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GROSS PREMIUM CALCULATIONS AND PROFIT MEASUREMENT FOR NONPARTICIPATING INSURANCE

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RATEMAKING for nonparticipating life insurance is a delightful mixture of science and art. In ratemaking, creative art begins where science leaves off. This paper will consider only the scientific aspects of ratemaking. Braver men may tamper with the mystical process which transforms actuaries into poets and calculated premiums into published rates. Final premium rates represent a blend of the sensations the actuary receives from examining the calculated rates, feeling the pulse of competition, smelling the sweet scent of low net cost, tasting the lotus of improving mortality, weighing the promise of increased new business, and hearing the distant rumble of the drums of war, epidemic, taxes and economic distress. Such activity transcends analysis:

> Weave a circle around him thrice,
> And close your eyes with holy dread,
> For he on honey-dew hath fed,
> And drunk the milk of Paradise.
>
> COLERIDGE

The specific purposes of this paper are:

1. To examine the relationship of gross premiums and profit margins, with particular reference to the probable effect of current developments
2. To suggest criteria for measuring contingency and profit margins and introducing these items into the calculation of premium rates
3. To discuss the types of assumptions suitable for the calculation of nonparticipating rates
4. To present the mathematical technique necessary for the determination of premiums according to the criteria and assumptions proposed
5. To develop a related technique suitable for evaluating the business in force and agency organization of a life insurance company
6. To demonstrate the practicality of the technique and to exhibit results derived by the presented method according to the proposed criteria and assumptions.

1. THE RELATIONSHIP OF GROSS PREMIUMS AND PROFIT MARGINS

Theoretical Premiums and Risk Classes

For each individual buyer of life insurance there exists a theoretical premium rate. It is uniquely defined by the following parameters:

(1) the probability of collecting premiums;
(2) the interest earned on accumulated funds;
(3) the benefits paid on survival, death and withdrawal;
(4) the expenses incurred, including taxes and reinsurance costs;
(5) the charges assessed for contingencies;
(6) the profit objectives adopted by the company; and
(7) the basis of liabilities established for future benefits.

To the extent that this premium rate differs from the premium rate actually paid by the policyholder and to the extent that estimates of these parameters are in error, the profit objective is distorted.

A discussion of the problem of determining premium rates for any form of insurance must begin with consideration of the concept of a "risk class." By definition, a risk class is one in which all members are subject to the same probabilities of risk occurrence. To determine the theoretical premium rate for nonparticipating life insurance, it is necessary to establish risk classes for the purpose of predicting mortality rates, interest rates, expense rates, withdrawal rates, and contingency margins. For each risk, the distribution of risk classes is continuous; that is, the probability of finding any two individuals in precisely the same risk class is infinitesimal. For practical reasons this continuous distribution must be approximated by a discrete distribution, in which individuals in *similar* risk classes are treated as individuals in the *same* risk class. The width of the risk classes defined by the discrete distribution is a matter of judgment; broad classes simplify the classification problem at the expense of precision, and narrow classes improve the approximation at the expense of simplicity. Within a given class the same premium is paid by all members, but the actual profit margin is subject to substantial variation among individuals, even if classes are narrowly defined; for example, among individuals of the same nearest integral age and sex, subject to the same underwriting requirements, there are significantly different probabilities of death and hence significantly different theoretical premiums. The determination of risk classes has, therefore, an important bearing on actual profit margins.

Current Developments

The broad averaging formerly applied in defining risk classes has been superseded, to a large extent, by a more sophisticated interpretation of

that term. The trend of the industry in this direction is evidenced by the now general recognition of policy size and the growing recognition of sex in rate structures for both participating and nonparticipating life insurance. However one may view these developments, there should be general agreement that rate structure refinement is a one-way street, the end of which is not yet in sight. Future developments may include:

(1) a system of premium rates graded by policy size which recognizes differences attributable to the length of the premium period;
(2) fractional premiums which recognize the per-collection expense, the effect of age and plan, and possibly the effect of fractional premiums on deficiency reserves;
(3) nonforfeiture values which recognize the effect of policy size and sex.

Because the method of defining discrete risk classes does not affect the theoretical premium for any individual, rate refinement can be viewed as a redistribution of the margins for profit in a way which reduces fluctuations within classes.

The trend of the life insurance industry towards more refined rate structures can be interpreted as an increase in the importance of price as a factor affecting the sale of life insurance. Whether or not this sentiment originated with the buying public is immaterial; if it was created by the industry itself, it will speedily be communicated to the public through life insurance agents. The ultimate results will likely be a more informed buyer and a consequent increase in the gravitational effect of low premium rates on the distribution of life insurance sales. The importance of the distribution of profit margins obviously increases as buyers of life insurance become more informed and as the distribution of sales is more substantially influenced by pricing. Another aspect of rate refinement and buyer awareness merits observation. With a few exceptions, such as special plans available only for amounts greater than some relatively high minimum, risk classes were formerly determined by the entire industry in a relatively uniform fashion, using nearest integral age as the only criterion of classification for standard risks. Risk classes are no longer uniform from company to company. The extent of the recognition of policy size and sex varies from company to company, giving the buyer the opportunity to select against the industry as a whole by buying from a company whose classification system favors him.

Perhaps this could be said more briefly: the cost of loss leaders has increased and is still increasing. If all life insurance purchases were made on the basis of price alone, aggregate industry profits would be materially reduced and might even vanish. One way to avoid shrinking margins is through the development of more sensitive methods of measuring profits

and calculating premium rates. It may also be necessary to refine continually the rate structure unless the industry adopts a uniform system of risk classification.

Control of Parameters and Rates

To what extent can a company control the factors involved in the equation which expresses the relationship between profit objectives and gross premiums? Among the expense items, commissions are subject to direct control within the limitations of section 213 of the New York Insurance Law, if applicable, and the limitations imposed by the competition for manpower. Direct control of the valuation basis may be exercised, but this is severely restricted by law in all jurisdictions of the United States and Canada and somewhat affected by competitive considerations in the United States due to the relationship of reserves and nonforfeiture values. Interest earnings, benefit payments, lapse rates, other expenses and margins for contingencies can be controlled only indirectly. The dependent variable in this equation is the gross premium, which is subject to legal limitations in Wisconsin[1] and Kansas[2] and to competitive limitations everywhere. The profit objective is therefore the only completely independent variable.

The Effect of Competition

To what extent is direct control of gross premiums limited by competitive considerations? Table 1 shows the premium rates for the whole life plan which have recently been adopted by ten prominent stock companies. This plan was selected to minimize differences attributable to cash values. Since the whole life plan is particularly sensitive to competitive considerations, the range of premium rates for any other plan would probably be much wider.

It is clear from Table 1 that competition has played a large part in the establishment of the rate patterns of these companies, especially at those points where sales volume is high. The ten companies have, in aggregate, premium rates which are quite comparable. The scale of weights used to determine the average premiums is arbitrary but in general agreement with industry averages. According to that scale, there is less than a 4% difference in the aggregate rates of the highest and lowest companies. At particular ages and for particular policy sizes, the average variation is 8%,

[1] Wisconsin law specifies a maximum gross premium; this has little effect on nonparticipating premiums.

[2] Kansas law specifies a minimum gross premium equal to the valuation net premium on the valuation basis selected; this has a material effect on rates, especially those for large amounts, high ages, and low premium plans.

