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INSUFFICIENT PREMIUMS

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INTRODUCTION

THIS paper analyzes the situation which would arise in a system which provided benefits based on life contingencies and where all participants paid insufficient premiums. We define an insufficient premium as any premium less than the sufficient premium, and the sufficient premium as the one computed by the usual actuarial methods so as to equate the present value of the premiums to be paid by an individual to the present value of the benefits he is to receive, as of the date he enters the system. Of course no private insurer would charge insufficient premiums even if it were legal to do so. The subject is of theoretical interest, however, and many social insurance and private pension plans do actually operate with insufficient premiums in their early years.

As an introduction, a trivial but interesting example will be given. A man pays a certain premium and one year later receives a benefit of one dollar, mortality and expenses being ignored. On an actuarial basis it is simply a one-year savings plan with sufficient premium of $(1+i)^{-1}$. Now let N persons enter the system at the beginning of the first year, $N(1+r)$ enter at the beginning of the second year, $N(1+r)^2$ at the beginning of the third year, etc. Suppose they pay a premium of $(1+r)^{-1}$ instead of $(1+i)^{-1}$. Since the number paying premiums would be $(1+r)$ times the number receiving benefits, premium income would equal benefit payments at the beginning of every year after the first. Now if r were greater than i the premium of $(1+r)^{-1}$ would be an insufficient premium. As long as the number of new entrants increased at a rate r greater than i, assets would never become exhausted with the insufficient premium of $(1+r)^{-1}$. In fact, a useless reserve would be created arising from the premiums paid by the initial group. To prevent this the initial group of N persons could be excused from paying premiums and still receive the benefit.

Now let the number of new entrants at the beginning of every year be a constant. But let the amount of benefit increase in geometric progression being 1 the first year, $(1+r)$ the second year, $(1+r)^2$ the third year, etc. If a premium of $(1+r)^{-1}$ times the amount of benefit to be received is paid instead of the sufficient premium of $(1+i)^{-1}$ times the amount of benefit, premium income will equal benefit payments at the beginning of every year after the first. If r is greater than i the premium of $(1 + r)^{-1}$ per dollar of benefit is insufficient. Although it is probably not possible for the number of new entrants to increase indefinitely at a rate greater than the interest rate unless the number were constant ($r = 0$) with a negative interest rate, it is possible for the amount of benefit to increase indefinitely at a rate greater than the interest rate in a perpetual inflation.

THE GENERAL SYSTEM

We now develop the theory for a very general system of benefits. Members enter at age x and pay continuously a premium which varies continuously such that the annual rate at exact age $x + s$ is a_{x+s} times the initial annual rate (thus $a_x = 1$). The amount of any premium always hereafter means the initial annual rate thereof. Upon death at exact age $x + s$ a benefit of B_{x+s} is paid immediately. A life annuity is also paid continuously with annual rate b_{x+s} at age $x + s$. The sufficient premium is thus

$$\frac{\int_0^\infty v^s {}_sp_x (B_{x+s}\mu_{x+s} + b_{x+s})\, d s}{\int_0^\infty v^s {}_sp_x a_{x+s} d s}. \tag{1}$$

In practice b_{x+s} would be zero up to a certain age and a_{x+s} would become zero at or before the time b_{x+s} becomes greater than zero. For convenience, however, we admit zero values of B_{x+s}, b_{x+s}, and a_{x+s}, and write integrals from zero to infinity. Almost any scheme of benefits based on single-life contingencies which is or could be offered is at least closely approximated by a special case of the general system.

Unless specifically stated otherwise, the following assumptions are made. Members enter the system at age x at an annual rate of u_t at time t. All pay the same premium. No one withdraws from the system except by death. Mortality table and interest rate are known and remain the same indefinitely. There are no expenses.

THE FUNDAMENTAL RELATION

If an insufficient premium is charged, the system is bankrupt in the usual sense from the moment it begins. Such bankruptcy we call actuarial bankruptcy, but we are concerned with bankruptcy in the sense of being unable to meet current obligations. This would happen only when the fund became exhausted, where the fund at a given time is defined as the accumulated excess of all premiums paid in the past over all benefits paid in the past.

