, the value of a will not be identical for every value of n in (36).

52. The solution of (37) would therefore yield a more significant value, since the results of several years may be taken into account. The determination of a from (37) results in the following:

$$a = \frac{\sum_{n=1}^{g} d_{[x]+n-1} - \sum_{n=1}^{g} d''_{[x]+n-1} + \sum_{n=1}^{g} (wd)_{[x]+n-1}}{\sum_{n=1}^{g} d'_{[x]+n-1} - \sum_{n=1}^{g} d''_{[x]+n-1}} \, . \qquad (39)$$

53. In the case of a hybrid class the composition of the two subclasses poses a problem requiring for solution the determination of set radixes for the two subclasses. One method of approaching this problem is to obtain by successive approximation values for the two subclasses entering into equation (39). The mortality of lives continuing under observation and rates of withdrawal being given, this method involves determining $\langle y' \rangle / x$ and $\langle y'' \rangle / x$ such that for two convenient values of the g of equation (39), the same value of a results.

54. That is to say, if it be assumed that the two values of g are g_1 and g_2, the y functions ($\langle y' \rangle$ and $\langle y'' \rangle$) desired will be those for which

$$a = \frac{\sum_{n=1}^{g_1} d_{[x]+n-1} - \sum_{n=1}^{g_1} d''_{[x]+n-1} + \sum_{n=1}^{g_1} (wd)_{[x]+n-1}}{\sum_{n=1}^{g_1} d'_{[x]+n-1} - \sum_{n=1}^{g_1} d''_{[x]+n-1}}$$

$$= \frac{\sum_{n=1}^{g_2} d_{[x]+n-1} - \sum_{n=1}^{g_2} d''_{[x]+n-1} + \sum_{n=1}^{g_2} (wd)_{[x]+n-1}}{\sum_{n=1}^{g_2} d'_{[x]+n-1} - \sum_{n=1}^{g_2} d''_{[x]+n-1}} . \qquad (40)$$

55. Assuming that a series of mortality rates, $q_{[x]+n-1}$, and a series of withdrawal rates, $(wq)_{[x]+n-1}$, are given, the characteristics of a hybrid table which will approximate certain desired features of the given experience may be determined by the following practical method:

(1) Decide what features are to be reproduced. A relatively uncomplicated function for this purpose is the number of deaths over the two different periods, g_1 and g_2. In view of the rapid changes frequently encountered in rates of mortality at the earliest durations, g_1 may very well be taken as 3 or 4. For g_2, the common 15 year period used in recent mortality investigations is a good choice. In the experiments described later, 3 and 15 are consistently used for g_1 and g_2 respectively.

(2) Select an age distribution $\langle y' \rangle / x$. The choice here should be an age somewhat younger than $[x]$ if it is thought that the mortality of the subclass subject to withdrawal is lighter than that of the hybrid class. On the basis of a subclass with a unit radix subject to rates of mortality appropriate to $\langle y' \rangle / x$, obtain the number of deaths over g_1 years and g_2 years, that is,

$$\sum_{n=1}^{g_1} d'_{[x]+n-1} \quad \text{and} \quad \sum_{n=1}^{g_2} d'_{[x]+n-1}\,.$$

(3) Keeping in mind that $\Sigma d_{[x]+n-1}$ and $\Sigma(wd)_{[x]+n-1}$ are, in effect, given in the basic assumptions of the problem, an age $\langle y''\rangle/x$ may be determined which will produce values of $\Sigma d''_{[x]+n-1}$ satisfying the condition that the second and third expressions in (40) be equal.

(4) $\langle y'\rangle$ and $\langle y''\rangle$ having been obtained, by means of equation (39), the relative proportions, a and $(1 - a)$, respectively, of the two subclasses may be derived.

(5) With the data now available, the d's of the hybrid class may be computed by means of equation (36). The q's may be developed by the usual methods.

(6) The q's so arrived at may be compared with the given rates. The usual criteria of closeness of fit may be employed to test the results. Incidentally, to the extent that the progression of deaths is smooth in the component basic sets, the resulting functions in the hybrid class will be smooth and require no further graduation.

(7) If the results are not satisfactory, a choice of other values of g_1, g_2 and $\langle y'\rangle$ may turn out better.

TABLES

56. The basic and some derived tables developed as described in this paper from an analysis of the *United States Life Tables 1949–51* for white males are given in the Appendix. Rates of mortality in the Census Tables diminish with age over some younger age spans. Where this occurs, the binomial distribution does not produce satisfactory rates of deterioration, so that although set radixes for the population distributions are shown for ages down to 10, no α's are shown for ages younger than 26. The youngest age ρ's may be of use in obtaining $\langle y\rangle$ ages.

57. In making calculations under the methods described, it will be found that frequent use is made of functions involving $\langle y\rangle/x$, especially the series of $d_{[x]+n-1}$. The latter function may be obtained by weighting the d's of the sets composing the class organized at age x by the population set radixes applicable to age y. A considerable saving in time and computation may be effected by obtaining by mechanical means tabulations of $d_{[x]+n-1}$ for all values of $\langle y\rangle$ likely to be needed. In order to keep the length of this paper within reasonable bounds, only some illustrations of this function for age 37 are shown. If any considerable volume of calculations is undertaken, a fairly full array of such functions is practically indispensable.

APPLICATIONS

58. Some experiments are here described illustrating the processes developed in this paper. What is done, generally, is to determine for each of several ages the mortality class which yields rates of mortality comparable with those shown in a select table constructed from experience in the vicinity of the year 1950. The select tables employed for this purpose include the 1946–1949 Basic Tables, the Benefit 1 Disabled Life Table included in *TSA* 1952 Reports and some data included in the annuity study prepared by the Joint Committee on Mortality in *TASA* XLIX, 112.

59. Withdrawal rates needed in the construction of hybrid tables are not available in connection with the comparisons made with the 1946–1949 Basic Table. (Such rates might have been obtained as a by-product of the investigation. Some consideration, it is suggested, might be given to deriving rates of termination as a natural concomitant of all mortality investigations.) In the absence of these data, it was assumed that Linton's "Medium" Termination Rates[3] were experienced at all ages. For the period of the study it is thought that this assumption is reasonable.

60. In the work done in connection with the disabled life study, the yearly rates of recovery given in the report were taken in all formulas to be a decrement precisely equivalent to the rate of withdrawal. No such decrement, of course, entered into the comparison made with the annuity study.

61. The several published tables used show rates of mortality for 5 or 10 year age groups. Where the age grouping covered 5 years, it was assumed that the rates represented those applicable to the age at the midpoint. However, where the range covered 10 years, the rates shown were taken to be those appropriate to an age one-half year older than the middle age of the group.

SINGLE PREMIUM NONREFUND ANNUITIES

62. The data used in this illustration are those given in the Report of the Joint Committee on Mortality on the mortality under individual annuities issued between 1931 and 1945 and observed between 1941 and 1946 anniversaries (*TASA* XLIX, 112). The application described is based on

[3] These rates are as follows:

Year	Med. Term. Rate	Year	Med. Term. Rate	Year	Med. Term. Rate
1	15 %	6	5.1%	11	3.3%
2	8.8	7	4.5	12	3.2
3	7.3	8	4.1	13	3.0
4	6.3	9	3.8	14	2.9
5	5.7	10	3.5	15	2.7

figures shown for age intervals 50–59, 60–69, and 70–79, and is limited to experience by number of annuity contracts issued on the nonrefund basis.

63. As a first step, simple male and female mortality classes at ages 55, 65 and 75 were obtained by determining y ages for which $2\frac{1}{2}\%$ annuities on the bases described in this paper would be equal to those shown in Table 8 (*TASA* XLIX, 125). Inasmuch as the commutation columns developed for this paper are based on $3\frac{1}{2}\%$, special calculations were employed in deriving the $2\frac{1}{2}\%$ values.

64. The y ages and $q_{[x]+n}$'s underlying the annuities so calculated are shown in Table 1.

65. In the study of annuity mortality, durations 6 through 15 (the maximum as measured by the interval between issuance in 1931 and the anniversary in 1946) were combined by attained age. In the light of the principles of this paper such a procedure suffers from two defects: first, the study sheds no light on the mortality at the attained ages beyond the fifteenth year; second, it obscures the gradation of death rates for durations between the fifth and fifteenth years. Since the attained age experience, it may be guessed, is weighted heavily by lives entering in the later 60's, the rates of mortality at the longer durations for the younger lives at entry are probably understated, while those for the durations following shortly after the fifth year in the case of the older lives, in all likelihood, are overstated.

66. In view of such considerations, it may be that the annuity premiums based on the study and shown in Table 8 of the Report are too high at the younger ages and too low at the older. Paradoxically, this is very much like saying that premiums based on the Standard Annuity Table may more closely represent the experience investigated, on a select basis, than the premiums shown.

67. Attention may be called to the ratios in Table 1 of this paper at the common attained ages included which show a well-marked progression with duration after the organization of the class. These rates it will be remembered are derived so as to reproduce the annuity premiums shown in Table 8 of the Report. If those annuity premiums had been closer to those based on the Standard Annuity Table, the death rates at the shorter durations produced by the present method would probably have been somewhat higher at entry age 55 and somewhat lower at entry age 75.

