# Advanced Portfolio Management Formula Package
# February 2010

The exam committee felt that by providing many key formulas, candidates would be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorizing formulas. The formula package was developed sequentially by reviewing the syllabus material. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not on the formula sheet.** In general, formulas not in the package are either relatively fundamental or uncomplicated, or can be derived from formulas that are in the package.

Candidates should carefully observe the subtle differences in similar formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply to a specific situation in the exam question.

Candidates will note that the formula package provides minimal information about where the formula occurs in the syllabus, and does not provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent mastering the learning objectives and learning outcomes provided as part of the syllabus.

**Tillman, Asset / Liability Management of Financial Institutions**

None

**Fabozzi, Handbook of Fixed Income Securities**

$$TIPS\ realized\ nominal\ yield = (1 + real\ yield) * \left(1 + inflation\right) - 1$$
$$Break\text{-}even\ \ inflation\ rate = \frac{1 + conventional\ \ nominal\ \ yield}{1 + TIPS\ \ real\ yield} - 1$$

inverser floater: $K - L * (reference\ rate)$

$${}^{O}\!/_{C} \cdot ratio \cdot\ for \cdot a \cdot tranche = \frac{principal(\ par) \cdot value \cdot of\ \cdot collateral \cdot\ portfolio}{principal \cdot\ for \cdot tranche + principal \cdot\ for \cdot all \cdot tranches \cdot senior \cdot to \cdot it}$$

$${}^{I}\!/_{C} \cdot ratio \cdot\ for \cdot a \cdot tranche = \frac{scheduled \cdot \mathrm{int} \cdot due \cdot on \cdot underlying \cdot collateral \cdot\ portfolio}{scheduled \cdot \mathrm{int} \cdot on \cdot that \cdot tranche + scheduled \cdot \mathrm{int} \cdot on \cdot all \cdot tranches \cdot senior}$$

$$class \cdot X \cdot OC \cdot ratio = \frac{cash \cdot collateral \cdot account \cdot balance}{notional \cdot amount \cdot class \cdot X \cdot notes \cdot and \cdot notes \cdot senior \cdot excluding \cdot \sup er \cdot senior}$$

$$dispersion = \frac{\sum (t_i - D)^2 PV(CF_i)}{\sum PV(CF_i)}$$

**Babbel and Fabozzi , Investment Management for Insurers,**

$$D_S = (D_A - D_L)\frac{A}{S} + D_L$$

where $D_S$ : duration of economic surplus

$D_A$ : duration of assets

$D_L$ : duration of liabilities

$A$ : market value of assets

$S$ : economic surplus = $A - L$ where $L$ present value of liabilities

$$P(j) \approx P(i)\left[1 - D(i)(j-i) + \frac{1}{2}C(i)(j-i)^2\right]$$

$$D(i) = \frac{-P'(i)}{P(i)} \qquad C(i) = \frac{P''(i)}{P(i)}$$

$$P'(i) \approx \frac{P(i+\triangle t) - P(i-\triangle t)}{2\triangle t}$$

$$P''(i) \approx \frac{P(i+\triangle t) - 2P(i) + P(i-\triangle t)}{(\triangle i)^2}$$

$$D = \frac{\sum t c_t v^{mt+1}}{p} \qquad C = \frac{\sum t(t+\frac{1}{m})c_t v^{mt+2}}{p}$$

$$P(j) = P(i)\exp\left[-\int_i^j D(s)ds\right]$$

$$D(j) \approx D(i) + \left[D^2(i) - C(i)\right](j-i)$$

$$P(j) \approx P(i)\exp\left[-D(i)(j-i)\right]$$

$$S(j) \approx S(i)\left[1 + C^S(j-i)^2\right]$$

$$D^S(j) \approx -C^S(i)(j-i)$$

$$di_t = \mu(t, i_t)dt + \sigma(t, i_t)dz_t$$

$$i_T = i_0 + \int_0^T \mu(t, i_t)dt + \int_0^T \sigma(t, i_t)dz_t$$

$$\frac{dP_t}{P_t} = \left(\frac{\partial_t P_t}{P_t} - D_t\mu_t + \frac{1}{2}C_t\sigma_t^2\right)dt - D_t\sigma_t dz_t$$

$$\frac{dP_t}{P_t} = \left(i_t - (T-t)\mu_t + \frac{1}{2}(T-t)^2\sigma_t^2\right)dt - (T-t)\sigma_t dz_t$$

$$dD_t = \left(\partial_t D_t + (D_t^2 - C_t)\mu_t + \frac{1}{2}\left[D_t(D_t^2 - C_t) - \partial_t C_t\right]\sigma_t^2\right)dt + (D_t^2 - C_t)\sigma_t dz_t$$

$$P(j) = P(i)\left[1 - D(i)\bullet\Delta i + \frac{1}{2}\Delta i^T C(i)\Delta\right]$$

$$D_k(i) = \frac{-\partial_k P(i)}{P(i)} \qquad C_{kl}(i) = \frac{-\partial_{kl} P(i)}{P(i)}$$

$$\partial_k P(i) \approx \frac{\left[P(i+\Delta i E_k) - P(i-\Delta i E_k)\right]}{\left[2\Delta i\right]}$$

$$\partial_{kl} P(i) \approx \frac{\left[P(i+\Delta i(E_j+E_k)) - P(i+\Delta i(E_l-E_k)) - P(i+\Delta i(E_k-E_l)) + P(i-\Delta i(E_k+E_l))\right]}{\left[2\Delta i\right]^2}$$

$$DVBP = \frac{Par \cdot amount \times (price + accrued) \times \text{mod}\,ified \cdot duration}{1{,}000{,}000}$$

$$DVBP = \frac{dollar \cdot par \cdot amount \times (change \cdot in \cdot constant - OAS \cdot price)}{yield \cdot curve \cdot shift \cdot in \cdot bps * 100}$$

$$\frac{\Delta P}{P} = -\sum_{i=1}^{n} D_i \Delta F_i$$

$$D_i = -\frac{1}{P}\frac{\partial P}{\partial F_i}$$

$$\Delta P = A - \sum_{i=1}^{n} D_i X_i + \frac{1}{2}\sum_{i=1}^{n} C_i X_i^2 + Y$$

