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A NEW ANNUITY MORTALITY TABLE
AND
A GRADED RATE SYSTEM FOR THE LIFE INCOME SETTLEMENT OPTIONS

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## INTRODUCTION

OF The many changes that have occurred in the life insurance field over the past quarter century, one of the most notable is the considerable growth in the annuity account. Annuity coverage, in the form of group, immediate and deferred annuities and life income settlements, has increased almost five times, ${ }^{1}$ both in number of contracts and in amount of benefit, far outstripping the percentage increase in life coverage during that period. It may be ascribed to a variety of causesattention focused on the need for such coverage by Federal Old-Age and Survivors Insurance, the tax incentive given to development of corporate retirement plans, the encouragement to individual insurance and annuity coverage programs from the same social security benefits and corporate retirement plans, etc. Whatever the cause, it seems apparent that there is now and will continue to be in future a growing demand for annuity coverage.

For the individual, the life insurance contract is an ideal vehicle for combining these two important risk coverages. A variety of plans, ranging from ordinary life to retirement endowment, permits varying the proportion of insurance and annuity coverage to fit the individual need, and at the same time assures the availability of annuity coverage at net rates for the policyholder's beneficiary.

It is not surprising then to find that the life income settlement options, which in the broader sense include deferred annuities with a death benefit prior to maturity and a cash value at maturity, have pre-empted the field of individual annuity coverage, leaving but a minor portion to immediate annuities. Table 1 shows the extent of this shift, as indicated by first year exposures in recent intercompany studies. ${ }^{2}$
${ }^{1}$ Institute of Life Insurance Fact Book, 1955, p. 30.
${ }^{2}$ TASA XLVIII, 133; TASA XLIX, 112; TSA 1951 Reports, 19; TSA 1954 Reports, 36.

TABLE 1
First Year Exposures

LIFE INCOME SETTLEMENTS AND DEFERRED ANNUITIES COMBINED

| $\begin{gathered} \text { Atralned } \\ \text { Ages } \end{gathered}$ | $1940-45$ <br> Experience |  | $1945-50$ <br> Experience |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Number of Contracts | Amount of Income | Number of Contracts | Amount of Income |
| Male |  |  |  |  |
| Under 60 | 3,523 | \$ 1,794,045 | 6,314 | \$ 3,710,499 |
| 60-69. | 9,780 | 5,271,435 | 20,079 | 11,725,309 |
| 70 and over. | 2,244 | 1,144,919 | 3,853 | 2,468,661 |
| All | 15,547 | \$ 8,210,399 | 30,246 | \$17,904,469 |
| Female 60 22 , 777 |  |  |  |  |
| Under 60 | 22,377 | \$ 9,781, 897 | 27,621 | \$13,237,750 |
| 60-69 | 29,741 | 9,614,516 | 40,276 | 15,249,465 |
| 70 and over . | 4,452 | 1,804,077 | 5,956 | 3,150,420 |
| All. | 56,570 | \$21,200,490 | 73,853 | \$31,637, 635 |
| Both sexes |  |  |  |  |
| Under 60 | 25,900 | \$11,575,942 | 33,935 | \$16,948, 249 |
| 60-69. | 39,521 | 14,885,951 | 60,355 | 26,974,774 |
| 70 and over. | 6,696 | 2,948,996 | 9,809 | 5,619,081 |
| All. | 72,117 | \$29,410,889 | 104,099 | \$49,542,104 |

INDIVIDUAL IMMEDIATE ANNUITIES

| $\begin{gathered} \text { Atrained } \\ \text { Ages } \end{gathered}$ | $\begin{gathered} \text { 1941-46 } \\ \text { ExpLRIENCE } \end{gathered}$ |  | $\begin{gathered} \text { 1948-53 } \\ \text { Experience } \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Number of Contracts | Amount of Income | Number of Contracts | Amount of Income |
| Male |  |  |  |  |
| $\begin{aligned} & \text { Under } 60 . . \\ & 60-69 . . . . \\ & 70 \text { and over. } \end{aligned}$ | 4,735 | \$ 1,412,794 | 1,617 | \$ 569,790 |
|  | 6,838 | 2,322,349 | 3,544 | 1,450,186 |
|  | 5,730 | 2,379,820 | 3,258 | 1,683,032 |
| All. | 17,303 | \$ 6,114,963 | 8,419 | \$ 3,703,008 |
| Female |  |  |  |  |
| Under 60 | 12,366 | \$ 2,864, 284 | 3,306 | \$ 1,194,724 |
| 60-69. | 18,526 | 4,107,566 | 7,219 | 2,242,625 |
| 70 and over. | 12,917 | 3,492,380 | 5,903 | 2,196,860 |
| All | 43,809 | \$10,464, 230 | 16,428 | \$ 5,634,209 |
| Both sexes |  |  |  |  |
| Under 60 | 17,101 | \$ 4, 277,078 | 4,923 | \$ 1,764,514 |
| 60-69 | 25,364 | 6,429,915 | 10,763 | 3,692,811 |
| 70 and over. | 18,647 | 5,872,200 | 9,161 | 3,879,892 |
| All | 61,112 | \$16,579,193 | 24,847 | \$ 9,337, 217 |

These figures are of course only indicative of the general trend, and the individual company can better measure the extent to which it has been affected from its own data. There can be no question, however, that a fundamental change has taken place in the Ordinary individual policy field. The life option provisions have become an indispensable part of that contract, and the net single premiums stipulated for the life options will have an increasingly important bearing on insurance costs.

When the life options were introduced at the beginning of this century, it was assumed they would be little more than a talking point for the agent. Apparently the low net single premium rates then used were not adopted in disregard of sound actuarial principle, but rather because of underestimating both the extent of use of the life options and the extent of the antiselection that would be exercised under them.

As stated in a 1924 discussion" of the subject, "selections by the beneficiary will probably always be a minor factor as compared with selection by the insured before death. For that reason the element of selection will probably never be as intense as it is in connection with immediate annuities." And further, "some feel that these annuity settlements are so seldom selected that it does not make much difference what table is used." As late as 1934,18 of the 25 then largest companies based their life option rates on the American Experience Table, and made no rate differential for sex.

The first report on mortality under the life options, ${ }^{4}$ which was published only 20 years ago in 1936, was undoubtedly hastened by the considerable increase in settlements that began with the depression in 1930. It showed, as did the 1941 report of the Joint Committee ${ }^{5}$ on the mortality under the life options, that mortality rates were considerably lighter than anticipated. Comparison in Table 2 of the mortality ratios taken from later reports of the Joint Committee shows that those for the life options are generally lower than for immediate annuities, and that there has been continuing and persistent decrease in the mortality rates of both groups.

The 1936 mortality report triggered fairly drastic increases in life option single premiums throughout the industry, and most companies have made two or three subsequent increases, in the main the result of adopting more conservative mortality assumptions and to a lesser degree lower interest assumptions. For the 25 largest U.S. companies, the last increase in life option single premium rates was made on an average of 8 years ago, and present rates are generally based on mortality and interest assumptions identical with or roughly equivalent to those being used for immediate annuities. Where that is the case, it means that the policy-

[^0]TABLE 2
Comparison of Experiences under Immediate annuities and Life income Settlement options by Number of Contracts

## Mortality Ratios on 1937 Standard annuity Table

| $\underset{\substack{\text { Attanned } \\ \text { Ages }}}{\substack{\text { and }}}$ | Male |  |  |  |  | Fimale |  |  |  |  | Both Sexis Combined |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ( $\begin{gathered}\text { lmm. } \\ \text { Ann. } \\ 1941-46 \\ \text { (1) }\end{gathered}$ | $\begin{gathered} \text { Life } \\ \text { Opt. } \\ 1940-45 \\ (2) \end{gathered}$ | $\begin{aligned} & \text { Imm. } \\ & \text { Ann. } \\ & 1946-48 \\ & (3) \end{aligned}$ | $\begin{gathered} \text { Life } \\ \text { Opt. } \\ 1945-50 \\ (4) \end{gathered}$ | $\begin{gathered} \text { Imm. } \\ \text { Ann. } \\ 1948-53 \end{gathered}$ (5) | $\begin{gathered} \text { Imm. } \\ \text { Ann- } \\ 1941-46 \\ (1) \end{gathered}$ | $\begin{gathered} \text { Life } \\ \text { Opt. } \\ 1940-45 \\ (2) \end{gathered}$ | $\begin{gathered} \text { Imm. } \\ \text { Ann. } \\ 1946-48 \end{gathered}$ <br> (3) | $\begin{gathered} \text { Life } \\ \text { Opt. } \\ 1945-50 \\ (4) \end{gathered}$ | $\underset{\substack{\text { Ann. } \\ 1948-53}}{\substack{\text { and }}}$ (5) | $\begin{gathered} \text { Imm. } \\ \text { Ann. } \\ \text { 1941-46 } \\ \text { (1) } \end{gathered}$ | $\begin{gathered} \text { Life } \\ \text { Opt. } \\ 1940-45 \end{gathered}$ (2) | $\begin{gathered} \text { Imm. } \\ \text { Ann. } \\ \text { 1946-48 } \end{gathered}$ (3) | $\left\lvert\, \begin{gathered} \mathrm{Life} \\ \text { Opt. } \\ 1945-50 \\ \text { (4) } \end{gathered}\right.$ | $\begin{gathered} \mathrm{Imm} . \\ \text { Ann. } \\ 1948-53 \\ (5) \end{gathered}$ |
|  | durations 1-5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Under 60. $60-69 \ldots$ $70-29$. 80 and over | $133 \%$ 81 91 110 | $83 \%$ 87 87 131 | $*$ $101 \%$ 88 68 | $66 \%$ 82 75 103 | $*$ $75 \%$ 79 67 | $50 \%$ 66 79 108 | $72 \%$ 63 80 110 | $69 \%$ 56 76 90 | $61 \%$ 63 74 90 | $*$ $53 \%$ 64 70 | $80 \%$ 70 83 109 | $74 \%$ 73 83 121 | $58 \%$ 69 80 80 | $62 \%$ 72 75 96 | $56 \%$ 60 70 69 |
| 80 and over | 94\% | 89\% | 83\% | $80 \%$ | $74 \%$ | 78\% | 70\% | 72\% | 66\% | 62\% | 84\% | 77\% | 76\% | 72\% | 67\% |
|  | all durations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Under $60 .$. $60-69 \ldots .$. $70-79 .$. 80 and over | $103 \%$ 96 98 113 | $86 \%$ <br> 87 <br> 91 <br> 125 | $101 \%$ <br> $101 \%$ <br> 98 <br> 109 | $71 \%$ <br> 83 <br> 93 <br> 98 | $74 \%$ <br> 73 <br> 98 <br> 95 <br> 9 | $61 \%$ 75 95 130 | $73 \%$ 68 86 120 | $83 \%$ <br> 69 <br> 92 <br> 126 | $66 \%$ <br> 66 <br> 87 <br> 121 | $58 \%$ <br> 66 <br> 90 <br> 116 | $75 \%$ <br> 81 <br> 96 <br> 124 | $75 \%$ <br> 74 <br> 88 <br> 122 | $90 \%$ <br> 79 <br> 94 <br> 121 | $67 \%$ <br> 72 <br> 89 <br> 112 | $64 \%$ <br> 68 <br> 92 <br> 110 |
| All. | 102\% | 92\% | 103\% | $87 \%$ | 93\% | 99\% | 76\% | 100\% | $77 \%$ | 99\% | 100\% | 82\% | 101\% | 81\% | 97\% |