TABLE 1—NONPARTICIPATING WHOLE LIFE PREMIUM RATES PER $1,000

Age	Amount	Weight	Company													Range	
			A	B	C	D*	E	F	G	H†	I	J	High	Mean	Low		
15	$ 3,000	.05	$14.38	$12.07	$12.05	$13.05	$12.64	$11.61	$12.57	$13.20	$11.46	$12.24	$14.38	$12.53	$11.46	$ 2.92	23.3%
	6,000	.02	11.38	12.07	11.22	11.80	10.64	10.77	11.41	11.70	9.80	10.99	12.07	11.18	9.80	2.27	20.3
	12,000	.01	10.53	10.57	10.38	10.78	9.89	10.36	10.82	10.95	8.96	10.37	10.95	10.36	8.96	1.99	19.2
	30,000	.01	10.08	10.07	9.88	10.40	9.64	10.11	10.47	10.50	8.46	9.99	10.50	9.96	8.46	2.04	20.5
25	3,000	.03	17.74	15.48	15.56	16.47	16.30	15.30	15.92	16.43	16.37	15.66	17.74	16.12	15.30	2.44	15.1
	6,000	.09	14.74	15.48	14.73	15.22	14.30	14.46	14.76	14.93	14.71	14.41	15.48	14.77	14.30	1.18	8.0
	12,000	.09	13.89	13.98	13.89	13.76	13.55	14.05	14.17	14.18	13.87	13.79	14.18	13.91	13.55	.63	4.5
	30,000	.04	13.44	13.48	13.39	13.38	13.30	13.80	13.82	13.73	13.37	13.41	13.82	13.51	13.30	.52	3.8
35	3,000	.03	23.45	21.25	21.26	21.95	22.10	20.82	21.15	21.30	21.98	21.26	23.45	21.65	20.82	2.63	12.1
	6,000	.09	20.45	21.25	20.43	20.70	20.10	19.98	19.99	19.80	20.32	20.01	21.25	20.30	19.80	1.45	7.1
	12,000	.12	19.60	19.75	19.59	19.07	19.35	19.57	19.40	19.05	19.48	19.39	19.75	19.43	19.05	.70	3.6
	30,000	.06	19.15	19.25	19.09	18.69	19.10	19.32	19.05	18.60	18.98	19.01	19.32	19.02	18.60	.72	3.8
45	3,000	.02	32.38	29.95	30.36	30.53	31.78	29.86	29.85	30.25	30.70	30.18	32.38	30.58	29.85	2.53	8.3
	6,000	.08	29.38	29.95	29.53	29.28	29.78	29.02	28.69	28.75	29.04	28.93	29.95	29.24	28.69	1.26	4.3
	12,000	.11	28.53	28.45	28.69	27.64	29.03	28.61	28.10	28.00	28.20	28.31	29.03	28.36	27.64	1.39	4.9
	30,000	.05	28.08	27.95	28.19	27.26	28.78	28.36	27.75	27.55	27.70	27.93	28.78	27.96	27.26	1.52	5.4
55	3,000	.01	46.77	44.36	44.60	45.62	48.84	44.09	44.65	44.25	45.36	44.48	48.84	45.30	44.09	4.75	10.5
	6,000	.02	43.77	44.36	43.77	44.37	46.84	43.25	43.49	42.75	43.70	43.23	46.84	43.95	42.75	4.09	9.3
	12,000	.02	42.92	42.86	42.93	42.33	46.09	42.84	42.90	42.00	42.86	42.61	46.09	43.03	42.00	4.09	9.5
	30,000	.01	42.47	42.36	42.43	41.95	45.84	42.59	42.55	41.55	42.36	42.23	45.84	42.63	41.55	4.29	10.1
65	3,000	.01	74.17	70.28	72.79	75.23	81.43	71.84	70.57	73.00	73.33	70.30	81.43	73.29	70.28	11.15	15.2
	6,000	.01	71.17	70.28	71.96	73.98	79.43	71.00	69.41	71.50	71.67	69.05	79.43	71.95	69.05	10.38	14.4
	12,000	.01	70.32	68.78	71.12	68.76	78.68	70.59	68.82	70.75	70.83	68.43	78.68	70.71	68.43	10.25	14.5
	30,000	.01	69.87	68.28	70.62	68.38	78.43	70.34	68.47	70.30	70.33	68.05	78.43	70.31	68.05	10.38	14.8
Weighted Average.....			$23.87	$23.69	$23.58	$23.50	$24.14	$23.42	$23.35	$23.36	$23.40	$23.26	$24.74	$23.56	22.85	$ 1.89	8.0%
Average Index (all companies=100.0) . .			101.3	100.6	100.1	99.7	102.5	99.4	99.1	99.2	99.3	98.7	102.5	100.0	98.7	3.8	3.8%

* Life Paid-up at 90 for $3,000 and $6,000, Whole Life for $12,000 and $30,000.

† Estimated waiver benefit subtracted.

or $1.89 per thousand. Even this narrowly defined competitive rate band is very wide in relation to the probable profit which is contemplated by these companies. The difference between adopting all of the "high" rates and all of the "low" rates is probably the difference between very comfortable profit margins and none at all.

Whenever the theoretical premium rate calculated by a particular company is outside of the competitive range, that company must choose between the following alternatives:

(1) controlling one or more of the factors involved in the relation between profit objectives and premium rates;
(2) revising its profit objectives;
(3) adopting premium rates which are competitively unrealistic;
(4) accepting the penalties of a loss leader.

In the case of a typical company, most of the theoretical premium rates will fall within the indicated competitive range. The competitive position of the company at particular points in the array of premium rates will then be largely determined by the distribution of profit margins adopted by the company. If there is a uniform distribution of profit throughout the rate structure, the realization of the profit objective cannot be thwarted by a shift in the distribution of sales.

Existing Methods of Expressing Profit Objectives and Calculating Gross Premiums

There is no single system for introducing profit margins into the calculation of gross premiums and, consequently, no unique scale of "uniform" profit margins. Frequently profit margins and contingency margins are introduced without specific identification. Traditionally, these margins are introduced, separately or jointly, in one or more of the following ways:

(1) as a function of premium;
(2) as a function of the amount of insurance;
(3) as a function of a specific reserve;
(4) as a redundant estimate of mortality costs;
(5) as an understatement of the probable future interest rate.

With contingency margins and profit objectives expressed in such a manner, with those factors fixed which can be directly controlled, and with estimates of other factors affecting the gross premium, the calculation of theoretical premium rates can be performed in a variety of ways. The three widely known and generally accepted methods are those associated with the names of Messrs. Cammack, Hoskins, and Jenkins; each method produces satisfactory premiums from the assumptions made and the

profit objectives and criteria established. One limitation shared by all three methods pertains to the establishment of profit objectives: the surplus depletion incurred at issue is recovered, together with interest at the assumed rate, and profit in excess of this interest on surplus depletion is realized, but the additional profit is not directly associated with the amount of surplus expended to produce it. The purpose of the technique to be proposed in this paper is to remove this limitation.

2. CONTINGENCY MARGINS AND PROFIT OBJECTIVES

A defense of each of the traditional methods of introducing contingency and profit margins into gross premium calculations can be mustered without great difficulty, but it cannot be conclusively shown that one is superior to all others. To make a meaningful analysis of contingency margins and profit objectives, each must be considered separately.

Contingency Margins

For what purpose are contingency margins required? Fundamentally, these are charges levied to meet the cost of unpredictable events of major financial consequence for which provision has not elsewhere been made. Contingency margins are not related to funds maintained for the purpose of absorbing statistical fluctuations; that latter purpose is properly performed by the surplus account of a company issuing nonparticipating life insurance. Contingency margins are required to meet the cost of events so different from expected experience that the statistical estimates are disqualified. For example, the use of mortality estimates derived from the experience of a period free from war or epidemic makes no provision for these contingencies and a separate charge must therefore be included. Other contingencies which must be considered in establishing premium rates for nonparticipating life insurance are capital losses (as distinguished from fluctuations in the book value of assets), the long-term effect of inflation on expense rates, and increased premium and federal income taxes.

It is suggested that provision be made for contingency margins in the calculation of premium rates by some kind of estimate, however crude, of the probable impact of the given contingency. For example, the hazard of war or epidemic might be assessed as twenty-five extra deaths per thousand per century at each attained age and introduced into the calculation by adding .00025 to each mortality rate. Similarly, the contingency margin for capital losses can be introduced by an adjustment to the assumed interest rate; the adjustment might be based on the contribution, ignoring capital gains and losses, to the Mandatory Security Valuation Reserve. The dollar effect of the inflation hazard on expense rates is

probably of minor consequence because the major portion of expenses are either incurred at issue or contractual. Provision for increased premium taxes can be made by an adjustment to the percentage expenses, possibly increasing by duration. The contingency of increased federal income taxes can be handled by a reappraisal of profit objectives. This list of contingencies is not exhaustive nor are the suggested methods of assessing charges unique; each represents one way to introduce the necessary charge. It is of great importance to recognize that contingency margins are *not* profit margins in the real sense despite the fact that, unless realistic reserves are established, they will emerge as such during the periods free from the occurrence of the hazards anticipated. Contingency margins are charges for real but deferred *costs*. Over a very extended period of time true profit may be realized from contingency margins, but only if charges made for contingencies prove to be redundant.

Profit Objectives

The profit objective is that introduced directly into the calculation of theoretical gross premiums. As stated in Section 1, the profit actually realized will differ from the profit objective on account of differences between the actual premium and the theoretical premium and differences between actual experience and estimated experience. Since the calculation of a theoretical gross premium is concerned only with the profit objective, these two differences need not be considered.

For what reason should profits be realized by stockholders on the sale of nonparticipating life insurance? Justification must be found in the service performed by stockholders for policyholders. Despite the dampening effect of the personal relationship between agents and prospects, life insurance sales are still subject to basic economic laws: a transaction is possible only if the lowest price at which the seller will furnish the service is less than the highest price which the buyer is prepared to pay; within this range, the actual price paid is greatly influenced by competitive considerations. Thus, the stockholders' viewpoint determines the minimum premium rate, the policyholders' viewpoint determines the maximum premium rate, and competitive considerations determine the final premium rate between these limits. In setting the profit *objective*, only the stockholders' viewpoint need be considered; the policyholders' viewpoint and competitive considerations affect only the *realization* of this objective. Two distinct profit objectives are worthy of separate consideration: the minimum *aggregate* profit which will be accepted on all business and the minimum *individual* profit which will be accepted at any particular point of the rate array. Premium rates based on the former objective will de-

termine the average level of gross premiums and premiums based on the latter objective will determine the minimum rate which can be charged at any point. Ideally, these two objectives would be equal.