$$D_i = -\frac{P_i^{'} - P_i^{"}}{2\Delta F_i}$$

$$C_i = \frac{P_i^{'} + P_i^{"} - 2P}{(\Delta F_i)^2}$$

$$A = \mu - \frac{1}{2}\sum_{i=1}^{n} C_i \sigma_i^2$$

$$\mu = E\Delta P$$

$$\sigma^2 = Var(\Delta P) = \sum_{i=1}^{n}\sum_{j=1}^{n} D_i D_j \sigma_{ij} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_i C_j \sigma_{ij}^2 + s^2$$

$$\mu_3 = E(\Delta P - \mu)^3 = 3\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} D_i D_j C_k \sigma_{ik}\sigma_{jk} + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} C_i C_j C_k \sigma_{ij}\sigma_{jk}\sigma_{ki}$$

$$h = \frac{\Delta V - \Delta S}{\Delta F}$$

$$h = \frac{-\Delta S}{\Delta F} = \frac{-\beta_S}{\beta_F}$$

probability density of stock price changing from $S_0$ to $S_f$ in time $T$ assuming log normal distribution

$$dS_f P_T\left(S_f / S_n\right) = \frac{dS_f}{S_f\sqrt{2\pi\sigma^2 T}} \exp\left( -\frac{\left(\ln\left(\frac{S_f}{S_0}\right) - \mu T\right)^2}{2\sigma^2 T} \right)$$

where

$S_0$ : initial stock price $S_f$ : value of stock on the expiry day of the option
$C_0$ : initial call price $C_f$ : value of call on the expiry day of the option
$W_0$ : initial investment $W_f$ : value of investment on the expiry day of the option
$D$ : dividend received over the time period of the option
$T$ : time to expiration
$\sigma^2$ : variance of log of stock price return ( $\ln(S_f / S_0)$ )
$\mu$ : mean per unit time of stock log price return

expected return from an investment in the combination of stock and option

$$\int dS_f P_T\left(\frac{S_f}{S_0}\right) \ln\left(\frac{W_f\left(S_f\right)}{W_0}\right)$$

for covered call position:

$$W_0 = S_0 - C_0$$

$$W_f = S_f - \max\left[0, S_f - E\right] + D$$

**Litterman, Modern Investment Management**

$$R_{L,t} - R_{f,t} = \beta(R_{B,t} - R_{f,t}) + \varepsilon_t$$

where $R_{L,t}$ : total return on liability index at time $t$

$R_{f,t}$ : risk-free rate of return

$R_{B,t}$ : total return on a bond index

$\varepsilon_t$ : noise term

$$SR_i = \frac{\mu_i - R_f}{\sigma_i}$$

$$RACS_t = \frac{E_t\left[S_{t+1} - S_t(1+R_f)\right]}{\sigma_t\left[S_{t+1}\right]}$$

$$RACS_t = \frac{E_t\left[A_t(1+R_{A,t+1}) - L_t(1+R_{L,t+1}) - (A_t - L_t)(1+R_f)\right]}{\sigma_t\left[A_t(1+R_{A,t+1}) - L_t(1+R_{L,t+1})\right]}$$

$$RACS_t = \frac{E_t\left[A_t(R_{A,t+1} - R_f)\right]}{\sigma_t\left[A_t(1+R_{A,t+1})\right]} = \frac{E_t\left[R_{A,t+1}\right] - R_f}{\sigma_t\left[R_{A,t+1}\right]}$$

$$E_t\left[F_{t+1}\right] = F_t E_t\left[\frac{1+R_{A,t+1}}{1+R_{L,t+a}}\right]\frac{1}{1-p} - \frac{p}{1-p}$$

$$E_0\left[F_t\right] = \left[\frac{1+E\left[R_x\right]}{1-p}\right]^t F_0 + p\,\frac{1-\left[\frac{1+E\left[R_x\right]}{1-p}\right]^t}{E\left[R_x\right]+p}$$

**Crouhy, Galai, and Mark, Risk Management,**

$$P_0 = -N\left(-d_1\right)V_0 + Fe^{-rT}N\left(-d_2\right)$$

$$d_1 = \frac{\ln\left(V_0/F\right) + \left(r + \tfrac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(V_0/Fe^{-rT}\right) + \tfrac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$y_T = -\frac{\ln\frac{B_0}{F}}{T} = -\frac{\ln\frac{Fe^{-rT} - P_0}{F}}{T}$$

$$\pi_T = y_T - r = -\frac{1}{T}\ln\left(N\left(d_2\right) + \frac{V_0}{Fe^{-rT}}N\left(-d_1\right)\right)$$

$$P_0 = \left[-\tfrac{N(-d_1)}{N(-d_2)}V_0 + Fe^{-rT}\right]N\left(-d_2\right)$$

$$EL_T = F\left(1 - N\left(d_2\right) - N\left(-d_1\right)\frac{1}{LR}\right)$$

$$\frac{1}{T}\ln\left(\frac{F}{F - EL_T}\right) = -\frac{1}{T}\ln\left(\frac{F\left(N\left(d_2\right) + N\left(-d_1\right)\frac{V_0}{Fe^{-rT}}\right)}{F}\right) = \pi_T$$

$$DD = \frac{\ln\frac{V_0}{DPT_T} + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

where $V_0$ : current market value of assets

$DPT_T$ : default point at time horizon $T$

$\mu$ : expected return on assets, net of cash outflows

$\sigma$ : annualized asset volatility

$$Q_T = N\left[N^{-1}\left(EDF\right) + \frac{\left(\mu - r\right)}{\sigma}\sqrt{T}\right]$$

$$Q_T = N\left[N^{-1}\left(EDF_T\right) + \rho_{V,M}\frac{\pi}{\sigma_M}\sqrt{T}\right]$$

$$Q_T = N\left[N^{-1}\left(EDF_T\right) + \rho_{v,m}\ SR\ T^{\theta}\right]$$

$$e^{-r_{v,i}t_i} = \left[\left(1 - LGD\right) + \left(1 - Q_i\right)LGD\right]e^{-r_i t_i}$$