TABLE 2-Continued
IMMEDIATE ANNUITIES VS. LIFE INCOME SETTLEMENTS

| $\begin{gathered} \text { Attained } \\ \text { Ages } \end{gathered}$ | Male |  |  |  |  | Female |  |  |  |  | Both Sexes Combined |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \mathrm{Imm} . \\ \text { Ann. } \\ 1941-46 \\ (1) \end{gathered}$ | Life Opt. 1940-45 $(2)$ | $\begin{gathered} \mathrm{Imm}_{1} \\ \mathrm{Ann}_{1} 94-48 \\ (3) \end{gathered}$ | $\begin{aligned} & \text { Life } \\ & \text { Opt. } \\ & 1945-50 \\ & (4) \end{aligned}$ | $\begin{gathered} \operatorname{Imm} . \\ \text { Ann. } \\ 1948-53 \\ (5) \end{gathered}$ | $\begin{gathered} \text { Imm. } \\ \text { Ann. } \\ 1941-46 \\ (1) \end{gathered}$ | $\begin{gathered} \text { Life } \\ \text { Opt. } \\ \text { 1940-45 } \\ \text { (2) } \end{gathered}$ | $\begin{gathered} \text { Imm. } \\ \text { Ann. } \\ \text { 1946-48 } \\ (3) \end{gathered}$ | $\begin{gathered} \text { Life } \\ \text { Opt. } \\ 1945-50 \\ (4) \end{gathered}$ | $\begin{gathered} \text { Imm. } \\ \text { Ann. } \\ \substack{948-53 \\ (5)} \end{gathered}$ | $\begin{gathered} \text { Imm, } \\ \text { Ann. } \\ 1941.46 \\ \text { (1) } \end{gathered}$ | $\begin{gathered} \text { Life } \\ \text { Opt. } \\ \substack{1940-45 \\ (2)} \end{gathered}$ | $\begin{gathered} \text { Imm. } \\ \text { Ann. } \\ 1946-48 \\ (3) \end{gathered}$ | Life Opt.- $1945-50$ <br> (4) |  |
|  | durations 1-5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { Under } 60 \ldots \\ & 60-69 . . . \\ & 70-79 . \\ & 80 \text { and over } \end{aligned}$ | $113 \%$ 97 97 108 | $100 \%$ 85 90 129 | $66 \%$ 97 91 103 | $72 \%$ 83 37 113 | $93 \%$ 92 92 86 | $66 \%$ 71 89 121 | $73 \%$ 69 92 117 | $73 \%$ 67 82 120 | $61 \%$ 64 82 103 | $48 \%$ 71 74 96 | $82 \%$ <br> 80 <br> 22 <br> 116 | $76 \%$ 74 91 122 | $70 \%$ 78 85 113 | 63\% 72 80 106 | $60 \%$ 79 81 91 |
| All. . . | 101\% | 90\% | 94\% | 81\% | 90\% | 88\% | 76\% | 84\% | 69\% | 77\% | 92\% | 80\% | $88 \%$ | $73 \%$ | 82\% |
|  | all durations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { Under } 60 . . \\ & 60-69 \ldots . . . \\ & 70-79 \\ & 80 \text { and over } \end{aligned}$ | 109\% <br> 105 <br> 103 <br> 116 <br> 10 | $102 \%$ 87 94 125 | 95\% 101 105 111 | $86 \%$ 84 93 99 | $92 \%$ 94 105 101 | $77 \%$ 77 98 132 | $75 \%$ <br> 75 <br> 98 <br> 136 | $77 \%$ <br> 78 <br> 97 <br> 129 | $66 \%$ <br> 69 <br> 94 <br> 128 | $73 \%$ 76 94 123 | $88 \%$ 80 100 126 | $78 \%$ 78 97 133 | $83 \%$ 85 99 123 | $69 \%$ <br> 73 <br> 93 <br> 120 | $81 \%$ 82 97 116 |
| All | 107\% | 95\% | 106\% | 89\% | 101\% | 102\% | 87\% | 103\% | 82\% | 103\% | 104\% | 89\% | 104\% | $84 \%$ | 103\% |

* Less than 25 contracts terminated by death.

Note.-Figures in italics indicate less than 100 but not less than 25 contracts terminated by death.
(1) Intercompany immediate annuity experience between 1941 and 1946 anniver2) saries; TASA XLVIII, 133.
2) Intercompany life option settlement experience between 1940 and 1945 anniversaries; TASA XLIX, 112.
(3) Intercompany immediate annuity experience between 1946 and 1948 anniversaries; TSA 1, 606.
(4) Intercompany'life option settlement experience between 1945 and 1950 annive -
saries; TSA 1951 Reports, 19.
(5) Intercompany immediate annuity experience between 1948 and 1953 anniversaries; TSA 1954 Reports, 36.
holder or his beneficiary has a contract right to purchase a life annuity at such future time as the policy matures by death, endowment or surrender, at premium rates comparable to those now currently charged for immediate annuities, less loading.

Failure to allow for future decrease in mortality rates over the period between policy issue and settlement, which may run to 50 years or more and has averaged around 25 years under recent Northwestern Mutual Life option settlements, "can involve and on occasion has resulted in large losses," as pointed out by Jenkins and Lew. ${ }^{6}$ Losses must, of course, be covered out of current or past surplus gains that would otherwise be available for dividends and contingency reserves. If they are charged against general surplus, the earlier series policies with the more favorable option rates will profit at the expense of the later series policies.

This inequity can be corrected under participating contracts by assessing the loss against the particular policy series from which it arose. However, such adjustment is at best crude, and does nothing to correct the inequity as between the two groups within each policy series-those who use the options and those who do not. Further, it should be recognized that to the extent use of the life options is increased and the proportion of policies under which they are elected becomes larger, both the charge against insurance gains and the distortion of equity as between users and nonusers of the life options will be correspondingly increased.

To put the life options in currently issued policies on a self-sustaining basis and avoid their becoming a lien against insurance gains requires that adequate allowance be made for future decrease in mortality rates. Under the flat rate system now generally used, with a single scale of rates to apply for all years of entry, the mortality assumption must be geared to the period of exposure for the average case. Starting with a mortality table based on current experience with a margin of safety for fluctuations, and adjusting it for the change in mortality expected over the average period between policy issue and settlement, a single scale of rates can be developed such that the expected loss from settlements that are made a longer-than-average time after issue will be balanced by gain from early settlements. This approach is described in detail in Mr. Harry Walker's paper, TSA VI, 85.

As pointed out by Mr. Hoskins ${ }^{7}$ in discussion of Mr. Walker's paper, the obvious flaw in this approach is that if net single premiums are fixed at a high enough level so that the "profit" from early settlements will offset the losses on later settlements, the life option net rates would probably equal or exceed the gross single premium rates currently quoted for non-
${ }^{5}$ TSA I, 369.
${ }^{7}$ Ibid., 546.
participating immediate annuities. This redundancy would be not only difficult to explain to the layman, but could tend to defeat its purpose, in that it would discourage life option settlements at early policy durations and hence reduce the anticipated gain from such settlements.

Regardless of the level at which any static scale of life option rates is established, however, such rates will be inequitable as between individual beneficiaries of the same sex and age who settle at different times, since the benefit value will vary with the mortality level, while the net single premium remains stationary. The spread in value of a given amount of income for beneficiaries of the same age and sex can be considerable over the 50 years or more between dates of the first and last settlement from a single year's issues.

The foregoing points up the fact that the flat rate system presently used is in itself a prime source of distortion of equity in the life option rates. To obtain rates that are both self-sustaining and equitable requires not only that adequate allowance be made for future improvement in mortality, but also that net single premiums be correlated with such mortality change so that they will approximate expected benefit values for each age and sex throughout the possible settlement period.

The practical problem to which this paper is directed is the development of a system that is detailed enough to do substantial equity, yet is simple enough so that it can be readily understood by the policyholder. A new mortality table and mortality projection basis are also offered for use with the graded rate system, although they are not an essential part thereof.

Grateful acknowledgment is due the Committee on Mortality under Ordinary Insurances and Annuities for making available the data on the 1948-53 Individual Immediate Annuity experience, and to my associates, William T. Chambers and Clair A. Lewis, for their able assistance.

## MORTALITY TABLE

The principal item of interest in the Joint Committee's report on the mortality experienced under individual immediate annuities between 1948 and 1953 anniversaries ${ }^{8}$ is the extent of the improvement in mortality at ages 80 and over, where, as shown in Table 3, mortality ratios on the Annuity Table for 1949 fell below $100 \%$ at a number of points.

With a current average entry age for life option settlements and individual annuities of around 60 for females and 65 for males, and the definite probability that these average ages will increase as longevity increases, the mortality rates assumed for the higher ages can have a considerable

[^1]- TABLE 3

COMPARISON OF 1941-46 Experience with 1948-53 Experience under Individual Immediate AnNuities at Attained Ages 80 and over by Number of Contracts

| Attained Ages | 1941-46 Experience |  | 1948-53 Experience |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Actual Deaths | Mortality Ratio Based on 1937 Standard Annuity Table | Actual <br> Deaths | Mortality Ratio Based on |  |
|  |  |  |  | 1937 Stand- <br> ard Annuity <br> Table | Annuity Table for 1949 |
|  | nonrefund-male |  |  |  |  |
| $\begin{aligned} & 80-84 \ldots \ldots \\ & 85-89 \ldots \ldots \\ & 90 \text { and over. } \end{aligned}$ | 802 | 108\% | 1,091 | 90\% | 89\% |
|  | 488 | 122 | 675 | 103 | 93 |
|  | 187 | 123 | 368 | 111 | 90 |
| 80 and over. | 1,477 | 114\% | 2,134 | 97\% | 91\% |
|  | nonrefund--pemale |  |  |  |  |
| $\begin{aligned} & 80-84 \ldots \ldots \\ & 85-89 \ldots \ldots \\ & 90 \text { and over. } \end{aligned}$ | 1,816 | 127\% | 3,058 | 109\% | 100\% |
|  | 1,017 | 140 | 1,954 | 123 | 97 |
|  | 297 | 148 | 972 | 142 | 95 |
| 80 and over. | 3,130 | 133\% | 5,984 | 118\% | 98\% |
|  | refund and monrefund combined-male |  |  |  |  |
| $\begin{aligned} & 80-84 \ldots \ldots \\ & 85-89 \ldots . . . \\ & 90 \text { and over } \end{aligned}$ | $\begin{aligned} & 2,208 \\ & 1,252 \\ & 398 \end{aligned}$ | $\begin{aligned} & 113 \% \\ & 134 \\ & 115 \end{aligned}$ | 3,0461,869 | 100\% | $98 \%$94 |
|  |  |  |  |  |  |
|  |  |  | 1,833 | 102 |  |
| 80 and over | 3,858 | 119\% | 5,748 | 102\% | 95\% |
|  | refund and nonrerund congined-fzuale |  |  |  |  |
| $\begin{aligned} & 80-84 \ldots . . \\ & 85-89 \ldots . . \\ & 90 \text { and over. } \end{aligned}$ | $\begin{array}{r} 4,630 \\ 2,774 \\ 815 \end{array}$ | 127\% | 8,086 | 115\% |  |
|  |  | 148 | 5,239 | 130 | 102 |
|  |  | 143 | 2,621 | 148 | 98 |
| 80 and over | 8,219 | 135\% | 15,946 | 124\% | 103\% |

effect on premiums and should hence be more conservative than those in the Annuity Table for 1949. A new annuity mortality table that will provide a cushion against mortality fluctuations throughout seems indicated for use under all forms of life annuities.

The intercompany 1948-53 immediate annuity experience is not only the most recent large-scale experience available on which to base a new table, but with over 22,000 deaths at ages 80 and up it furnishes much more extensive information than has hitherto been available on annuitant mortality here at the extreme upper ages. The experience by number of contracts, rather than by amounts of annual income, was used as the basis of the Proposed 1955 American Annuity Table. The experience rates under the former not only were more stable, but also were generally lower than those by amounts.

Rough graded experience rates were developed for both sexes for ages 60 and over from the nonrefund annuity experience, excluding the first two contract years. Since some extension in the limiting age beyond 110 as used for both the 1937 Standard Annuity Table and the Annuity Table for 1949 seemed indicated by the recent improvement in mortality at the higher ages, it was arbitrarily increased 5 years to age 115.

After some study of these experience rates and of the derived annuity values, the question whether to develop separate tables for each sex or a dual reference table with constant age adjustment for sex was resolved in favor of the latter. The desirability, if not necessity, of using a single table under a graded rate system is obvious, to keep the multiplicity of values at a minimum. Using the male table as a base, with 5 year setback in age for females, the effect is to understate the mortality rates for females at the older ages and overstate those at the younger ages. The result in terms of annuity net single premiums is to overstate female values by a percentage that increases with increase in age.

The female experience table on which the comparative values in Tables 6 and 7 are based was constructed for the purpose by combining the graded female nonrefund annuity experience rates for ages 60 and above with the Annuity Table for 1949 female rates for ages below 60 ; these were smoothed graphically, and adjusted so as to give a table roughly equivalent in conservatism to the Proposed 1955 American Annuity Table for males.

The overstatement in value of the single life annuity option with 10 year certain period, which is the type used in the large majority of life option settlements, increases from $0.1 \%$ at age 20 to $1 \%$ at age 55 and reaches a maximum of $6.1 \%$ at age 80 . In view of the higher rate of decrease in the mortality for the U.S. white female population in recent


Comparison of Mortality Rates-Males

-     - . - 1937 Standard Annuity Table
-     -         - Annuity Table for 1949
———Proposed 1955 American Annuity Table


Comparison of Mortality Rates--Females
--- - - 1937 Standard Annuity Table

- . - Annuity Table for 1949
———Proposed 1955 American Annuity Table
years, and the probable increase in proportion of payee-elected settlements of death claims (over $95 \%$ of which are on female lives), this added safety margin for fluctuation in female mortality does not seem unreasonable.

TABLE 4
Comparison of Mortality Rates Proposed 1955 American annuity Table, Annuity Table for 1949 Ultimate, and 1937 Standard Annuity Table

| $\underset{x}{\operatorname{Age}}$ | Males |  |  | Females |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Proposed 1955 American Annuity Table | Annuity Table for 1949 Ultimate | 1937 Standard Annuity Table | Proposed 1955 American Annuity Table | Annuity Table for 1949 Ultimate | 1937 Standard Annuity Table |
|  | 1,000 $q_{x}$ |  |  |  |  |  |
| 10. | . 420 | . 483 | 1.257 | . 370 | . 191 | 1.234 |
| 20. | . 553 | . 624 | 1.331 | . 475 | . 376 | 1.262 |
| 30. | . 893 | 1.004 | 2.065 | . 679 | . 677 | 1.561 |
| 40. | 1.925 | 2.025 | 4.356 | 1.273 | 1.355 | 2.981 |
| 50. | 4.550 | 6.557 | 9.288 | 2.975 | 3.109 | 6.362 |
| 60. | 12.027 | 15.662 | 19.753 | 7.073 | 7.504 | 13.554 |
| 70. | 32.367 | 35.092 | 41.758 | 20.192 | 20.964 | 28.751 |
| 80 | 76.847 | 85.503 | 87.161 | 50.417 | 61.415 | 60.464 |
| 90. | 169.202 | 208.485 | 177.138 | 114.977 | 176.161 | 124.837 |
| 100. | 342.402 | 463.415 | 362.122 | 243.947 | 449.400 | 248.059 |

After considerable experimentation, Makeham's formula was selected as the most suitable for graduation of the male rates for ages 60 and over, using the following constants for colog $10 p_{x}=A+B c^{x}$ :

| 1,000 A | -1.93615 |
| :---: | :---: |
| 1,000 B | 0.054435 |
| $\log _{10} c$. | 0.0353435 |

The rates above age 109 as derived by formula were modified so as to give an orderly transition to the rate of unity at age 114.