What then is the lowest aggregate and individual profit for which the stockholder is willing to perform this service and to what is it best related? A life insurance company can be viewed as a vehicle for the investment of stockholders' capital and surplus. The investment of capital must be confined to certain types of securities permitted by law, but surplus may be invested in one of three ways:

(1) in tangible assets such as real estate and in stocks, bonds, mortgages and other intangible assets, subject to state regulation of life insurance company investments;
(2) in absorbing the surplus drains which result from the issuance of new business in prospect of future profits on that business;
(3) in expanding the agency plant by means of which additional amounts of new business will be produced, additional surplus invested and additional profit realized.

In investments of the first type, the yield is measured in relation to the amount invested and expressed as a rate which varies from one type of security to another. It is suggested that the same two criteria be applied to the measurement of profit resulting from investments of types 2 and 3 mentioned above; that is:

(1) that the amount of profit be related to the amount of surplus which must be invested to acquire that profit and expressed as a yield rate on the investment; and
(2) that the yield rate be associated with the degree of risk incurred on the type of investment made.

When a new policy is sold and when new agents are recruited, the surplus account of the life insurance company is depleted in virtually every case. It is proposed that the profit objective be defined by the criterion that the present value of the profits which will be received in the future be equal to the present value of the surplus depletion, with both present values based on a yield rate or yield rates which represent adequate return to the stockholders for the degree of risk incurred in expending surplus in the expectation of receiving future profits. That is, the present value of the entire series of profits and losses is zero.

The return on funds invested in an agency organization emerges in a more obscure fashion than that attributable to investment in new business. To introduce profit associated with this type of surplus investment, it is necessary to analyze the development of an agency from its date of organization for the purpose of estimating the costs incurred in that de-

velopment and the volume of business which will result. Such an analysis will differ vastly from company to company. Those companies operating under the general agency system will experience different patterns of cost and return than those companies operating under a branch office system; those companies with a relatively high contractual compensation scheme will incur smaller development costs than those companies with a relatively low contractual compensation scheme. For the purpose of this analysis, it is necessary to make estimates of the following items:

(1) the rate of growth of the agency, in terms of new manpower recruited;
(2) turnover rates of agents;
(3) average production per agent for each contract year;
(4) financing costs per agent by contract year;

TABLE 2

MODEL AGENCY PROJECTION

Agency Year	Commission Value Produced	Financing Costs Incurred	Excess Expenses Incurred	Unearned Renewals Recovered	Net Development Outlay
1	$ 21,675	$5,338	$10,247	$ 0	$15,585
2	36,525	7,198	6,990	142	14,046
3	47,225	7,813	4,644	611	11,846
4	55,775	7,813	2,769	1,368	9,214
5	62,650	7,813	1,261	2,306	6,768
6	68,400	7,813	0	3,343	4,470
7	73,375	7,813		4,428	3,385
8	77,725	7,813		5,532	2,281
9	81,625	7,813		6,652	1,161
10	85,200	7,813		7,772	41
11	88,475	7,813		8,667	− 854
12	91,525	7,813		9,421	−1,608
13	94,450	7,813		10,076	−2,263
14	97,275	7,813		10,720	−2,907
15	99,925	7,813		11,353	−3,540
16	102,525	7,813		11,680	−3,867
17	105,050	7,813		11,809	−3,996
18	107,550	7,813		11,809	−3,996
19	109,975	7,813		11,809	−3,996
20 & subs.	112,300	7,813		11,809	−3,996

Yield Rate	Present Value of Production	Present Value of Outlay	Ratio
15%	$491,928	$46,344	9.42%
20	349,507	45,470	13.01

(5) the excess of actual agency expenses over assumed agency expenses during the early years of the agency (attributable to branch manager's salary, financing of general agents, etc.); and
(6) recoveries of unearned renewal commissions from terminated agents.

From these data estimates can be made of agency development costs in excess of those introduced directly into the premium rate calculation, and of the volume of production which will be realized from the agency. Table 2 traces the development of a model agency from its date of organization to its assumed date of maturity 20 years later. After the 20th year the size of the organization is assumed to be stationary and the net development outlay after that time is the excess of the financing cost for agents hired to replace ones that terminate over the amount recovered from the unearned renewal commissions of agents who have previously terminated.[3] Under a fully vested agency contract this latter amount would be zero. The aggregate production of the agency is measured in terms of the present value of commissions (including contractual expense allowances) on business produced. Since commissions are the basis of determining the compensation of an agency organization, that organization will measure its own performance and effectiveness in accordance with such an index. To make the objectives of the company coincide with those of its agency organization, it is submitted that this index is the best available measure of the aggregate business activity of an agency. The suggested method could also be applied if performance were to be measured in terms of premium volume or amount of insurance, should these indexes be deemed more appropriate. The assumptions underlying the model agency are stated below; the variations of such assumptions from company to company would be very great and it is unlikely that the illustration could be applied to any particular company.

Assumptions:

(1) Four new men hired at beginning of each year
(2) McConney-Guest Modified Agents Survival Table
(3) Average production—$10,000 total commission value per agent, reduced to 80%, 90% and 95% in first three contract years and to 25% in year of termination
(4) Financing costs to produce $4,800 stable income:
1st Year—$1,700 per agent completing year, $850 per terminated agent
2d Year—$1,000 per agent completing year, $500 per terminated agent
3d Year—$500 per agent completing year, $250 per terminated agent

[3] The evaluation of the renewal commissions on agents who have previously terminated could, instead, be included directly in the premium calculations as a modification to the commission assumptions.

(5) Estimated agency costs of $15,000 in 6th year, as provided in premium calculations; expenses for years 1 through 5 assumed to be $15,000; excess expenses based on ratio of production to that of year 6
(6) Agents renewal commission 5% for policy years 2–10, 3% for policy years 11–15; no vesting on production of contract years 1–3, full vesting thereafter
(7) Persistency and other assumptions in accordance with the standard assumptions shown in Appendix A

To provide adequate return to the stockholders for their investment, it is necessary that the value of profits generated by an agency be equal to the value of the investment made in that agency, with both present values based on a yield rate which reflects the degree of risk incurred on amounts invested in this manner. This is somewhat complicated by the fact that additional investment will be made as new business is issued, and provision must be made for adequate return on such amounts. The basic condition of the equality of the value of profits and investment can be satisfied in the following manner:

(1) express the agency development costs as a percentage of commissions;
(2) determine the theoretical gross premium by the criterion that the value of profits be equal to that percentage of the value of commissions.

The rate used for valuation in (1) above is the yield on surplus invested in agency development; the rate used for valuation in (2) above is the yield on surplus invested in new business. These two yield rates are independent.

3. ESTIMATING THE PARAMETERS WHICH AFFECT GROSS PREMIUMS

With the inclusion of specific contingency margins and profit objectives, it is proposed that other assumptions necessary to calculate gross premiums be introduced on the basis of "best estimates" rather than "conservative estimates."

Five items must be established or estimated: mortality, interest, expenses, persistency, and reserves. The nature of the benefits must include a definition of the nonforfeiture values and, in the case of term insurance, the inclusion of a conversion privilege may somewhat alter the kind of mortality estimates which must be made.

A specific set of assumptions is stated in Appendix A. These assumptions have been used to determine the premium rates which are illustrated later in this paper. At this point it is appropriate only to discuss the rationale of certain of the assumptions, and the considerations affecting the choice of specific estimates.

Mortality

The technique which will be described in Section 4 uses mortality rates without the construction of intermediate functions. This allows great flexibility in the selection of mortality assumptions. It is practical to give recognition to a number of factors which have a significant effect on mortality but which are generally ignored because of the complications introduced in calculating premiums. The factors which affect mortality in some degree are: attained age, selection age, selection standards, sex, broad plan groups (such as permanent plans, term plans and possibly decreasing term plans) and amount of insurance. The extent to which each of these factors is reflected is a matter of judgment, strongly influenced by the calculation facilities available: if a computer is used the storage space necessary to retain the mortality table or tables must not surpass the capacities of the machine. The mortality assumptions for the various risk classes can be wholly independent or can be related to one set of mortality rates by some simple formula such as:

$$q' = aq + b\,.$$

Two items which, technically, are not mortality costs should be considered at this point. These items are contingency margins for catastrophic mortality costs and reinsurance costs. Both of these can most conveniently be included as modifications of the mortality rates, although such inclusion incorrectly affects the probability of survival. The effect of this theoretical error is very small.

