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SUPPLEMENTARY DISCUSSION OF PAPER PRESENTED AT THE WESTERN SPRING MEETING

THE EFFECT OF VARYING INTEREST RATES

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ALEXANDER T. BROOKS:

In this paper the author shows how by varying interest rate assumptions it is possible to reduce net premiums without being required to alter scales of cash values. Members of the Society may be interested to hear of the practical application of a somewhat similar approach, but making use of a combined variation in interest and mortality rates.

When I first joined my present Company in the Philippines, I was faced with the problem of reducing nonparticipating premium rates (mainly for competitive reasons), while if possible maintaining the existing scales of cash values and reserves. At that time premium rates, reserves and cash values were based on the American Experience Table at $3\frac{1}{2}\%$, and my problem was accentuated by the fact that the only complete tables of basic values and reserves immediately available were on that table at that rate of interest.

On investigation I found that the margin in the gross premium on account of loading was negligible, but that there was a good mortality margin and also a reasonable interest margin. Further study of experience mortality showed that the margin was greatest at the younger ages and reduced with increase in age. It therefore followed that lower premium rates could be justified by assuming mortality better than American Experience and an interest rate higher than $3\frac{1}{2}\%$, and the best results would be obtained if the assumed mortality improvement were larger at the younger ages.

I finally obtained the desired result by the simple expedient of replacing the standard net level premium P (based on A. E. at $3\frac{1}{2}\%$) in the premium formula by a special net level premium P′, where $P' = P - c$, and c is a constant independent of age or plan. If c is taken as equal to $d' - d$ (where d' and d are the discount rates corresponding respectively to the special and standard rates of interest i' and i),

$$P' = P - c = P - (d' - d) \tag{1}$$

and, in all cases where the relationship $P + d = 1/\ddot{a}$ holds,

$$\ddot{a}' = \ddot{a} \, . \tag{2}$$

It follows from (2) that

$$v'p'_x = vp_x \tag{3}$$

and

$$p'_x = \frac{1+i'}{1+i}\,p_x\,. \tag{4}$$

It is seen, therefore, that P′ is the net premium at interest rate i' on a mortality table such that the ratio of the special to the standard probability of surviving one year is constant at all ages, and equal to $(1 + i')/(1 + i)$. This satisfies the requirement stated above, that the ratio of q' to q should increase with increase in age. Furthermore, if $l'_0 = l_0$,

$$D'_0 = D_0$$

and

$$D'_x = D_x \tag{5}$$

and

$$\begin{aligned} N'_x &= N_x \\ M'_x &= D'_x - d'N'_x \\ &= D_x - dN_x - (d' - d)\,N_x \\ &= M_x - cN_x\,. \end{aligned} \tag{6}$$

Now if it is assumed that the special mortality and interest apply throughout the premium term and that ordinary mortality and interest apply thereafter in cases where the risk extends beyond the premium term (a convenient assumption, providing additional margins in such cases to meet overhead expenses after the cessation of premiums), it can be shown that for all normal plans of insurance $P' = P - c$ and $V' = V$. In the general form,

$$\begin{aligned} {}_kP'_{x:\overline{n|}} &= (M'_x - M'_{x+k} + M_{x+k} - M_{x+n} + D_{x+n}) \,/\, (N'_x - N'_{x+k}) \\ &= [M_x - M_{x+n} + D_{x+n} - c\,(N_x - N_{x+k})\,] \,/\, (N_x - N_{x+k}) \\ &= {}_kP_{x:\overline{n|}} - c\,. \end{aligned} \tag{7}$$

$$\left.\begin{aligned} {}^k_tV'_{x:\overline{n|}} &= [{}_kP'_{x:\overline{n|}}\,(N'_x - N'_{x+t}) - (M'_x - M'_{x+t})\,] \,/D'_{x+t} \\ &= [\,({}_kP_{x:\overline{n|}} - c)\,(N_x - N_{x+t}) - (M_x - M_{x+t}) + c(N_x - N_{x+t})\,] \,/D_{x+t} \\ &= {}^k_tV_{x:\overline{n|}} \qquad (t < k) \\ &\text{or} \\ {}^k_tV'_{x:\overline{n|}} &= A_{x+t:\overline{n-t|}} = {}^k_tV_{x:\overline{n|}} \qquad (t \geq k) \end{aligned}\right\} \tag{8}$$

In actual practice c varied by plan, being smaller for the longer term plans in order to retain larger interest and mortality margins with increasing term, but in no case did it exceed 4.65 per 1,000, corresponding to $i' = .04$ and $i' - i = .005$, at which level it was found that the special mortality rates still retained a satisfactory, though small, margin at all ages.

While this approach cannot, of course, be of general application, it is interesting to note that it can possibly be applied using mortality tables other than the American Experience Table. For instance, if it is applied to premium calculations on the American Men mortality table, using $i' - i = .0025$, the special mortality rates approximate very closely to those of the X_{17} Table at younger ages and retain an increasing margin over the X_{17} rates with advance in age.