Lacking any up-to-date group annuity mortality data on which to base rates at ages under 60, the male rates in the Annuity Table for 1949 for ages 10 to 40 were taken as a starting point, and the above formula for colog ${ }_{10} p_{x}$ was used with $A$ modified as follows:

| Ages | $1,000 A$ for Age $x$ |
| :---: | :---: |
| $5-35$ | $-1.93615+1.5526-.0304(x-35)-.0005(x-35)^{2}$ |
| $35-50$ | $-1.93615+.7321-.08665(x-50)-.00264(x-50)^{2}$ |
| $50-60$ | $-1.93615+.013298(x-60)^{2}+.0005977(x-60)^{3}$ |

As a check on the reasonableness of these rates below age 50 , they were compared with the 1948-53 mortality experience of the Northwestern Mutual under medically examined Ordinary insurance, excluding war deaths, by number of policies (Table 5).

Joint life annuities for two lives at equal ages, and a table of uniform seniority, are given in the last section of the paper.

TABLE 5
Aggregate Experience on Ordinary Insurance between Anniversaries in 1948 and 1953

Excluding War Deaths

| Attained Age Group | Ratios of Actual to Expected on Proposed 1955 American Annuity Table | Attained Age Group | Ratios of Actual to Expected on Proposed 1955 American Annuity Table |
| :---: | :---: | :---: | :---: |
| 13-17. | 157\% | 33-37. | 93\% |
| 18-22. | 170 | 38-42. | 103 |
| 23-27. | 124 | 43-47. | 112 |
| 28-32. | 98 | 48-52. | 129 |

GRADED RATES
The twin objectives of the proposed graded rate system are understandability and equity. Optimum realization of either objective requires, in the author's opinion, that life option rates be correlated as closely as possible with the anticipated future decrease in mortality, which for all practical purposes may be considered to be continuous. A discontinuous gradation such as a step-rate system, with rates changing periodically by substantial amounts, would not only distort equities but because of its arbitrariness might well impair policyholders' acceptance and understanding. Accordingly, rates in the system described herein are varied yearly, i.e., are based on the year of settlement as well as the sex and age.

It is equally as important that the policy rate table take a minimum of space, yet permit calculation of exact rates for any age, sex and year of settlement by simple arithmetic, without interpolation or rounding. This is accomplished by stating the rates in terms of the net single premium value of $\$ 10$ monthly income for any particular year of settlement that is

TABLE 6
Comparison of Annuity Values-Females
Immediate Nonrefund Annuities, $2 \frac{1}{2} \%$ Interest

| Age | Proposed 1955 American Annuity Table | Female Experience Table | Ratio of Value on Proposed 1955 American Annuity Table to Value on Female Experience Table |
| :---: | :---: | :---: | :---: |
| 15. | 31.453 | 31.444 | 100.0\% |
| 20. | 30.403 | 30.369 | 100.1 |
| 25. | 29.223 | 29.167 | 100.2 |
| 30. | 27.900 | 27.826 | 100.3 |
| 35. | 26.420 | 26.332 | 100.3 |
| 40. | 24.778 | 24.676 | 100.4 |
| 45. | 22.972 | 22.852 | 100.5 |
| 50. | 21.010 | 20.861 | 100.7 |
| 55. | 18.900 | 18.709 | 101.0 |
| 60. | 16.653 | 16.409 | 101.5 |
| 65. | 14.330 | 14.005 | 102.3 |
| 70. | 12.048 | 11.569 | 104.1 |
| 75. | 9.883 | 9.197 | 107.5 |
| 80. | 7.890 | 7.005 | 112.6 |
| 85. | 6.117 | 5.122 | 119.4 |

TABLE 7
Comparison of Annuity Values--Females
Life income Setilement Options with 10 Year Certain Period, $2 \frac{1}{2} \%$ Interest Single Life, First Payment Immediate

| Age | Proposed 1955 American Annuity Table | Female Experience Table | Ratio of Value on Proposed 1955 American Annuity Table to Value on Female Experience Table |
| :---: | :---: | :---: | :---: |
| 15. | 32.470 | 32.455 | 100.0\% |
| 20. | 31.423 | 31.385 | 100.1 |
| 25. | 30.247 | 30.188 | 100.2 |
| 30. | 28.930 | 28.855 | 100.3 |
| 35. | 27.462 | 27.373 | 100.3 |
| 40. | 25.840 | 25.736 | 100.4 |
| 45. | 24.067 | 23.942 | 100.5 |
| 50. | 22.154 | 21.998 | 100.7 |
| 55. | 20.121 | 19.921 | 101.0 |
| 60. | 18.012 | 17.748 | 101.5 |
| 65. | 15.927 | 15.561 | 102.3 |
| 70. | 13.993 | 13.487 | 103.8 |
| 75. | 12.311 | 11.690 | 105.3 |
| 80. | 10.958 | 10.330 | 106.1 |
| 85. | 9.986 | 9.479 | 105.3 |

TABLE 8
Intercompany Experience under Individual Immediate annuities
between 1948 and 1953 Anniversaries, Issues of 1931 to 1952
Mortality Ratios on (1) Proposed 1955 american Annuity Table
(2) Annuity table for 1949
(3) 1937 Standard Annuity Table


[^2]TABLE 8-Continued


* Less than 10 contracts terminated by death.

Note,-Mortality ratios in italics where less than 50 but not less than 10 contracts terminated by death.

TABLE 9
Life Income Settlements
Aggregate Experience of Northwestern Mutual
Life Insurance Company
between 1950 and 1955 AnNiversaries

| $\begin{gathered} \text { Attained } \\ \text { Ages } \end{gathered}$ | Actual deaths |  | Mortality Ratios Based on |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Proposed 1955 American Annuity Table |  | Annuity Table for 1949 |  | 1937 Standard Annuity Table |  |
|  | Number of Contracts | Amounts of Annual Income | Number of Contracts | Amounts of Annual Income | Number of Contracts |  | Number of Contracts | Amounts of Annual Income |
| Under 60 <br> 60-69 <br> 70-79 <br> 80 and over.. <br> All Ages. | male-payee elections |  |  |  |  |  |  |  |
|  | 25 | 12,643 | 171\% | 146\% | 121\% | 104\% | 93\% | 79\% |
|  | 198 | 97,564 | 96 | 75 | 85 | 66 | 69 | 54 |
|  | 253 | 131,365 | 98 | 90 | 91 | 83 | 81 | 74 |
|  | 116 | 53,435 | 110 | 125 | 95 | 109 | 100 | 114 |
|  | 592 | 295,007 | 102\% | 90\% | 90\% | 80\% | 80\% | 70\% |
|  | female-payee elections |  |  |  |  |  |  |  |
| Under 60 <br> 60-69 <br> 70-79 <br> 80 and over. <br> All Ages. | 45 | 21,778 | 126\% | 102\% | 120\% | 98\% | 61\% | 50\% |
|  | 168 | 113,367 | 96 |  | 93 | 104 |  | 66 |
|  | 294 | 179,307 | 105 | 106 | 95 | 96 | 80 | 81 |
|  | 224 | 150,486 | 150 | 163 | 113 | 124 | 130 | 141 |
|  | 731 | 464,938 | 114\% | 120\% | 101\% | 106\% | 82\% | 86\% |
|  | male-all elections |  |  |  |  |  |  |  |
| Under 60 . <br> 60-69 <br> 70-79. <br> 80 and over. | 29 | 18,080 |  |  |  |  |  | 100\% |
|  | 202 | 99,261 | 97 | 76 | 85 | 67 | 69 | 54 |
|  | 255 | 132,259 | 98 | 90 | 91 | 83 | 81 | 73 |
|  | 124 | 56,312 | 113 | 127 | 98 | 111 | 103 | 115 |
| All Ages. . | 610 | 305,912 | 103\% | $92 \%$ | $91 \%$ | 82\% | 80\% | 71\% |
|  | female-all elections |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { Under } 60 \ldots . \\ & 60-69 \ldots \\ & 70-79 \ldots \ldots \\ & 80 \text { and over. } \end{aligned}$ | 74 | 39,294 | 140\% | 124\% | $134 \%$ | 118\% | 68\% | 60\% |
|  | 217 | 146,772 | 98 | 106 | 94 | 102 | 60 | 65 |
|  | 367 | 236,233 | 103 | 103 | 94 | 93 | 79 | 78 |
|  | 310 | 217,736 | 148 | 153 | 110 | 114 | 128 | 133 |
| All Ages. . | 968 | 640,035 | 115\% | 118\% | 101\% | 103\% | 82\% | 85\% |

TABLE 10
Proposed 1955 American annuity table
Elementary Functions and annuity Values

| Age |  | $l_{x}$ | $d_{x}$ | $1,000 q_{x}$ | $\begin{gathered} a_{x} \text { AT } 2 f \% \\ \text { INTEREST } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Male | Female |  |  |  |  |
| 5 | 10 | 1,000.0000 | . 3700 | . 370 | 32.386 |
| 6 | 11 | 999.6300 | . 3799 | . 380 | 32.208 |
| 7 | 12 | 999.2501 | . 3897 | . 390 | 32.026 |
| 8 | 13 | 998.8604 | . 3995 | . 400 | 31.839 |
| 9 | 14 | 998.4609 | 4094 | . 410 | 31.648 |
| 10 | 15 | 998.0515 | 4192 | . 420 | 31.453 |
| 11 | 16 | 997.6323 | . 4290 | . 430 | 31.253 |
| 12 | 17 | 997.2033 | . 4398 | . 441 | 31.048 |
| 13 | 18 | 996.7635 | . 4505 | . 452 | 30.838 |
| 14 | 19 | 996.3130 | . 4613 | . 463 | 30.623 |
| 15 | 20 | 995.8517 | .4730 | . 475 | 30.403 |
| 16 | 21 | 995.3787 | . 4857 | . 488 | 30.178 |
| 17 | 22 | 994.8930 | 4994 | . 502 | 29.948 |
| 18 | 23 | 994.3936 | 5141 | 517 | 29.712 |
| 19 | 24 | 993.8795 | . 5307 | . 534 | 29.470 |
| 20 | 25 | 993.3488 | . 5493 | . 553 | 29.223 |
| 21 | 26 | 992.7995 | . 5699 | . 574 | 28.970 |
| 22 | 27 | 992.2296 | . 5924 | . 597 | 28.712 |
| 23 | 28 | 991.6372 | . 6168 | . 622 | 28.447 |
| 24 | 29 | 991.0204 | . 6432 | . 649 | 28.176 |
| 25 | 30 | 990.3772 | 6725 | 679 | 27.900 |
| 26 | 31 | 989.7047 | 7047 | . 712 | 27.617 |
| 27 | 32 | 989.0000 | . 7408 | . 749 | 27.327 |
| 28 | 33 | 988.2592 | . 7817 | . 791 | 27.031 |
| 29 | 34 | 987.4775 | . 8285 | . 839 | 26.729 |
| 30 | 35 | 986.6490 | . 8811 | . 893 | 26.420 |
| 31 | 36 | 985.7679 | . 9394 | . 953 | 26.105 |
| 32 | 37 | 984.8285 | 1.0045 | 1.020 | 25.783 |
| 33 | 38 | 983.8240 | 1.0773 | 1.095 | 25.455 |
| 34 | 39 | 982.7467 | 1.1587 | 1.179 | 25.120 |
| 35 | 40 | 981.5880 | 1.2496 | 1.273 | 24.778 |
| 36 | 41 | 980.3384 | 1.3509 | 1.378 | 24.430 |
| 37 | 42 | 978.9875 | 1.4636 | 1.495 | 24.075 |
| 38 | 43 | 977.5239 | 1.5875 | 1.624 | 23.714 |
| 39 | 44 | 975.9364 | 1.7245 | 1.767 | 23.346 |
| 40 | 45 | 974.2119 | 1.8754 | 1.925 | 22.972 |
| 41 | 46 | 972.3365 | 2.0409 | 2.099 | 22.592 |
| 42 | 47 | 970.2956 | 2.2220 | 2.290 | 22.206 |
| 43 | 48 | 968.0736 | 2.4192 | 2.499 | 21.813 |
| 44 | 49 | 965.6544 | 2.6333 | 2.727 | 21.414 |
| 45 | 50 | 963.0211 | 2.8650 | 2.975 | 21.010 |
| 46 | 51 | 960.1561 | 3.1147 | 3.244 | 20.599 |
| 47 | 52 | 957.0414 | 3.3831 | 3.535 | 20.183 |
| 48 | 53 | 953.6583 | 3.6706 | 3.849 | 19.761 |
| 49 | 54 | 949.9877 | 3.9776 | 4.187 | 19.333 |
| 50 | 55 | 946.0101 | 4.3043 | 4.550 | 18.900 |
| 51 | 56 | 941.7058 | 4.6549 | 4.943 | 18.461 |
| 32 | 57 | 937.0509 | 5.0395 | 5.378 | 18.016 |
| 53 | 58 | 932.0114 | 5.4690 | 5.868 | 17.567 |
| 54 | 59 | 926.5424 | 5.9558 | 6.428 | 17.112 |
| 55 | 60 | 920.5866 | 6.5113 | 7.073 | 16.653 |
| 56 | 61 | 914.0753 | 7.1462 | 7.818 | 16.191 |
| 57 | 62 | 906.9291 | 7.8703 | 8.678 | 15.727 |
| 58 | 63 | 899.0588 | 8.6903 | 9.666 | 15.261 |
| 59 | 64 | 890.3685 | 9.6000 | 10.782 | 14.795 |

