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DISCUSSION OF PRECEDING PAPER

JAMES E. HOSKINS:

Mr. Connolly refers to the type of varying interest rate which he describes as "an artificial interest assumption." Some might feel, however, that under current conditions the assumption of an interest rate which decreases with the passage of time may be more realistic than the assumption of a level rate. A sharp decrease in the interest rate at the end of n years is, of course, to be regarded as an approximation to a decrease of a continuous nature.

If, as in Mr. Connolly's illustration, the period n is chosen so that the net premium at 3% for n years and $2\frac{1}{2}$% thereafter approximates that at $2\frac{3}{4}$% throughout, then the period n will vary with the issue age. It will also vary with plan. For example, on 20 Payment Life at age 35 n would equal 22, as against 23 for Ordinary Life. It might be more realistic to assume a value of n which is independent of age and is derived by Mr. Connolly's method for a representative plan and age. The net premium at the varying rates would then be higher than the $2\frac{3}{4}$% net premium at some ages and lower at others. This is illustrated in the accompanying table. Whether this would be a satisfactory result from the standpoint of minimizing deficiency reserves depends on the shape of the gross premium scale.

From this illustration it appears that the year r after which the reserve at the varying interest rates exceeds that at $2\frac{1}{2}$% varies less widely when a common value of n is used than when n is chosen so that the net premium at each age approximates the $2\frac{3}{4}$% net premium.

The method is evidently intended for use where a company, if not hampered by deficiency reserve requirements, would base its gross premiums on a conservative interest rate such as $2\frac{1}{2}$%, but would recognize mortality improvement which has occurred since the compilation of the CSO Table, and where it wishes to give cash values not materially less than it would have derived on the basis of CSO $2\frac{1}{2}$%. There are perhaps three principal types of cash value scale, namely, minimum values, values greater than the minimum but with a single nonforfeiture factor for the premium periods (except on plans with varying gross premiums), and values which reach the full reserve before the end of the premium period, thus requiring two nonforfeiture factors.

Where the former scale of cash values at $2\frac{1}{2}$% which the company wishes to approximate at varying interest rates consists of minimum

Full Level Premium Terminal Reserves

	Interest Basis			
	Varying $n=31$	Varying $n=23$	$2\frac{3}{4}\%$	$2\frac{1}{2}\%$
Age 20 Net Premium:	$11.87	$12.10	$11.87	$12.49
Duration				
1.........	9.82	10.06	9.79	10.40
3.........	30.19	30.93	30.02	31.83
5.........	51.57	52.84	51.15	54.11
10.........	109.57	112.33	107.93	113.60
15.........	174.19	178.73	170.31	178.33
20.........	245.41	252.09	237.99	247.93
23.........	291.21	299.22	280.87	291.72
25.........	322.96	328.81	310.28	321.62
30.........	406.31	404.60	386.04	398.23
31.........	423.63	419.98	401.48	413.77
40.........	560.86	558.08	541.19	553.34
60.........	814.99	813.81	804.47	811.82
75.........	924.42	923.94	919.64	923.13

	Interest Basis			
	Varying $n=16$	Varying $n=23$	$2\frac{3}{4}\%$	$2\frac{1}{2}\%$
Age 50 Net Premium:	$36.18	$35.80	$36.16	$36.90
Duration				
1.........	25.26	24.86	25.14	25.82
3.........	76.26	75.02	75.71	77.62
5.........	127.76	125.58	126.47	129.46
10.........	257.49	252.45	252.71	257.76
15.........	386.52	377.37	374.97	381.19
16.........	411.91	401.85	398.59	404.95
20.........	501.86	497.64	489.36	495.96
23.........	564.43	567.16	552.73	559.27
25.........	603.48	605.97	592.40	598.79
30.........	690.95	692.89	681.52	687.29
40.........	821.15	822.27	814.95	819.03
45.........	873.75	874.54	869.12	872.25

values, it appears from the illustrative figures that a formula could be chosen whereby the new values approximate the old at the shorter durations and exceed them at the longer durations. In view of the relatively low withdrawal rates at the durations where values would be increased, the increase would probably not add much to the gross premium.