68. If the annuity experience had been large enough to minimize chance fluctuations and had indicated mortality results on a select basis for all 15 durations (as in the continuous intercompany insurance mortality investigation), mortality classes could have been described on the present method which might have had greater credibility. Where there are no withdrawals, theoretically even five years of select experience

TABLE 1

RATES OF MORTALITY UNDERLYING 2½% NONREFUND LIFE ANNUITIES, COMPUTED BY PRESENT METHOD, EQUAL TO CORRESPONDING ANNUITIES SHOWN IN REPORT OF JOINT COMMITTEE ON MORTALITY (*TASA* XLIX, 125)

(Ratios to 1937 Standard Annuity Table)

MALE

Age [x]:	55		65		75	
⟨y⟩:	48.20		57.99		69.83	
n	1,000 $q_{[x]}$	Ratio	1,000 $q_{[x]+n}$	Ratio	1,000 $q_{[x]+n}$	Ratio
0	8.61	64%	20.39	71%	49.65	82%
1	10.05	69	23.78	77	58.84	90
2	11.49	73	27.11	81	67.26	96
3	13.01	77	30.60	85	75.54	100
4	14.60	80	34.28	88	83.93	104
5	16.29	82	38.15	91	92.69	106
10	26.76	93	63.75	105		
15	42.52	102	99.91	115		
20	67.37	111				
25	102.88	118				

FEMALE

Age [x]:	55		65		75	
⟨y⟩:	44.34		55.36		68.00	
n	1,000 $q_{[x]+n}$	Ratio	1,000 $q_{[x]+n}$	Ratio	1,000 $q_{[x]+n}$	Ratio
0	3.52	38%	8.81	45%	27.64	66%
1	4.10	41	10.65	50	34.97	78
2	4.77	44	12.74	55	42.23	87
3	5.51	47	15.17	61	49.87	96
4	6.35	51	17.92	67	59.97	103
5	7.27	54	21.04	73	66.62	110
10	13.53	69	42.69	102		
15	25.83	90	77.21	128		
20	47.57	114				
25	81.79	135				

should be sufficient to describe a mortality class. However, in such a case the volume of data should be large enough to rule out material fluctuations.

STANDARD MEDICALLY EXAMINED INSURED LIVES

69. The study made by the Joint Committee on Mortality of intercompany experience in the period 1946–1949 on insured lives under standard policies issued subject to examination is much more useful for present purposes than the annuity study just referred to. The advantages, as may be expected, lie in the volume of data and the completeness of the select data exhibited.

70. The 1946–1949 experience, however, did not separate the experience on male and female lives. The rates of mortality shown in the Report, accordingly, it may be assumed with confidence, are lower than they would be on a true male experience. But the preponderance of lives were male and it is thought that, for purposes of illustration, no great error will be committed if the data are treated as if they were composed exclusively of males. At any rate, such a procedure is consistent with the methods of the 1946–1949 investigation, and any results developed are comparable with those given in the Report.

71. In an investigation of insured lives some of whom withdraw, only the lives continuing in the experience and the deaths among them yield the death rates finally arrived at. If, as is assumed, the mortality among those withdrawing is lighter than among those who remain, a progressively decreasing proportion of the lives is subject to the lighter mortality rates and the slope of the death curve, therefore, is determined not only by the increasing hazard of death with age but also by the increasing concentration of the less healthy lives. These considerations, of course, suggest a hybrid class such as has been described.

72. Hybrid classes, accordingly, were formed at ages 27, 37, 47 and 57 of such a nature that the number of deaths over 15 years among the survivors remaining within the experience was equal to those arising among such survivors under a double decrement table. The decrements were those occasioned by death according to the crude 1946–1949 death rates (given in *TSA* II, Table 11, page 510) and by withdrawal in accordance with the Linton Medium Annual Rates of Termination (par. 59, footnote). The numbers of deaths over the first three years were also equated (since g_1 was taken as 3, par. 55, (1)).

73. The assumptions employed in completing the hybrid classes at the four ages are shown in the tabulation at the top of page 67.

74. Table 2 shows the number of deaths among the persisting lives in each year of the 15 year select period under the present method and on the

bases of the crude 1946–1949 Table and the Linton Medium Rates of Termination. It should be observed that about the same numbers of deaths occur over years 1–3 and years 1–15 under the two methods.

Age $[x]$	27	37	47	57
$\langle y' \rangle$	12	13	19	25
$\langle y'' \rangle$	16.8600	18.6420	38.6640	50.1848
a	.8055	.4927	.4344	.5945
g_1	3	3	3	3
g_2	15	15	15	15

75. Comparative rates of mortality on the bases in Table 2 are given in Table 3. The processes by which the death rates were produced on the present method, it will be noted, serve also as a method of graduation.

TABLE 2

NUMBER OF DEATHS AMONG LIVES PERSISTING OF 1,000 ORIGINAL ENTRANTS

Subject to Withdrawal According to Linton Medium Rates of Withdrawal and Mortality According to (1) Present Method (Paragraph 73) (2) Crude 1946–49 Rates

Age:	27			37			47			57		
Year	(1)	(2)	(1)−(2)	(1)	(2)	(1)−(2)	(1)	(2)	(1)−(2)	(1)	(2)	(1)−(2)
1	.79	.69	.10	1.13	.96	.17	2.60	2.32	.28	5.19	4.99	.20
2	.71	.74	−.03	1.15	1.21	−.06	2.78	2.53	.25	5.80	5.70	.10
3	.68	.76	−.08	1.24	1.36	−.12	3.05	3.61	− .56	6.47	6.77	− .30
4	.67	.85	−.18	1.35	1.50	−.15	3.36	3.95	− .59	7.16	8.53	−1.37
5	.67	.71	−.04	1.48	1.61	−.13	3.69	3.53	.16	7.86	10.59	−2.73
6	.69	.69	.00	1.63	1.70	−.07	4.07	3.98	.09	8.59	8.88	− .29
7	.71	.64	.07	1.78	1.71	.07	4.47	4.00	.47	9.31	8.71	.60
8	.75	.86	−.11	1.96	2.03	−.07	4.91	5.35	− .44	10.02	9.87	.15
9	.80	.73	.07	2.14	2.24	−.10	5.35	4.95	.40	10.71	10.38	.33
10	.86	.88	−.02	2.34	2.16	.18	5.82	5.36	.46	11.42	11.02	.40
11	.92	.85	.07	2.56	2.50	.06	6.28	6.51	− .23	12.15	11.16	.99
12	1.00	.93	.07	2.82	2.83	−.01	6.76	6.92	− .16	12.86	11.58	1.28
13	1.08	1.09	−.01	3.05	2.89	.16	7.20	6.36	.84	13.52	13.16	.36
14	1.19	1.24	−.05	3.30	3.17	.13	7.65	7.21	.44	14.11	15.07	− .96
15	1.29	1.15	.14	3.58	3.63	−.05	8.06	9.50	−1.44	14.67	13.42	1.25
Yrs. 1–3	2.18	2.19	−.01	3.52	3.53	−.01	8.43	8.46	− .03	17.46	17.46	.00
Yrs. 1–15	12.81	12.81	.00	31.51	31.50	.01	76.05	76.08	− .03	149.84	149.83	.01

The graduation effects considerable smoothness, perhaps at some sacrifice of fidelity to the crude data. This feature is well illustrated at age 57 where the large number of deaths noted in the crude data in policy years 4 and 5 results in a relatively high series of q's in that vicinity in the 1946–1949 graduated table.

76. The period over which the policies entering into the 1946–1949 investigation were in force was a period of mixed economic conditions. The 1940's, especially after the entry of the United States into the war, were years of full employment and economic activity and, we know, favorable

TABLE 3

Application to 1946–1949 Select Experience
Mortality Rates per 1,000
Comparison of Rates from Hybrid Table with Crude and Graduated Rates (*TSA* II, 506, 510)
(Linton Medium Rates of Termination Assumed)

Age:	27					37				
⟨y′⟩:	12					13				
⟨y″⟩:	16.8600					18.6420				
a:	.8055					.4927				
Policy Year	Present Method	1946–1949 Experience		(1) −	(3) −	Present Method	1946–1949 Experience		(1) −	(3) −
		Crude	Grad.				Crude	Grad.		
	(1)	(2)	(3)	(2)	(2)	(1)	(2)	(3)	(2)	(2)
1	.79	.69	.66	.10	− .03	1.13	.96	.97	.17	.01
2	.84	.87	.82	− .03	− .05	1.35	1.42	1.39	− .07	− .03
3	.88	.98	.97	− .10	− .01	1.60	1.76	1.78	− .16	.02
4	.93	1.18	1.04	− .25	− .14	1.90	2.10	2.07	− .20	− .03
5	1.00	1.06	1.12	− .06	.06	2.21	2.41	2.43	− .20	.02
6	1.09	1.09	1.17	.00	.08	2.59	2.70	2.75	− .11	.05
7	1.19	1.06	1.23	.13	.17	2.99	2.88	3.00	.11	.12
8	1.31	1.50	1.35	− .19	− .15	3.46	3.58	3.52	− .12	− .06
9	1.46	1.34	1.45	.12	.11	3.94	4.13	3.91	− .19	− .22
10	1.64	1.68	1.58	− .04	− .10	4.51	4.16	4.37	.35	.24
11	1.82	1.68	1.78	.14	.10	5.13	5.01	5.02	.12	.01
12	2.05	1.90	1.97	.15	.07	5.88	5.91	5.60	− .03	− .31
13	2.29	2.30	2.22	− .01	− .08	6.61	6.26	6.16	.35	− .10
14	2.60	2.72	2.56	− .12	− .16	7.42	7.14	6.96	.28	− .18
15	2.92	2.61	2.95	.31	.34	8.36	8.46	8.21	− .10	− .25