TABLE 10-Continued

| $\underset{x}{A_{G E}}$ |  | $l_{x}$ | $d_{x}$ | $1,000 g x$ | $a_{x} \text { at } 2 \frac{1}{2} \%$ <br> INTEREST |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Male | Female |  |  |  |  |
| 60 | 65 | 880.7685 | 10.5930 | 12.027 | 14.330 |
| 61 | 66 | 870.1755 | 11.6612 | 13.401 | 13.868 |
| 62 | 67 | 858.5143 | 12.7927 | 14.901 | 13.407 |
| 63 | 68 | 845.7216 | 13.9764 | 16.526 | 12.950 |
| 64 | 69 | 831.7452 | 15.2093 | 18.286 | 12.497 |
| 65 | 70 | 816.5359 | 16.4875 | 20.192 | 12.048 |
| 66 | 71 | 800.0484 | 17.8059 | 22.256 | 11.604 |
| 67 | 72 | 782.2425 | 19.1579 | 24.491 | 11.165 |
| 68 | 73 | 763.0846 | 20.5354 | 26.911 | 10.731 |
| 69 | 74 | 742.5492 | 21.9282 | 29.531 | 10.304 |
| 70 | 75 | 720.6210 | 23.3243 | 32.367 | 9.883 |
| 71 | 76 | 697.2967 | 24.7094 | 35.436 | 9.468 |
| 72 | 77 | 072.5873 | 26.0668 | 38.756 | 9.062 |
| 73 | 78 | 646.5205 | 27.3776 | 42.346 | 8.663 |
| 74 | 79 | 619.1429 | 28.6205 | 46.226 | 8.272 |
| 75 | 80 | 590.5224 | 29.7724 | 50.417 | 7.890 |
| 76 | 81 | 560.7500 | 30.8082 | 54.941 | 7.516 |
| 77 | 82 | 529.9418 | 31.7016 | 59.821 | 7.152 |
| 78 | 83 | 498.2402 | 32.4260 | 65.081 | 6.797 |
| 79 | 84 | 465.8142 | 32.9550 | 70.747 | 6.452 |
| 80 | 85 | 432.8592 | 33.2639 | 76.847 | 6.117 |
| 81 | 86 | 399.5953 | 33.3306 | 83.411 | 5.792 |
| 82 | 87 | 366.2647 | 33.1363 | 90.471 | 5.477 |
| 83 | 88 | 333.1284 | 32.6669 | 98.061 | 5.172 |
| 84 | 89 | 300.4615 | 31.9141 | 106.217 | 4.878 |
| 85 | 90 | 268.5474 | 30.8768 | 114.977 | 4.594 |
| 86 | 91 | 237.6706 | 29.5615 | 124.380 | 4.321 |
| 87 | 92 | 208.1091 | 27.9836 | 134.466 | 4.058 |
| 88 | 93 | 180.1255 | 26.1676 | 145.274 | 3.806 |
| 89 | 94 | 153.9579 | 24.1469 | 156.841 | 3.564 |
| 90 | 95 | 129.8110 | 21.9643 | 169.202 | 3.333 |
| 91 | 96 | 107.8467 | 19.6701 | 182.389 | 3.112 |
| 92 | 97 | 88.17660 | 17.32062 | 196.431 | 2.901 |
| 93 | 98 | 70.85598 | 14.97577 | 211.355 | 2.700 |
| 94 | 99 | 55.88021 | 12.69520 | 227.186 | 2.510 |
| 95 | 100 | 43.18501 | 10.53485 | 243.947 | 2.329 |
| 96 | 101 | 32.65016 | 8.54321 | 261.659 | 2.157 |
| 97 | 102 | 24.10695 | 6.75819 | 280.342 | 1.994 |
| 98 | 103 | 17.34876 | 5.20489 | 300.015 | 1.841 |
| 99 | 104 | 12.14387 | 3.89449 | 320.696 | 1.695 |
| 100 | 105 | B. 249380 | 2.824604 | 342.402 | 1. 558 |
| 101 | 106 | 5.424776 | 1.980857 | 365.150 | 1.428 |
| 102 | 107 | 3.443919 | 1.339533 | 388.956 | 1. 306 |
| 103 | 108 | 2.104386 | . 870871 | 413.836 | 1.190 |
| 104 | 109 | 1.233515 | . 542507 | 439.806 | 1.081 |
| 105 | 110 | . 6910080 | . 3226185 | 466.881 | . 978 |
| 106 | 111 | 3683895 | . 1823808 | 495.076 | . 881 |
| 107 | 112 | . 1860087 | . 0975441 | 524.406 | . 789 |
| 108 | 113 | 08846460 | . 04908777 | 554.886 | . 700 |
| 109 | 114 | 03937683 | . 02313590 | 587.551 | . 611 |
| 110 | 115 | . 01624093 | . 01014386 | 624.586 | . 520 |
| 111 | 116 | . 00609707 | . 00408724 | 670.361 | . 419 |
| 112 | 117 | . 00200983 | . 00147445 | 733.619 | 302 |
| 113 | 118 | . 00053538 | . 00044535 | 831.839 | . 164 |
| 114 | 119 | . 00009003 | . 00009003 | 1000.000 |  |

TABLE 10-Continued
Proposed 1955 American Annuity Table Commutation Columns at $2 \frac{1}{2} \%$ Interest

| $\underset{x}{A g E}$ |  | $D_{x}$ | $N_{x}$ | $C_{x}$ | $M_{x}$ | $\boldsymbol{R}_{x}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Male | Female |  |  |  |  |  |
| 5 | 10 | 883.8543 | 29508.3870 | .319050 | 164.137544 | 10605.072963 |
| 6 | 11 | 861.9778 | 28024.5327 | . 319597 | 163.818494 | 10440.935419 |
| 7 | 12 | 840.6344 | 27762.5549 | . 319845 | 163.498897 | 10277.116925 |
| 8 | 13 | 819.8112 | 26921.9205 | . 319891 | 163.179052 | 10113.618028 |
| 9 | 14 | 799.4960 | 26102.1093 | . 319823 | 162.859161 | 9950.438976 |
| 10 | 15 | 779.6762 | 25302.6133 | . 319491 | 162.539338 | 9787.579815 |
| 11 | 16 | 760.3403 | 24522.9371 | . 318985 | 162.219847 | 9625.040477 |
| 12 | 17 | 741.4764 | 23762.5968 | . 319040 | 161.900862 | 9462.820630 |
| 13 | 18 | 723.0726 | 23021.1204 | . 318831 | 161.581822 | 9300.919768 |
| 14 | 19 | 705.1178 | 22298.0478 | . 318512 | 161.262991 | 9139.337946 |
| 15 | 20 | 687.6013 | 21592.9300 | . 318625 | 160.944479 | 8978.074955 |
| 16 | 21 | 670.5119 | 20905.3287 | . 319200 | 160.625854 | 8817.130476 |
| 17 | 22 | 653.8388 | 20234.8168 | . 320198 | 160.306654 | 8656.504622 |
| 18 | 23 | 637.5713 | 19580.9780 | 321584 | 159.986456 | 8496.197968 |
| 19 | 24 | 621.6992 | 18943.4067 | . 323871 | 159.664872 | 8336.211512 |
| 20 | 25 | 606.2119 | 18321.7075 | . 327046 | 159.341001 | 8176.546640 |
| 21 | 26 | 591.0992 | 17715.4956 | . 331035 | 159.013955 | 8017.205639 |
| 22 | 27 | 576.3511 | 17124.3964 | . 335711 | 158.682920 | 7858.191684 |
| 23 | 28 | 561.9581 | 16548.0453 | . 341014 | 158.347209 | 7699.508764 |
| 24 | 29 | 547.9108 | 15986.0872 | . 346936 | 158.006195 | 7541.161555 |
| 25 | 30 | 534.2001 | 15438.1764 | . 353893 | 157.659259 | 7383.155360 |
| 26 | 31 | 520.8170 | 14903.9763 | . 361793 | 157.305366 | 7225.496101 |
| 27 | 32 | 507.7523 | 14383.1593 | . 371050 | 156.943573 | 7068.190735 |
| 28 | 33 | 494.9971 | 13875.4070 | . 381987 | 156.572523 | 6911.247162 |
| 29 | 34 | 482,5420 | 13380.4099 | 394981 | 156. 190536 | 6754.674639 |
| 30 | 35 | 470.3777 | 12897.8679 | . 409813 | 155.795555 | 6598.484103 |
| 31 | 36 | 458.4953 | 12427.4902 | . 426272 | 155.385742 | 6442.688548 |
| 32 | 37 | 446.8862 | 11968.9949 | . 444695 | 154.959470 | 6287.302806 |
| 33 | 38 | 435.5418 | 11522.1087 | . 465292 | 154.514775 | 6132.343336 |
| 34 | 39 | 424.4536 | 11086.5669 | . 488243 | 154.049483 | 5977.828561 |
| 35 | 40 | 413.6128 | 10662.1133 | . 513703 | 153.561240 | 5823.779078 |
| 36 | 41 | 403.0110 | 10248.5005 | . 541801 | 153.047537 | 5670.217838 |
| 37 | 42 | 392.6396 | 9845.4895 | . 572685 | 152.505736 | 5517.170301 |
| 38 | 43 | 382.4904 | 9452.8499 | . 606014 | 151.933051 | 5364.664565 |
| 39 | 44 | 372.5553 | 9070.3595 | . 642257 | 151.327037 | 5212.731514 |
| 40 | 45 | 362.8263 | 8697.8042 | . 681421 | 150.684780 | 5061.404477 |
| 41 | 46 | 353.2955 | 8334.9779 | . 723468 | 150.003359 | 4910.719697 |
| 42 | 47 | 343.9551 | 7981.6824 | . 768454 | 149.279891 | 4760.716338 |
| 43 | 48 | 334.7975 | 7637.7273 | . 816247 | 148.511437 | 4611.436447 |
| 44 | 49 | 325.8154 | 7302.9298 | . 866815 | 147.695190 | 4462.925010 |
| 45 | 50 | 317.0019 | 6977.1144 | . 920083 | 146.828375 | 4315.229820 |
| 46 | 51 | 308.3501 | 6600.1125 | . 975876 | 145.908292 | 4168.401445 |
| 47 | 52 | 299.8535 | 6351.7624 | 1.034116 | 144.932416 | 4022.493153 |
| 48 | 53 | 291.5058 | 6051.9089 | 1.094631 | 143.898300 | 3877.560737 |
| 49 | 54 | 283.3013 | 5760.4031 | 1.157252 | 142.803669 | 3733.662437 |
| 50 | 55 | 275.2343 | 5477.1018 | 1.221759 | 141.646417 | 3590.858768 |
| 51 | 56 | 267.2995 | 5201.8675 | 1.289049 | 140.424658 | 3449.212351 |
| 52 | 57 | 259.4909 | 4934.5680 | 1.361515 | 139.135609 | 3308.787693 |
| 53 | 58 | 2518004 | 4675.0771 | 1.441515 | 137.774094 | 3169.652084 |
| 54 | 59 | 244.2174 | 4423.2767 | 1.531537 | 136.332579 | 3031.877990 |
| 55 | 60 | 236.7293 | 4179.0593 | 1.633546 | 134.801042 | 2895.545411 |
| 56 | 61 | 229.3219 | 3942.3300 | 1.749101 | 133.167496 | 2760.744369 |
| 57 | 62 | 221.9796 | 3713.0081 | 1.879348 | 131.418395 | 2627.576873 |
| 58 | 63 | 214.6861 | 3491.0285 | 2.024542 | 129.539047 | 2496. 158478 |
| 59 | 64 | 207.4253 | 3276.3424 | 2.181922 | 127.514505 | 2366.619431 |