Contingency margins can be introduced in the manner suggested in Section 2 by increasing each mortality rate by an amount representing the estimated cost of mortality contingencies. In the past, the impact of the war hazard was much greater on male mortality in the age range 18 through 40. To the extent that this is expected to continue, the margin for contingencies might be redistributed to increase the charges at these ages. The distribution of mortality contingency charges is subject to the judgment of each individual actuary.

Reinsurance costs can also be estimated by an adjustment to the mortality rates. Although experience refund formulas and premium rates on yearly renewable term reinsurance vary somewhat from company to company, a satisfactory approximation can be made. Typical reinsurance premium rates are quite close to the mortality rates according to the 1941 CSO Mortality Table; typical experience refund formulas return to the ceding company one half of the difference between 90% of the reinsurance premiums paid and the claim costs incurred. If k represents the pro-

portion of the net amount at risk which is reinsured and q the expected mortality, the following relationship defines the mortality rate, q', which will approximate combined mortality and reinsurance costs:

$$q' = .55k\,q^{\text{CSO}} + (1 - .50k)\,q\,.$$

In a particular case, the exact reinsurance premiums and experience refund formula could be applied to develop a better approximation.

Interest

The proposed premium calculation technique allows complete freedom in selecting interest assumptions because no intermediate functions are involved. The reliability of an estimate of future interest earnings decreases as the period to which it applies becomes more remote. This is especially significant if current interest rates are materially different from expected long-range average yields. An interest assumption varying by duration is probably the most satisfactory estimate when such a condition prevails.

Interest earnings are affected only gradually by the yield rate on new investments; the earnings of a given interval are determined by an average of yields from investment made in many prior years. An estimate of the trend of this aggregate yield can be made with reference to six items:

(1) the aggregate yield on the present holdings of each risk class;
(2) the estimated net new money available each year on account of each risk class;
(3) the yield rate on expected maturities and sales;
(4) the yield rate currently available on new investments;
(5) the yield rate ultimately expected on new investments; and
(6) the rate of change of the yield rate on new investments from current to ultimate levels.

The notion of risk classes for investment purposes deserves some attention. It has been customary for life insurance companies to include all policyholders in one investment risk class. Without violating any law pertaining to segregation of assets, a company might establish several risk classes to take account of major lines of business and, possibly, groups of calendar issue years.

The method of estimating future interest earnings is unaffected by the number of risk classes, although the results of one approach will usually differ from those of another. Today, the yield rate on new investments is higher than the yield on the aggregate holdings of an established company; that aggregate yield rate is higher than the long-term yield rate on

which most companies are prepared to gamble. In this situation, for companies following the single risk class philosophy, the estimated interest earnings would begin at a rate equal to the aggregate yield and continue as a level or slightly increasing rate for a few years, then decline gradually for many years and approach the expected long-term yield rate. For companies establishing new issues as a risk class apart from past issues, the interest estimate would begin at a somewhat higher level, equal to the current rate available on new investments, and decline monotonically to approach the same ultimate rate.

As stated in Section 3, the hazard of capital losses can be conveniently and appropriately reflected by a reduction in the assumed interest rate. For the purpose of assessing a charge for this contingency, a single risk class is clearly indicated in the absence of special circumstances. The amount of the charge should reflect the investment policy of the company and the current distribution of assets.

Expenses

For the purpose of calculating gross premiums, estimates of future insurance expenses are usually expressed as *rates* related to premium volume, amount of insurance or number of policies. Expense rates are determined by policy year of incidence for each policy status (premium paying, paid-up, terminated by death and otherwise terminated).

Many of the expenses of a life insurance company are of an *indirect* nature, comparable to those expenses of a commercial venture which are referred to as "overhead." Such expenses are related only vaguely, if at all, to the usual indexes; it is more appropriate to express them as a function of *volume, as measured by stockholders' profit objectives.* If profit objectives are expressed as a yield on surplus investment, these expenses might be allocated as a charge against *gross* yield; this would determine *net* yield. This would treat indirect expenses in a manner analogous to the allocation of investment expenses and is probably the most precise theoretically. Practical difficulties arise for two reasons:

(1) the adequacy of the expense estimate depends upon the volume of business *and* the volume of surplus invested; the latter may be difficult to control or estimate;
(2) some indirect expenses are more properly related to the amount of business activity rather than total surplus investment; indirect agency expense is a suitable example.

To reduce these practical difficulties, all or part of the indirect expenses might be related to volume as measured by the present value of commissions (and, hence, profit).

The proposed method of introducing expenses allows allocation in the following ways:

(1) expenses varying by policy year per policy in force, expenses per claim and expenses per other termination are used together with a stipulated policy size; for term conversions a conversion expense per policy converted is also used;
(2) percentage-of-premium expenses are divided into commissions (including contractual expense allowances) and other percentage expenses, with complete flexibility by duration;
(3) indirect first year expenses are introduced as a percentage of the present value of commissions; any indirect renewal expenses may either be expressed as a percentage of premium renewal expense, perhaps related to renewal commissions, or included in the first year indirect expense at discounted amount.

Indirect expenses may also be included, in whole or in part, by using a *gross* yield rate as the profit objective. In this case, the premium rate will properly reflect these expenses, but the series of annual profits will not.

The difference between the proposed method of expense allocation and the customary method is the handling of indirect expenses. Clearly, items such as advertising, general research, home office agency department, and association dues are indirect expenses; it is also clear that items such as commissions are direct expenses. For another class of expenses, the distinction is not clear because more than one philosophy can be applied. This class consists primarily of per-policy expenses. To illustrate the philosophic difference, the following example is cited:

> The issue expense of 2,000 policies of a given risk class is $140,000; if only 1,000 such policies were issued, the expense would be $90,000.

The issue expense of this company can be expressed in many ways, each representing a different mixture of the following two extremes:

(1) express the issue expense as the *average cost* of $70 per policy;
(2) express the issue expense as the *marginal cost* of $50 per policy and include the difference between aggregate marginal cost and total expense as an indirect expense of $40,000, which would be merged with other indirect expenses.

Evidently, the marginal cost philosophy substantially increases the importance of indirect expenses. The effect of marginal cost expense rates is especially marked for a company with a low volume of business. Theoretical premium rates for such a company will show spectacular variations for various policy sizes if the average cost philosophy is adopted.

The use of marginal cost expense rates sharply reduces the effect of policy size on premium rates and produces a premium rate pattern by policy size which more closely resembles that of a large company because the expenses allocated on a per-policy basis are reduced.

One obvious objection to the use of marginal cost expense rates is the practical difficulty of determining such rates. Although this problem is outside the scope of the subject under discussion, it might be observed that marginal cost expense rates are made up of that portion of total expense which is sometimes designated as the variant. The invariant expense is immediately part of the indirect expense. Marginal cost expense rates might, therefore, be more easily and more precisely determined than average cost expense rates, since the invariant is the most awkward item to handle in an expense study.

Persistency

Any scale of probabilities of lapse can be used in the proposed method since the probabilities are introduced directly. Despite the fact that significant studies of persistency can be made on considerably smaller groups than significant studies of mortality, it is difficult to find authentic statistical data to support suspected differences in persistency among various groups or risk classes. Broad plan classes, sex, issue age, mode of payment, and amount of insurance probably have significant effects on persistency which should be recognized. Because the extent to which each of these factors affects persistency varies substantially between companies because of differences in underwriting policy, agency objectives and other factors, industry experience is not necessarily a reliable guide for a particular company. Where differences on account of any of these possible classifications are known to be significant, the differences should be recognized in estimating persistency rates for the purpose of calculating gross premiums.

It is difficult to make an accurate estimate of persistency because changes in economic conditions can produce massive shifts in probabilities. Such changes might properly be regarded as contingencies for which provisions should be made, but because the effect of such shifts is greatly different for given plans, ages and durations, it is no easy matter to introduce charges which effectively reflect the true financial impact. The use of conservative estimates of persistency is probably the only practical device for including a contingency margin.

Reserve Basis

The proposed profit objective takes account of realized profit; realized profit is profit released to surplus and available for distribution to or re-

investment by stockholders. Unless the yield rate on invested surplus is exactly equal to the assumed interest rate, the reserve basis will have a significant effect on the value of realized profit. The use of net level premium reserves rather than modified preliminary term reserves increases the amount of invested surplus at issue by an amount which probably averages $20 per $1,000 of insurance. Changes in the valuation interest rates have a less spectacular effect on invested surplus, unless deficiency reserves are involved. There is, for all practical purposes, no choice of mortality tables at this time, but this situation will be changed when the 1958 CSO Table is approved for use in all jurisdictions of the United States. In any case, the selection of a mortality table would have a relatively minor effect on the invested surplus unless, again, the question of deficiency reserves is involved.