Age:	47					57				
⟨y′⟩:	19					25				
⟨y″⟩:	38.6640					50.1848				
a:	.4344					.5945				
Policy Year	Present Method	1946–1949 Experience		(1) −	(3) −	Present Method	1946–1949 Experience		(1) −	(3) −
		Crude	Grad.				Crude	Grad.		
	(1)	(2)	(3)	(2)	(2)	(1)	(2)	(3)	(2)	(2)
1	2.60	2.32	2.14	.28	− .18	5.19	4.99	4.66	.20	− .33
2	3.28	2.98	3.16	.30	.18	6.86	6.74	6.84	.12	.10
3	3.96	4.68	4.27	− .72	− .41	8.46	8.85	9.52	− .39	.67
4	4.73	5.56	5.04	− .83	− .52	10.19	12.14	11.62	−1.95	− .52
5	5.57	5.33	5.95	.24	.62	12.08	16.31	13.92	−4.23	−2.39
6	6.55	6.42	6.75	.13	.33	14.18	14.76	15.42	− .58	.66
7	7.64	6.84	7.43	.80	.59	16.43	15.49	16.42	.94	.93
8	8.85	9.64	8.72	− .79	− .92	18.82	18.69	18.68	.13	− .01
9	10.15	9.39	9.80	.76	.41	21.41	20.90	20.63	.51	− .27
10	11.60	10.69	11.15	.91	.46	24.26	23.57	23.05	.69	− .52
11	13.13	13.59	12.97	− .46	− .62	27.42	25.35	26.65	2.07	1.30
12	14.82	15.16	14.57	− .34	− .59	30.87	27.94	30.30	2.93	2.36
13	16.56	14.63	16.08	1.93	1.45	34.64	33.78	34.13	.86	.35
14	18.45	17.34	18.21	1.11	.87	38.64	41.30	38.50	−2.66	−2.84
15	20.41	23.98	21.17	−3.57	−2.81	43.10	39.56	43.60	3.54	4.04

policy persistency. However, the 1930's, for the most part, were a time in which life insurance experienced relatively poor persistency. It has been assumed that the Linton Medium Rates were representative, on the average, of the rate of lapse during the entire period. Since the close of World War II, persistency of business has been remarkably good. In order to test the effect which different rates of lapse would have on mortality rates, assuming all other things are unchanged, the rates shown in Table 4

TABLE 4

EFFECT OF CHANGE IN RATES OF WITHDRAWAL ON MORTALITY RATES PER 1,000
HYBRID CLASS—AGE 37

$\langle y' \rangle$: 13
$\langle y'' \rangle$: 18.6420
a: .4927

Withdrawal according to (1) Linton Medium Termination Rates
(2) Linton Termination Rates A
(3) Linton Termination Rates B

Year	Mortality Rates per 1,000				
	(1)	(2)	(3)	(2)−(1)	(3)−(1)
1	1.13	1.13	1.13	.00	.00
2	1.35	1.32	1.38	−.03	.06
3	1.60	1.55	1.67	−.05	.12
4	1.90	1.82	1.98	−.08	.16
5	2.21	2.11	2.34	−.10	.23
6	2.59	2.45	2.77	−.14	.32
7	2.99	2.83	3.24	−.16	.41
8	3.46	3.26	3.76	−.20	.50
9	3.94	3.70	4.32	−.24	.62
10	4.51	4.22	4.98	−.29	.76
11	5.13	4.80	5.69	−.33	.89
12	5.88	5.50	6.55	−.38	1.05
13	6.61	6.17	7.35	−.44	1.18
14	7.42	6.93	8.31	−.49	1.38
15	8.36	7.80	9.37	−.56	1.57

for age 37 at issue were prepared. These rates were computed by using exactly the assumptions set forth in paragraph 73 for the characterization of the hybrid class. However, in lieu of the Linton Medium Rates, it was assumed that in one case the Linton Termination Rates A[4] and, in the other, the Linton Termination Rates B[4] were experienced.

77. An interesting speculation is suggested by the changes brought about in the rate of mortality by altering the assumed rate of withdrawal. Is some part of the mortality improvement which has generally

[4] M. A. Linton, "Returns under Agency Contracts," *RAIA* XIII, 287.

been observed among insured lives since the 1946–1949 investigation attributable to the more favorable persistency enjoyed in recent years? Then, too, would the mortality of insured lives show a retrogression if rates of withdrawal worsened? No implication is intended that mortality generally has not experienced a genuine decline, since investigations not limited to insured lives undeniably show a diminution in death rates. What is offered as a possibility is the likelihood that variations in the mortality of insured lives may appear independently by reason of changes in the level of withdrawals.

DISABLED LIFE MORTALITY

78. The 1952 Report of the Committee on Disability and Double Indemnity contained a comprehensive intercompany investigation of experience during the period from 1930 to 1950 in the case of disability benefits offered by a number of companies. Included in the Report (*TSA* 1952 Reports, 102–104) was a study on a 15 year select basis of the annual rates of termination by death and recovery among persons who had become disabled. These termination rates were separately reported for three types of benefit, namely Benefit 1, Benefits 2 and 3 combined and Benefit 5.

79. In the case of each type of benefit, the tables indicated that very sizable proportions of lives qualifying for disability benefit recover in the course of several years. Since the rate of death among those recovering is unquestionably lower than that among those remaining disabled, and inasmuch as the data were in select form and, furthermore, provided an experience on withdrawal (by recovery), these tables seemed to lend themselves quite naturally to analysis as a hybrid class.

80. The waiting periods involved in Benefits 2, 3 and 5 seemed likely to give rise to special problems with regard to first year death rates, which it was felt would not obtain in the case of Benefit 1. Accordingly, hybrid tables based on the data given for Benefit 1 experience (Report, page 102) were developed at entry ages 37, 47 and 57.

81. The assumptions used in these instances were as follows:

Age $[x]$	37	47	57
$\langle y' \rangle$	44	55	59
$\langle y'' \rangle$	94.4498	94.8023	85.2977
a	.6230	.5188	.2270
g_1	3	3	3
g_2	15	15	15

82. The rates of mortality assumed experienced among the lives continuing to be classed as disabled according to the present method and as given in the 1952 Report are shown in Table 5.

COMMENTS ON THE MORTALITY OF IMPAIRED LIVES

83. The impact of withdrawal experience on the level of death rates and on their incidence cannot be overemphasized. The effects on the progression of mortality rates produced by rates of withdrawal are illustrated in connection with standard lives in the figures shown in Table 4, and in the case of disabled lives by the rates shown in the 1952 Disability Report (and in the hybrid class analysis described above). It must be assumed that the effects on impaired lives are not any less important.

TABLE 5

PROBABILITIES OF DEATH (× 1,000)—BENEFIT 1 DISABLED LIVES COMPARED WITH RATES SHOWN IN *TSA* 1952 REPORTS, PAGE 102

$1{,}000\; q_{[x]+n-1}$

Age [x]: ⟨y′⟩: ⟨y″⟩: a:	37 44 94.4498 .6230			47 55 94.8023 .5188			57 59 85.2977 .2270		
Disability Year	Present Method	1930–1950 Grad. Death Rates	Diff.	Present Method	1930–1950 Grad. Death Rates	Diff.	Present Method	1930–1950 Grad. Death Rates	Diff.
1......	115.4	115.0	.4	153.7	157.5	−3.8	134.0	138.1	−4.1
2......	62.7	61.1	1.6	90.2	85.6	4.6	99.6	98.3	1.3
3......	47.0	49.3	−2.3	68.8	69.2	− .4	88.1	85.0	3.1
4......	41.3	41.7	− .4	60.4	59.3	1.1	83.4	78.6	4.8
5......	38.5	37.3	1.2	56.3	54.1	2.2	81.0	77.0	4.0
6......	36.7	35.2	1.5	53.6	52.0	1.6	79.5	78.2	1.4
7......	35.1	34.3	.8	51.6	51.5	.1	78.7	80.4	−1.7
8......	33.7	34.0	− .3	49.9	51.4	−1.5	78.2	82.3	−4.1
9......	32.4	33.7	−1.3	48.7	50.9	−2.2	78.1	83.1	−5.0
10......	31.3	33.1	−1.8	47.8	49.9	−2.1	78.6	82.8	−4.2
11......	30.4	32.1	−1.7	47.2	48.4	−1.2	79.7	81.9	−2.2
12......	29.8	30.8	−1.0	47.0	46.9	.1	81.2	81.1	.1
13......	29.3	29.3	0	47.0	46.1	.9	83.2	81.3	1.9
14......	29.1	28.1	1.0	47.4	46.5	.9	85.3	82.8	2.5
15......	29.1	27.5	1.6	47.9	48.2	− .3	88.1	86.1	2.0

84. No information, however, appears to be published with respect to the proportion of policies placed on impaired lives, according to impairment, which terminate by surrender or lapse. Some contributions to experience, moreover, may treat reduction or removal of rating as a withdrawal from experience. It may perhaps fairly be suspected that the better lives with a given rating exercise a strong antiselection by giving up policies subject to extra charge.

85. Some attempts were made to apply the present methods to some of

the impaired life groups included in the *1951 Impairment Study*. The methods of this paper required data in select form and by age at entry, but the material in the study, although select in form, grouped all ages at entry together. Any application, therefore, involved either assuming that the ratios indicated were common to all ages or estimating possible differences in the progressions of ratios by age. Neither of these approaches was fruitful of result, and inasmuch as the absence of withdrawal figures interposed, in itself, a barrier to success, no further effort was made in this direction.