Let π_n be a premium such that the fund at time n is zero. Then the present value of premiums of π_n for the first n years equals the present value of benefits paid during the first n years. For the u_t new entrants at time t we count the present value of benefits and premiums up to time n, which is for the first $n - t$ years they are in the system. Thus we have

$$\pi_n = \frac{\int_0^n v^t u_t \int_0^{n-t} v^s {}_s p_x \left(B_{x+s}\mu_{x+s} + b_{x+s}\right) d\,s\,dt}{\int_0^n v^t u_t \int_0^{n-t} v^s {}_s p_x a_{x+s} d\,s\,dt}. \tag{2}$$

For specific functions B_{x+s}, b_{x+s}, a_{x+s}, and u_t, numerical values could be computed. Some examples are given later. We shall also use the following formula for π_n, which is obtained from (2) by changing the order of integration,

$$\pi_n = \frac{\int_0^n v^s {}_s p_x \left(B_{x+s}\mu_{x+s} + b_{x+s}\right) \int_0^{n-s} v^t u_t dt\,d\,s}{\int_0^n v^s {}_s p_x a_{x+s} \int_0^{n-s} v^t u_t dt\,d\,s}. \tag{3}$$

There is another approach to finding π_n. If the sufficient premium were paid, there would at the end of n years be a fund consisting of the total reserves for members then in the system. Let us distribute these reserves as an equal reduction in premium for all premiums paid in the first n years. Denoting the sufficient premium by P and the individual reserve at the end of $n - t$ years by ${}_{n-t}\mathrm{V}$,

$$\pi_n = \mathrm{P} - \frac{v^n \int_0^n u_t \cdot {}_{n-t}p_x \cdot {}_{n-t}\mathrm{V} dt}{\int_0^n v^t u_t \int_0^{n-t} v^s {}_s p_x a_{x+s} d\,s\,dt}. \tag{4}$$

This approach shows that π_n is less than the sufficient premium and is an insufficient premium if reserves are positive. Substituting for ${}_{n-t}\mathrm{V}$ with the retrospective formula

$$\frac{(1+i)^{n-t}}{{}_{n-t}p_x}\left[\mathrm{P}\int_0^{n-t} v^s {}_s p_x a_{x+s} d\,s - \int_0^{n-t} v^s {}_s p_x \left(B_{x+s}\mu_{x+s} + b_{x+s}\right) d\,s\right] \tag{5}$$

and simplifying, the equivalence of (2) and (4) is proved algebraically.

Under a premium of π_n it is possible, although very unlikely, that the system might continue to meet current obligations after time n. The fund might remain at zero or even increase after time n. Also it might become negative but then increase to at least zero at some future time, and until that time current obligations might be met by borrowing. The theory to provide for these unlikely possibilities is developed in a later section.

PREMIUM REQUIRED FOR PERPETUAL OPERATION

Consider the limit, π_∞, of π_n as n increases without limit. Under a premium of π_∞ the system could operate in perpetuity except in unusual cases when the fund became temporarily exhausted and current obligations could not be met by borrowing. Divide numerator and denominator of (3) by

$$\int_0^n v^t u_t dt .$$

The upper limit of integration with respect to s may be changed to $\omega - x$. Then

$$\pi_\infty = \frac{\int_0^{\omega - x} v^s {}_s p_x (B_{x+s}\mu_{x+s} + b_{x+s}) L_s d s}{\int_0^{\omega - x} v^s {}_s p_x a_{x+s} L_s d s}, \tag{6}$$

where

$$L_s = \lim_{n\to\infty} \left[\int_0^{n-s} v^t u_t dt \Big/ \int_0^n v^t u_t dt \right] .$$

If $\lim_{t\to\infty} v^t u_t = 0$, the difference between denominator and numerator of L_s approaches a limit of zero and $L_s = 1$. If $v^t u_t$ does not approach a limit of zero but does not increase without limit, both the denominator and the numerator of L_s increase without limit but the difference between them does not, and still $L_s = 1$. But if $v^t u_t$ increases without limit we have by l'Hospital's Rule, $L_s = (1 + i)^s \lim_{n\to\infty} (u_{n-s}/u_n)$ provided $\lim_{n\to\infty} (u_{n-s}/u_n)$ exists for s between 0 and $\omega - x$.