The reserves to be used for the purpose of calculating gross premiums according to the suggested profit objective should include the deficiency reserves, if any, based on the calculated premium since this affects markedly the amount of surplus invested in the sale of a given piece of new business. At certain points in the rate array of some nonparticipating companies, deficiency reserves in excess of $100 per $1,000 are required at issue. This factor has a substantial effect on gross premiums. The proposed method takes direct account of the valuation basis and the deficiency reserves produced by the calculated premium.

4. GROSS PREMIUM CALCULATION TECHNIQUE

Definition of Symbols:

The following items are defined by the policy benefits:

$_tD$ = Death benefit, policy year t

$_tS$ = Survival benefit payable to those completing policy year t

$_tCV$ = Cash value at end of policy year t

$_tV$ = Terminal reserve, policy year t

$_tV'$ = Terminal reserve, policy year t, including any deficiency reserve arising from the calculated premium

$_tTA$ = Conversion allowance, end of policy year t, in excess of cash value

T_t = Conversion cost (excess mortality) per $1,000 converted at the end of policy year t

A = Policy size

P = Valuation net renewal premium

n = Premium payment period

m = Frequency of premium payment

The following items are yield rates; each applies to policy year t:

i_t = Interest earned on invested assets

j_t = Profit required on surplus invested in new business

The following items are expense rates and expense related items:

C_t = Commission rate, policy year t

P_t = Other percentage expenses, policy year t

E_t = Expense per policy in force at beginning of policy year t

Q = Claim expense per policy

W = Other termination expense per policy

TE = Conversion expense per policy

a = Indirect expense as a multiple of present value of commissions

b = Required present value of profit as a multiple of present value of commissions

The following items are probabilities of occurrence during policy year t among entrants of that year:

q_t = Probability of death

w_t = Probability of voluntary withdrawal, excluding conversion

c_t = Probability of conversion.

The following items are calculated:

P' = Calculated gross premium

${}_tB$ = Book (realized) profit, as of beginning of policy year t, based on a gross premium equal to the valuation net premium

Z = Present value at issue of book profits, ${}_tB$

X = Present value of commissions based on a gross premium equal to the valuation net premium

Y = Present value of increased profits for each \$1 increase in gross premium from P

Y' = Present value of decreased profits for each \$1 decrease in gross premium from P (algebraically positive)

Formulas

$$
\begin{aligned}
{}_tB = {}_{t-1}\mathrm{V} + \frac{\mathrm{P}}{1+\frac{m-1}{2m}\cdot i_t}\left[1-\frac{m-1}{2m}(w_t+q_t)\right](1-C_t-P_t) \\
-\frac{E_t}{A}-\frac{{}_tD+Q/A}{1+i_t/2}\cdot q_t-\frac{\frac{m+1}{2m}\cdot {}_tCV+\frac{m-1}{2m}\cdot {}_{t-1}CV+W/A}{1+\frac{m+1}{2m}\cdot i_t}\cdot w \\
-\frac{{}_tCV+TA_t+T_t-(E_1-TE)/A}{1+i_t}\cdot c_t \\
-\frac{{}_tS}{1+i_t}\left(1-\frac{m-1}{m}\cdot w_t-q_t\right)-\frac{{}_t\mathrm{V}}{1+i_t}(1-q_t-w_t-c_t)
\end{aligned}
$$

where $t \leq n$; where $t > n$, apply the same formula with $\mathrm{P} \equiv 0$.

$$Z = \sum_{1}^{\infty} {}_{t}B\, F_t$$

where $F_1 \equiv 1$ and

$$F_{t+1} = \frac{1 - q_t - w_t - c_t}{1 + j_t} \cdot F_t$$

$$X = \mathrm{P} \cdot \sum_{1}^{n} C_t F_t$$

$$Y = \sum_{1}^{n} \frac{\left[1 - \frac{m-1}{2m}(w_t + q_t)\right](1 - C_t - P_t)}{1 + \frac{m-1}{2m} \cdot i_t} \cdot F_t$$

$$Y' = Y + \sum_{1}^{n} \left[\frac{\ddot{a}_t (1 - q_t - w_t - c_t)}{1 + i_t} - \ddot{a}_{t-1}\right] F_t,$$

whence

$$Y' = Y + \sum_{1}^{n} \frac{\ddot{a}_t}{1 + i_t} (j_t - i_t) F_{t+1},$$

where $\ddot{a}_t$ represents the present value of a life annuity due for the remainder of the premium period on the valuation basis and where $\ddot{a}_0 = 0$. Let

$$Z' = Z + (\mathrm{P}' - \mathrm{P})\, Y \qquad \text{where} \qquad \mathrm{P}' \geq \mathrm{P},$$

and

$$Z' = Z + (\mathrm{P}' - \mathrm{P})\, Y' \qquad \text{where} \qquad \mathrm{P}' < \mathrm{P}.$$

Let

$$X' = \frac{\mathrm{P}'}{\mathrm{P}} \cdot X.$$

The required relationship is then:

$$Z' = (a + b)\, X';$$

i.e.,

$$+ (\mathrm{P}' - \mathrm{P})\, Y = (a + b) \frac{\mathrm{P}'}{\mathrm{P}} \cdot X$$

or

$$\mathrm{P}' = \frac{Y\mathrm{P}^2 - Z\mathrm{P}}{Y\mathrm{P} - (a + b)\, X}, \qquad \text{if } \mathrm{P}' \geq \mathrm{P};$$

similarly,

$$\mathrm{P}' = \frac{Y'\mathrm{P}^2 - Z\mathrm{P}}{Y'\mathrm{P} - (a + b)\, X}, \qquad \text{if } \mathrm{P}' < \mathrm{P}.$$

The actual book profit, ${}_tB'$, can be determined by the formula for ${}_tB$, substituting ${}_tV'$ for ${}_tV$ and P' for P, and introducing the first year indirect expense of aX' and the results may be checked by:

$$bX' = \sum_{1}^{\infty} {}_tB' F_t$$

Description

A trial calculation is made using the valuation net premium as the test gross premium. This trial premium is then adjusted to satisfy the criterion that the present value of profit and indirect expense equal the sum of:

a — the indirect expense and
b — the required profit,

both expressed as a multiple of the present value of commissions. The use of the valuation net premium as a trial premium simplifies the problem of defining the deficiency reserve.

Comments

The calculation technique will produce a premium which exactly satisfies the condition imposed, subject only to the limitation of determining a premium rate to the nearest cent. Interest functions for fractional parts of a year are approximated by simple interest methods.

In practice, the discount function F_t becomes quite small after twenty years and the summation can be stopped at any desired duration by specifying:

$$w_r = 1 - q_r - c_r .$$

This is equivalent to the assumption that all policies surrender at the end of r years. It is also equivalent to the assumption that the difference between the reserve and cash value at duration r is equal to the then present value of profits after duration r.

If the frequency of payment is annual and if no conversion benefits are involved, numerous simplifications develop. Despite these simplifications and that of terminating the calculation prior to maturity, the arithmetic is sufficiently extensive to be practical only with a computer of the capacity of the IBM 650. The specimen calculations exhibited later were performed by such a machine.

It should be noted that for the calculation of other than annual premiums the deficiency reserve is defined as the difference between the fractional premium multiplied by the frequency of payment and the valuation net premium. In such cases, it might be appropriate to use the fractional valuation premium. Note, too, that the fractional premium is

a *true* one as distinguished from the apportionable or installment type. Either could be calculated by altering the term:

$$\frac{m-1}{2m} \cdot q_t .$$

For apportionable premium, this would become:

$$\frac{k-1}{2k} \cdot q_t ,$$

where k is the annual number of adjustment intervals (12 for months, 52 for weeks, or 365 for days).

For installment premiums, this would become zero.

5. DETERMINING THE AGGREGATE VALUE OF A LIFE INSURANCE COMPANY

The value of a life insurance company is not adequately represented by the total of its capital and surplus. A more realistic value of an entire company must take account of its business in force and agency organization. A value of these nonledger assets is frequently needed for one or more of the following purposes:

(1) determining an equitable basis of merger with another company;
(2) establishing a fair price for an outright sale;
(3) calculating the amount of assets to be transferred as a result of reinsuring a block of business;
(4) testing the reasonableness of the offering price of an additional issue of stock;
(5) demonstrating the soundness of a plan of mutualization;
(6) tracing *real* earnings by taking account of changes in the aggregate *real* worth of the company.

The proposed technique for calculating gross premiums for nonparticipating life insurance suggests a method of determining the aggregate worth of a block of nonparticipating business and of an agency organization engaged in marketing this product. The suggested method operates in the following manner:

(1) the worth of a block of nonparticipating business would be the present value of unrealized profits on business now in force, discounted at a rate representing adequate return to the investor on the total value; and
(2) the worth of an agency organization would be the present value of profits on business expected to be produced in the future, less the present value of net development outlay.