$$r_{v,i} - r_i = -\frac{1}{t_i}\ln\left[1 - Q_i LGD\right]$$

$$r_{v,i} - r_i = -\frac{1}{t_i}\ln\left[1 - N\left(N^{-1}\left(EDF_{t_i}\right) + \rho_{v,m}\ SR\ T^{\theta}\right)LGD\right]$$

$$PV = \left(1 - LGD\right)\sum_{i=1}^{n}\frac{C_i}{\left(1 + R_i\right)^{t_i}} + LGD\sum_{i=1}^{n}\frac{\left(1 - Q_i\right)C_i}{\left(1 + R_i\right)^{t_i}}$$

$$PV = \left(1 - LGD\right)\sum_{i=1}^{n}C_i e^{-i^{t_i}} + LGD\sum_{i=1}^{n}\left(1 - Q_i\right)C_i e^{-i^{t_i}}$$

$$dr = \beta\left(m - r\right)dt + \eta dZ_r$$
$$dV = \mu V dt + \sigma V dZ_V$$
$$corr\left(dZ_r, dZ_V\right) = \rho dt$$

$$G_j(z)=\sum_{n=0}^{\infty}\frac{e^{-\bar{n}_j}\bar{n}_j^{\,n}}{n!}z^{nL_j}=e^{-\bar{n}_j+\bar{n}_j z^{L_j}}$$

$$G(z)=\prod_{j=1}^{m}e^{-\bar{n}_j+\bar{n}_j z^{L_j}}=e^{-\sum_{j=1}^{m}\bar{n}_j+\sum_{j=1}^{m}\bar{n}_j z^{L_j}}$$

Note: on the right, the first sum in the exponent, text has n bar **times** j. Should be n bar **sub** j. Full credit for either.

$$\frac{1}{n!}\frac{d^n G(z)}{dz^n}\bigg|_{z=0}$$

$$Y=\frac{R+\lambda LGD}{1-\lambda+\lambda(1-LGD)}$$

$$Y\Delta t=\frac{r\Delta t+\lambda\Delta tLGD}{1-\lambda\Delta t+\lambda\Delta t(1-LGD)}$$

$$Y=r+\lambda LGD$$

$$V(t,T)=E^*\left[\exp\left(-\int_t^T Y(s)\,ds\right)CF\right]$$

$$Y(t)=r(t)+\lambda(t)LGD+l$$

$$\lambda(t)=\lambda_0+\lambda_1 r(t)+\lambda_2 W_M(t)$$

$$dM(t)=\left[r(t)dt+\sigma_M dW_M(t)\right]M(t)$$

$$l(t)=l_0+l_1 r(t)+l_2 M(t)+l_3\left[M_H(t)-M_L(t)\right]^2$$

$$dr=(\alpha-\beta r)dt+\sigma_r dZ_r$$
$$dU=(a-bX)dt+\sigma_u dZ_u$$
$$corr(dZ_r,dZ_u)=\rho$$

$$ARAROC=\frac{RAROC-R_F}{\beta_E}$$

**Maginn & Tuttle, Managing Investment Portfolios, A Dynamic Process**

$$IR \approx IC\sqrt{Breadth}$$

where $IR =$ information ratio, $IC =$ information coefficient, $Breadth =$ investment discipline's breadth ( # of independent active investment decision made each year)

$$\underset{by\,choice\,of\ managers}{maximize}\ U_A = r_A - \lambda_A \sigma_A^2$$

where $U_A =$ expected utility of active return of the manager mix

$r_A =$ expected return of the manager mix

$\lambda_A =$ the investor's trade-off between active risk and active return, measure risk aversion in active risk terms

$\sigma_A^2 =$ variance of the active return

$$portfolio\,active\,return = \sum_{i=1}^{n} h_{A_i} r_{A_i}$$

where $h_{A_i}$ : weight assigned to the ith manager

$r_{A_i}$ : active return of the ith manager

$$portfolio\,active\,risk = \sqrt{\sum_{i=1}^{n} h_{A_i}^2 \sigma_{A_i}^2}$$

where $\sigma_{A_i}$ : active risk of the ith manager

$$manager's\,total\,active\,risk = \left[\left(manager's\,"true"\,active\,risk\right)^2 + \left(manager's\,"misfit"\,active\,risk\right)^2\right]^{1/2}$$

$$total\,return\,on\,commodity\,index = collateral\,return + roll\,return + spot\,return$$

$$rate\,of\ return = \frac{\left[\left(ending\,value\,of\ portfolio\right) - \left(beginning\,value\,of\ portfolio\right)\right]}{\left(beginning\,value\,of\ portfolio\right)}$$

$$RR_{n,t} = \frac{\left(R_t + R_{t-1} + R_{t-2} + ... + R_{t-n}\right)}{n}$$ where $RR_{n,t} =$ rolling return

$$downside\,deviation = \sqrt{\frac{\sum_{i=1}^{n}\left[\min\left(r_t - r^*, 0\right)\right]^2}{n-1}}$$

where $r^* =$ specified return

$$sharpe\,ratio = \frac{(annualized\,rate\,of\ return - annualized\,risk - free\,rate)}{annualized\,standard\,deviation}$$

$$gain-to-loss\ ratio = \left(\frac{number\ months\ with\ positive\ returns}{number\ months\ with\ negative\ returns}\right) * \left(\frac{average\ up-month\ return}{average\ down-month\ return}\right)$$

external cash flow at the beginning of the period

$$r_t = \frac{MV_1 - (MV_0 + CF)}{MV_0 + CF}$$

external cash flow at the end of period

$$r_t = \frac{(MV_1 - CF) - MV_0}{MV_0}$$

$$MV_1 = MV_0(1+R)^m + CF_1(1+R)^{m-L(1)} + .... + CF_n(1+R)^{m-L(n)}$$

where $m$ : number of time units in the evaluation period

$CF_i$ : $i$ th cash flow

$L(i)$ : number of time units by which the $i$ th cash flow is separated from the beginning of the evaluation period