Substituting for L_s in (6) gives π_∞ equal to the sufficient premium if $v^t u_t$ does not increase without limit. If $v^t u_t$ increases without limit with $\lim_{n\to\infty} (u_{n-s}/u_n)$ existing,

$$\pi_\infty = \frac{\int_0^{\omega - x} [\lim_{n\to\infty} (u_{n-s}/u_n)]\, {}_s p_x (B_{x+s}\mu_{x+s} + b_{x+s})\, d s}{\int_0^{\omega - x} [\lim_{n\to\infty} (u_{n-s}/u_n)]\, {}_s p_x a_{x+s} d s}. \tag{7}$$

If the annual rate of increase in u_t is never greater than i after some finite future date, then $v^t u_t$ cannot increase without limit, and π_∞

is the sufficient premium. On the other hand, if the annual rate of increase in u_t approaches a limit greater than i, π_∞ is the insufficient premium equal to the sufficient premium at a special interest rate equal to the limit of the annual rate of increase in u_t. It is even possible for π_∞ to equal the sufficient premium at a special infinite interest rate and thus π_∞ could be very small and perhaps zero. This would happen if the annual rate of increase in u_t increased without limit as, for example, when $u_t = e^{t^2}$. The annual rate of increase in u_t means $(u_{t+1} - u_t)/u_t$.

NEW ENTRANTS IN GEOMETRIC PROGRESSION WITH NUMERICAL EXAMPLES

The formula for π_n may be simplified and numerical examples readily computed when u_t is in geometric progression. Let $u_t = (1 + r)^t$ and substitute in (3). Performing the integration with respect to t and simplifying,

$$\pi_n = \frac{\int_0^n v^s {}_sp_x (B_{x+s}\mu_{x+s} + b_{x+s})\,ds - v^n(1+r)^n \int_0^n (1+r)^{-s} {}_sp_x (B_{x+s}\mu_{x+s} + b_{x+s})\,ds}{\int_0^n v^s {}_sp_x a_{x+s}\,ds - v^n(1+r)^n \int_0^n (1+r)^{-s} {}_sp_x a_{x+s}\,ds} \quad \text{for } r \neq i\,,$$

$$= \frac{\int_0^n (n-s)\, v^s {}_sp_x (B_{x+s}\mu_{x+s} + b_{x+s})\,ds}{\int_0^n (n-s)\, v^s {}_sp_x a_{x+s}\,ds} \quad \text{for } r = i\,. \tag{8}$$

If $r \neq i$, π_n is the present value of benefits under an individual contract minus $v^n(1 + r)^n$ times the present value of benefits at the special interest rate r, all divided by the present value of a unit premium minus $v^n(1 + r)^n$ times the present value of a unit premium at the special interest rate r, benefits and premiums after n years being excluded. If $r = i$ the insufficient premium becomes the sufficient premium for a contract which differs from the actual one by the fact that benefits and premiums at age $x + s$ are multiplied by $n - s$ and there are no benefits or premiums after n years.

Table 1 gives numerical examples on a continuous basis for an ordinary life policy issued at age 20, and for a deferred life annuity beginning at 65 issued at age 20 with premium payable to 65 and no death benefit. An interest rate of 3% is used, and values are shown for $r = -1$

(no new entrants after the initial group), $r = 0$, $r = .03$, and $r = .06$. For the ordinary life policy the formula becomes

$$\frac{\bar{A}^{\,1}_{20:\overline{n}|} - v^n (1+r)^n \bar{A}^{1\,(r)}_{20:\overline{n}|}}{\bar{a}_{20:\overline{n}|} - v^n (1+r)^n \bar{a}^{(r)}_{20:\overline{n}|}} \quad \text{for } r \neq i \,,$$

$$\frac{(\bar{D}\bar{A})^{\,1}_{20:\overline{n}|}}{(\bar{D}\,\bar{a})_{20:\overline{n}|}} \quad \text{for } r = i \,. \qquad (9)$$

TABLE 1

INSUFFICIENT PREMIUMS WHICH PRODUCE BANKRUPTCY AT SELECTED TIMES AS PERCENTAGE OF SUFFICIENT PREMIUM

Years to Bankruptcy	Annual Percentage Increase in Number of New Entrants: −100%	0%	3%	6%
	Ordinary Life Issued at Age 20*			
10	16.79%	16.71%	16.69%	16.68%
25	23.70	19.35	18.99	18.65
50	66.11	40.82	35.68	30.57
75	99.62	73.25	58.88	43.70
100	100.00	88.81	71.61	49.60
150	100.00	97.66	82.47	53.16
250	100.00	99.88	90.06	54.13
500	100.00	100.00	95.23	54.18
1,000	100.00	100.00	97.67	54.18
∞	100.00	100.00	100.00	54.18
	Deferred Life Annuity Beginning at 65 Issued at Age 20†			
50	42.22%	5.23%	3.36%	1.93%
60	87.53	28.77	18.47	10.15
75	99.86	58.95	38.58	20.15
100	100.00	82.54	57.02	28.11
125	100.00	92.08	66.95	31.44
150	100.00	96.31	73.15	32.96
250	100.00	99.81	84.66	34.26
500	100.00	100.00	92.60	34.34
1,000	100.00	100.00	96.36	34.34
∞	100.00	100.00	100.00	34.34

* Premiums payable continuously and benefit payable at moment of death. Based on 1949–51 U.S. Life Table for white males and 3% interest. Sufficient premium is $10.19 per $1,000 of insurance.