Valuation of Business

Consider, first, the problem of evaluating a block of business in force; suppose that ${}_tB$ represents the book profit, based on the actual gross premium, realized in policy year t and ${}_kZ$ is the value of unrealized profits on business at duration k:

$$ {}_kZ = \frac{\sum_k^\infty {}_tBF_t}{F_k}, $$

where F_t is the discount factor previously defined. By constructing a model office and calculating appropriate values of ${}_kZ$, the value of a block of business could be estimated.

This technique is superior to a gross premium valuation because future earnings can be capitalized at *any* desired yield rate. The difference between gross and net premium valuations represents the value of future profits *capitalized at the interest rate assumed* in the gross premium valuation; it is probable that this value would be viewed as excessive by the sophisticated investor, who would expect a higher return on his investment.

The values ${}_kZ$ can be developed as a by-product of the premium calculation. This is convenient for a company that wants to maintain an inventory of the value of its business in force. Specimen values are illustrated in Section 6.

Valuation of Agency Organization

To assign a value to an agency organization, two viewpoints might be taken:

(1) regard present agents as a closed group and estimate future production and net development outlay on this basis; or
(2) regard the present organization as perpetual and estimate future production and net development outlay on this basis.

Future production must be estimated according to some index; again, it is suggested that production be measured as the discounted value of all commissions on the issue date, but other indexes might also be used. Next, an examination of current premium rates must be made to determine a relationship between the unit of production and the present value of profits at issue, discounted at a rate representing adequate return to stockholders on surplus invested in issued business. Finally, the net development outlay is estimated; this is the excess of amounts spent financing replacements for terminated agents over amounts recovered on account of unearned renewal commissions.

Let ${}^nP$ denote production in year n; let a denote the present value of profit at issue per unit of production; let ${}^nD$ denote net development outlay in year n; then H, the value of the agency organization, is given by:

$$H = \sum_{n=0}^{\infty} (a\,{}^nP - {}^nD)\; v^{n-1/2},$$

where v is the usual present value function at an interest rate, r, representing adequate return to stockholders on surplus invested in future production.

If the agency organization is viewed as perpetual and stationary then ${}^nP$ and ${}^nD$ would become constant and the following simplification can be made:

$$H = \frac{a\,{}^nP - {}^nD}{r\,v^{1/2}}.$$

$$= a'\,{}^nP\,,$$

where

$$a' = \frac{a - {}^nD/{}^nP}{r\,v^{1/2}}.$$

This last formula offers a practical method of maintaining a current approximation to the value of an agency organization.

Although the suggested method is aimed primarily at nonparticipating life insurance, it might also be applied to the problem of valuing participating insurance. It would first be necessary to establish an estimated dividend scale or a relationship between estimated book profit and estimated dividends. Because unfavorable experience can be reflected in dividends, a more optimistic assumption regarding future experience might be warranted. Alternately, the discount rate applied to the profits might be lower than that deemed appropriate for nonparticipating insurance.

6. EXHIBIT OF RESULTS

Premium Calculations

Specimen premium rates were calculated by the proposed method in accordance with the assumptions stated in Appendix A. These assumptions were selected as representative of a general industry experience but apply to no specific company.

The calculated premium rates are confined to the Whole Life plan, decennial issue ages and four policy sizes. Table 3 illustrates the complete output of the calculation for age 35; the calculated items are:

(1) discount factor, F_t;

(2) total reserve, if a deficiency reserve is required;

TABLE 3

SPECIMEN CALCULATION EXHIBIT: WHOLE LIFE—AGE 35

(*a*) POLICY AMOUNT $3,000

Issue Age	Plan Code	Policy Amount	Per-Policy Expense				Level % Expense	Linton A Factor	
			Initial	Renewal	Death	Lapse		Initial	Renewal
35	1	$3,000	$40.00	$4.00	$25.00	$5.00	2.0%	100%	100%

Policy Year	Discount Factor	Commission Rate	Cash Value	Terminal Reserve	Book Profit per $1,000		
					In Force	Issued	Value of
1	1.0000000	78.0%			$−13.84	$−13.84	$ 2.42
2	.7815913	7.5%		$ 15.69	+ 3.26	+ 2.93	20.80
3	.6379484	7.5%	$ 14.42	31.65	3.32	2.80	21.49
4	.5261243	7.5%	30.94	47.88	3.30	2.64	22.04
5	.4365185	7.5%	47.71	64.35	3.29	2.51	22.58
6	.3635630	7.5%	64.73	81.08	3.19	2.33	23.16
7	.3039355	7.5%	82.00	98.04	3.25	2.28	23.89
8	.2550706	7.5%	99.50	115.24	3.29	2.23	24.60
9	.2146585	7.5%	117.23	132.66	3.34	2.19	25.32
10	.1809608	7.5%	135.17	150.28	3.40	2.16	26.07
11	.1528128	5.0%	153.31	168.11	4.00	2.47	26.85
12	.1291241	5.0%	171.63	186.11	4.06	2.44	27.04
13	.1091716	5.0%	190.13	204.29	4.06	2.37	27.18
14	.0923525	5.0%	208.79	222.62	4.06	2.31	27.33
15	.0781616	5.0%	227.58	241.08	4.03	2.23	27.50
16	.0661784	2.0%	246.50	259.67	4.65	2.50	27.72
17	.0559944	2.0%	265.52	278.36	4.60	2.41	27.26
18	.0473420	2.0%	284.63	297.14	4.54	2.31	26.80
19	.0399954	2.0%	303.80	315.97	4.49	2.22	26.35
20	.0337606	2.0%	323.02	334.85	4.41	2.12	25.90
21	.0284713	2.0%	342.25	353.75	4.47	2.08	25.48
22	.0239862	2.0%	361.49	372.65	4.53	2.05	24.94
23	.0201847	2.0%	380.70	391.53	4.58	2.00	24.25
24	.0169644	2.0%	399.87	410.36	4.62	1.95	23.41
25	.0142383	2.0%	418.96	429.12	4.67	1.90	22.39
26	.0119325	2.0%	437.96	447.78	4.71	1.85	21.14
27	.0099836	2.0%	456.84	466.33	4.76	1.80	19.64
28	.0083382	2.0%	475.56	484.73	4.82	1.75	17.81
29	.0069507	2.0%	494.13	502.97	4.88	1.70	15.59
30	.0057820	2.0%	512.49	521.01	12.87	4.28	12.87

X_{18} Factor	X_{18} Constant	Indirect Expenses	Yield Rate	Present Value at Issue of			Net Premium	Gross Premium
				Book Profit	$1.00 Premium	Commissions		
100%	.00025	$3.66	15.0%	$2.42	$4.46	$24.41	$19.88	$22.35

TABLE 3—*Continued*

(*b*) POLICY AMOUNT $6,000

Issue Age	Plan Code	Policy Amount	Per-Policy Expense				Level % Expense	Linton A Factor	
			Initial	Renewal	Death	Lapse		Initial	Renewal
35	1	$6,000	$45.00	$5.00	$30.00	$6.00	2.0%	100%	100%

Policy Year	Discount Factor	Commission Rate	Cash Value	Terminal Reserve	Book Profit per $1,000		
					In Force	Issued	Value of
1	1.0000000	78.0%			$− 8.02	$−8.02	$ 2.25
2	.7815913	7.5%		$ 15.69	+ 2.00	+1.80	13.14
3	.6379484	7.5%	$ 14.42	31.65	2.05	1.73	13.65
4	.5261243	7.5%	30.94	47.88	2.03	1.62	14.06
5	.4365185	7.5%	47.71	64.35	2.01	1.53	14.50
6	.3635630	7.5%	64.73	81.08	1.91	1.40	15.00
7	.3039355	7.5%	82.00	98.04	1.97	1.38	15.66
8	.2550706	7.5%	99.50	115.24	2.01	1.36	16.31
9	.2146585	7.5%	117.23	132.66	2.06	1.35	17.00
10	.1809608	7.5%	135.17	150.28	2.13	1.36	17.72
11	.1528128	5.0%	153.31	168.11	2.67	1.65	18.46
12	.1291241	5.0%	171.63	186.11	2.73	1.64	18.68
13	.1091716	5.0%	190.13	204.29	2.73	1.59	18.87
14	.0923525	5.0%	208.79	222.62	2.73	1.55	19.08
15	.0781616	5.0%	227.58	241.08	2.70	1.49	19.32
16	.0661784	2.0%	246.50	259.67	3.27	1.76	19.63
17	.0559944	2.0%	265.52	278.36	3.21	1.68	19.33
18	.0473420	2.0%	284.63	297.14	3.16	1.61	19.07
19	.0399954	2.0%	303.80	315.97	3.11	1.54	18.83
20	.0337606	2.0%	323.02	334.85	3.03	1.46	18.62
21	.0284713	2.0%	342.25	353.75	3.10	1.44	18.49
22	.0239862	2.0%	361.49	372.65	3.16	1.43	18.27
23	.0201847	2.0%	380.70	391.53	3.21	1.40	17.95
24	.0169644	2.0%	399.87	410.36	3.27	1.38	17.54
25	.0142383	2.0%	418.96	429.12	3.32	1.35	17.00
26	.0119325	2.0%	437.96	447.78	3.37	1.32	16.33
27	.0099836	2.0%	456.84	466.33	3.41	1.29	15.49
28	.0083382	2.0%	475.56	484.73	3.49	1.27	14.46
29	.0069507	2.0%	494.13	502.97	3.56	1.24	13.16
30	.0057820	2.0%	512.49	521.01	11.54	3.84	11.54