86. In interpreting the results of the *Impairment Study* (*TSA* VI, 291), three types of experience were noted. These types differed one from the other with respect to the relation percentagewise which the progression of mortality rates bore to the standard base. The most common type showed a relationship in that respect tending to diminish with duration; not as common were impairments which displayed a tendency either to a constant relationship or to an increasing one.

87. The first type (diminishing relationship) is what normally would be expected in any class which contained a larger proportion of lives in the higher mortality strata than are found in the standard group. The class of impaired lives tends early to lose by death its larger contingent of members in the poorest health. Others take their places, it is true, but the rate of increase in the mortality of the class survivors is slowed down as the poorest lives originally included in the class disappear.

88. In the class of lives comprising the standard group, on the other hand, the rate of increase in mortality is steeper, as the larger component of better lives evidences the relatively heavier effects of deterioration. All simple classes, in fact, having a y-age distribution where y is greater than $[x]$ generate death rates which approach those characteristic of age $[x]$.

89. Impaired lives considered as hybrid classes will also produce death rates representing a progressively smaller ratio to those of a standard, provided rates of withdrawal among the lives in better health are comparable in magnitude with the Linton rates, say. In order not to have the ratios decrease with duration, unusual rates of withdrawal must be experienced—the character of the rates depending on the level of mortality and on the age. From the common sense point of view, the rate of increase in the probability of death necessary to show a constant or increasing relationship to the base can be obtained only by experiencing an appropriate loss by withdrawal among the better lives, and such losses may be abnormally large. The availability of statistics with regard to withdrawal is much to be desired for a better understanding of this kind of mortality experience.

GENERAL OBSERVATIONS

90. Mortality is determined by many forces and this paper is an attempt to isolate some of these forces and indicate the effects they exert on its course. Basic, of course, as the practice of actuaries in the study of mortality has traditionally recognized, is the process of selection in the broadest sense, that is, the quality of lives (as measured by their prospects for longevity) whose rates of death are investigated. That this elemental influence fundamentally controls and determines the incidence of death in any collection of lives for many years (not merely for five years or fifteen) is the major thesis of this paper.

91. Of almost equal importance appear to be the generalized effects of "antiselection" as represented by the withdrawal from observation of lives subject, on the average, to a lighter rate of mortality than those which continue. While this concept is well understood by actuaries, the measurement of its effects has not received the attention its importance seems to deserve. It might be well to repeat the suggestion that statistics on this subject be compiled as a natural concomitant of all mortality studies.

92. The concepts here developed would also suggest periodic analysis of the mortality of the whole population to gauge the changes occurring in the rates of deterioration brought about by all the complex forces of the general environment. Knowledge of the extent of such changes should enable actuaries to measure the effects on the body of "existing" lives in any exposure, assuming that the distribution of those lives into the several strata of mortality was determinable from the rates of deterioration previously operating.

93. A wider knowledge by actuaries of the influences bearing on the course of mortality may enable them to determine which features of experience are characteristic and which accidental. And from the practical standpoint, in life insurance, such knowledge may permit more accurate determination of the level of surrender charges (as far as any differential in mortality might enter into consideration of them), of the magnitude of reserves, of the evaluation of some benefits which involve the rate of deterioration, etc.

94. The author's thanks are due to several of his business associates who assisted in various ways in the completion of this paper. Mention cannot be made of all who helped, but especial appreciation must be expressed for the invaluable aid received from William A. Bailey, A.S.A., and Edwin L. Luippold in calculating the very considerable volume of functions needed through the medium of an IBM 650 electronic computer. For

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APPENDIX

TABLE A

Factors for Obtaining Rates of Deterioration—White Males Derived from U.S. Life Tables 1949–51

x	${}_1b_x$	${}_2b_x$	${}_3b_x$	${}_4b_x$	${}_5b_x$
26......	.99924	.99892	.99795	.99468	.98152
27......	.99826	.99793	.99695	.99368	.98048
28......	.99778	.99745	.99646	.99313	.97967
29......	.99686	.99652	.99551	.99211	.97847
30......	.99600	.99565	.99460	.99111	.97711
31......	.99477	.99440	.99331	.98969	.97519
32......	.99409	.99371	.99256	.98877	.97360
33......	.99310	.99269	.99148	.98749	.97163
34......	.99263	.99219	.99090	.98667	.96993
35......	.99187	.99140	.99002	.98555	.96789
36......	.99090	.99039	.98892	.98416	.96548
37......	.99009	.98954	.98796	.98289	.96306
38......	.98972	.98912	.98742	.98200	.96093
39......	.98915	.98850	.98667	.98088	.95852
40......	.98868	.98798	.98601	.97982	.95610
41......	.98783	.98706	.98495	.97835	.95323
42......	.98779	.98695	.98468	.97764	.95098
43......	.98755	.98665	.98421	.97671	.94854
44......	.98716	.98618	.98357	.97561	.94590
45......	.98664	.98558	.98279	.97436	.94308
46......	.98619	.98506	.98208	.97314	.94024
47......	.98598	.98475	.98157	.97212	.93753
48......	.98578	.98446	.98108	.97109	.93482
49......	.98545	.98403	.98044	.96992	.93196
50......	.98488	.98336	.97955	.96848	.92883
51......	.98385	.98224	.97820	.96657	.92517
52......	.98321	.98147	.97718	.96494	.92169
53......	.98273	.98087	.97632	.96343	.91826
54......	.98229	.98030	.97548	.96193	.91481
55......	.98177	.97964	.97454	.96031	.91122
56......	.98156	.97929	.97389	.95897	.90786
57......	.98124	.97883	.97313	.95751	.90441
58......	.98109	.97853	.97252	.95620	.90109
59......	.98058	.97786	.97155	.95453	.89746
60......	.97983	.97695	.97033	.95260	.89358
61......	.97896	.97591	.96896	.95050	.88948
62......	.97849	.97526	.96797	.94873	.88564
63......	.97807	.97466	.96701	.94700	.88182
64......	.97716	.97357	.96556	.94478	.87756
65......	.97575	.97197	.96360	.94202	.87275
66......	.97397	.96998	.96122	.93881	.86741
67......	.97257	.96834	.95915	.93584	.86213
68......	.97120	.96673	.95708	.93282	.85671
69......	.97028	.96555	.95542	.93015	.85153
70......	.96798	.96298	.95238	.92612	.84509

x	${}_1b_x$	${}_2b_x$	${}_3b_x$	${}_4b_x$	${}_5b_x$
71....	.96566	.96037	.94923	.92190	.83831
72....	.96349	.95787	.94617	.91768	.83135
73....	.96152	.95556	.94324	.91353	.82431
74....	.95950	.95317	.94020	.90921	.81704
75....	.95733	.95061	.93696	.90465	.80947
76....	.95504	.94791	.93355	.89987	.80161
77....	.95334	.94577	.93066	.89552	.79406
78....	.95159	.94357	.92768	.89108	.78644
79....	.94889	.94041	.92375	.88572	.77801
80....	.94524	.93629	.91884	.87935	.76862
81....	.94072	.93126	.91298	.87197	.75817
82....	.93542	.92541	.90623	.86361	.74662
83....	.93047	.91984	.89965	.85525	.73478
84....	.92636	.91505	.89376	.84741	.72318
85....	.92304	.91100	.88855	.84015	.71202
86....	.92045	.90766	.88400	.83351	.70145
87....	.91792	.90437	.87951	.82696	.69111
88....	.91460	.90031	.87429	.81978	.68042
89....	.91066	.89563	.86847	.81204	.66937
90....	.90617	.89039	.86208	.80376	.65793
91....	.90109	.88456	.85509	.79489	.64601
92....	.89529	.87798	.84734	.78527	.63347
93....	.88942	.87129	.83943	.77544	.62071
94....	.88364	.86466	.83154	.76557	.60791
95....	.87806	.85821	.82378	.75580	.59521
96....	.87267	.85192	.81617	.74617	.58270
97....	.86662	.84498	.80793	.73598	.56990
98....	.85986	.83730	.79898	.72515	.55668
99....	.85316	.82976	.79013	.71440	.54358
100....	.84652	.82232	.78137	.70374	.53066
101....	.84061	.81518	.77288	.69336	.51808
102....	.83437	.80786	.76424	.68289	.50563
103....	.82915	.79973	.75499	.67194	.49299
104....	.82376	.79112	.74528	.66063	.48024
105....	.81683	.78314	.73499	.64882	.46727
106....	.75786	.76981	.72364	.63639	.45399
107....	....	.76599	.71184	.62319	.44024

$${}_{s+1}^{\ t}a_x = C_t^{6-s}({}_sb_x)^{6-s-t}(1-{}_sb_x)^t\,,\quad (6-s)\geq t\geq 0$$

Note references to ${}_sb_x$ in paragraphs 36, 37, 38.