† Annuity payable continuously and premiums payable continuously from 20 to 65 with no death benefit. Based on 1949–51 U.S. Life Table for white males and 3% interest. Sufficient premium is $75.87 per $1,000 of annual income.

For the deferred annuity the formula is

$$\frac{{}_{45|n-45}\bar{a}_{20} - v^n (1+r)^n{}_{45|n-45}\bar{a}^{(r)}_{20}}{\bar{a}_{20:\overline{45|}} - v^n (1+r)^n \bar{a}^{(r)}_{20:\overline{45|}}} \quad \text{for } r \neq i\,,$$

$$\frac{{}_{45}E_{20} (\bar{D}\bar{a})_{65:\overline{n-45|}}}{(n-45)\,\bar{a}_{20:\overline{45|}} + (\bar{D}\bar{a})_{20:\overline{45|}}} \quad \text{for } r = i\,, \tag{10}$$

except that π_n is zero for n less than 45. The superscript (r) indicates that a function is computed at the special interest rate r. The symbol $(\bar{D}\bar{A})^{1}_{x:\overline{n|}}$ denotes the present value of an insurance payable at moment of death with amount decreasing continuously and uniformly from n at age x to 0 at age $x+n$. $(\bar{D}\bar{a})_{x:\overline{n|}}$ denotes the present value of a continuous annuity with annual rate of payment decreasing continuously and uniformly from n at age x to 0 at age $x+n$. The continuous functions were computed on the assumption of a uniform distribution of deaths over each year of age.[1]

PAY-AS-YOU-GO COST

Let β_n be the annual rate at which benefits are paid out at time n, and γ_n times the premium be the annual rate at which premium income is being received at time n. Then,

$$\beta_n = \int_0^n u_{n-s} \cdot {}_sp_x \,(B_{x+s}\mu_{x+s} + b_{x+s})\, ds$$

$$\gamma_n = \int_0^n u_{n-s} \cdot {}_sp_x \, a_{x+s}\, ds\,. \tag{11}$$

If benefits payable at time n are provided entirely by contributions paid at that time with persons aged $x+s$ paying at an annual rate of $\phi_n a_{x+s}$,

[1] The formulas are as follows:

$$\bar{A}^{1}_{x:\overline{n|}} = \frac{i}{\delta} A^{1}_{x:\overline{n|}}$$

$$\bar{a}_{x:\overline{n|}} = \frac{\delta - d}{\delta^2}\ddot{a}_{x:\overline{n|}} + \frac{i-\delta}{\delta^2} a_{x:\overline{n|}}$$

$$(\bar{D}\bar{A})^{1}_{x:\overline{n|}} = \frac{i}{\delta}(DA)^{1}_{x:\overline{n|}} - \frac{i-\delta}{\delta^2} A^{1}_{x:\overline{n|}}$$

$$(\bar{D}\bar{a})_{x:\overline{n|}} = \frac{\delta - d}{\delta^2}(D\ddot{a})_{x:\overline{n|}} + \frac{i-\delta}{\delta^2}(Da)_{x:\overline{n|}} - \frac{2(\delta-d) - d\delta}{\delta^3}\ddot{a}_{x:\overline{n|}} - \frac{2(i-\delta) - \delta^2}{\delta^3} a_{x:\overline{n|}}\,.$$

then $\phi_n = \beta_n/\gamma_n$. Dividing numerator and denominator by u_n, this pay-as-you-go cost may be written as

$$\phi_n = \frac{\int_0^n (u_{n-s}/u_n)\ {}_sp_x\,(B_{x+s}\mu_{x+s} + b_{x+s})\,ds}{\int_0^n (u_{n-s}/u_n)\ {}_sp_x\,a_{x+s}ds}. \qquad (12)$$

The pay-as-you-go cost at any time n may thus be regarded as the sufficient premium, with no benefits or premiums payable after n years, computed at a special variable interest rate such that the interest discount factor for benefits and premiums payable at age $x + s$ is the ratio of the annual rate of new entrants at time $n - s$ to the annual rate of new entrants at time n. In other words, the interest rate for the first year is $(u_n - u_{n-1})/u_{n-1}$, for the second year $(u_{n-1} - u_{n-2})/u_{n-2}$, etc. If this rate of increase in u_t is greater than the actual interest rate of i for a sufficient part of the preceding n (or $\omega - x$, if less) years, the pay-as-you-go cost at time n is less than the sufficient premium.