TABLE 10-Continuted

| $\underset{x}{A G E}$ |  | $D_{x}$ | $N_{x}$ | $C_{x}$ | $M_{x}$ | $\boldsymbol{R}_{\boldsymbol{x}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Male | Female |  |  |  |  |  |
| 60 | 65 | 200.1842 | 3068.9171 | 2.348893 | 125.332583 | 2239.104926 |
| 61 | 66 | 192.9528 | 2868.7329 | 2.522688 | 122.983690 | 2113.772343 |
| 62 | 67 | 185.7239 | 2675.7801 | 2.699968 | 120.461002 | 1990.788653 |
| 63 | 68 | 178.4941 | 2490.0562 | 2.877849 | 117.761034 | 1870.327651 |
| 64 | 69 | 171.2628 | 2311.5621 | 3.055329 | 114.883185 | 1752.566617 |
| 65 | 70 | 164.0303 | 2140.2993 | 3.231318 | 111.827856 | 1637.683432 |
| 66 | 71 | 156.7982 | 1976.2690 | 3.404591 | 108.596538 | 1525.855576 |
| 67 | 72 | 149.5693 | 1819.4708 | 3.573757 | 105. 191947 | 1417.259038 |
| 68 | 73 | 142.3475 | 1669.9015 | 3.737287 | 101.618190 | 1312.067091 |
| 69 | 74 | 135.1383 | 1527.5540 | 3.893430 | 97.880903 | 1210.448901 |
| 70 | 75 | 127.9488 | 1392.4157 | 4.040305 | 93.987473 | 1112.567998 |
| 71 | 76 | 120.7878 | 1264.4669 | 4.175840 | 89.947168 | 1018.580525 |
| 72 | 77 | 113.6659 | 1143.6791 | 4.297794 | 85.771328 | 928.633357 |
| 73 | 78 | 106.5958 | 1030.0132 | 4.403818 | 81.473534 | $842.862029{ }^{\circ}$ |
| 74 | 79 | 99.59209 | 923.41742 | 4.491458 | 77.009716 | 761.388495 |
| 75 | 80 | 92.67156 | 823.82533 | 4.558270 | 72.578258 | 684318779 |
| 76 | 81 | 85.85301 | 731.15377 | 4.601810 | 68.019988 | 611.740521 |
| 77 | 82 | 79.15722 | 645.30076 | 4.619763 | 63.418178 | 543.720533 |
| 78 | 83 | 72.60679 | 566.14354 | 4.610075 | 58.798415 | 480.302355 |
| 79 | 84 | 66.22582 | 493.53675 | 4.571009 | 54.188340 | 421.503940 |
| 80 | 85 | 60.03955 | 427.31093 | 4.501322 | 49.617331 | 367.315600 |
| 81 | 86 | 54.07385 | 367.27138 | 4.400339 | 45.116009 | 317.698269 |
| 82 | 87 | 48.35463 | 313.19753 | 4.267988 | 40.715670 | 272.582260 |
| 83 | 88 | 42.90726 | 264.84290 | 4.104906 | 36.447682 | 231.866590 |
| 84 | 89 | 37.75584 | 221.93564 | 3.912497 | 32.342776 | 195.418908 |
| 85 | 90 | 32.92247 | 184.17980 | 3.693005 | 28.430279 | 163.076132 |
| 86 | 91 | 28.42648 | 151.25733 | 3.449453 | 24.737274 | 134.645853 |
| 87 | 92 | 24.28370 | 122.83085 | 3. 185689 | 21.287821 | 109.908579 |
| 88 | 93 | 20.50572 | 98.54715 | 2.906296 | 18.102132 | 88.620758 |
| 89 | 94 | 17.09929 | 78.04143 | 2.616456 | 15.195836 | 70.518626 |
| 90 | 95 | 14.06577 | 60.94214 | 2.321911 | 12.579380 | 55.322790 |
| 91 | 96 | 11.40079 | 46.87637 | 2.028668 | 10.257469 | 42.743410 |
| 92 | 97 | 9.094058 | 35.475576 | 1.742786 | 8.228801 | 32.485941 |
| 93 | 98 | 7.129466 | 26.381518 | 1.470096 | 6.486015 | 24.257140 |
| 94 | 99 | 5.485481 | 19.252052 | 1.215829 | 5.015919 | 17.771125 |
| 95 | 100 | 4.135860 | 13.766571 | . 9843222 | 3.8000899 | 12.7552058 |
| 96 | 101 | 3.050663 | 9.630711 | . 7787645 | 2.8157677 | 8.9551159 |
| 97 | 102 | 2.197492 | 6.580048 | . 6010237 | 2.0370032 | 6.1393482 |
| 98 | 103 | 1.542871 | 4.382556 | . 4515948 | 1.4359795 | 4. 1023450 |
| 99 | 104 | 1.053645 | 2.839685 | . 3296583 | . 9843847 | 2.6663655 |
| 100 | 105 | . 6982883 | 1.7860395 | . 2332637 | . 6547264 | 1.6819808 |
| 101 | 106 | . 4479932 | 1.0877512 | . 1595948 | . 4214627 | 1.0272544 |
| 102 | 107 | . 2774717 | . 6397580 | . 1052920 | . 2618679 | . 6057917 |
| 103 | 108 | . 1654121 | . 3622863 | . 06678391 | . 15657585 | . 34392379 |
| 104 | 109 | . 09459375 | . 19687416 | . 04058817 | . 08979194 | . 18734794 |
| 105 | 110 | . 05169841 | . 10228041 | . 02354830 | . 04920377 | . 09755600 |
| 106 | 111 | . 02688918 | . 05058200 | . 01298750 | . 02565547 | . 04835223 |
| 107 | 112 | . 01324584 | . 02369282 | . 00677678 | . 01266797 | . 02269676 |
| 108 | 113 | . 00614599 | . 01044698 | . 003332715 | . 00589119 | . 01002879 |
| 109 | 114 | . 00266894 | . 00430099 | . 00152989 | . 00256404 | . 00413760 |
| 110 | 115 | . 00107395 | . 00163205 | . 00065442 | . 00103415 | . 00157356 |
| 111 | 116 | . 00039334 | . 00055810 | . 00025725 | . 000037973 | . 000053941 |
| 112 | 117 | . 00012650 | . 00016476 | . 00009054 | . 00012248 | . 00015968 |
| 113 | 118 | . 00003287 | . 00003826 | . 00002668 | . 00003194 | . 00003720 |
| 114 | 119 | .00000539 | . 00000539 | . 00000526 | . 00000526 | . 00000526 |

TABLE 11
Comparison of annuity Values
Immediate Nonrefund Annurties, $2 \frac{2}{2} \%$ Interest

| Age | Proposed 1955 American Annuity Table | Annuity <br> Table for 1949 | 1937 Standard Annuity Table Set Back 2 Years | Ratio of Value on Proposed 1955 American Annuity Table to Value on Annuity Table for 1949 | Ratio of Value on Proposed 1955 American Annuity <br> Table to Value on 1937 Standard Annuity Table Set Back 2 Years |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | males |  |  |  |  |
| 15. | 30.403 | 29.932 | 29.334 | 101.6\% | 103.6\% |
| 25. | 27.900 | 27.317 | 26.768 | 102.1 | 104.2 |
| 35. | 24.778 | 24.057 | 23.588 | 103.0 | 105.0 |
| 45. | 21.010 | 20.112 | 19.905 | 104.5 | 105.6 |
| 55. | 16.653 | 15.837 | 15.887 | 105.2 | 104.8 |
| 60. | 14.330 | 13.676 | 13.832 | 104.8 | 103.6 |
| 65. | 12.048 | 11.496 | 11.806 | 104.8 | 102.0 |
| 70. | 9.883 | 9.351 | 9.855 | 105.7 | 100.3 |
| 75. | 7.890 | 7.323 | 8.029 | 107.7 | 98.3 |
| 80. | 6.117 | 5.492 | 6.371 | 111.4 | 96.0 |
| 85. | 4.594 | 3.923 | 4.911 | 117.1 | 93.5 |
|  | females |  |  |  |  |
| 15. | 31.453 | 31.222 | 30.393 | 100.7\% | 103.5\% |
| 25. | 29.223 | 28.890 | 28.129 | 101.2 | 103.9 |
| 35. | 26.420 | 25.988 | 25.251 | 101.7 | 104.6 |
| 45. | 22.972 | 22.433 | 21.800 | 102.4 | 105.4 |
| 55. | 18.900 | 18.215 | 17.924 | 103.8 | 105.4 |
| 60. | 16.653 | 15.882 | 15.887 | 104.9 | 104.8 |
| 65. | 14.330 | 13.455 | 13.832 | 106.5 | 103.6 |
| 70. | 12.048 | 11.010 | 11.806 | 109.4 | 102.0 |
| 75. | 9.883 | 8.642 | 9.855 | 114.4 | 100.3 |
| 80. | 7.890 | 6.459 | 8.029 | 122.2 | 98.3 |
| 85. | 6.117 | 4.560 | 6.371 | 134.1 | 96.0 |

TABLE 12
Comparison of annuity Values
Life Income Settlement Options with 10 Year Certain Period, $2 \frac{3}{3} \%$ Interest Single Life, First Payment Immediate

| Age | Proposed 1955 American Annuity Table | $\begin{gathered} \text { Annuity Table } \\ \text { for } 1949 \end{gathered}$ | 1937 Standard Annuity Table Set Back 2 Years | Ratio of Value on Proposed 1955 American Annuity Table to Value on Annuity Table for 1949 | Ratio of Value on Proposed 1955 American Annuity Table to Value on 1937 Standard Annuity Table Set Back |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | males |  |  |  |  |
| 15. | 31.423 | 30.955 | 30.383 | 101.5\% | 103.4\% |
| 25. | 28.930 | 28.351 | 27.831 | 102.0 | 103.9 |
| 35. | 25.840 | 25.124 | 24.709 | 102.8 | 104.6 |
| 45. | 22.154 | 21.306 | 21.160 | 104.0 | 104.7 |
| 55 | 18.012 | 17.327 | 17.419 | 104.0 | 103.4 |
| 60. | 15.927 | 15.386 | 15.595 | 103.5 | 102.1 |
| 65. | 13.993 | 13.533 | 13.886 | 103.4 | 100.8 |
| 70. | 12.311 | 11.881 | 12.366 | 103.6 | 99.6 |
| 75. | 10.958 | 10.557 | 11.104 | 103.8 | 98.7 |
| 80. | 9.986 | 9.655 | 10.149 | 103.4 | 98.4 |
| 85. | 9.389 | 9.177 | 9.515 | 102.3 | 98.7 |
|  | females |  |  |  |  |
| 15. | 32.470 | 32.235 | 31.442 | 100.7\% | 103.3\% |
| 25. | 30.247 | 29.913 | 29.182 | 101.1 | 103.6 |
| 35. | 27.462 | 27.032 | 26.335 | 101.6 | 104.3 |
| 45 | 24.067 | 23.531 | 22.976 | 102.3 | 104.7 |
| 55. | 20.121 | 19.447 | 19.294 | 103.5 | 104.3 |
| 60. | 18.012 | 17.256 | 17.419 | 104.4 | 103.4 |
| 65. | 15.927 | 15.071 | 15.595 | 105.7 | 102.1 |
| 70. | 13.993 | 13.023 | 13.886 | 107.4 | 100.8 |
| 75 | 12.311 | 11.282 | 12.366 | 109.1 | 99.6 |
| 80 | 10.958 | 10.012 | 11.104 | 109.4 | 98.7 |
| 85. | 9.986 | 9.293 | 10.149 | 107.5 | 98.4 |

chosen as the "anchor point," with yearly adjustment amounts which are to be added to or subtracted from the stated net single premiums to give the values for other years of settlement. For example, if 1965 were taken as the base year, the formula and rate table for the single life option might be given as shown in the following table.

## LIFE INCOME WITH INSTALLMENTS CERTAIN

The amount of policy proceeds required to provide monthly installments of $\$ 10$ is the amount in Column (i) where settlement is effected during the year 1965; where settlement is effected prior to 1965, it is the amount in Column (i) minus the amount of the Yearly Adjustment multiplied by the difference between 1965 and the year of settlement; where settlement is effected subsequent to 1965, it is the amount in Column (i) plus the amount of the Yearly Adjustment multiplied by the difference between the year of settlement and 1965.