TABLE B

Set Radixes—White Males
Distribution of Lives in Population—Based on U.S. Life Tables 1949–51

s:	1	2	3	4	5	6	q_x
x	$_sp(x)$						
10	.88502	.10944	.00541	.00013			.00060
11	.87581	.11769	.00633	.00017			.00062
12	.85411	.13684	.00877	.00028			.00067
13	.81905	.16680	.01359	.00055	.00001		.00076
14	.77252	.20462	.02168	.00115	.00003		.00090
15	.73067	.23663	.03065	.00199	.00006		.00105
16	.69492	.26236	.03962	.00299	.00011		.00120
17	.66769	.28089	.04727	.00398	.00017		.00133
18	.64872	.29325	.05302	.00479	.00022		.00143
19	.63116	.30424	.05866	.00566	.00027	.00001	.00153
20	.61643	.31314	.06363	.00646	.00033	.00001	.00162
21	.60561	.31948	.06741	.00711	.00038	.00001	.00169
22	.59818	.32373	.07008	.00759	.00041	.00001	.00174
23	.59527	.32537	.07114	.00778	.00043	.00001	.00176
24	.59818	.32373	.07008	.00759	.00041	.00001	.00174
25	.60261	.32121	.06848	.00730	.00039	.00001	.00171
26	.60712	.31860	.06688	.00702	.00037	.00001	.00168
27	.60561	.31948	.06741	.00711	.00038	.00001	.00169
28	.60111	.32206	.06902	.00740	.00040	.00001	.00172
29	.59527	.32537	.07114	.00778	.00043	.00001	.00176
30	.58680	.33009	.07427	.00836	.00047	.00001	.00182
31	.57598	.33596	.07838	.00914	.00053	.00001	.00190
32	.56190	.34330	.08390	.01025	.00063	.00002	.00201
33	.54639	.35104	.09022	.01159	.00074	.00002	.00214
34	.52874	.35936	.09770	.01328	.00090	.00002	.00230
35	.51050	.36739	.10576	.01522	.00110	.00003	.00248
36	.49110	.37528	.11471	.01753	.00134	.00004	.00269
37	.47024	.38297	.12476	.02032	.00166	.00005	.00294
38	.44854	.39006	.13568	.02360	.00205	.00007	.00323
39	.42717	.39607	.14690	.02724	.00253	.00009	.00355
40	.40577	.40108	.15858	.03135	.00310	.00012	.00391
41	.38468	.40496	.17052	.03590	.00378	.00016	.00431
42	.36325	.40775	.18308	.04110	.00461	.00021	.00477
43	.34311	.40923	.19525	.04658	.00556	.00027	.00526
44	.32385	.40957	.20720	.05241	.00663	.00034	.00579
45	.30523	.40880	.21902	.05867	.00786	.00042	.00637
46	.28709	.40694	.23075	.06542	.00927	.00053	.00701
47	.26959	.40405	.24222	.07261	.01088	.00065	.00771
48	.25306	.40022	.25318	.08008	.01266	.00080	.00846
49	.23748	.39558	.26355	.08780	.01462	.00097	.00926
50	.22267	.39013	.27342	.09581	.01679	.00118	.01012
51	.20836	.38388	.28290	.10424	.01920	.00142	.01106
52	.19415	.37659	.29220	.11336	.02199	.00171	.01212
53	.18050	.36852	.30095	.12289	.02509	.00205	.01328
54	.16761	.35981	.30898	.13267	.02848	.00245	.01453
55	.15548	.35060	.31624	.14262	.03216	.00290	.01587
56	.14404	.34090	.32273	.15276	.03615	.00342	.01731
57	.13350	.33101	.32831	.16281	.04037	.00400	.01882
58	.12372	.32095	.33305	.17280	.04483	.00465	.02041

TABLE B—*Continued*

s:	1	2	3	4	5	6	q_x
x	$_sp[x]$						
59	.11475	.31091	.33693	.18258	.04947	.00536	.02206
60	.10634	.30069	.34010	.19233	.05439	.00615	.02381
61	.09834	.29021	.34257	.20218	.05966	.00704	.02569
62	.09072	.27945	.34432	.21212	.06534	.00805	.02772
63	.08366	.26874	.34531	.22186	.07127	.00916	.02985
64	.07715	.25818	.34561	.23130	.07740	.01036	.03207
65	.07098	.24750	.34517	.24071	.08393	.01171	.03445
66	.06500	.23643	.34401	.25027	.09104	.01325	.03707
67	.05914	.22486	.34199	.26008	.09889	.01504	.04000
68	.05358	.21313	.33913	.26978	.10731	.01707	.04319
69	.04837	.20138	.33536	.27926	.11627	.01936	.04664
70	.04361	.18994	.33088	.28820	.12551	.02186	.05027
71	.03901	.17812	.32533	.29710	.13566	.02478	.05434
72	.03463	.16610	.31870	.30576	.14667	.02814	.05887
73	.03054	.15409	.31104	.31393	.15842	.03198	.06384
74	.02680	.14235	.30248	.32137	.17072	.03628	.06921
75	.02340	.13095	.29308	.32799	.18351	.04107	.07499
76	.02034	.11993	.28290	.33366	.19676	.04641	.08121
77	.01758	.10933	.27200	.33832	.21042	.05235	.08789
78	.01517	.09945	.26075	.34182	.22406	.05875	.09487
79	.01307	.09026	.24926	.34418	.23761	.06562	.10214
80	.01120	.08150	.23728	.34541	.25141	.07320	.10993
81	.00949	.07299	.22457	.34546	.26573	.08176	.11848
82	.00793	.06466	.21096	.34414	.28072	.09159	.12802
83	.00651	.05653	.19641	.34121	.29637	.10297	.13875
84	.00527	.04887	.18136	.33649	.31217	.11584	.15053
85	.00423	.04194	.16641	.33011	.32741	.12990	.16304
86	.00338	.03587	.15205	.32230	.34159	.14481	.17596
87	.00271	.03066	.13864	.31344	.35433	.16022	.18898
88	.00218	.02622	.12621	.30379	.36560	.17600	.20202
89	.00175	.02236	.11454	.29334	.37562	.19239	.21528
90	.00139	.01899	.10350	.28210	.38445	.20957	.22890
91	.00110	.01602	.09303	.27009	.39208	.22768	.24299
92	.00087	.01341	.08306	.25729	.39850	.24687	.25766
93	.00067	.01110	.07357	.24369	.40360	.26737	.27305
94	.00051	.00911	.06465	.22946	.40720	.28907	.28906
95	.00039	.00740	.05640	.21485	.40921	.31175	.30553
96	.00029	.00598	.04891	.20014	.40950	.33518	.32227
97	.00022	.00479	.04218	.18556	.40816	.35909	.33911
98	.00016	.00382	.03614	.17111	.40511	.38366	.35617
99	.00012	.00300	.03069	.15676	.40038	.40905	.37357
100	.00008	.00234	.02585	.14273	.39398	.43502	.39113
101	.00006	.00181	.02163	.12923	.38602	.46125	.40866
102	.00004	.00139	.01799	.11641	.37667	.48750	.42600
103	.00003	.00106	.01487	.10433	.36603	.51368	.44312
104	.00002	.00080	.01218	.09289	.35412	.53999	.46014
105	.00001	.00060	.00988	.08210	.34097	.56644	.47710
106	.00001	.00044	.00792	.07194	.32660	.59309	.49402
107	.00001	.00031	.00626	.06239	.31098	.62005	.51100
108	.00000	.00022	.00485	.05343	.29406	.64744	.52810

TABLE C

Number Living of 100,000 Entrants Originally in Strata 1–6—White Males Derived from U.S. Life Tables 1949–51

$l_{[x(s)]+n}$

n	s	Age at Entry 27	37	47	57	67	77	87
1	1	99960	99960	99960	99960	99960	99960	99960
	2	99820	99820	99820	99820	99820	99820	99820
	3	99190	99190	99190	99190	99190	99190	99190
	4	96355	96355	96355	96355	96355	96355	96355
	5	83597	83597	83597	83597	83597	83597	83597
	6	26189	26189	26189	26189	26189	26189	26189
2	1	99919	99913	99909	99905	99897	99875	99819
	2	99635	99612	99598	99580	99546	99460	99250
	3	98361	98280	98221	98138	97991	97649	96881
	4	92686	92409	92124	91720	91088	89805	87301
	5	68948	68112	66887	65298	63269	60002	55061
	6	6859	6859	6859	6859	6859	6859	6859
3	1	99876	99857	99845	99830	99797	99698	99375
	2	99444	99373	99327	99264	99128	98754	97663
	3	97508	97260	97066	96777	96222	94862	91534
	4	89003	88205	87352	86113	84108	80104	72577
	5	56594	54967	52679	49797	46202	40768	33286
	6	1796	1796	1796	1796	1796	1796	1796
4	1	99831	99790	99764	99728	99639	99342	98191
	2	99244	99096	98996	98845	98497	97452	94165
	3	96624	96117	95700	95051	93740	90465	82624
	4	85314	83812	82200	79845	76051	68778	56300
	5	46344	44146	41191	37572	33194	26972	19274
	6	470	470	470	470	470	470	470
5	1	99783	99710	99662	99591	99391	98661	95646
	2	99032	98776	98591	98295	97561	95269	88148
	3	95700	94844	94107	92924	90467	84434	71056
	4	81629	79305	76822	73228	67512	57186	41366
	5	37886	35341	32059	28164	23595	17534	10854
	6	123	123	123	123	123	123	123
6	1	99730	99615	99534	99403	99006	97453	91168
	2	98804	98404	98096	97582	96225	91960	79619
	3	94726	93429	92269	90384	86399	76996	58304
	4	77946	74740	71337	66511	58959	46226	29068
	5	30913	28205	24847	21002	16623	11228	5978
	6	32	32	32	32	32	32	32
7	1	99672	99501	99372	99152	98422	95480	84471
	2	98557	97975	97495	96674	94397	87367	69153
	3	93695	91872	90183	87447	81595	68525	45758
	4	74282	70180	65853	59893	50726	36407	19552
	5	25181	22451	19183	15588	11612	7083	3226
	6	8	8	8	8	8	8	8
8	1	99608	99366	99167	98820	97567	92505	75687
	2	98287	97482	96772	95546	92006	81465	57666
	3	92599	90175	87852	84143	76166	59481	34433
	4	70642	65674	60458	53518	43045	27976	12829
	5	20472	17825	14755	11519	8043	4399	1707
	6	2	2	2	2	2	2	2
9	1	99535	99206	98911	98385	96364	88357	65350
	2	97991	96918	95914	94171	89002	74393	46151
	3	91433	88340	85289	80510	70252	50351	24896
	4	67044	61257	55220	47479	36053	21000	8106
	5	16614	14114	11306	8473	5523	2690	885
	6	1	1	1	1	1	1	1
10	1	99453	99016	98595	97823	94733	82979	54258
	2	97664	96277	94905	92523	85366	66437	35461
	3	90191	86368	82507	76585	64010	41579	17328
	4	63495	56954	50189	41825	29819	15418	4967
	5	13456	11143	8630	6199	3758	1620	450
	6	0	0	0	0	0	0	0

