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PAYMENT OF RESERVE IN ADDITION TO FACE AMOUNT

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THE problem of determining the level annual premium for a policy providing for payment of the reserve in addition to the face amount has long been part of the reading required of the actuarial student. The problem was first solved by Marshall (*RAIA* XXII, 216) and a similar treatment is given by Jordan (*Life Contingencies*, p. 120). These authors use what is, in essence, a difference-equation approach, taking as the starting point the formula connecting successive terminal reserves. The purpose of this note is to obtain a slightly more general result by a more direct route.[1]

In general, the net annual premium for a life insurance contract may for any year be analyzed into a one-year term premium for the net amount at risk in that year and a deposit into a savings fund. A basic characteristic of an insurance contract providing for payment of the reserve in addition to the face amount is that *the net amount at risk in every year is equal to the face amount.* Then, for purposes of actuarial analysis, such a contract may be considered as a combination of two contracts:

(1) A one-year renewable term insurance for the nominal face amount, on which, of course, no terminal reserves are accumulated.

(2) A savings fund, accumulating at compound interest, into which is deposited at the beginning of each year that portion of the net premium not needed to pay the cost of that year's insurance protection. Eventually it might become necessary to make withdrawals from the savings fund to pay a portion of the cost of each year's insurance protection.

Thus, the formulas to be developed will also indicate the financial results that can be achieved through a combination of a level amount of pure insurance and an investment program, when the total annual outlay is a level amount. Of course, in the usual type of life insurance contract, the amount of pure insurance decreases in such a way that the sum of the savings fund and the amount of pure insurance is a constant (the face amount of the policy). A somewhat similar analysis involving an increasing annual outlay has been given by Baillie (*TSA* VII, 382).

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[1] A similar approach has been used by Prof. C. J. Nesbitt in study notes distributed to students of actuarial mathematics at the University of Michigan.

We shall consider the general case of a policy on a life aged x which provides during the first n years for an annual net level premium π and a death benefit consisting of a nominal face amount F plus the terminal reserve. No assumption is made as to the premiums or benefits after n years, except that of course they must be such as to produce equality between the prospective and retrospective reserves at duration n. Since all terminal reserves under the insurance contract (1) are zero, the terminal reserve for the total policy is merely the accumulated amount in the savings fund (2) on the same date. Therefore, for any $k \leq n$,

$$
\begin{aligned}
{}_kV &= \sum_{t=0}^{k-1} (1+i)^{k-t} (\pi - F v q_{x+t}) \\
&= \pi \ddot{s}_{\overline{k}|} - F (1+i)^{x+k} (M'_x - M'_{x+k}) ,
\end{aligned}
\tag{1}
$$

where, as used by Jordan,

$$
M'_x = \sum_{z=x}^{\infty} v^{z+1} q_z .
$$

It will be noted that the net level premium π does not bear any necessary relationship to the nominal face amount F. Subject to the usual desire to have all terminal reserves positive, the premium may be either chosen arbitrarily or determined so as to produce a stipulated amount of reserve at some specified duration. Usually it would be determined so as to produce a stipulated reserve at the end of the nth year.

Thus, setting $k = n$ in equation (1) and solving for π gives

$$
\pi = \frac{{}_nV + F (1+i)^{x+n} (M'_x - M'_{x+n})}{\ddot{s}_{\overline{n}|}} . \tag{2}
$$

The first $n - 1$ reserves may then be computed from formula (1). If ${}_nV = {}_nV_x$ and $F = 1$, as in the specific example considered by Jordan (p. 120), we immediately obtain his result on substituting these values in formula (2). In Jordan's Exercise 32 (p. 125), we have, on equating prospective and retrospective reserves,

$$
A_{x+n} - \pi \ddot{a}_{x+n} = \pi \ddot{s}_{\overline{n}|} - (1+i)^{x+n} (M'_x - M'_{x+n}) ,
$$

which gives his answer on solving for π. The solution to Exercise 33 is obtained by setting ${}_{20}V = F = 1$.