$$R_p = a_p + \beta_p R_I + \varepsilon_p$$

$$r_V = \sum_{i=1}^{n}\left[w_{Vi} r_i\right] = \sum_{i=1}^{n}\left[(w_{pi} - w_{Bi}) r_i\right] = \sum_{i=1}^{n} w_{pi} r_i - \sum_{i=1}^{n} w_{Bi} r_i = r_p - r_B$$

where $r_V$ : value-added return

$r_P$ : portfolio return

$r_B$ : benchmark return

$$r_{AC} = \sum_{i=1}^{A} w_i (r_{Ci} - r_f)$$

$$r_{IS} = \sum_{i=1}^{A}\sum_{j=1}^{M} w_i w_{ij} (r_{Bij} - r_{Ci})$$

$$r_{IM} = \sum_{i=1}^{A}\sum_{j=1}^{M} w_i w_{ij} (r_{Aij} - r_{Bij})$$

$$r_V = \sum_{i=1}^{n}\left[(w_{pi} - w_{Bi})(r_i - r_B)\right]$$

$$r_V = \sum_{j=1}^{S}(w_{pj} - w_{Bj})(r_{Bj} - r_B) + \sum_{j=1}^{S}(w_{pj} - w_{Bj})(r_{pj} - r_{Bj}) + \sum_{j=1}^{S} w_{Bj}(r_{pj} - r_{Bj})$$

where $\sum_{j=1}^{S}(W_{Pj} - W_{Bj})(r_{Bj} - r_B)$ : pure sector allocation

$$\sum_{j=1}^{S}(W_{Pj}-W_{Bj})(r_{Pj}-r_{Bj})$$ : allocation / selection interaction

$$\sum_{j=1}^{S}W_{Bj}(r_{Pj}-r_{Bj})$$ : within-sector selection

$$R_{At}-r_{ft}=\alpha_A+\beta_A(R_{Mt}-r_{ft})+\varepsilon_t$$

$$T_A=\frac{\bar{R}_A-\bar{r}_f}{\hat{\beta}_A}$$

$$S_A=\frac{\bar{R}_A-\bar{r}_f}{\hat{\sigma}_A}$$

$$M_A^2=\bar{r}_f+\left[\frac{\bar{R}_A-\bar{r}_f}{\hat{\sigma}_A}\right]\hat{\sigma}_M$$

$$IR_A=\frac{\bar{R}_A-\bar{R}_B}{\hat{\sigma}_{A-B}}$$

where $\hat{\sigma}_{A-B}$: standard deviation of the difference between the return on the account and the return on the benchmark

**V-C107-07**

None

**V-C108-07**

None

**V-C109-07**

None

**V-C111-07**

None

**V-C114-07**

$$k=(1+p)(1+r'+r'')-1$$

where $k$ : nominal, required rate of return

$p$ : inflation rate

$r'$ : real risk-free rate

$r''$ : risk premium

$$E(R_i)=R_f+B_i\times[E(R_m)-R_f]+e_i$$

where $E(R_i)$: required rate of return on ith asset

$R_f$ : risk-free rate of return

$B_i$ : sensitivity of the ith asset's return to the market return

$E(R_m)$ : expected return on the market

$e_i$ : error or non-market-related (unsystematic) return

$$R_i = b_{i,0} + b_{i,1} \times F_1 + b_{i,2} \times F_2 + \ldots + b_{i,n} \times F_n + e_i$$

where $R_i$ : return on the ith asset

$b_{i,0}$ : constant (or intercept) term

$b_{i,n}$ : sensitivity of the asset's return to factor $n$

$F_n$ : value of factor $n$

$e_i$ : error term, or non-factor-related (unsystematic) return

$$V_0 = \sum_{n=1}^{N} \frac{CF_n}{(1+k)^n}$$

where $V_0$ : current intrinsic value

$CF_n$ : cash flow (including sales price) in period $n$

$N$ : holding period

$n$ : equal, discrete time period

$k$ : discount rate

$$V_0 = \frac{CF_0(1+g_1)}{(1+k)^1} + \frac{CF_1(1+g_2)}{(1+k)^2} + .. + \frac{CF_{N-1} \, {}^{(1+g_N)}\!\big/\!{}_{(k-g_N)}}{(1+k)^N}$$

where $g_n$ : growth in cash flow in period $n$

$k - g_N$ : reversionary capitalization rate (assuming long-term equilibrium)

**V-C119-07**

None

**V-C120-07**

$$r = \frac{D}{P} + g$$

where $r$ : rate of return

$\frac{D}{P}$ : (expected) dividend yield

$g$ : long-term growth rate

**V-C122-07**

None

**V-C124-07**

None

**V-C126-09 or FET-127-07 or 8V-114-00**

None

**V-C127-09 or FET-124-07 or 8V-323-05**

$$L_0 R_{S(L)} = A_0 R_A - L_0 R_L$$

where $L_0$ : current liabilities

$R_{S(L)}$ : liability-relative return of the surplus

$A_0$ : current asset

$R_A$ : return on assets

$R_L$ : return on liability

$$R_{S(L)} = \left( \frac{A_0}{L_0} R_A \right) - R_L$$

$$R_A = R_f + \beta_A r_Q + \alpha$$

where $R_A$ : asset portfolio return

$R_f$ : risk-free rate of return

$r_Q$ : excess return of the total investable market (portfolio $Q$) over cash

$$\sigma_A^2 = \beta_A \sigma_Q^2 + \omega_A^2$$

where $\sigma_A^2$ : variance of the asset portfolio

$\sigma_Q^2$ : variance of the market risk premium on the relevant benchmark

$\omega_A^2$ : variance of the alpha

$$\max\left(U_S\right) = R_S - \lambda \sigma_S^2$$

where $U_S$ : surplus utility

$\lambda$ : a constant representing the degree of risk aversion

$$\max\left(U_S\right) = \left( \frac{A_0}{L_0} - 1 \right) R_F + \beta_S \mu_Q - \lambda_\beta \beta_S^2 \sigma_Q^2 + \left( \frac{A_0}{L_0} \alpha_A - \alpha_L \right) - \lambda_\omega \left[ \left( \frac{A_0}{L_0} \right)^2 \omega_A^2 - 2 \frac{A_0}{L_0} \omega_A \omega_L + \omega_L^2 \right]$$

where $\mu_Q$ : the equilibrium or consensus, expected return of the total market across all asset classes