If $u_t = (1 + r)^t$, $u_{n-s}/u_n = (1 + r)^{-s}$, and after $\omega - x$ years ϕ_n is the sufficient premium (1) at the special interest rate of r. When $r \geq i$ it follows from (7) that π_∞ is also the sufficient premium at the special interest rate r, and thus $\phi_n = \pi_\infty$ for $n \geq \omega - x$. When $r \geq i$, a premium of π_∞, although needed to prevent ultimate bankruptcy, would lead to the accumulation of a useless fund built up from the amounts by which π_∞ exceeded ϕ_n during the first $\omega - x$ years. To prevent this, the system could be operated on a pay-as-you-go basis for the first $\omega - x$ years, and this could be done in two ways. The premium could be reduced to the pay-as-you-go cost; or the premium could be kept at π_∞, and, at every age over x, $(1 + r)^{-s}{}_sp_xds$ persons of exact age $x + s$ could be admitted to the system at the time it begins.

TYPES OF BANKRUPTCY

We have called the usual sort of bankruptcy, which arises immediately if insufficient premiums are paid, actuarial bankruptcy. We now define a time of current bankruptcy under a given insufficient premium as a time at which the fund becomes negative but such that it would become at least equal to zero at some future time, and the time of total bankruptcy under a given insufficient premium as the time at which the fund becomes negative and would always remain negative thereafter. There could be more than one time of current bankruptcy each followed by a recovery, but only one time of total bankruptcy which definitely terminates the system.

Under a premium of π_{n_1} the fund at any time n is positive, zero, or negative according as π_n is less than, equal to, or greater than π_{n_1}. Thus a premium of π_{n_1} produces current bankruptcy at time n_1 if and only if there exists a time n_2 after n_1 such that $\pi_{n_2} = \pi_{n_1}$ and π_n is greater than π_{n_1} for all n between n_1 and n_2 (time n_2 would be the time of recovery). A premium of π_{n_1} produces total bankruptcy at time n_1 if and only if π_n is greater than π_{n_1} for all $n > n_1$. If π_n is a monotonic increasing function of n then, for all values of n, a premium of π_n produces total bankruptcy at time n.

We now consider the problem of determining under what conditions π_n always increases. For this purpose we write π_n in the form

$$\pi_n = \frac{\int_0^n v^y \beta_y dy}{\int_0^n v^y \gamma_y dy}, \tag{13}$$

where, as in (11), β_y is the annual rate of benefit payments and $\pi_n \gamma_y$ the annual rate of premium income at time y. The equivalence of (3) and (13) may be proved analytically by letting $t = y - s$ in (3), changing the order of integration, and substituting with (11). Since $\phi_n = \beta_n / \gamma_n$, we may write

$$\pi_n = \frac{\int_0^n v^y \gamma_y \phi_y dy}{\int_0^n v^y \gamma_y dy}. \tag{14}$$

This shows that π_n is a weighted average of all the pay-as-you-go costs of the first n years with the weight at time y being $v^y \gamma_y$. Thus π_n is a monotonic increasing function and all bankruptcies are total if and only if ϕ_n is always greater than π_n. This condition may be proved analytically, upon differentiating (14) with respect to n, from the inequality $\pi_n' > 0$.

From the interpretation of π_n as a weighted average of ϕ_y for $y < n$, it also follows that a sufficient condition for π_n to be monotonic increasing is that ϕ_n be nondecreasing and that it increase initially. In practice the pay-as-you-go cost almost always would increase for many years after the system began. From the interpretation of ϕ_n as a sufficient premium it follows that ϕ_n is nondecreasing if the rate of increase in u_t is nonincreasing provided, as would almost always be the case, a decrease or no change in the interest rate at all ages produces an increase or no change in the sufficient premium. In particular, if u_t is in geometric progression ϕ_n is nondecreasing and all bankruptcies are total.