X_{18} Factor	X_{18} Constant	Indirect Expenses	Yield Rate	Present Value at Issue of			Net Premium	Gross Premium
				Book Profit	$1.00 Premium	Commissions		
100%	.00025	$3.34	15.0%	$2.25	$4.46	$22.23	$19.88	$20.35

TABLE 3—*Continued*

(*c*) POLICY AMOUNT $12,000

Issue Age	Plan Code	Policy Amount	Per-Policy Expense				Level % Expense	Linton A Factor	
			Initial	Renewal	Death	Lapse		Initial	Renewal
35	1	$12,000	$55.00	$7.00	$40.00	$8.00	2.0%	100%	100%

Policy Year	Discount Factor	Commission Rate	Cash Value	Terminal Reserve	Book Profit per $1,000		
					In Force	Issued	Value of
1	1.0000000	78.0%		$ 3.26	$− 7.90	$−7.90	$ 2.24
2	.7815913	7.5%		18.90	+ 2.06	+1.85	12.97
3	.6379484	7.5%	$ 14.42	34.81	2.06	1.74	13.37
4	.5261243	7.5%	30.94	50.99	2.02	1.62	13.71
5	.4365185	7.5%	47.71	67.40	2.00	1.53	14.09
6	.3635630	7.5%	64.73	84.08	1.88	1.37	14.51
7	.3039355	7.5%	82.00	100.98	1.93	1.36	15.11
8	.2550706	7.5%	99.50	118.13	1.95	1.32	15.71
9	.2146585	7.5%	117.23	135.49	2.00	1.31	16.35
10	.1809608	7.5%	135.17	153.05	2.05	1.31	17.02
11	.1528128	5.0%	153.31	170.83	2.57	1.59	17.73
12	.1291241	5.0%	171.63	188.77	2.61	1.57	17.94
13	.1091716	5.0%	190.13	206.89	2.63	1.54	18.13
14	.0923525	5.0%	208.79	225.16	2.60	1.48	18.32
15	.0781616	5.0%	227.58	243.56	2.58	1.43	18.57
16	.0661784	2.0%	246.50	262.09	3.13	1.69	18.89
17	.0559944	2.0%	265.52	280.72	3.06	1.60	18.63
18	.0473420	2.0%	284.63	299.43	3.02	1.54	18.41
19	.0399954	2.0%	303.80	318.20	2.97	1.47	18.22
20	.0337606	2.0%	323.02	337.02	2.88	1.38	18.06
21	.0284713	2.0%	342.25	355.86	2.95	1.37	18.00
22	.0239862	2.0%	361.49	374.70	3.01	1.36	17.87
23	.0201847	2.0%	380.70	393.52	3.05	1.33	17.66
24	.0169644	2.0%	399.87	412.29	3.11	1.31	17.38
25	.0142383	2.0%	418.96	430.98	3.18	1.30	17.00
26	.0119325	2.0%	437.96	449.58	3.21	1.26	16.49
27	.0099836	2.0%	456.84	468.07	3.27	1.24	15.88
28	.0083382	2.0%	475.56	486.41	3.34	1.21	15.09
29	.0069507	2.0%	494.13	504.59	3.41	1.19	14.10
30	.0057820	2.0%	512.49	522.57	12.85	4.28	12.85

X_{18} Factor	X_{18} Constant	Indirect Expenses	Yield Rate	Present Value at Issue of			Net Premium	Gross Premium
				Book Profit	$1.00 Premium	Commissions		
100%	.00025	$3.23	15.0%	$2.24	$13.96	$21.58	$19.88	$19.72

TABLE 3—*Continued*

(*d*) POLICY AMOUNT $30,000

Issue Age	Plan Code	Policy Amount	Per-Policy Expense				Level % Expense	Linton A Factor	
			Initial	Renewal	Death	Lapse		Initial	Renewal
35	1	$30,000	$65.00	$13.00	$70.00	$14.00	2.0%	100%	100%

Policy Year	Discount Factor	Commission Rate	Cash Value	Terminal Reserve	Book Profit per $1,000		
					In Force	Issued	Value of
1	1.0000000	78.0%		$ 8.16	$− 9.73	$−9.73	$ 2.09
2	.7815913	7.5%		23.72	+ 2.54	+2.28	15.12
3	.6379484	7.5%	$ 14.42	39.55	2.50	2.11	15.41
4	.5261243	7.5%	30.94	55.65	2.41	1.93	15.65
5	.4365185	7.5%	47.71	71.99	2.37	1.81	15.96
6	.3635630	7.5%	64.73	88.58	2.24	1.64	16.32
7	.3039355	7.5%	82.00	105.40	2.25	1.58	16.84
8	.2550706	7.5%	99.50	122.46	2.25	1.53	17.38
9	.2146585	7.5%	117.23	139.74	2.28	1.50	17.98
10	.1809608	7.5%	135.17	157.22	2.32	1.48	18.63
11	.1529128	5.0%	153.31	174.90	2.84	1.76	19.31
12	.1291241	5.0%	171.63	192.75	2.88	1.73	19.49
13	.1091716	5.0%	190.13	210.78	2.87	1.68	19.65
14	.0923525	5.0%	208.79	228.97	2.86	1.63	19.84
15	.0781616	5.0%	227.58	247.27	2.83	1.57	20.06
16	.0661784	2.0%	246.50	265.71	3.34	1.80	20.35
17	.0559944	2.0%	265.52	284.25	3.28	1.72	20.10
18	.0473420	2.0%	284.63	302.88	3.23	1.65	19.90
19	.0399954	2.0%	303.80	321.55	3.19	1.58	19.73
20	.0337606	2.0%	323.02	340.28	3.10	1.49	19.59
21	.0284713	2.0%	342.25	359.02	3.18	1.48	19.56
22	.0239862	2.0%	361.49	377.77	3.21	1.45	19.44
23	.0201847	2.0%	380.70	396.50	3.27	1.43	19.28
24	.0169644	2.0%	399.87	415.17	3.33	1.41	19.05
25	.0142383	2.0%	418.96	433.78	3.38	1.38	18.73
26	.0119325	2.0%	437.96	452.29	3.43	1.35	18.32
27	.0099836	2.0%	456.84	470.69	3.47	1.31	17.80
28	.0083382	2.0%	475.56	488.94	3.56	1.29	17.15
29	.0069507	2.0%	494.13	507.03	3.63	1.26	16.31
30	.0057820	2.0%	512.49	524.92	15.24	5.07	15.24

X_{18} Factor	X_{18} Constant	Indirect Expenses	Yield Rate	Present Value at Issue of			Net Premium	Gross Premium
				Book Profit	$1.00 Premium	Commissions		
100%	.00025	$3.19	15.0%	$2.09	$13.96	$21.29	$19.88	$19.48

(3) book profit per $1,000 in force, ${}_tB$;
(4) book profit per $1,000 issued;
(5) present value of unrealized profits by duration;
(6) indirect first year expenses per $1,000;
(7) present value of book profits at issue;
(8) present value of additional profit at issue per $1 additional premium;
(9) present value of commissions at issue;
(10) calculated gross premium.

The proof of the calculation is the criterion that the indirect expense and present value of profit equal 15% and 10%, respectively, of the present value of commissions. With allowance for the limitation of determining annual book profits and the gross premium to the nearest cent, this condition is satisfied. The lapse and mortality factors shown are a result of the particular program used for these calculations; this program was designed to accommodate any multiple of a standard set of lapse and death probabilities.

The discount factors and present value of additional profit per $1 additional premium are useful for testing the effect of variations in expense assumptions; for example, commissions may be redistributed, in such a way that their present value is unchanged, and hence the premium is unchanged. Any number of approximate adjustments to the premium can be made without recalculating the book profit series. Table 4 compares the entire set of premium rates to the mean and range of the premiums of ten stock companies presented in Table 1.