$\beta_S = (\frac{A}{L} \beta_A - \beta_L)$, surplus beta, the weighted relative betas of the assets and liabilities

$\omega$ : the standard deviation of the alphas, subscripted to indicate the assets and the liabilities, residual risk

$$P_{TIPS} = \frac{F}{(1+r)^T}$$

where $P_{TIPS}$ : the price of TIPS bond
$F$ : face value of bond
$i$ : inflation rate
$r$ : real interest rate
$T$ : time

$$P_{EQUITY} = \sum_{t=0}^{\infty} \frac{Dvd_0 (1+g_r)^t}{(1+r)^t}$$

Where $Dvd_0$ : beginning dividend
$g$ : growth rate of dividends

**V-C130-07**

$$r = r_f + OAS - D_{OAS}\Delta OAS - \sum D(i)\Delta r(i)$$

$$r = r_f + OAS - D_{OAS}\Delta OAS - \sum D(i)\Delta r(i) + \frac{r}{c}$$

$$r = r_f + OAS - D_{oas}\Delta OAS - \sum D(i)\Delta r(i) + r/c + pa - e_a$$

$$r = r_f + ROAS - \sum D_l(i)\Delta r(i) + e_l$$

$$r_i = r_f + NOAS - D_{noas}\Delta NOAS - \sum D(i)\Delta r(i)$$

$$r_a - r_i = OAS - NOAS - D_{oas}\Delta OAS + D_{noas}\Delta NOAS - \sum D_a(i)\Delta r(i) + \sum D_l(i)\Delta r(i) + r/c + pa - e_a$$

$$r_i - r_l = NOAS - D_{noas}\Delta NOAS - ROAS - e_l$$

**V-C133-07**
None

**V-C135-08**
None

**V-C136-09 or FET-128-07 or 6-31-00**
None

**V-C137-09 or FET-125-07 or 6-28-00**
None

**V-C138-09 or FET-126-07 or 8V-120-03**

None

**V-C139-09 or FE-C113-07**

None

**V-C140-09 or FET-115-07**

$$E[D] = \sum_{n} (DthBen - actuarialreserve)_t * {}_t p_x * q_{x+t} * v_t$$

$$E[D^2] = \sum_{n} [(DthBen - actuarialreserve)_t * v^t]^2 *_t p_x * q_{x+t}$$

$$Var[D] = E[D^2] - (E[D])^2$$

**V-C141-09 or FET-123-07 or 8V-126-04**

$$\min\left( E\left[ DK\left( \alpha + \frac{1}{\alpha}(P-\beta)^2 \right) \right] \right)$$

$$\beta = \frac{E[DP]}{E[D]}$$

$$\alpha = \frac{1}{E[D]}\sqrt{E\left[DP^2\right]E[D] - E[DP]^2}$$

$$D_t = D_0 \exp\left\{ \frac{1}{2\sigma^2}\left( \mu^2 - \left( r + \frac{\sigma^2}{2} \right)^2 \right) t + \frac{1}{\sigma^2}\left( r - \mu - \frac{\sigma^2}{2} \right) X_t \right\}$$

$$D_t = D_0 \left( \frac{S_t}{S_0} \right)^{-\alpha} \exp\left\{ -r(1-\alpha)t + \frac{1}{2}\sigma^2 \alpha (\alpha - 1) t \right\}$$

$$S_t = S_0 e^{X_t}, X_t \sim N\left( \left( r - \frac{\sigma^2}{2} \right)(T-t), \sigma^2 (T-t) \right)$$

**V-C142-09 or FET-110-07 or 8FE-320-01**

market value of surplus = market value of assets – market value of liabilities
= PV (assets cash flows) – PV (liabilities cash flows)
= PV of net cash flows

Total return = income + realized returns + unrealized returns

**V-C143-09**
None

**V-C144-09**
None

**V-C145-09**
None

**V-C146-09**
None

**V-C147-09**
None

**V-C148-09**

investor's utility function $U(W) = \left[\frac{1}{(1-A)}\right] W^{(1-A)}$

where $A =$ coefficient of relative risk aversion
$W =$ investor's wealth

arithmetic equity premium $EP \approx A(\sigma^2)$

where $\sigma =$ standard deviation of return on investor's portfolio

**V-C149-09**
None

**V-C150-09**

$$S_p = \frac{\bar{R}_p - \bar{R}_f}{\sigma_p}$$

where $S_p =$ Sharpe ratio for a portfolio
$\bar{R}_p =$ mean return on the portfolio
$\bar{R}_f =$ mean return on the U.S. T-bill (proxy for risk-free rate of interest)
$\sigma_p =$ sample standard deviation of returns

approximate Sharpe ratio for multi-period investment horizon

$$S_n = \frac{(1+R_1)^n - (1+R_f)^n}{\left\{\left[\sigma_1^2 + (1+R_1)^2\right]^n - (1+R_1)^{2n}\right\}^{1/2}}$$

where $R_1$ *and* $\sigma_1$ are one period expected return and standard deviation

$R_n = (1+R_1)^n - 1$ n-period expected return

$\sigma_n = \left\{\left[\sigma_1^2 + (1+R_1)^2\right]^n - (1+R_1)^{2n}\right\}^{1/2}$ n-period standard deviation

$$HPR_n = \prod_{i=1}^{n}(1+R_i)$$ n-year holding period return

**V-C151-09**

$$d = e - \frac{v}{t}$$

where $d =$ desirability of the portfolio for the investor
$e =$ portfolio expected return
$v =$ portfolio variance of return
$t =$ investor's risk tolerance

$$eu = \sum_s \pi_s u(R_{ps})$$

where $u(R_{ps}) =$ utility of total portfolio return $R_p$ in state $s$
$\pi_s =$ probability that state $s$ will occur

marginal expected utility: $meu(R_{ps}) = \pi_s m(R_{ps})$

where $m(R_{ps}) =$ marginal utility of total portfolio return $R_p$ in state $s$, the 1st derivative of $u(R_{ps})$