n	s	Age at Entry						
		27	37	47	57	67	77	87
11	1	99360	98792	98206	97104	92598	76460	43275
	2	97300	95549	93735	90575	81107	57981	26178
	3	88866	84261	79530	72402	57598	33520	11630
	4	60000	52789	45409	36584	24359	11085	2955
	5	10872	8772	6562	4510	2533	961	224
	6	0	0	0	0	0	0	0
12	1	99253	98530	97735	96197	89902	69019	33137
	2	96895	94728	92394	88307	76281	49456	18579
	3	87452	82029	76383	68012	51182	26401	7533
	4	56567	48783	40904	31774	19662	7813	1707
	5	8764	6885	4971	3261	1692	562	109
	6	0	0	0	0	0	0	0
13	1	99128	98222	97170	95070	86614	60955	24348
	2	96442	93810	90879	85711	70972	41168	12679
	3	85947	79681	73096	63467	44915	20324	4711
	4	53210	44950	36694	27401	15686	5400	958
	5	7048	5388	3752	2344	1119	324	52
	6	0	0	0	0	0	0	0
14	1	98986	97866	96498	93693	82717	52609	17165
	2	95937	92790	89183	82789	65277	33472	8324
	3	84350	77228	69694	58830	38910	15293	2846
	4	49939	41300	32783	23468	12365	3659	523
	5	5654	4204	2821	1674	733	183	24
	6	0	0	0	0	0	0	0
15	1	98820	97452	95708	92031	78208	44325	11610
	2	95375	91659	87302	79536	59294	26559	5258
	3	82660	74676	66200	54143	33254	11246	1661
	4	46760	37836	29168	19951	9624	2431	277
	5	4525	3270	2112	1188	474	102	11
	6	0	0	0	0	0	0	0
16	1	98630	96971	94784	90046	73104	36416	7538
	2	94747	90409	85231	75953	53127	20552	3198
	3	80874	72025	62636	49451	28008	8077	937
	4	43678	34555	25841	16828	7387	1582	143
	5	3611	2534	1374	836	303	56	5
	6	0	0	0	0	0	0	0
17	1	98410	96417	93712	87707	67444	29136	4699
	2	94051	89032	82973	72058	46885	15495	1873
	3	78998	69287	59030	44803	23216	5662	512
	4	40704	31455	22798	14077	5584	1008	71
	5	2874	1956	1168	584	191	30	2
	6	0	0	0	0	0	0	0
18	1	98157	95778	92485	84989	61309	22675	2813
	2	93281	87524	80534	67877	40696	11371	1057
	3	77036	66471	55410	40251	18917	3870	270
	4	37847	28541	20029	11676	4153	628	35
	5	2282	1504	863	404	119	16	1
	6	0	0	0	0	0	0	0
19	1	97867	95048	91088	81880	54836	17146	1617
	2	92434	85882	77918	63451	34708	8117	575
	3	74993	63592	51798	35843	15141	2578	138
	4	35109	25813	17521	9599	3037	382	16
	5	1807	1153	635	278	73	8	0
	6	0	0	0	0	0	0	0
20	1	97536	94216	89507	78375	48204	12588	892
	2	91505	84104	75127	58828	29063	5632	301
	3	72872	60663	48209	31624	11899	1673	68
	4	32491	23267	15255	7818	2182	227	7
	5	1426	880	465	189	44	4	0
	6	0	0	0	0	0	0	0
21	1	97157	93278	87723	74482	41617	8967	472
	2	90486	82193	72159	54063	23887	3797	151
	3	70676	57702	44654	27633	9182	1057	32
	4	29996	20904	13214	6307	1542	132	3
	5	1123	669	338	127	26	2	0
	6	0	0	0	0	0	0	0

TABLE C—*Continued*

NUMBER LIVING OF 100,000 ENTRANTS ORIGINALLY IN STRATA 1–6—WHITE MALES DERIVED FROM U.S. LIFE TABLES 1949–51

$l_{[x(s)]+n}$

n	s	Age at Entry 27	37	47	57	67	77
22	1	96727	92224	85722	70230	35269	6193
	2	89377	80151	69022	49228	19267	2486
	3	68414	54724	41152	23907	6958	650
	4	27626	18720	11383	5038	1072	75
	5	881	507	244	85	15	1
	6	0	0	0	0	0	0
23	1	96240	91052	83493	65669	29321	4143
	2	88174	77985	65727	44396	15246	1579
	3	66095	51748	37722	20475	5176	388
	4	25382	16710	9750	3985	732	41
	5	690	383	176	56	9	0
	6	0	0	0	0	0	0
24	1	95692	89755	81034	60845	23893	2682
	2	86875	75699	62298	39629	11828	972
	3	63726	48787	34390	17349	3778	225
	4	23264	14866	8303	3119	491	22
	5	538	288	125	37	5	0
	6	0	0	0	0	0	0
25	1	95075	88325	78330	55087	19067	1679
	2	85476	73298	58741	34982	8989	580
	3	61312	45851	31163	14531	2704	127
	4	21269	13180	7026	2413	324	12
	5	419	215	89	24	3	0
	6	0	0	0	0	0	0
26	1	94381	86755	75373	50607	14885	1017
	2	83969	70782	55072	30498	6686	335
	3	58853	42948	28054	12015	1896	70
	4	19389	11642	5903	1843	209	6
	5	324	161	63	15	2	0
	6	0	0	0	0	0	0
27	1	93602	85040	72160	45304	11353	595
	2	82351	68162	51310	26223	4861	188
	3	56355	40093	25076	9793	1301	37
	4	17623	10245	4921	1388	133	3
	5	250	119	44	10	1	0
	6	0	0	0	0	0	0
28	1	92730	83181	68698	39978	8448	336
	2	80620	65451	47484	22205	3451	102
	3	53828	37299	22247	7857	872	19
	4	15970	8981	4070	1029	82	1
	5	193	88	30	6	0	0
	6	0	0	0	0	0	0
29	1	91759	81174	65001	34733	6127	184
	2	78777	62655	43627	18498	2389	53
	3	51281	34575	19580	6201	571	10
	4	14427	7842	3338	750	50	1
	5	148	65	21	4	0	0
	6	0	0	0	0	0	0
30	1	90683	79011	61092	29682	4327	97
	2	76822	59780	39775	15151	1612	27
	3	48723	31924	17089	4811	365	5
	4	12993	6818	2714	538	30	0
	5	113	47	14	2	0	0
	6	0	0	0	0	0	0
31	1	89496	76682	56997	24939	2973	49
	2	74760	56825	35963	12197	1059	13
	3	46167	29348	14782	3669	227	2
	4	11664	5898	2186	379	17	0
	5	86	34	10	1	0	0
	6	0	0	0	0	0	0

TABLE C—*Continued*

n	s	Age at Entry				
		27	37	47	57	67
32	1	88195	74185	52763	20592	1986
	2	72597	53800	32236	9649	678
	3	43626	26854	12669	2751	138
	4	10438	5075	1744	263	10
	5	65	25	6	1	0
	6	0	0	0	0	0
33	1	86777	71519	48442	16699	1288
	2	70341	50717	28636	7498	421
	3	41111	24451	10757	2027	82
	4	9312	4343	1378	180	5
	5	49	18	4	0	0
	6	0	0	0	0	0
34	1	85241	68692	44077	13291	810
	2	67997	47597	25194	5719	254
	3	38631	22150	9041	1467	47
	4	8280	3696	1077	120	3
	5	37	13	3	0	0
	6	0	0	0	0	0
35	1	83579	65699	39710	10372	494
	2	65570	44445	21931	4279	149
	3	36193	19952	7516	1042	26
	4	7338	3125	833	79	1
	5	28	9	2	0	0
	6	0	0	0	0	0
36	1	81789	62542	35380	7927	292
	2	63062	41272	18866	3136	85
	3	33800	17860	6172	725	14
	4	6479	2624	636	51	1
	5	21	6	1	0	0
	6	0	0	0	0	0
37	1	79869	59226	31127	5926	167
	2	60485	38093	16014	2249	47
	3	31464	15879	4998	494	7
	4	5700	2187	478	32	0
	5	15	4	1	0	0
	6	0	0	0	0	0
38	1	77821	55768	27002	4327	92
	2	57849	34931	13394	1576	25
	3	29193	14017	3986	329	4
	4	4996	1808	354	20	0
	5	11	3	0	0	0
	6	0	0	0	0	0
39	1	75644	52187	23071	3082	49
	2	55162	31807	11027	1078	13
	3	26991	12279	3128	214	2
	4	4361	1483	258	12	0
	5	8	2	0	0	0
	6	0	0	0	0	0
40	1	73335	48508	19399	2140	26
	2	52428	28746	8931	720	6
	3	24861	10668	2415	136	1
	4	3791	1205	185	7	0
	5	6	1	0	0	0
	6	0	0	0	0	0
41	1	70885	44759	16045	1448	13
	2	49647	25771	7114	468	3
	3	22802	9190	1833	85	0
	4	3279	970	131	4	0
	5	4	1	0	0	0
	6	0	0	0	0	0
42	1	68294	40982	13050	953	0
	2	46828	22911	5571	296	0
	3	20819	7845	1368	51	0
	4	2821	774	91	2	0
	5	3	1	0	0	0
	6	0	0	0	0	0