| Age of Beneficiary |  | With Payments Certain for |  |  |  | Installment Refund |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 10 years |  | 20 years |  |  |  |
| Male | Female | (i) | Yearly Adjustment | (i) | Yearly Adjustment | (i) | Yearly Adjustment |
|  | 10* | \$3,974 | \$2.00 | \$3,981 | \$1.95 | \$3,993 | \$1.90 |
|  | 11 | 3,953 | 2.05 | 3,960 | 2.00 | 3,972 | 1.95 |
|  | 12 | 3,932 | 2.10 | 3,939 | 2.05 | 3,951 | 2.00 |
|  | 13 | 3,910 | 2.15 | 3,917 | 2.10 | 3,930 | 2.05 |
|  | 14 | 3,888 | 2.20 | 3,895 | 2.15 | 3,908 | 2.10 |
| $10^{*}$ | 15 | 3,865 | 2.25 | 3,872 | 2.20 | 3,886 | 2.15 |
| 11 | 16 | 3,842 | 2.30 | 3,849 | 2.25 | 3,863 | 2.20 |
| 12 | 17 | 3,818 | 2.35 | 3,825 | 2.30 | 3,840 | 2.25 |
| 13 | 18 | 3,794 | 2.40 | 3,801 | 2.35 | 3,816 | 2.30 |
| 14 | 19 | 3,769 | 2.45 | 3,776 | 2.40 | 3,791 | 2.35 |
| 15 | 20 | 3,743 | 2.50 | 3,751 | 2.45 | 3,767 | 2.40 |
| 16 | 21 | 3,717 | 2.55 | 3,725 | 2.50 | 3,741 | 2.45 |
| 17 | 22 | 3,690 | 2.60 | 3,699 | 2.55 | 3,716 | 2.50 |
| 18 | 23 | 3,663 | 2.65 | 3,672 | 2.60 | 3,689 | 2.55 |
| 19 | 24 | 3,635 | 2.70 | 3,644 | 2.65 | 3,662 | 2.60 |
| 20 | 25 | 3,606 | 2.75 | 3,616 | 2.70 | 3,635 | 2.65 |
| 21 | 26 | 3,577 | 2.80 | 3,587 | 2.75 | 3,607 | 2.70 |
| 22 | 27 | 3,547 | 2.85 | 3,558 | 2.80 | 3,578 | 2.75 |
| 23 | 28 | 3,517 | 2.90 | 3,528 | 2.85 | 3,549 | 2.80 |
| 24 | 29 | 3,486 | 2.95 | 3,497 | 2.90 | 3,519 | 2.85 |
| 25 | 30 | 3,454 | 3.00 | 3,466 | 2.95 | 3,489 | 2.90 |
| 26 | 31 | 3,421 | 3.05 | 3,434 | 3.00 | 3,458 | 2.95 |
| 27 | 32 | 3,388 | 3.12 | 3,401 | 3.07 | 3,426 | 3.00 |
| 28 | 33 | 3,353 | 3.20 | 3,368 | 3.15 | 3,394 | 3.05 |
| 29 | 34 | 3,317 | 3.30 | 3,333 | 3.25 | 3,362 | 3.10 |
| 30 | 35 | 3,281 | 3.40 | 3,298 | 3.35 | 3,328 | 3.15 |
| 31 | 36 | 3,244 | 3.50 | 3,262 | 3.43 | 3,294 | 3.22 |
| 32 | 37 | 3,206 | 3.60 | 3,226 | 3.50 | 3,259 | 3.30 |
| 33 | . 38 | 3,167 | 3.70 | 3,190 | 3.55 | 3,223 | 3.40 |
| 34 | 39 | 3,128 | 3.80 | 3,153 | 3.60 | 3,186 | 3.50 |
| 35 | 40 | 3,087 | 3.90 | 3,116 | 3.65 | 3,149 | 3. 58 |
| 36 | 41 | 3,046 | 4.00 | 3,078 | 3.70 | 3,112 | 3.65 |
| 37 | 42 | 3,005 | 4.10 | 3,040 | 3.75 | 3,074 | 3.70 |
| 38 | 43 | 2,963 | 4.18 | 3,001 | 3.80 | 3,036 | 3.75 |
| 39 | 44 | 2,920 | 4.25 | 2,962 | 3.85 | 2,997 | 3.80 |
| 40 | 45 | 2,878 | 4.30 | 2,922 | 3.90 | 2,958 | 3.85 |
| 41 | 46 | 2,834 | 4.35 | 2,881 | 3.95 | 2,918 | 3.90 |
| 42 | 47 | 2,790 | 4.40 | 2,841 | 4.00 | 2,878 | 3.95 |
| 43 | 48 | 2,746 | 4.45 | 2,800 | 4.03 | 2,837 | 4.00 |
| 44 | 49 | 2,700 | 4.50 | 2,759 | 4.05 | 2,795 | 4.05 |

* 10 years and under.


## LIFE INCOME WITH INSTALLMENTS CERTAIN-Continued

The amount of policy proceeds required to provide monthly installments of $\$ 10$ is the amount in Column (i) where settlement is effected during the year 1965; where settlement is effected prior to 1965, it is the amount in Column (i) minus the amount of the Yearly Adjustment multiplied by the difference between 1965 and the year of settlement; where settlement is effected subsequent to 1965, it is the amount in Column (i) plus the amount of the Yearly Adjustment multiplied by the difference between the year of settlement and 1965.

| Age of Beneficiary |  | With Payments Certain for |  |  |  | Installment Refund |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 10 years |  | 20 years |  |  |  |
| Male | Female | (i) | Yearly Adjustment | (i) | Yearly Adjust- <br> ment | (i) | Yearly Adjustment |
| 45 | 50 | \$2,654 | \$4.55 | \$2,719 | \$4.05 | \$2,754 | \$4.10 |
| 46 | 51 | 2,607 | 4.60 | 2,678 | 4.05 | 2,711 | 4.15 |
| 47 | 52 | 2,559 | 4.65 | 2,637 | 4.05 | 2,668 | 4.20 |
| 48 | 53 | 2,511 | 4.70 | 2,597 | 4.05 | 2,625 | 4.25 |
| 49 | 54 | 2,463 | 4.75 | 2,556 | 4.05 | 2,581 | 4.30 |
| 50 | 55 | 2,414 | 4.80 | 2,516 | 4.03 | 2,537 | 4.35 |
| 51 | 56 | 2,364 | 4.85 | 2,477 | 4.00 | 2,492 | 4.40 |
| 52 | 57 | 2,314 | 4.90 | 2,438 | 3.95 | 2,446 | 4.45 |
| 53 | 58 | 2,263 | 4.95 | 2,400 | 3.88 | 2,400 | 4.50 |
| 54 | 59 | 2,213 | 4.98 | 2,363 | 3.80 | 2,355 | 4.55 |
| 55 | 60 | 2,162 | 5.00 | 2,327 | 3.70 | 2,309 | 4.60 |
| 56 | 61 | 2,112 | 5.00 | 2,292 | 3.60 | 2,263 | 4.63 |
| 57 | 62 | 2,062 | 5.00 | 2,257 | 3.50 | 2,217 | 4.65 |
| 58 | 63 | 2,011 | 5.00 | 2,224 | 3.40 | 2,170 | 4.65 |
| 59 | 64 | 1,961 | 4.98 | 2,192 | 3.30 | 2,124 | 4.65 |
| 60 | 65 | 1,912 | 4.95 | 2,162 | 3.18 | 2,078 | 4.65 |
| 61 | 66 | 1,863 | 4.90 | 2,133 | 3.05 | 2,031 | 4.65 |
| 62 | 67 | 1,815 | 4.85 | 2,106 | 2.90 | 1,985 | 4.65 |
| 63 | 68 | 1,768 | 4.78 | 2,081 | 2.75 | 1,937 | 4.65 |
| 64 | 69 | 1,722 | 4.70 | 2,057 | 2.58 | 1,891 | 4.65 |
| 65 | 70 | 1,678 | 4.60 | 2,036 | 2.40 | 1,844 | 4.65 |
| 66 | 71 | 1,634 | 4.50 | 2,017 | 2.20 | 1,798 | 4.65 |
| 67 | 72 | 1,591 | 4.40 | 1,999 | 2.02 | 1,752 | 4.65 |
| 68 | 73 | 1,550 | 4.28 | 1,983 | 1.85 | 1,704 | 4.65 |
| 69 | 74 | 1,511 | 4.15 | 1,968 | 1.70 | 1,659 | 4.65 |
| 70 | 75 | 1,473 | 4.00 | 1,955 | 1.55 | 1,614 | 4.63 |
| 71 | 76 | 1,437 | 3.85 | 1,943 | 1.40 | 1,568 | 4.60 |
| 72 | 77 | 1,402 | 3.70 | 1,933 | 1.25 | 1,524 | 4.55 |
| 73 | 78 | 1,368 | 3.55 | 1,925 | 1.10 | 1,480 | 4.50 |
| 74 | 79 | 1,337 | 3.40 | 1,918 | .95 | 1,436 | 4.45 |
| 75 | $80 \dagger$ | 1,307 | 3.25 | 1,913 | . 80 | 1,393 | 4.40 |
| 76 |  | 1,279 | 3.08 | 1,909 | . 65 | 1,349 | 4.35 |
| 77 |  | 1,253 | 2.90 | 1,906 | . 52 | 1,308 | 4.30 |
| 78 |  | 1,229 | 2.70 | 1,903 | . 40 | 1,265 | 4.25 |
| 79 |  | 1,208 | 2.50 | 1,901 | . 30 | 1,224 | 4.20 |
| $80 \dagger$ |  | 1,189 | 2.30 | 1,900 | . 20 | 1,182 | 4.15 |

Using the policy table to give，for example，the rate for settlement in 1980 under the 10 year certain life option with a female beneficiary aged 60 ，the policy proceeds required to provide $\$ 10$ a month would be：

$$
\$ 2,162+\$ 5.00(1980-1965)=\$ 2,237
$$

Similarly，for a female beneficiary aged 40 in 1959，the net single premium would be：

$$
\$ 3,087-\$ 3.90(1965-1959)=\$ 3,063.60 .
$$

The values in this table，intended solely for illustration，are based on the Proposed 1955 American Annuity Table and 2⿳亠口冋2$\%$ interest，assuming a one－tenth year setback in age per calendar year elapsed from 1955. The average yearly increases in net single premiums between the years 1975 and 2000，after smoothing and minor adjustment to assure consist－ ency，were taken as the yearly adjustments．Net single premiums for 1965 were then found by extrapolation from the premiums for the year 1975， using these smoothed yearly adjustment factors．

The use of 1965 as the base year or anchor point in the table is of no real significance in the system．Normally it would be taken a few years subse－ quent to current date，so as to avoid the possible misunderstanding that might arise if the basic life option rates in Column（i）were stated for a year prior to the policy issue year．

Because of its built－in adjustment，a graded rate scale based on reason－ ably conservative mortality and interest assumptions should be expected to apply unchanged for upward of 15 years，or double the average duration of flat rate scales．Since life option settlements tend to con－ centrate in the ten year period 20 to 30 years following the policy issue date，the rates that are of primary importance in a graded scale currently adopted for use over a 15 year period are those that will apply for settle－ ment effected 20 to 45 years in the future．The adjustment factors to be used in the graded scale should accordingly be based on the average yearly increase between the rates that are estimated to be appropriate for settlements to be made 20 years hence and 45 years hence．Net single premiums for whatever base year has been chosen can then be determined by extrapolation from the 20th year rates，using the assumed yearly adjustment factors．

Some smoothing or correction of the adjustment factors may be neces－ sary to prevent inconsistencies in rates from developing between the vari－ ous options，particularly at the points where the refund period under the installment refund option approximately equals the payment certain period of the period certain options．

Variations from the straight line projection may be introduced as a transition device to go from a scale of rates based on one mortality table to a scale based on another table, by using two sets of yearly adjustment factors, one set to apply prior to and the other subsequent to the base year. It is questionable, however, whether such added refinement would justify the sacrifice in simplicity.

Other adjustments that have been used with flat rate systems, such as an age set back or forward to distinguish between settlements for particular types of beneficiaries or elections, can be applied with equal facility under this graded rate system.

The only serious complication introduced by the use of graded rates, other than the larger reference tables that would be necessary for field and home office use, is in the retirement income type contracts which provide for a retirement income at maturity of a set percentage such as $1 \%$ of the face amount of insurance. With life option rates varying by year of settlement, the retirement income maturity value for a given sex and maturity age not only would vary with age at issue, but also should change each calendar year, i.e., vary with the year of issue.

The variation in maturity value by age at issue obviously poses no problem, particularly where business machines are used to calculate premiums and nonforfeiture values, but changing the maturity values and consequently premiums and nonforfeiture values for each year of issue would be unduly expensive to administer. This can be avoided by continuing the initial year's maturity values unchanged for a period of 4 or 5 years, and deducting the small deficiency in life option net single premium from the final year's dividend.

## MORTALITY PROJECTION

The recent decrease in annuitant mortality shown in the 1954 Committee Report ${ }^{9}$ is, as would be expected, fairly consistent with the decrease in the U.S. population mortality for white males and females. As shown in Table 13, there has been a fairly steady improvement for all age groups and both sexes, with average yearly rates of decrease over the period 1940 to 1952 of $1 \%$ or more except for the male age groups 55-64 and 65-74.

In light of the comprehensive treatment in Messrs. Jenkins and Lew's paper ${ }^{10}$ of the probability and possible extent of the decrease in future mortality according to cause of death, it is of interest to note the source of this improvement in population mortality at the higher ages, as shown

[^3]in Table 14. Decrease in the death rate from cardiovascular-renal disease accounts for roughly $40 \%$ of the total reduction, influenza and pneumonia for $25 \%$, and "other causes" for $25 \%$; the cancer death rate has remained relatively stationary.