marginal expected utility of asset $i$: $meu_i = \sum_s R_{is} meu(R_{ps})$

HARA (hyperbolic absolute risk aversion) : $u(R) = \frac{(R-b)^{1-c}}{1-c}$

where $c$ : risk aversion coefficient
$b$ : investor's minimum required level of return

$$m(R) = (R-b)^{-c}$$

$$Expected\ returns + Risks + Correlations + Risk\ tolerance \rightarrow Optimal\ portfolio$$

$$R_{is} = R_{is}^{O} + d_i$$

where $R_{is}^{O}$ old return for each asset in each state
$d_i$ difference
$R_{is}$ new return for each asset in each state

$$E(R_i) = E(R_i^{O}) + d_i$$

$$R_{ms} = \sum_i x_{im} R_{is} = \sum_i x_{im} R_{is}^{0} + \sum_i x_{im} d_i$$

$$d_m = \sum_i x_{im} d_i$$

$$E(R_m) = E(R_m^O) + d_m$$

marginal expected utility of market portfolio $meu_m = \sum_s R_{ms}\pi_s m(R_{ms})$

marginal expected utility of risk-free asset $meu_f = R_f \sum_s \pi_s m(R_{ms})$

assets' marginal expected utility $\sum_s R_{ms}\pi_s m(R_{ms}) = R_f \sum_s \pi_s m(R_{ms})$

$$\sum_s R_{is}\pi_s m(R_{ms}) = R_f \sum_s \pi_s m(R_{ms})$$

$$\sum_s (R_{is}^O + d_i)\pi_s m(R_{ms}) = R_f \sum_s \pi_s m(R_{ms})$$

$$d_i = \frac{R_f \sum_s \pi_s m(R_{ms}) - \sum_s R_{is}^O \pi_s m(R_{ms})}{\sum_s \pi_s m(R_{ms})}$$

$$\sum_s R_{ms}\pi_s \left[(R_{ms} - b)^{-c}\right] = R_f \sum_s \pi_s \left[(R_{ms} - b)^{-c}\right]$$

$$mrs_s = \frac{\pi_s m(R_{ps})}{\sum_s \pi_s m(R_{ps})}$$

where $R_{ps}$ return on investor's portfolio in state $s$

state price for the state claim: $p_s = \dfrac{\pi_s m(R_{ps})}{R_f \sum_s \pi_s m(R_{ps})}$

**V-C153-09**

None

**V-C154-09**

None

**V-C155-09 or FET-117-07 or 8V-316-02**

None

**V-C156-09 or FE-C128-07**

None

**V-C157-09 or FET-103-07 or 8V-324-05 or 8E-712-05**

None

**V-C158-09 or FET-140-07 or 8V-319-04**

$$dr_t = a\left[b - r_t\right]dt + \sigma dW_t$$

$$\frac{dP(t,T)}{P(t,T)} = r_t dt - \sigma_P\left(T-t\right)dW_t$$

$$\sigma_P\left(t,T\right) = \frac{\sigma}{a}\left(1 - e^{-a(T-t)}\right)$$

$$P(t,T) = G(T-t)\ \exp\left(-H\left(T-t\right)r_t\right)$$

$$H\left(T-t\right) = \frac{1-e^{-a(T-t)}}{a}$$

$$G(T-t) = \exp\left[\left(\frac{\sigma^2}{2a^2} - b\right)(T-t) + \left(b - \frac{\sigma^2}{a^2}\right)H\left(T-t\right) + \frac{\sigma^2}{4a^2}H\left(2\left(T-t\right)\right)\right]$$

$$\frac{dA_t}{A_t} = \mu dt + \sigma_A\left[\rho dW_t + \sqrt{1-\rho^2}\ dZ_t\right]$$

$$B_T = \max\left[0, \delta\left(\frac{L_0}{A_0}\left(A_T - A_0\right) - \left(L_T^* - L_0\right)\right)\right] = \delta\alpha \max\left[0, A_T - \frac{L_T^*}{\alpha}\right]$$

$$L_T^* = L_0 e^{r^* T}$$

$$L_T = L_T^* + B_T = \delta\alpha A_T + \left(1-\delta\right)L_T^* \text{ if } L_T^* \le A_T$$

$$E_T = \max\left[0, A_T - L_T^*\right] - \delta\alpha \max\left[0, A_T - \frac{L_T^*}{\alpha}\right]$$

$$E_t = C_E\left(A_t, L_T^*\right) - \delta\alpha C_E\left(A_t, \frac{L_T^*}{\alpha}\right)$$

$$C_E\left(A_t, L_T^*\right) = A_t N\left(d_1\right) - P\left(t,T\right) L_T^* N\left(d_2\right)$$

$$C_E\left(A_t, \frac{L_T^*}{\alpha}\right) = A_t N\left(d_3\right) - P\left(t,T\right) \frac{L_T^*}{\alpha} N\left(d_4\right)$$

$$d_1 = \frac{\ln A_t / P\left(t,T\right) L_T^* + \bar{\sigma}\left(t,T\right)^2 \left(T-t\right)/2}{\bar{\sigma}\left(t,T\right)\sqrt{\left(T-t\right)}} = d_2 + \bar{\sigma}\left(t,T\right)\sqrt{\left(T-t\right)}$$

$$d_3 = \frac{\ln \alpha A_t / P\left(t,T\right) L_T^* + \bar{\sigma}\left(t,T\right)^2 \left(T-t\right)/2}{\bar{\sigma}\left(t,T\right)\sqrt{\left(T-t\right)}} = d_4 + \bar{\sigma}\left(t,T\right)\sqrt{\left(T-t\right)}$$

$$\bar{\sigma}\left(t,T\right)^2 = \frac{1}{T-t}\int_t^T \left[\left(\rho\sigma_A + \sigma_P\left(u,T\right)\right)^2 + \left(1-\rho^2\right)\sigma_A^{\ 2}\right] du$$

$$E_t = A_t\left[N\left(d_1\right) - \delta\alpha N\left(d_3\right)\right] - P\left(t,T\right) L_T^*\left[N\left(d_2\right) - \delta N\left(d_4\right)\right]$$