Where the old scale is of the second type, with cash values in excess of minimum values but not reaching the full reserve until the end of the premium period, it appears likely that the present scale could be even more closely approximated. In both cases it might be necessary to use more than one nonforfeiture factor in the new formula to achieve a close approximation.

Likewise where it is desired to approximate a former scale which reaches the full $2\frac{1}{2}\%$ reserve at a certain duration, it would appear that this could readily be done; that is, the old values would tend to lie between the minimum values and the full reserve on the new basis.

If a company should want to compute its premiums on the basis of $2\frac{3}{4}\%$ and up-to-date mortality, thereby producing some rates below the CSO $2\frac{3}{4}\%$ net, then it could not use the analogous procedure of valuing at a combination of $3\frac{1}{4}\%$ and $2\frac{3}{4}\%$ if it operates in New York, since the maximum interest rate permitted there for valuation is 3%.

One drawback to the use of a varying interest rate is that the amount of paid-up insurance purchased by a given cash value at a given attained age would vary according as the duration is greater or less than that at which the interest rate changes. This could be relieved by using the ultimate interest rate on paid-up policies, whether arising from full payment of premiums or from nonforfeiture values. In other words, the interest assumption in the illustration would become 3% to the end of n years or prior discontinuance of premiums, and $2\frac{1}{2}\%$ thereafter.

The thought underlying Mr. Connolly's paper has an interesting corollary. There has been suggested[1] as a partial solution to the deficiency reserve "problem" an amendment to the statutes so as not to require a deficiency reserve if the basic reserve actually carried by the company exceeds the sum of the basic reserve and accompanying deficiency reserve on the minimum standard of valuation permitted. At first glance this seems to be a logical proposal. Taking illustrative figures from Mr. Connolly's table, a company which finds itself able to sell Ordinary Life at age 35 at a premium of \$19.80 can do so without deficiency reserve if it values at $2\frac{3}{4}\%$ (or a higher rate), and in that event its first year terminal reserve would be \$15.83 or less. If, however, it chooses for reason of conservatism the $2\frac{1}{2}\%$ rate, it must increase this particular reserve not

[1] *E.g.*, W. M. Anderson, *TSA* VII, 101.

merely to $16.49, which it is willing to do, but to $31.83, including a deficiency reserve of $15.34.

It can be argued that if the company voluntarily reserves more than the minimum legal requirement it should not be penalized by being forced to put up a further deficiency reserve. (Since a $2\frac{1}{2}\%$ valuation may be considered to imply the expectation of gross premiums at least equal to the $2\frac{1}{2}\%$ net, the $2\frac{1}{2}\%$ reserve maintained under the suggested amendment should probably be labeled as a $2\frac{1}{2}\%$ reserve *without deficiency reserve.*)

Nevertheless there are certain conditions under which the maintenance of a reserve higher than CSO Commissioners Method $3\frac{1}{2}\%$ (3% in New York) is not voluntary but is a legal requirement. Such a situation occurs if (1) all of the company's business[2] issued under the Standard Valuation Law has been valued on the CSO Table by the Commissioners Method, (2) all such policies have been valued at the same interest rate as that on which nonforfeiture values are calculated, and (3) the interest rate used for nonforfeiture values in the category of policies under consideration is less than the minimum valuation interest rate (*e.g.*, if it is $2\frac{1}{2}\%$). This follows from the provisions in the Standard Valuation Law that "Reserves for any category of policies . . . may be calculated . . . according to any standards which produce greater aggregate reserves for such category than those calculated according to the minimum standard herein provided, but the rate or rates of interest used shall not be higher than the corresponding rate or rates of interest used in calculating any nonforfeiture benefits provided for therein" and that "In no event shall a company's aggregate reserves for all life insurance policies, excluding disability and accidental death benefits, issued on or after the effective date of this Act, be less than the aggregate reserves calculated in accordance with [the Commissioners Method] and the mortality table or tables and rate or rates of interest used in calculating non-forfeiture benefits for such policies."

Under these conditions the maintenance of reserve as high as CSO $2\frac{1}{2}\%$ is mandatory rather than voluntary; and the argument mentioned above for exempting the company from such deficiency reserve as would attach to a $2\frac{1}{2}\%$ valuation would not apply.