TABLE C—*Continued*

Number Living of 100,000 Entrants Originally in Strata 1–6—White Males Derived from U.S. Life Tables 1949–51

$l_{[x(s)]+n}$

n	s	Age at Entry			
		27	37	47	57
43	1	65565	37218	10431	609
	2	43983	20190	4288	183
	3	18916	6636	1004	30
	4	2414	611	62	1
	5	2	0	0	0
	6	0	0	0	0
44	1	62707	33502	8188	378
	2	41127	17626	3242	109
	3	17102	5559	724	17
	4	2054	478	41	1
	5	2	0	0	0
	6	0	0	0	0
45	1	59721	29864	6306	228
	2	38267	15229	2404	64
	3	15376	4606	512	10
	4	1737	369	27	0
	5	1	0	0	0
	6	0	0	0	0
46	1	56607	26331	4759	133
	2	35410	13007	1748	36
	3	13739	3771	355	5
	4	1458	282	18	0
	5	1	0	0	0
	6	0	0	0	0
47	1	53375	22927	3515	75
	2	32571	10963	1244	20
	3	12194	3045	242	3
	4	1215	212	11	0
	5	1	0	0	0
	6	0	0	0	0
48	1	50040	19687	2537	41
	2	29766	9108	866	10
	3	10747	2422	161	1
	4	1005	157	7	0
	5	0	0	0	0
	6	0	0	0	0

n	s	Age at Entry			
		27	37	47	57
49	1	46623	16654	1788	22
	2	27014	7450	588	5
	3	9399	1896	104	1
	4	824	115	4	0
	5	0	0	0	0
	6	0	0	0	0
50	1	43147	13868	1229	11
	2	24335	5996	390	3
	3	8155	1460	66	0
	4	669	82	2	0
	5	0	0	0	0
	6	0	0	0	0
51	1	39639	11362	823	6
	2	21747	4747	252	1
	3	7015	1106	41	0
	4	539	58	1	0
	5	0	0	0	0
	6	0	0	0	0
52	1	36137	9157	537	0
	2	19274	3697	159	0
	3	5981	824	25	0
	4	430	40	1	0
	5	0	0	0	0
	6	0	0	0	0
53	1	32677	7255	340	0
	2	16934	2830	97	0
	3	5053	604	15	0
	4	340	27	0	0
	5	0	0	0	0
	6	0	0	0	0

n	s	Age at Entry		
		27	37	47
54	1	29291	5647	210
	2	14740	2128	58
	3	4228	434	8
	4	265	18	0
	5	0	0	0
	6	0	0	0
55	1	26001	4313	125
	2	12700	1571	34
	3	3499	307	5
	4	205	12	0
	5	0	0	0
	6	0	0	0
56	1	22830	3230	73
	2	10816	1137	19
	3	2862	213	2
	4	157	8	0
	5	0	0	0
	6	0	0	0
57	1	19797	2368	41
	2	9093	806	10
	3	2309	144	1
	4	118	5	0
	5	0	0	0
	6	0	0	0
58	1	16932	1697	22
	2	7534	558	5
	3	1835	96	1
	4	87	3	0
	5	0	0	0
	6	0	0	0
59	1	14266	1188	12
	2	6147	378	3
	3	1435	62	0
	4	64	2	0
	5	0	0	0
	6	0	0	0

TABLE C—*Continued*

n	s	Age at Entry 27	Age at Entry 37	Age at Entry 47
60	1	11833	811	6
	2	4936	250	1
	3	1104	39	0
	4	46	1	0
	5	0	0	0
	6	0	0	0
61	1	9659	540	3
	2	3899	161	1
	3	836	24	0
	4	32	1	0
	5	0	0	0
	6	0	0	0
62	1	7756	350	0
	2	3029	101	0
	3	622	15	0
	4	22	0	0
	5	0	0	0
	6	0	0	0
63	1	6123	221	0
	2	2314	62	0
	3	456	9	0
	4	15	0	0
	5	0	0	0
	6	0	0	0
64	1	4749	135	0
	2	1737	37	0
	3	328	5	0
	4	10	0	0
	5	0	0	0
	6	0	0	0
65	1	3616	80	0
	2	1279	21	0
	3	231	3	0
	4	7	0	0
	5	0	0	0
	6	0	0	0

n	s	Age at Entry 27	Age at Entry 37
66	1	2699	46
	2	924	12
	3	160	1
	4	4	0
	5	0	0
	6	0	0
67	1	1972	26
	2	654	6
	3	109	1
	4	3	0
	5	0	0
	6	0	0
68	1	1409	14
	2	452	3
	3	72	0
	4	2	0
	5	0	0
	6	0	0
69	1	984	7
	2	306	2
	3	47	0
	4	1	0
	5	0	0
	6	0	0
70	1	670	4
	2	202	1
	3	30	0
	4	1	0
	5	0	0
	6	0	0
71	1	445	2
	2	130	0
	3	18	0
	4	0	0
	5	0	0
	6	0	0

n	s	Age at Entry 27
72	1	288
	2	81
	3	11
	4	0
	5	0
	6	0
73	1	181
	2	50
	3	6
	4	0
	5	0
	6	0
74	1	111
	2	29
	3	4
	4	0
	5	0
	6	0
75	1	66
	2	17
	3	2
	4	0
	5	0
	6	0
76	1	38
	2	10
	3	1
	4	0
	5	0
	6	0
77	1	21
	2	5
	3	1
	4	0
	5	0
	6	0

n	s	Age at Entry 27
78	1	11
	2	3
	3	0
	4	0
	5	0
	6	0
79	1	6
	2	1
	3	0
	4	0
	5	0
	6	0
80	1	3
	2	1
	3	0
	4	0
	5	0
	6	0
81	1	1
	2	0
	3	0
	4	0
	5	0
	6	0

TABLE D

3½% Commutation Columns—White Males
Sets in Strata 1–6
Derived from U.S. Life Tables 1949–51

Age 27 at Entry

n	*s*	*M**	*N**	*D**	*n*	*s*	*M**	*N**	*D**
0	1	8540	915561	39501	30	1	6840	175130	12762
	2	11067	840833	39501		2	6546	126127	10811
	3	15815	700436	39501		3	4695	63936	6857
	4	23975	459126	39501		4	1414	12251	1828
	5	33044	190943	39501		5	14	58	16
	6	37713	52882	39501		6	0	0	0
5	1	8463	731116	33187	33	1	6328	138612	11015
	2	10723	656903	32937		2	5695	95613	8929
	3	14285	518807	31829		3	3694	45070	5218
	4	17423	287595	27149		4	930	7466	1182
	5	10639	57996	12600		5	5	22	6
	6	39	55	41		6	0	0	0
10	1	8365	576208	27850	35	1	5942	117143	9904
	2	10315	503731	27349		2	5120	78345	7770
	3	12637	373171	25256		3	3101	35113	4289
	4	11974	171714	17781		4	691	5268	870
	5	3204	16687	3768		5	3	11	3
	6	0	0	0		6	0	0	0
15	1	8206	446336	23300	38	1	5306	89041	8317
	2	9740	376957	22487		2	4266	56664	6183
	3	10741	258718	19490		3	2326	23475	3120
	4	7735	97286	11025		4	432	3026	534
	5	914	4527	1067		5	1	4	1
	6	0	0	0		6	0	0	0
20	1	7935	337925	19363	40	1	4851	72913	7317
	2	8922	273355	18165		2	3716	44786	5231
	3	8663	171627	14466		3	1886	17568	2480
	4	4689	52089	6450		4	309	2042	378
	5	244	1148	283		5	1	2	1
	6	0	0	0		6	0	0	0
25	1	7498	248215	15892					
	2	7847	190454	14287					
	3	6594	108066	10248					
	4	2669	26197	3555					
	5	61	270	70					
	6	0	0	0					

Age 37 at Entry

n	*s*	*M**	*N**	*D**	*n*	*s*	*M**	*N**	*D**
0	1	7863	595561	28003	25	1	6140	127919	10466
	2	9801	538276	28003		2	5644	89942	8685
	3	13079	441336	28003		3	3907	45118	5433
	4	18155	291218	28003		4	1241	9471	1562
	5	23660	128445	28003		5	23	89	26
	6	26735	37489	28003		6	0	0	0
5	1	7791	464821	23510	28	1	5572	98113	8890
	2	9494	407944	23289		2	4776	65613	6995
	3	11785	312784	22362		3	2961	30333	3986
	4	12941	170257	18699		4	776	5443	960
	5	7137	35362	8333		5	8	31	9
	6	28	39	29		6	0	0	0
10	1	7645	355203	19657	30	1	5149	80841	7883
	2	8967	300030	19113		2	4201	52148	5964
	3	9991	211589	17146		3	2415	22776	3185
	4	8177	92533	11307		4	556	3674	680
	5	1909	8972	2212		5	4	15	5
	6	0	0	0		6	0	0	0
15	1	7367	263821	16289	35	1	3955	46249	5519
	2	8145	212186	15321		2	2821	26994	3733
	3	7901	135452	12482		3	1333	10135	1676
	4	4739	46867	6324		4	221	1240	263
	5	475	2110	547		5	1	2	1
	6	0	0	0		6	0	0	0
20	1	6884	188543	13259	40	1	2654	22964	3431
	2	7012	142667	11836		2	1629	11948	2033
	3	5790	81243	8537		3	626	3804	755
	4	2531	21992	3275		4	73	348	85
	5	108	454	124		5	0	0	0
	6	0	0	0		6	0	0	0
23	1	6469	150484	11558					
	2	6209	109125	9899					
	3	4618	57670	6569					
	4	1667	13416	2121					
	5	43	172	49					
	6	0	0	0					

* For age $[x(s)] + n$, where x is age at entry.