If there were negative reserves another type of bankruptcy could be distinguished. The fund could become exhausted even when sufficient premiums were charged and the system was not actuarially bankrupt. This might be called nominal bankruptcy.

CHANGE IN BENEFIT AMOUNTS

It would probably not be possible for the number of new entrants to increase indefinitely at a rate greater than the interest rate, unless the latter were negative. In practice, under the assumptions we have made, any insufficient premium would eventually lead to total bankruptcy unless a negative interest rate or infinite population is assumed. We can, however, generalize the theory to cover cases when an actual system could operate forever with insufficient premiums. Let u_t be the annual rate at which new contract units are issued at time t to persons age x, where one contract unit provides a death benefit of B_{x+s}, payable immediately upon death at age $x + s$, a life annuity with annual rate of b_{x+s}, and premium payable continuously with annual rate at age $x + s$ of a_{x+s} times the initial annual rate. π_n is the initial annual premium rate per contract unit which would result in a fund of zero n years after the system begins. u_t is the product of the number of new entrants at time t and their average number of contract units (previously the average was taken as one). All the formulas are the same in this more general theory. With a constant price level it would not be possible for the average number of contract units to increase indefinitely. In a perpetual inflation where prices increased faster than the interest rate, however, the average number of contract units could likewise increase faster than the interest rate and the system could operate forever with insufficient premiums. But the insufficient premium would become sufficient if it and the benefits were expressed in units of constant purchasing power.

CHANGE IN SUFFICIENT PREMIUM

Hitherto we have assumed that the interest rate and mortality, and thus the sufficient premium, remain unchanged. Now assume that the interest rate and mortality change in a way that is known in advance so that the sufficient premium for new entrants at any future time is determined. We will assume that the ratio of insufficient premium to sufficient premium is the same for all new entrants. If the sufficient premium increases, it seems reasonable to think that the ratio of insufficient premium to sufficient premium which would produce bankruptcy at a given time would be less than it would be with a constant sufficient premium. We shall prove this in the case of a single-premium life annuity with new entrants at a constant rate.

The ratio of insufficient to sufficient premium which would produce bankruptcy in n years is

$$\frac{\int_0^n v^t \bar{a}^{(t)}_{x:\overline{n-t|}}\, dt}{\int_0^n v^t \bar{a}^{(t)}_x\, dt} \tag{15}$$

where v^t represents the present value of a unit payment at time t at an interest rate which may now vary, and the superscript (t) indicates that the function is computed with the interest and mortality to be experienced by a new entrant at time t. If the interest rate changes from a constant to a decreasing rate, the interest discount factor increases relatively more as duration increases, $\bar{a}^{(t)}_x$ increases relatively more than $\bar{a}^{(t)}_{x:\overline{n-t|}}$, and the ratio decreases. If mortality at every attained age changes from a constant to a decreasing rate, ${}_sp^{(t)}_x$ will increase relatively more as s increases, and again $\bar{a}^{(t)}_x$ will increase relatively more than $\bar{a}^{(t)}_{x:\overline{n-t|}}$, and the ratio decreases. Thus an increasing sufficient premium, resulting from either a decreasing interest rate or decreasing mortality, produces a lower ratio of insufficient to sufficient premium. If the interest rate is positive, the ratio approaches a limit of one as n increases without limit, since the integrands of the two integrals are equal except for the last $\omega - x$ values of t and the integrand approaches a limit of zero no matter how much mortality decreases.

With a positive interest rate the sufficient premium for an annuity must remain below the present value of a perpetuity. Also it seems unlikely that the interest rate would decrease or mortality increase so as to produce a very large increase in the sufficient premium for an insurance. Thus changes in the sufficient premium offer relatively limited possibilities for operating with insufficient premiums.

CONCLUSION

We have studied the relation between the time of bankruptcy and the amount of insufficient premium in a very general system of benefits. The number of new entrants is a major factor and the rate at which it increases may, in a sense, take the place of the interest rate. The paper is intended as a theoretical analysis. As to how an actual system should be financed, only the following will be stated. When participation is not compulsory, as in a private insurance company, the only sound method of financing is that which is actually followed—everyone pays a sufficient premium. In systems where participation is compulsory, insufficient premiums are usually paid in the early years, often because middle-aged people are included for whom the sufficient premium for any appreciable benefit would be prohibitively high. In planning for the financing of such systems it is suggested that the rate at which new entrants will come in, difficult as it is to predict, is a very important factor and should receive more consideration than it is often given.