Although the over-all rate of decrease at these ages is somewhat higher than had previously been experienced, the data by cause of death do not

TABLE 13*
U.S. White population Death Rates $\dagger$ per 1,000

| Year $\dagger$ | $\begin{aligned} & \text { Ages } \\ & 45-54 \end{aligned}$ | $\begin{gathered} \text { Ages } \\ 55-64 \end{gathered}$ | $\begin{aligned} & \text { Ages } \\ & 65-74 \end{aligned}$ | $\begin{aligned} & \text { Ages } \\ & 75-84 \end{aligned}$ | Ages <br> 85 and over |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | males |  |  |  |  |
| 1940 | 11.3 | 24.7 | 52.9 | 119.1 | 242.4 |
| 1941 | 11.2 | 24.7 | 52.3 | 116.9 | 237.4 |
| 1942 | 11.2 | 24.6 | 51.7 | 115.8 | 236.0 |
| 1943 | 11.0 | 24.6 | 51.0 | 114.6 | 234.0 |
| 1944. | 10.9 | 24.6 | 50.4 | 113.6 | 234.0 |
| 1945. | 10.6 | 24.1 | 48.8 | 109.3 | 227.3 |
| 1946 | 10.5 | 24.1 | 48.4 | 107.9 | 229.2 |
| 1947 | 10.4 | 23.9 | 48.3 | 106.9 | 231.4 |
| 1948 | 10.3 | 23.8 | 48.4 | 106.6 | 228.9 |
| 1949 | 10.0 | 23.4 | 48.3 | 105.6 | 224.4 |
| 1950. | 9.9 | 23.1 | 48.3 | 105.2 | 218.3 |
| 1951 | 9.8 | 23.0 | 48.3 | 105.0 | 211.9 |
| 1952 | 9.7 | 23.0 | 48.1 | 105.1 | 205.8 |
|  | females |  |  |  |  |
| 1940 | 7.5 | 16.7 | 40.6 | 102.3 | 225.2 |
| 1941 | 7.2 | 16.1 | 39.6 | 99.4 | 219.6 |
| 1942 | 7.1 | 15.9 | 38.9 | 97.9 | 217.6 |
| 1943 | 6.9 | 15.7 | 38.3 | 96.8 | 217.4 |
| 1944. | 6.8 | 15.4 | 37.5 | 95.8 | 217.2 |
| 1945. | 6.5 | 14.9 | 36.0 | 91.9 | 211.3 |
| 1946. | 6.3 | 14.5 | 35.1 | 90.2 | 211.5 |
| 1947 | 6.0 | 14.1 | 34.4 | 88.8 | 212.1 |
| 1948 | 5.8 | 13.7 | 33.7 | 87.4 | 208.7 |
| 1949. | 5.6 | 13.3 | 32.9 | 85.7 | 202.6 |
| 1950. | 5.5 | 12.9 | 32.3 | 84.8 | 196.4 |
| 1951 | 5.4 | 12.7 | 31.7 | 84.3 | 192.2 |
| 1952 | 5.2 | 12.4 | 31.2 | 84.1 | 189.7 |

Average Yeably Rates of Decrease, 1940-52, Geometric Basis

| Males...... | $1.3 \%$ | $0.6 \%$ | $0.8 \%$ | $1.0 \%$ | $1.4 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Females... | 3.0 | 2.5 | 2.2 | 1.6 | 1.4 |

* Derived from data given in Bureau of the Census publications.
$\dagger$ Average of annual rates for 3 year period shown for central year of period.

TABLE 14*
Decrease in Mortality of U.S. White population
by Cause of Death-1940 to 1952
Attained Ages 75 and over


* Derived from data given in Bureau of the Census publications.
indicate that the anticipated improvement in mortality that must inevitably result from the extensive medical research on heart disease and cancer has as yet been realized. In the author's opinion, it would be unsound to assume for projection purposes that the future rate of decrease at ages 80 and over would be much less than the rates assumed for ages under 80.

An excellent statement of the rationale in selection of a scale of mortality decrease rates was given by Mr. Wilmer A. Jenkins in 1946:

Equally unproductive would be argument as to whether or not the yearly mortality decrease should be a constant percentage at all ages. At some times in the past, it is true, changes have been far from that, but the very diversity of changes that have occurred suggests that an assumption of a simple percentage change may be as good an assumption as any other reasonable one. The objective of any scheme of projecting mortality rates far into the future is to produce premiums and reserves which, as far as is known, are safe yet not redundant as a whole, and which, as far as is known, are as likely to result in as complete equity at the various ages as would calculations based on any other equally justifiable assumption. Opinions as to the future trends of mortality will naturally differ, but it is quite likely that several opinions will yield actual annuity premiums, reserves, etc., that are much the same. ${ }^{11}$
Certainly the more recent immediate annuity experience would point to use of a fairly level percentage decrease at all ages.

A device that gives reasonably level rates of decrease under the Proposed 1955 American Annuity Table for ages 40 and above, as shown in Table 15, is a constant setback in age per calendar year. The assumption of $1 / 10$ year age setback per calendar year is roughly comparable in conservatism to the Projection Scale A rates suggested by Messrs. Jenkins and Lew, ${ }^{12}$ and the $1 / 8$ year age setback per calendar year corresponds to their Projection Scale B rates. So far as the effect on annuity premiums is concerned, the principal difference between the two sets of tables lies in the rate of decrease assumed at the higher ages; as demonstrated by Messrs. Jenkins and Lew in their paper, ${ }^{12}$ the rates of decrease assumed for ages 40 and under are of no real significance.

This approach, originally suggested by Mr. Duncan C. Fraser ${ }^{13}$ in 1924 and further developed by Mr. Jenkins in his 1946 paper, ${ }^{14}$ offers a considerable advantage in the convenience of being able to derive values for any age and calendar year from the mortality table for the base year. If the age setback is one-tenth year per calendar year elapsed from the base year, nine additional sets of the elementary functions for the fractional
"TASA XLVII, 265.
${ }^{22}$ TSA I, 369.
${ }^{13}$ JLA LV, 160.
14 TASA XLVII, 265.
ages $x+n / 10, n=1$ to 9 , taken together with the basic table will permit derivation of values for any age and calendar year based on either the "year of issue" or "year of exposure" hypothesis. Similarly, where an age setback of one-eighth year is used, seven additional sets of the elementary functions for fractional ages $x+n / 8, n=1$ to 7 , are all that would be needed.

TABLE 15
average Yearly Rates of Decrease in Mortality
(Geometric Basis)

| Male Age | Profosed 1955 American <br> Annuity Table |  | Annuty Table POR 1949 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | One-Tenth Year Age Setback per Calendar Year | One-Eighth Year Age Setback per Calendar Year | Projection <br> Scale A | Projection Scale B |
| 20 | . $35 \%$ | 44\% | $2.8 \%$ | 1.25\% |
| 25 | . 45 | . 56 | 2.6 | 1.25 |
| 30. | . 62 | . 78 | 2.4 | 1.25 |
| 35 | . 76 | . 95 | 2.2 | 1.25 |
| 40 | . 85 | 1.06 | 2.0 | 1.25 |
| 45 | . 87 | 1.08 | 1.8 | 1.25 |
| 50 | . 83 | 1.03 | 1.6 | 1.25 |
| 55. | . 95 | 1.19 | 1.4 | 1.225 |
| 60 | 1.09 | 1.36 | 1.2 | 1.20 |
| 65 | 99 | 1.23 | 1.0 | 1.10 |
| 70. | . 91 | 1.14 | 8 | . 95 |
| 75 | . 86 | 1.08 | 6 | 75 |
| 80. | . 82 | 1.03 | 4 | . 50 |
| 85 | 79 | 99 | 2 | 25 |
| 90 | 76 | 94 | 0 | 0 |

Nonrefund immediate annuity and life income settlement option values are given on the Proposed 1955 American Annuity Table projected with one-tenth year age setback per calendar year elapsed from 1955, and on the Annuity Table for 1949 with projection according to Scale B, in Tables 16 to 19 . Using the apt terminology coined by Mr. Jenkins to distinguish between the two projection bases, the mortality rates on which these values are based are according to the "year of exposure" hypothesis, which is that the mortality rate at a given attained age is a function of and varies with the calendar year of exposure. For comparison, values are also given on the Proposed 1955 American Annuity Table without projection, i.e., on the "year of issue" hypothesis, with mortality rates adjusted for decrease in mortality only up to the year of issue of the annuity.

TABLE 16
Comparison of Annuity values
Immediate Nonrefund annuities, $2 \frac{1}{3} \%$ Interest
1955 Year of Issue

| Age | Proposfed 1955 <br> Americin Annuity T.lale with Projection* | Proposed 1955 <br> American Anhuty Table witholt Projection | Annelity <br> Table for 1949 Proi- <br> jected on <br> Scale B $\dagger$ | Ratio of Value on Proposed 1955 Amertcan Annuty Tible with Projection to Value on |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Proposed 1955 American Annuity Table without Projection | Annuity Table for 1949 Projected on Scale B $\dagger$ |
|  | males |  |  |  |  |
| 35. | 25.835 | 24.778 | 25.140 | 104.3\% | 102.8\% |
| 40. | 23.981 | 22.972 | 23.185 | 104.4 | 103.4 |
| 45. | 21.947 | 21.010 | 21.057 | 104.5 | 104.2 |
| 50 | 19.746 | 18.900 | 18.829 | 104.5 | 104.9 |
| 55. | 17.384 | 16.653 | 16.550 | 104.4 | 105.0 |
| 60. | 14.933 | 14.330 | 14.250 | 104.2 | 104.8 |
| 65. | 12.524 | 12.048 | 11.944 | 104.0 | 104.9 |
| 70. | 10.240 | 9.883 | 9.641 | 103.6 | 106.2 |
| 75. | 8.146 | 7.890 | 7.491 | 103.2 | 108.7 |
| 80. | 6.290 | 6.117 | 5.574 | 102.8 | 112.8 |
| 85. | 4.704 | 4.594 | 3.947 | 102.4 | 119.2 |
|  | females |  |  |  |  |
| 35. | 27.506 | 26.420 | 26.794 | 104.1\% | 102.7\% |
| 40. | 25.835 | 24.778 | 25.072 | 104.3 | 103.0 |
| 45. | 23.981 | 22.972 | 23.151 | 104.4 | 103.6 |
| 50 | 21.947 | 21.010 | 21.057 | 104.5 | 104.2 |
| 55. | 19.746 | 18.900 | 18.798 | 104.5 | 105.0 |
| 60. | 17.384 | 16.653 | 16.358 | 104.4 | 106.3 |
| 65 | 14.933 | 14.330 | 13.832 | 104.2 | 108.0 |
| 70. | 12.524 | 12.048 | 11.274 | 104.0 | 111.1 |
| 75 | 10.240 | 9.883 | 8.798 | 103.6 | 116.4 |
| 80. | 8.146 | 7.890 | 6.537 | 103.2 | 126.1 |
| 85. | 6.290 | 6.117 | 4.583 | 102.8 | 137.2 |

[^4]TABLE 17
Comparison of Annuity Values
Life Income Settlement Options with 10 Year Certain Period,
$2 \frac{1}{2} \%$ INTEREST
1955 Year of Settlement
Single Life, First Payment Immediate

| Ages | $\begin{gathered} \text { Proposed } \\ 1955 \\ \text { Ayericin } \\ \text { Annuity Table } \\ \text { With Pro- } \\ \text { Jection** } \end{gathered}$ | $\begin{gathered} \text { Proposed } \\ 1955 \\ \text { American } \\ \text { Annctity Table } \\ \text { mithont } \\ \text { Projection } \end{gathered}$ | Annuity Table <br> for 1949 <br> Projected <br> on Scale B $\dagger$ | Ratto of Value on Proposed 1955 American Annuity Table with Projection to Value on |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Proposed 1955 American Annuity Table without Projection | Annuity Table for 1949 Projected on Scale B $\dagger$ |
|  | males |  |  |  |  |
| 35. | 26.895 | 25.840 | 26.229 | 104.1\% | 102.5\% |
| 40. | 25.072 | 24.067 | 24.320 | 104.2 | 103.1 |
| 45. | 23.088 | 22.154 | 22.265 | 104.2 | 103.7 |
| 50. | 20.961 | 20.121 | 20.158 | 104.2 | 104.0 |
| 55. | 18.732 | 18.012 | 18.020 | 104.0 | 104.0 |
| 60. | 16.513 | 15.927 | 15.909 | 103.7 | 103.8 |
| 65. | 14.446 | 13.993 | 13.898 | 103.2 | 103.9 |
| 70. | 12.638 | 12.311 | 12.071 | 102.7 | 104.7 |
| 75. | 11.176 | 10.958 | 10.631 | 102.0 | 105.1 |
| 80. | 10.114 | 9.986 | 9.674 | 101.3 | 104.5 |
| 85. | 9.452 | 9.389 | 9.177 | 100.7 | 103.0 |
|  | females |  |  |  |  |
| 35. | 28.546 | 27.462 | 27.870 | $103.9 \%$ | $102.4 \%$ |
| 40. | 26.895 | 25.840 | 26.146 | 104.1 | 102.9 |
| 45. | 25.072 | 24.067 | 24.284 | 104.2 | 103.2 |
| 50. | 23.088 | 22.154 | 22.221 | 104.2 | 103.9 |
| 55. | 20.961 | 20.121 | 20.030 | 104.2 | 104.6 |
| 60. | 18.732 | 18.012 | 17.722 | 104.0 | 105.7 |
| 65. | 16.513 | 15.927 | 15.403 | 103.7 | 107.2 |
| 70. | 14.446 | 13.993 | 13.218 | 103.2 | 109.2 |
| 75. | 12.638 | 12.311 | 11.372 | 102.7 | 111.1 |
| 80. | 11.176 | 10.958 | 10.032 | 102.0 | 111.4 |
| 85. | 10.114 | 9.986 | 9.293 | 101.3 | 108.8 |

* One-tenth year age setback per calendar year.
$\dagger$ See TSA I, 417.