TABLE D—*Continued*

Age 47 at Entry

n	s	M*	N*	D*	n	s	M*	N*	D*
0	1	6834	384959	19852	20	1	5596	98600	8930
	2	8199	344590	19852		2	5146	69482	7495
	3	10384	279967	19852		3	3610	35479	4810
	4	13580	165484	19852		4	1243	8246	1522
	5	16960	85527	19852		5	41	151	46
	6	18953	26577	19852		6	0	0	0
5	1	6774	292282	16658	25	1	4596	58671	6580
	2	7950	252223	16479		2	3674	37271	4934
	3	9340	188951	15730		3	2072	16148	2618
	4	9452	100217	12841		4	496	2791	590
	5	4659	20687	5359		5	0	23	7
	6	20	28	21		6	0	0	0
10	1	6616	214687	13876	30	1	3295	30352	4321
	2	7400	176130	13356		2	2237	17052	2813
	3	7602	118581	11612		3	1000	6168	1209
	4	5426	48423	7063		4	165	783	192
	5	1064	4451	1214		5	1	3	1
	6	0	0	0		6	0	0	0
13	1	6429	174617	12334	35	1	1930	12845	2365
	2	6873	137889	11536		2	1094	6274	1306
	3	6366	86121	9278		3	384	1869	448
	4	3650	29 11	4658		4	44	173	50
	5	419	1691	476		5	0	0	0
	6	0	0	0		6	0	0	0
15	1	6253	150448	11341	40	1	835	4067	973
	2	6442	115416	10345		2	390	1700	448
	3	5535	68295	7844		3	107	412	121
	4	2742	21133	3456		4	8	27	9
	5	221	869	250		5	0	0	0
	6	0	0	0		6	0	0	0
18	1	5898	117890	9884					
	2	5694	85135	8607					
	3	4341	46750	5922					
	4	1723	12196	2141					
	5	82	309	92					
	6	0	0	0					

Age 57 at Entry

n	s	M*	N*	D*	n	s	M*	N*	D*
0	1	5887	242080	14073	20	1	4110	42386	5543
	2	6801	215063	14073		2	3255	26782	4161
	3	8189	174015	14073		3	1839	11752	2237
	4	10129	116649	14073		4	476	2264	553
	5	12162	56535	14073		5	12	37	13
	6	13436	18841	14073		6	0	0	0
3	1	5865	201289	12672	25	1	2675	19176	3323
	2	6705	174334	12600		2	1731	10401	2083
	3	7768	133561	12284		3	741	3673	865
	4	8312	77424	10931		4	127	501	144
	5	5559	22516	6321		5	1	4	1
	6	218	305	228		6	0	0	0
5	1	5836	176386	11801	30	1	1269	6473	1488
	2	6588	149612	11647		2	660	2953	760
	3	7304	109619	11011		3	213	829	241
	4	6760	56701	8677		4	24	80	27
	5	2945	11587	3337		5	0	0	0
	6	14	20	15		6	0	0	0
8	1	5751	142237	10561	35	1	388	1476	438
	2	6285	116099	10212		2	162	562	181
	3	6335	78588	8993		3	40	125	44
	4	4577	33784	5720		4	3	8	3
	5	1092	4121	1231		5	0	0	0
	6	0	0	0		6	0	0	0
10	1	5650	121516	9760	40	1	69	204	76
	2	5979	96163	9231		2	23	65	26
	3	5568	61281	7641		3	4	12	5
	4	3390	23161	4173		4	0	1	0
	5	550	2015	619		5	0	0	0
	6	0	0	0		6	0	0	0
15	1	5136	76736	7731					
	2	4819	55066	6681					
	3	3551	29501	4548					
	4	1406	7997	1676					
	5	90	302	100					
	6	0	0	0					

* For age $[x(s)] + n$, where x is age at entry.

TABLE E

ENTRANTS AT AGE 37—WHITE MALES
SPECIMEN DISTRIBUTIONS OF SETS CORRESPONDING TO DIFFERENT y-AGES OF POPULATION

$\langle y \rangle$: / n	13	17	18	19	27	37	44	47	57	67	77	87	94	95
	Deaths per Class of 1,000—$d_{[x]+n-1}$ for $\langle y \rangle/x$													
1	.76	1.33	1.43	1.53	1.69	2.94	5.79	7.71	18.82	40.00	87.89	188.98	289.06	305.53
2	.88	1.51	1.62	1.73	1.90	3.22	6.05	7.84	17.19	32.09	58.76	99.53	128.59	132.63
3	1.02	1.72	1.84	1.95	2.14	3.53	6.37	8.10	16.57	28.76	47.58	69.36	78.49	79.20
4	1.19	1.96	2.09	2.21	2.41	3.87	6.74	8.44	16.33	26.89	41.75	55.90	58.74	58.51
5	1.39	2.23	2.37	2.50	2.72	4.25	7.15	8.83	16.25	25.60	37.78	47.75	48.05	47.54
6	1.62	2.55	2.70	2.84	3.07	4.69	7.63	9.28	16.32	24.68	34.78	41.82	40.75	40.12
7	1.88	2.89	3.05	3.21	3.45	5.14	8.12	9.75	16.43	23.91	32.26	37.01	35.01	34.33
8	2.19	3.28	3.45	3.62	3.88	5.63	8.64	10.24	16.59	23.29	30.16	33.03	30.34	29.62
9	2.52	3.70	3.88	4.06	4.33	6.16	9.19	10.76	16.79	22.80	28.39	29.72	26.49	25.76
10	2.91	4.18	4.37	4.56	4.84	6.73	9.78	11.33	17.06	22.43	26.91	26.95	23.30	22.56
11	3.36	4.70	4.91	5.10	5.41	7.35	10.41	11.93	17.36	22.15	25.65	24.61	20.64	19.90
12	3.85	5.28	5.49	5.70	6.01	8.01	11.06	12.55	17.68	21.92	24.54	22.60	18.40	17.65
13	4.39	5.90	6.12	6.33	6.66	8.69	11.73	13.17	18.00	21.72	23.56	20.86	16.49	15.75
14	4.98	6.55	6.78	7.00	7.34	9.41	12.41	13.81	18.33	21.56	22.69	19.36	14.88	14.15
15	5.64	7.28	7.52	7.75	8.09	10.18	13.14	14.48	18.69	21.45	21.94	18.06	13.51	12.79
	Rates of Mortality—$1{,}000q_{[x]+n-1}$ for $\langle y \rangle/x$													
1	.76	1.33	1.43	1.53	1.69	2.94	5.79	7.71	18.82	40.00	87.89	188.98	289.06	305.53
2	.88	1.51	1.62	1.73	1.90	3.23	6.08	7.91	17.52	33.42	64.42	122.72	180.88	190.98
3	1.02	1.72	1.84	1.96	2.15	3.55	6.44	8.23	17.19	30.99	55.76	97.48	134.79	140.97
4	1.19	1.97	2.10	2.23	2.43	3.91	6.87	8.65	17.24	29.91	51.81	87.06	116.58	121.24
5	1.39	2.24	2.38	2.52	2.74	4.31	7.34	9.12	17.46	29.35	49.46	81.45	107.96	112.09
6	1.63	2.57	2.72	2.87	3.11	4.77	7.89	9.68	17.84	29.15	47.90	77.67	102.62	106.54
7	1.90	2.92	3.09	3.25	3.50	5.26	8.46	10.26	18.29	29.09	46.67	74.52	98.25	102.02
8	2.21	3.32	3.50	3.68	3.94	5.79	9.08	10.89	18.80	29.18	45.75	71.86	94.42	98.05
9	2.55	3.77	3.96	4.14	4.43	6.37	9.74	11.58	19.40	29.42	45.13	69.66	91.04	94.52
10	2.95	4.27	4.47	4.67	4.97	7.01	10.47	12.33	20.10	29.82	44.80	67.90	88.11	91.43
11	3.41	4.83	5.04	5.25	5.58	7.71	11.26	13.14	20.88	30.36	44.70	66.52	85.60	88.75
12	3.92	5.44	5.67	5.89	6.23	8.46	12.10	14.01	21.71	30.99	44.78	65.44	83.42	86.40
13	4.50	6.11	6.35	6.59	6.95	9.26	12.99	14.91	22.60	31.69	45.00	64.64	81.59	84.40
14	5.12	6.84	7.09	7.34	7.71	10.12	13.92	15.87	23.54	32.48	45.38	64.12	80.13	82.78
15	5.83	7.65	7.92	8.18	8.57	11.06	14.94	16.91	24.58	33.40	45.96	63.94	79.10	81.60