DISCUSSION OF PRECEDING PAPER

CECIL J. NESBITT:

Time has not permitted me to make a detailed study of the author's paper, so my discussion will consist of general observations in relation to the paper. It is always a pleasure for a teacher to note the achievements of former students and particularly so when the accomplishment is a paper such as this with its display of skillful analysis. Also, I am very partial to continuous models for the investigation of actuarial problems, and am happy to see such a model in use here.

Occasionally, in the back of my mind, there have been problems similar to those studied in the paper. One meets such problems when considering a pension system which has a low rate of funding but a presumably permanent character. For such a system, one may take recourse to some modified form of "aggregate funding." In fact, in our course on pension fund mathematics, when discussing funding methods, as defined by C. L. Trowbridge in his paper "Fundamentals of Pension Funding," *TSA* IV, 17, we consider a whole scale of "modified aggregate funding" methods which, according to the adjustment factor used, will tend to yield almost any level of ultimate fund, from bankruptcy to an extremely large fund.

That brings me to the observation that the author in his paper has studied what I would call an "open immature group" which initially has no members and is built up from a continuous flow of new entrants at a fixed age of entrance x. Other possible groups are a "closed mature group" which has a distribution of members over the age range at the initial time but receives no new entrants in the future, and an "open mature group" which would be the combination of an open immature group and a closed mature group. The introduction of a mature membership at the initial time would lead to the usual problem of unfunded initial accrued liability, and as such would be more realistic, but perhaps the author was wise to leave this complication aside and to demonstrate what analysis can be made for the open immature situation. That he has been so successful in his case suggests that something might be accomplished in the other cases.

Somewhat related to the foregoing is the remark that the author's contribution rates appear to be determined in respect to *individuals* entering at age x, although there is an element of the aggregate or group approach in his determination of τ_n and ϕ_n. It would be interesting, though perhaps very difficult, to investigate aggregate funding for open immature groups, using some of the author's techniques for handling the flow of new entrants.

In his concluding remarks, the author indicates that in compulsory systems, insufficient premiums are usually paid in the early years. If, for future new entrants, one compares the required contribution rate with the entry age normal cost, one may find the required contribution rate to be *oversufficient*, and examples are not hard to find. There is then the problem of balancing the oversufficient premiums for future new entrants against the ins fficient premiums for initial members at the higher ages. This is the sort of problem one gets into when one considers an open mature group.

Our thanks are due to the author for an original and stimulating paper.

<center>(AUTHOR'S REVIEW OF DISCUSSION)</center>

<center>PAUL W. NOWLIN:</center>

It is true that the paper deals with an "open immature group." This would have been clearer if I had written the paper in terms of a private insurance company. In such terms, the paper is confined to the study of the relation, for a given policy issued at a given age, between the amount of insufficient premium and the time it takes after the policy is first issued for the fund arising therefrom to be exhausted.

I would be pleased if some of my work could be of use in the study of an "open mature group," since this would have practical value. The term "open mature group" might be misleading because, although the group has members at all ages at time 0, they should not constitute a stationary population but in practice would be relatively younger. To obtain such a group the function u_t could be extended to negative values of t, and at any time t after the beginning of the plan at time 0 there would be $u_{t-s} \cdot {}_s p_x ds$ members at exact age $x + s$. Preferably also, the member's benefits should be related to his average earnings, and average earnings should be a general function.

The great importance of the number of new entrants and average earnings may be seen in the following way. Suppose the unfunded accrued liability is to be paid for as a level percentage of all future payrolls. This cost would be infinitesimal in the extreme case of payroll increasing indefinitely at a rate greater than the interest rate (possible in a perpetual inflation). Thus in this case the "open mature group" is equivalent in cost to the "open immature group"—a fact which in a different, less general way I tried to show in the section "Pay-as-You-Go Cost" of my paper. On the other hand, the cost of the unfunded accrued liability would be very high in the case of a "closed mature group" with decreasing average earnings.