However, Mr. Connolly's paper suggests that it is often possible to approximate closely to cash values based on $2\frac{1}{2}\%$ interest, by a combination of interest rates which produce a lower net premium than that at $2\frac{1}{2}\%$. If a company should express its cash values as based on such a combination of interest rates, and if the suggested amendment should be

[2] Assuming that it is all Ordinary business.

adopted, then the minimum standard for that company and category of business would be the combination of varying interest rates and, even if it actually valued at $2\frac{1}{2}\%$, no deficiency reserve would be required if the gross premium were not less than the net at the combined rates.

This procedure—and for that matter the use of varying interest rates for nonforfeiture benefits—is subject to the interpretation of the Standard Nonforfeiture Law that the provision in Section 5 for calculating minimum values on "the rate of interest, not exceeding three and one-half per cent ($3\frac{1}{2}\%$) per annum, specified in the policy . . ." does not limit the company to a single rate of interest.

W. HAROLD BITTEL:

During the presentation and oral discussion of this paper the suggestion was made that it would be possible to use the ultimate interest rate for the calculation of paid-up nonforfeiture benefits where premiums are discontinued prior to the policy anniversary on which this ultimate rate would otherwise become applicable. Since it was implied that this procedure would be permissible under the Standard Nonforfeiture Law, I feel it would be desirable to have my views to the contrary added to the discussion of the paper. The definition in the policy of the interest rate used for calculating such paid-up nonforfeiture benefits could be as follows: i' to the end of the nth policy year or prior discontinuance of premiums and i'' thereafter.

The Standard Nonforfeiture Law requires that "all adjusted premiums and present values . . . shall be calculated on the basis of . . . the rate of interest . . . specified in the policy for calculating cash surrender values and paid-up nonforfeiture benefits." Although I would not be inclined to construe the word "rate" as precluding the use of two interest rates varying by definite periods of time after a policy is issued, nevertheless it seems clear the law contemplates that the interest periods selected should depend only on time after issuance of a policy and should be the same for cash surrender values and paid-up nonforfeiture benefits. In other words, the definition of the interest rate in the policy would have to be the following for all calculations thereunder: i' to the end of the nth policy year and i'' thereafter.

The section of the law quoted above also requires that the rate of interest be "specified in the policy." If this rate varies according to some future contingency, such as nonpayment of premiums, then it is not definite at the time the policy is issued. The word "specify" means "to state definitely" and this requirement is not complied with if the rate depends on a future contingency. Moreover, if a different future rate of interest is assumed for paid-up nonforfeiture benefits than that assumed for cash surrender values, we are specifying two inconsistent rates of interest covering the same period of time.

In addition to the above points, which seem to rule out completely the use of the date of forfeiture as a measure of the period for which an interest rate is effective, the following provision of the Standard Valuation Law may restrict some of the methods described by the author in the paper: "Reserves for any category of policies, contracts or benefits, as established by the Commissioner may be calculated, at the option of the company, according to any standards which produce greater aggregate reserves for such category than those calculated according to the minimum standard herein provided, but the rate or rates of interest used shall not be higher than the corresponding rate or rates of interest used in calculating any nonforfeiture benefits provided for therein."

One of the basic elements in Mr. Connolly's proposal is the use of a number of different periods for which the initial rate of interest is guaranteed. The actual period to be used would vary with age at issue. It would be necessary for a Commissioner to give a very strained construction of the word "category" to permit such variations. It would hardly appear that a single age would be a "category of policies" under the statute quoted above.

Another suggested method about which there would be a question is that under the heading "Control of Cash Values" in the paper. This appears to contemplate the use of a rate or rates of interest for the calculation of reserves which would be higher than the corresponding rate or rates of interest used in calculating nonforfeiture benefits. Such reserves would not appear to comply with the requirements of the Standard Valuation Law.

LLOYD K. FRIEDMAN:

Mr. Connolly has developed a method of such great power and flexibility that there will almost surely be adaptations of it for use in other situations. It certainly displays great ingenuity in solving or at least in mitigating the problem to which it has been applied. In that connection it is at least a partial answer to those who think that legal barriers can be an effective substitute for enlightened company management.