TABLE 18

## Comparison of annuity Values <br> Immediate Nonrefund Annuities, $2 \frac{1}{2} \%$ Interest 1980 Year of Issue

| Agr: | Propgelo 1955 American Annuity Table yith Prodection* | Proposed 1955 American Anncity Table Set back $2 \frac{1}{2}$ Years without Projection | ```Ansuity Table for 1949 Projected on Scale, B \dagger``` | Ratio of Value on Proposed 1955 Anerican Anncity Tabue with Projection to Valte on |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Proposed 1955 American Annuity Table Set Back 21) Years without Projection | Annuity Table for 1949 Projected on Scale B $\dagger$ |
|  | miles |  |  |  |  |
| 35. | 26.693 | 25.621 | 26.193 | 104.2\% | 101.9\% |
| 40. | 24.931 | 23.897 | 24.306 | 104.3 | 102.6 |
| 45. | 22.986 | 22.012 | 22.222 | 104.4 | 103.4 |
| 50. | 20.867 | 19.975 | 20.000 | 104.5 | 104.3 |
| 55. | 18.583 | 17.795 | 17.684 | 104.4 | 105.1 |
| 60. | 16.162 | 15.498 | 15.285 | 104.3 | 105.7 |
| 65. | 13.716 | 13.183 | 12.812 | 104.0 | 107.1 |
| 70. | 11.362 | 10.953 | 10.341 | 103.7 | 109.9 |
| 75. | 9.165 | 8.869 | 7.978 | 103.3 | 114.9 |
| 80. | 7.186 | 6.982 | 5.843 | 102.9 | 123.0 |
| 85. | 3.462 | 5.334 | 4.041 | 102.4 | 135.2 |
|  | females |  |  |  |  |
| 35. | 28.273 | 27.181 | 27.560 | $104.0 \%$ | $102.6 \%$ |
| 40. | 26.693 | 25.621 | 25.873 | 104.2 | 103.2 |
| 45. | 24.931 | 23.897 | 23.993 | 104.3 | 103.9 |
| 50. | 22.986 | 22.012 | 21.914 | 104.4 | 104.9 |
| 55. | 20.867 | 19.975 | 19.636 | 104.5 | 106.3 |
| 60. | 18.583 | 17.795 | 17.168 | 104.4 | 108.2 |
| 65. | 16.162 | 15.498 | 14.552 | 104.3 | 111.1 |
| 70. | 13.716 | 13.183 | 11.868 | 104.0 | 115.6 |
| 75. | 11.362 | 10.953 | 9.230 | 103.7 | 123.1 |
| 80. | 9.165 | 8.869 | 6.781 | 103.3 | 135.2 |
| 85. | 7.186 | 6.982 | 4.669 | 102.9 | 153.9 |

* One-tenth year age setback per calendar year.
$\dagger$ See TSA I, 417.

TABLE 19

## Comparison of Annuity Values

Life Income Settlement Options with 10 Year Certain Period, $2 \frac{1}{2} \%$ Interest
1980 Year of Settlement Single Life, First Payment Immediate

*One-tenth year age setback per calendar year.
$\dagger$ See TSA I, 417.

The law of uniform seniority can of course be applied under the Proposed 1955 American Annuity Table for ages 60 and above, where Makeham's formula was used, to give the equal ages to be substituted for two different ages in obtaining joint life annuity values. However, in testing its application where one or both lives were under age 60 , it was found that although the errors in value were within acceptable limits, the values were at some points inconsistent. Where the difference in ages was 30 years or more and the younger life was age 35 or under, joint annuity values for the equal ages were in some cases larger than the value of the single life annuity at the older age.

The age adjustments were accordingly modified so as to give consistent values, as given in Table 21 for use with the joint life annuity values in Table 20.

TABLE 20
Joint Life Annuities for Two Lives at Equal Ages on the
Proposed 1955 American Annuity Table at $2 \frac{1}{2} \%$ Interest

| $\begin{gathered} \mathrm{Age} \\ x \end{gathered}$ |  | $a_{x x}$ | Agr |  | $a_{x x}$ | Age$x$ |  | $a_{x x}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Male | Female |  | Male | Female |  | Male | Female |  |
| 5 | 10 | 30.774 | 40 | 45 | 20.016 | 75 | 80 | 5.118 |
| 6 | 11 | 30.566 | 41 | 46 | 19.595 | 76 | 81 | 4.818 |
| 7 | 12 | 30.354 | 42 | 47 | 19.170 | 77 | 82 | 4.529 |
| 8 | 13 | 30.137 | 43 | 48 | 18.739 | 78 | 83 | 4.252 |
| 9 | 14 | 29.916 | 44 | 49 | 18.304 | 79 | 84 | 3.986 |
| 10 | 15 | 29.689 | 45 | 50 | 17.864 | 80 | 85 | 3.732 |
| 11 | 16 | 29.456 | 46 | 51 | 17.420 | 81 | 86 | 3.488 |
| 12 | 17 | 29.219 | 47 | 52 | 16.972 | 82 | 87 | 3.256 |
| 13 | 18 | 28.976 | 48 | 53 | 16.520 | 83 | 88 | 3.034 |
| 14 | 19 | 28.727 | 49 | 54 | 16.064 | 84 | 89 | 2.823 |
| 15 | 20 | 28.472 | 50 | 55 | 15.605 | 85 | 90 | 2.622 |
| 16 | 21 | 28.212 | 51 | 56 | 15.141 | 86 | 91 | 2.432 |
| 17 | 22 | 27.945 | 52 | 57 | 14.675 | 87 | 92 | 2.251 |
| 18 | 23 | 27.673 | 53 | 58 | 14.205 | 88 | 93 | 2.080 |
| 19 | 24 | 27.394 | 54 | 59 | 13.732 | 89 | 94 | 1.918 |
| 20 | 25 | 27.109 | 55 | 60 | 13.258 | 90 | 95 | 1.765 |
| 21 | 26 | 26.817 | 56 | 61 | 12.784 | 91 | 96 | 1.622 |
| 22 | 27 | 26.519 | 57 | 62 | 12.311 | 92 | 97 | 1.487 |
| 23 | 28 | 26.215 | 58 | 63 | 11.840 | 93 | 98 | 1.360 |
| 24 | 29 | 25.904 | 59 | 64 | 11.374 | 94 | 99 | 1.241 |
| 25 | 30 | 25.586 | 60 | 65 | 10.914 | 95 | 100 | 1.130 |
| 26 | 31 | 25.261 | 61 | 66 | 10.461 | 96 | 101 | 1.026 |
| 27 | 32 | 24.930 | 62 | 67 | 10.016 | 97 | 102 | . 929 |
| 28 | 33 | 24.591 | 63 | 68 | 9.579 | 98 | 103 | . 839 |
| 29 | 34 | 24.246 | 64 | 69 | 9.152 | 99 | 104 | . 755 |
| 30 | 35 | 23.894 | 65 | 70 | 8.733 | 100 | 105 | . 677 |
| 31 | 36 | 23.535 | 66 | 71 | 8.324 | 101 | 106 | 605 |
| 32 | 37 | 23.169 | 67 | 72 | 7.925 | 102 | 107 | . 537 |
| 33 | 38 | 22.797 | 68 | 73 | 7.536 | 103 | 108 | . 475 |
| 34 | 39 | 22.418 | 69 | 74 | 7.158 | 104 | 109 | . 418 |
| 35 | 40 | 22.033 | 70 | 75 | 6.790 | 105 | 110 | . 365 |
| 36 | 41 | 21.641 | 71 | 76 | 6.433 | 106 | 111 | . 316 |
| 37 | 42 | 21.244 | 72 | 77 | 6.087 | 107 | 112 | 271 |
| 38 | 43 | 20.840 | 73 | 78 | 5.753 | 108 | 113 | . 230 |
| 39 | 44 | 20.431 | 74 | 79 | 5.430 | 109 | 114 | . 191 |
|  |  |  |  |  |  | 110 | 115 | . 153 |
|  |  |  |  |  |  | 111 | 116 | . 114 |
|  |  |  |  |  |  | 112 | 117 | . 071 |
|  |  |  |  |  |  | 113 | 118 | . 028 |
|  |  |  |  |  |  | 114 | 119 | . 000 |

TABLE 21
Age adjustments to Give Equal Ages FOR JOINT LIFE ANNUITIES ON THE
Proposed 1955 American Annuity Table

| Years Difference in Age* | Addition to Younger Age in Years | Years Difference in Age* | Addition to Younger Age in Years |
| :---: | :---: | :---: | :---: |
| 1 | 510 | 31 | 24.300 |
| 2 | 1.043 | 32 | 25.262 |
| 3 | 1.597 | 33 | 26.224 |
| 4 | 2.171 | 34 | 27.186 |
| 5 | 2.765 | 35 | 28.148 |
| 6 | 3.379 | 36 | 29.110 |
| 7 | 4.013 | 37 | 30.071 |
| 8 | 4.667 | 38 | 31.032 |
| 9 | 5.341 | 39 | 31.993 |
| 10 | 6.034 | 40 | 32.954 |
| 11 | 6.747 | 41 | 33.915 |
| 12 | 7.480 | 42 | 34.876 |
| 13 | 8.232 | 43 | 35.837 |
| 14 | 9.003 | 44 | 36.798 |
| 15 | 9.793 | 45 | 37.760 |
| 16 | 10.602 | 46 | 38.722 |
| 17 | 11.429 | 47 | 39.684 |
| 18 | 12.273 | 48 | 40.646 |
| 19 | 13.133 | 49 | 41.608 |
| 20 | 14.008 | 50 | 42.570 |
| 21 | 14.897 | 51 | 43.532 |
| 22 | 15.799 | 52 | 44.494 |
| 23 | 16.713 | 53 | 45.456 |
| 24 | 17.638 | 54 | 46.418 |
| 25 | 18.572 | 55 | 47.380 |
| 26 | 19.514 | 56 | 48.342 |
| 27 | 20.463 | 57 | 49.304 |
| 28 | 21.418 | 58 | 50.266 |
| 29 | 22.377 | 59 | 51.228 |
| 30 | 23.338 | 60 | 52.190 |

* For one male and one female, the age of the female is taken as the actual attained age minus 5 .

TABLE 22
Test of Proposed age adjustments for Joint life annuities on the Proposed 1955 american annuity Table at $2 \frac{1}{3} \%$ Interest

| Younger <br> Life <br> Age <br> $\boldsymbol{x}$ |  | Older <br> Life <br> Age <br> » |  | Exact <br> Value <br> $a_{x y}$ | Approximatr Value ahow | Error |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Value of $a_{x y}-a_{y w x}$ | Percentage of $a_{x y}$ |  |  |
|  |  |  |  |  |  |  |  |  |  |
| 15 |  | 25 | 30 | 26.786 | 26.807 | $-.021$ | $-0.1 \%$ |
|  |  | 35 | 40 | 24.202 | 24.243 | $-.041$ | $-0.2$ |
|  |  | 45 | 50 | 20.702 | 20.702 | . 000 | 0.0 |
|  |  | 55 | 60 | 16.510 | 16.541 | $-.031$ | -0.2 |
|  |  | 65 | 70 | 11.967 | 12.043 | $-.076$ | -0.6 |
|  |  | 75 | 80 | 7.856 | 7.851 | . 005 | 0.1 |
| 25 | 30 | 35 | 40 | 23.559 | 23.522 | . 037 | 0.2 |
|  |  | 45 | 50 | 20.364 | 20.428 | $-.064$ | $-0.3$ |
|  |  | 55 | 60 | 16.361 | 16.366 | $-.005$ | -0.0 |
|  |  | 65 | 70 | 11.908 | 11.862 | . 046 | 0.4 |
|  |  | 75 | 80 | 7.836 | 7.703 | . 133 | 1.7 |
|  |  | 85 | 90 | 4.578 | 4.476 | . 102 | 2.2 |
| 35 | 40 | 45 | 50 | 19.570 | 19.580 | $-.010$ | -0.1 |
|  |  | 55 | 60 | 15.983 | 16.060 | $-.077$ | -0.5 |
|  |  | 65 | 70 | 11.752 | 11.682 | . 070 | 0.6 |
|  |  | 75 | 80 | 7.780 | 7.554 | . 226 | 2.9 |
|  |  | 85 | 90 | 4.561 | 4.371 | . 190 | 4.2 |
| 45 | 50 |  | 60 | 15.060 | 15.125 | $-.065$ | $-0.4$ |
|  |  | 65 | 70 | 11.349 | 11.370 | $-.021$ | -0.2 |
|  |  | 75 | 80 | 7.635 | 7.408 | . 227 | 3.0 |
|  |  | 85 | 90 | 4.514 | 4.265 | . 249 | 5.5 |
| 55 | 60 | 65 | 70 | 10.425 | 10.445 | $-.020$ | $-0.2$ |
|  |  | 75 | 80 | 7.264 | 7.155 | . 109 | 1.5 |
|  |  | 85 | 90 | 4.393 | 4.162 | . 231 | 5.3 |
| 65 | 70 | 75 | 80 | 6.437 | 6.421 | . 016 | 0.2 |
|  |  | 85 | 90 | 4.070 | 3.984 | . 086 | 2.1 |
| 75 | 80 | 85 | 90 | 3.491 | 3.480 | . 011 | 0.3 |


[^0]:    ${ }^{3}$ RAIA XIII, 103.

    - TASA XXXVII, 207.
    ${ }^{5}$ TASA XLII, 172.

[^1]:    ${ }^{8}$ TSA 1954 Reports, 36.

[^2]:    * Less than 10 contracts terminated by death.

    Nore.-Mortality ratios in italics where less than 50 but not less than 10 contracts terminated by death.

[^3]:    - TSA 1954 Reports, 36.
    ${ }^{10}$ TSA I, 369.

[^4]:    * One-tenth year age setback per calendar year.
    $\dagger$ See TSA I, 417.