$$L_t = L_T^* P\left(t,T\right) - P_E\left(A_t, L_T^*\right) + \delta\alpha C_E\left(A_t, \frac{L_T^*}{\alpha}\right)$$

$$P_E\left(A_t, L_T^*\right) = -A_t N\left(-d_1\right) + P\left(t,T\right) L_T^* N\left(-d_2\right)$$

$$L_t = A_t\left[N\left(-d_1\right) + \delta\alpha N\left(d_3\right)\right] + P\left(t,T\right) L_T^*\left[N\left(d_2\right) - \delta N\left(d_4\right)\right]$$

$$\left(1-\alpha\right) A_0 = C_E\left(A_0, L_T^*\right) - \delta\alpha C_E\left(A_0, \frac{L_T^*}{\alpha}\right)$$

$$\delta = \frac{C_E\left(A_0, L_T^*\right) - \left(1-\alpha\right) A_0}{\alpha C_E\left(A_0, \frac{L_T^*}{\alpha}\right)}$$

$$\eta_P\left(t,T\right) = H\left(T-t\right)$$

$$\eta_A\left(t,T\right) = -\frac{\rho\sigma_A}{\sigma}$$

$$\eta_L\left(t,T\right) = \eta_P\left(t,T\right) - \frac{A_t}{L_t}\left[\eta_P\left(t,T\right) - \eta_A\right]\left[N\left(-d_1\right) + \delta\alpha N\left(d_3\right)\right]$$

$$D_L = \frac{\ln\left(1 - a\eta_L\left(0,T\right)\right)}{a}$$

$$\eta_A(t,T) = \frac{E_t}{A_t}\eta_E(t,T) + \frac{L_t}{A_t}\eta_L(t,T)$$

$$\eta_E(t,T) = \eta_P(t,T) - \frac{A_t}{E_t}\left[\eta_P(t,T) - \eta_A\right]\left[N(d_1) - \delta\alpha N(d_3)\right]$$

$$\hat{P}(t,T,\hat{r}_t) = G(T-t)\exp\left(-H(T-t)\hat{r}_t\right)$$

$$\hat{A}_t(\hat{r}_t) = A_t \exp\left(-\eta_A(\hat{r}_t - r_t)\right)$$

**V-C159-09 or FET-121-07 or 8V-322-05**

None

**V-C160-09 or FET-136-07 or 8E-704-04**

None

**V-C161-09 or FET-130-07**

None

**V-C164-09**

$$x_S + x_L = 1$$
$$x_S D_S + x_L D_L = D_B$$

$$\bar{r} = \sum_{s=1}^{4} p_i r_i$$

$$\sigma^2 = \sum_{s=1}^{4} p_i (r_i - \bar{r})^2$$

$$p_i^{PERFECT} = \begin{cases} 1/n_W & \text{if } i \text{ is correct decision} \\ 0 & \text{otherwise} \end{cases}$$

where $n_W$ are correct decisions among $n$ choices, $n_L = n - n_W$ are incorrect decisions

$$p_i(s) = (1-s)\,p_i^{RANDOM} + s p_i^{PERFECT} = \begin{cases} \dfrac{(n_W + s n_L)}{n_W(n_W + n_L)} & \text{if } i \text{ is correct decision} \\ \dfrac{(1-s)}{(n_W + n_L)} & \text{otherwise} \end{cases}$$

$$r = \sum_j w_j r_j$$

where $w_j$ percentage market capitalization of the index in cell $j$

$r_j$ strategy outperformance of the index within cell $j$

$$\bar{r} = \sum_j w_j \bar{r}_j$$
$$\sigma^2 = \sum_j w_j^2 \sigma_j^2$$

$$r = \frac{1}{n}\sum_{i=1}^{n} r_i$$

where $r_i$ outperformance due to decision $i$
$r$ overall portfolio outperformance

$$\mu_{strategy} = \mu_{decision} \qquad \sigma_{strategy} = \sigma_{decision} / \sqrt{n}$$

$$R_{S,b} = bR_S + (1-b)R_B = R_B + b(R_S - R_B)$$

where $R_B$ benchmark performance $R_S$ strategy performance
$b$ portion of portfolio assets is committed to strategy

$$\mu_{S,b} = E(R_{S,b} - R_B) = E(b(R_S - R_B)) = bE(R_S - R_B) = b\mu_s$$
$$\sigma_{S,b}^2 = Var(R_{S,b} - R_B) = Var(b(R_S - R_B)) = b^2 Var(R_S - R_B) = b^2\sigma_S^2$$

strategy information ratio $IR_S = \dfrac{\mu_S}{\sigma_S}$

$$IR_{S,b} = \frac{\mu_{S,b}}{\sigma_{S,b}} = \frac{b\mu_S}{b\sigma_S} = \frac{\mu_S}{\sigma_S} = IR_S$$

$$E(y) = E(E(y|x))$$

$$Var(y) = Var(E(y|x)) + E(Var(y|x))$$

**V-C165-09**

None

**V-C166-09**

asset value $x_i = a_i M + \sqrt{1-a_i^2} Z_i$

$$q_i(t|M) = H_i\left(\frac{\bar{x}_i - a_i M}{\sqrt{1-a_i^2}}\right)$$

where $q_i(t) =$ risk-neutral probability that credit i default before t
$\bar{x}_i =$ the threshold amount of normalized asset value $x_i$

$$p^0(0,t|M)=1$$
$$p^{K+1}(0,t|M)=p^K(0,t|M)(1-q_{K+1}(t|M))$$
$$p^{K+1}(l,t|M)=p^K(l,t|M)(1-q_{K+1}(t|M))+p^K(l-1,t|M)q_{K+1}(t|M)\quad l=1,...,K$$
$$p^{K+1}(K+1,t|M)=p^K(K,t|M)q_{K+1}(t|M)$$

$$p(l,t)=\int_{-\infty}^{\infty}p^N(l,t|M)g(M)dM$$
where $g(t)$ is probability density of $M$

expected loss on tranche up to payment date $T_i$
$$EL_i=\sum_{l=0}^{N}p(l,T_i)\max(\min(lA(1-R),H)-L,0)$$
where $p(l,t)$ default probability distribution
$A(1-R)$ loss from any default