Two phases of Mr. Connolly's paper will be considered. In some cases even on the basis of variable interest rates or for that matter on the maximum legal interest rate of $3\frac{1}{2}\%$, net premiums are obtained which are in excess of gross premiums imposed by competitive conditions. Deficiency reserves then occur on the basis of the variable interest rate, the amount at the end of the mth policy year being

$$(P_x^{\text{var}} - \pi_x)\, \ddot{a}_{x+m}^{i''} ,$$

where π is the gross premium. The amounts of the deficiency reserves will be found to be tolerable even for a young company.

If the gross premium has been calculated on the assumption of total liability at the end of m years equal to the net level reserve at interest rate i'', a value of n can be determined by the following obvious modification of Mr. Connolly's formula (7)

$$P_x^{\text{var}} = \frac{{}_mV_x^{i''} - (P_x^{\text{var}} - \pi_x)\, \ddot{a}_{x+m}^{i''} + {}_mk_x^{i'}}{{}_mu_x^{i'}} , \tag{7'}$$

which may be solved for P^{var} to yield

$$P_x^{\text{var}} = \frac{{}_mV_x^{i''} + \pi_x \ddot{a}_{x+m}^{i''} + {}_mk_x^{i'}}{{}_mu_x^{i'} + \ddot{a}_{x+m}^{i''}} .$$

On the basis of n so determined, the variable interest reserve at year n will be less than ${}_mV_x^{i''}$ by exactly the amount of the deficiency reserve.

Agreement or disagreement with Mr. Connolly's statement that the calculations would become tedious without high-speed electronic calculators necessarily depends on one's preconceived standards of tedium. Calculations for an ordinary life policy on interest rates $2\frac{1}{2}\%$ and $3\frac{1}{2}\%$, by the use of desk calculators, were found to be no more laborious than for a regular retirement income plan. The values of n followed a fairly uniform pattern, so that not too many trials were required for their determination.

(AUTHOR'S REVIEW OF DISCUSSION)

CHARLES H. CONNOLLY:

The fact that each of those discussing this paper represents a distinct and separate actuarial field is appreciated.

Mr. Hoskins points out the desirability of the selection of a value of n varying only by plan. Mr. Bittel indicates the desirability of such a selection from a legal standpoint keyed to the definition of the word "category." It certainly seems desirable to make such a selection, although the legal necessity does not seem to me to be obvious. "Category" and "class" are used synonymously, and the recent approval by the various state regulatory bodies of rate gradation by amount has been based in many states upon the extension of the word "class" to cover not only plan, sex, age, and condition of health, but also the amount of insurance being purchased. With the background of such an extension of the definition of "class" an extension of the word "category" to allow differentials in n by age seems reasonable.

Mr. Bittel's discussion of the suggestion made by Mr. Hoskins that the ultimate interest rate be used for the calculation of paid-up nonforfeiture benefits brings out clearly the various objections from the standpoint of the Standard Nonforfeiture Law. Considerations of equity seem to me to lead to the same conclusions that Mr. Bittel reached. The use of the ultimate interest rate for calculation of paid-up nonforfeiture benefits would lead to a double forfeiture. First, there would be a forfeiture equal to the difference between the cash value and the reserve on the contract, and second, there would be a forfeiture of potential interest to be earned on the cash value. This could amount to considerably more than the first forfeiture if the nth year were remote.

Mr. Friedman presents an interesting modification on the thoery which should be quite useful in making a stop-gap use of varying interest when the use of the results of previously prepared asset shares seems advisable. Apparently, our "preconceived standards of tedium" differ.

Mr. Bittel points out in the last paragraph of his discussion an ambiguity in the section of the paper entitled "Control of Cash Values." The exact control is not possible unless reserves and cash values are calculated on different assumptions, but approximate control can be gained by the methods described with the same interest basis for both.

On several occasions since the delivery of the paper, we have heard of other companies adopting its methods. This seems to indicate the acceptance of these methods by the various state regulatory authorities.