$$Contingent=\sum_{i=1}^{n}D_i(EL_i-EL_{i-1})$$
where $D_i$ risk-free discount factor at payment date $i$

$$Fee=s\sum_{i=1}^{n}D_i\Delta_i\{(H-L)-EL_i\}$$
where $\Delta_i\approx T_i-T_{i-1}$ accrual factor for payment date $i$
$s$ the spread per annum paid to tranche investor
$(H-L)-EL_i$ expected tranche principal outstanding on payment date $T_i$

mark-to-market value of the tranche $MTM=Fee-Contingent$

$$s_{Par}=\frac{Contingent}{\sum_{i=1}^{n}D_i\Delta_i\{(H-L)-EL_i\}}$$

standard deviation of loss up to payment date $T_i$
$$SD_i=\left(\sum_{l=0}^{N}p(l,T_i)\left[\max(\min(lA(1-R),H)-L,0)-EL_i\right]^2\right)^{1/2}$$
unexpected loss $UL_i=EL_i+SD_i$

**V-C168-09**

$$Total\ return=income\ return+price\ return+currency\ return$$

$$duration\ return=roll\ down+shift+twist+shape\ return$$

$$term\ structure\ effect=DU^{roll}+DU^{shift}+DU^{twist}+DU^{shape}$$

**V-C169-09**

total return $TR = (1+TR_t)(1+TR_{t+1})...(1+TR_n)-1$

$$TR_f = \sum_{f=1}^{F} RF_{f,t}$$

where $RF_{f,t}$ = fth attribution effect obtained at time t

**V-C170-09**

None

**V-C171-09**

None

**V-C172-09**

None

**V-C173-09**

$$CF_0 + CF_1^*(1+IRR)^{-1} + ... + CF_t^*(1+IRR)^{-t} = 0$$

where $CF$ = capital flow, $IRR$ = dollar-weighted return on stock investment

$$Distributions_t = MV_{t-1}^*(1+r_t) - MV_t$$

where $MV$ = market capitalization,

$r_t$ = total value-weighted return for that period (including dividends)

**V-C174-09**

None

**V-C175-09**

$$Risky\,note = (Default - free\,note) - asset\,insurance$$

**V-C176-09**

None

**V-C17-09**

None

**V-C178-09**

None

**V-C180-10**

$$\Pr ob(H) + \Pr ob(H^C) = 1$$

$$\sum_{k=1}^{\infty} 2^{-k} \log(2^k) = 2\log 2 \simeq 4 \quad \sum_{k=1}^{\infty} 2^{-k} \log(2^k) = 2\log 2 \simeq 4$$

**V-C181-10**

None

**V- C183-10**

Equation 1: CDS Spread as a Function of Default Probability (PD) and Recovery Rate (R)

$S = PD \times (1 - R)$

Equation 2: CDS Pricing Equation – From upfront plus running to full running, using the CDS risky annuity (RA) and accrued interest (AI)

$$Full\ Running = \frac{Upfront\ \text{-}\ AI}{RA} + Fixed\ Coupon$$

Equation 3: Par Asset Swap Spread Calculation

$$\text{Asset swap spread} = \frac{PV[\text{Coupon} + \text{Principal}] - \text{Bond Price}}{\text{Risk free annuity}}$$

Equation 4: Basis Trade Profit on Default

$$\text{CDS Notional x (100} - \text{Recovery} - \text{CDS Upfront} - \text{CDS Coupons Paid} - \text{CDS Funding Costs Paid)}$$
$$+ \text{Bond Notional x (Recovery} + \text{Bond Coupons Received} - \text{Bond Price} - \text{Bond Funding Costs Paid)}$$

Note: Bond Price refers to the dirty bond price.

Equation 5: Basis Trade Profit on Maturity

$$\text{Bond Notional x (100} + \text{Bond Coupons Received} - \text{Bond Price} - \text{Bond Funding Costs Paid)}$$
$$- \text{CDS Notional x ( CDS Upfront} + \text{CDS Coupons Paid} + \text{CDS Funding Costs Paid)}$$

Note: Bond Price refers to the dirty bond price.

Equation 6: Basis Trade Profit on Default splitting the trade cash flows into running and one-off payments

$$\underline{\text{From one - off payments}}:$$
$$\text{CDS Notional } x \text{ (100} - \text{Recovery} - \text{CDS Upfront)} + \text{Bond Notional } x \text{ (Recovery} - \text{Bond Price)}$$
$$\underline{\text{From running payments}}:$$
$$\text{Bond Notional } x \text{ (Bond Coupons Received} - \text{Bond Funding Costs Paid )}$$
$$- \text{CDS Notional } x \text{ (CDS Coupons Paid} + \text{CDS Funding Costs Paid)}$$

Equation 7: CDS Notional in a “Capital-at-Risk” Basis Trade

$$\text{CDS Notional} = \frac{\text{Bond Price} - \text{Recovery}}{100 - \text{Recovery} - \text{CDS Upfront}} x \text{ Bond Notional}$$

Equation 8: Equal Notional Basis Trade Profit on Default or Maturity (Ignoring risk-free discounting and funding costs)

$$(100 - \text{Bond Price} + \text{Bond Coupons Received} - \text{CDS Upfront} - \text{CDS Coupons Paid})$$

Note: Bond Price refers to the dirty bond price.

**CIA : An Overview of an Investment Policy Statement in an Asset / Liability Management Context**

None

**AAA Monograph: Fair Valuation of Insurance Liabilities Principles and Methods**

$$r_L = r_A - e\left(\frac{r_E}{1-t} - r_A\right)$$

$$MVM_t = L_{t-1}\left(r_f - r_L\right)$$

**CIA Educational Note: Liquidity Risk Measurement**

None

**RSA, Vol 27, No. 2: Liquidity Modeling and Management**

None

**Byrne & Brooks, "Behavioral Finance: Theory and Evidence"**

None

**A. Lo, "The Three P's of Total Risk Management"**

$$\Pr ob(H) + \Pr ob(H^C) = 1$$

$$\sum_{k=1}^{\infty} 2^{-k} \log(2^k) = 2\log 2 \simeq 4$$