The weighted average of the calculated premiums is 2.5% higher than the composite average of all ten companies illustrated. With three exceptions, the calculated rates are within the competitive range. The characteristics of the calculated rates are:

(1) higher relative rates for smaller policies, due to the larger surplus drain at issue and the consequent increase in required profit;
(2) higher relative rates at points where deficiency reserves are required, especially ages 45 and up and larger policy sizes;
(3) approximate quantity discounts of $2 from $3,000 to $6,000, $.75 from $6,000 to $12,000, and $.30 from $12,000 to $30,000, compared to approximate average quantity discounts of $1.35, $.90, and $.40 respectively;
(4) premium rate decreases by policy size which do not follow a reciprocal pattern—that is, the kind of pattern which could be closely approximated by a policy fee;
(5) substantial variation in quantity discounts by age of issue;

(6) more favorable competitive position for amounts of $6,000 and $12,000.

One general comment might be made with regard to the profit objective: the invested surplus might be regarded as "secured" by the difference between the reserve and the cash value, and the yield rate might be adjusted downward when this "security" is substantial. This would be a practical application of the suggested association of gain and risk. The adoption of such a philosophy would increase the effect of policy size on premiums, particularly at those points where deficiency reserves are required.

TABLE 4

COMPARISON OF CALCULATED AND TYPICAL NONPARTICIPATING WHOLE LIFE PREMIUM RATES

Age	Amount	Calculated Premium	Typical Nonparticipating Premiums		
			High	Mean	Low
15	$ 3,000	$13.16	$14.38	$12.53	$11.46
	6,000	11.38	12.07	11.18	9.80
	12,000	10.39	10.95	10.36	8.96
	30,000	9.90	10.50	9.96	8.46
25	3,000	16.60	17.74	16.12	15.30
	6,000	14.82	15.48	14.77	14.30
	12,000	13.82	14.18	13.91	13.55
	30,000	13.65	13.82	13.51	13.30
35	3,000	22.35	23.45	21.65	20.82
	6,000	20.35	21.25	20.30	19.80
	12,000	19.72	19.75	19.43	19.05
	30,000	19.48	19.32	19.02	18.60
45	3,000	32.33	32.38	30.58	29.85
	6,000	29.85	29.95	29.24	28.69
	12,000	29.37	29.03	28.36	27.64
	30,000	29.04	28.78	27.96	27.26
55	3,000	48.11	48.84	45.30	44.09
	6,000	46.02	46.84	43.95	42.75
	12,000	45.34	46.09	43.03	42.00
	30,000	44.90	45.84	42.63	41.55
65	3,000	75.21	81.43	73.29	70.28
	6,000	73.27	79.43	71.95	69.05
	12,000	72.25	78.68	70.71	68.43
	30,000	71.58	78.43	70.31	68.05
Weighted Average.....		$24.15	$24.74	$23.56	$22.85

Value of Business

In Table 3, there is a year by year tabulation of the present value of remaining profits discounted in accordance with the assumptions made for calculating premiums for age 35. Table 5 summarizes the results at sample durations for other issue ages.

TABLE 5

VALUE PER $1,000 WHOLE LIFE IN FORCE

AGE	AMOUNT	VALUE AT BEGINNING OF POLICY YEAR				
		2	5	10	15	20
15	$ 3,000	$17.23	$19.21	$22.31	$24.48	$24.47
	6,000	10.62	12.18	14.91	17.14	17.88
	12,000	6.78	8.14	10.66	12.91	14.02
	30,000	5.44	6.63	9.02	11.26	12.61
25	3,000	17.87	20.51	24.25	25.65	24.52
	6,000	11.31	13.56	16.93	18.43	18.03
	12,000	7.45	9.44	12.61	14.17	14.21
	30,000	9.28	11.09	14.07	15.54	15.62
35	3,000	20.80	22.58	26.07	27.50	25.90
	6,000	13.14	14.50	17.72	19.32	18.62
	12,000	12.97	14.09	17.02	18.57	18.06
	30,000	15.12	15.96	18.63	20.06	19.59
45	3,000	27.40	27.26	31.38	33.01	30.53
	6,000	17.44	16.92	20.92	23.00	21.75
	12,000	20.27	19.35	22.90	24.77	23.49
	30,000	22.51	21.29	24.51	26.16	24.85
55	3,000	35.31	27.83	32.40	35.81	36.44
	6,000	33.25	25.26	29.36	32.81	34.01
	12,000	36.64	28.19	31.70	34.80	35.84
	30,000	38.78	30.00	33.10	36.00	36.92
65	3,000	64.84	36.59	43.25	47.44	45.22
	6,000	70.75	41.81	47.25	50.76	48.25
	12,000	73.79	44.47	49.25	52.38	49.73
	30,000	75.67	46.14	50.50	53.39	50.62

For durations above 20, the results are unreliable because of the assumption that all policies surrender after 30 years. At the young ages at issue, this assumption has a significant effect even on the 20th year values. The extent of the error could be estimated and appropriate adjustment made if a more precise value were required.

The values per $1,000 of business in force shown in Table 5 exhibit the following characteristics:

(1) much smaller values on larger policies, unless deficiency reserves are involved;
(2) higher values at higher ages;
(3) rapid reduction in value at upper ages due to select mortality gains;
(4) large and erratic variations, difficult to relate to a simple index;
(5) irregular change from durations 2 to 5, consistent increase from durations 5 to 10, and stable, usually slightly increasing values from durations 10 through 20.

The values illustrated must be interpreted with care to avoid unwarranted conclusions. It is clear, however, that some of the broad rules-of-thumb such as "$20 per $1,000" and "one year's premium" may be dangerously crude estimates of the value of a particular block of business.

Effect of Variation of Factors

To illustrate the effect of varying certain assumptions, additional premiums were calculated with certain assumptions changed in each case. To avoid altering the indirect expense and present value of profit because of alteration of the gross premium, the amount of both items was fixed at that amount determined from the "standard" assumptions. The additional premiums are confined to age at issue 35. The following alterations were made:

(1) Less Favorable Mortality: Assumed mortality was changed to 125% of the select modification of Mortality Table X_{18}, plus the catastrophe allowance of .00025; other assumptions were unchanged.
(2) 100% Y.R.T. Reinsurance: The entire risk was assumed reinsured and mortality rates modified in the following way:

$$q' = .55\,q^{\text{CSO}} + .50\,q^{X_{18}};$$

the assumed per-policy expenses were increased $5 first year and $1 renewal; other assumptions were unchanged.
(3) Less Favorable Interest: The interest rate was assumed to be $3\frac{1}{2}\%$ for policy year 1, declining .05% in each of the subsequent 10 years, reaching 3% in policy year 11 and remaining constant thereafter; other assumptions were unchanged.
(4) Less Favorable Persistency: Linton B termination rates were assumed as probabilities of voluntary withdrawal; other assumptions were unchanged.
(5) Higher Values and Reserves: 1941 CSO $2\frac{1}{4}\%$ minimum values and CRVM reserves were assumed; other assumptions were unchanged.

(6) Monthly Payment: Monthly payment of premiums was assumed, with true monthly premiums; assumed first year and renewal per-policy expenses were increased $3; Linton B termination rates were assumed; the deficiency reserve was determined by comparison of the total yearly premium with the valuation net premium based on monthly payment of premiums; other assumptions were unchanged.

Premium rates developed in accordance with these assumptions are shown in Table 6.

The effect of increased mortality rates is surprisingly small. Where premiums are not deficient, the rate increase resulting from 25% extra mortality is less than 3½%; deficient premiums increase less than 1½%.

The effect of reinsurance is, on the other hand, surprisingly great. Because the illustration is quite artificial and because the assumption of 100% Y.R.T. reinsurance is unrealistic, the results require careful scrutiny

TABLE 6

EFFECT OF VARIATION OF CERTAIN ASSUMPTIONS
WHOLE LIFE AGE 35 PREMIUM PER $1,000

Policy Size	Standard Assumptions	Less Favorable Mortality	100% Y.R.T. Reinsurance	Less Favorable Interest	Less Favorable Persistency	Higher Values and Reserves	Monthly Payment
$ 3,000	$22.35	$23.06	$25.59	$22.66	$23.21	$22.78	$25.67
6,000	20.35	21.05	23.20	20.66	20.67	20.79	22.27
12,000	19.72	20.08	22.03	19.82	19.74	20.32	20.61
30,000	19.48	19.70	21.16	19.59	19.43	20.08	20.04

if false conclusions are to be avoided. One conclusion is nevertheless evident: retention limits, whether current or projected, have an appreciable impact on premium rates and profit. The calculations suggest that the additional premium on account of reinsurance is $2.46 per $1,000 plus $2.34 per policy; both amounts are smaller if deficiency reserves are involved, since a premium increase reduces the required deficiency reserve. This statement of the effect of reinsurance can be used to determine a premium for any retention.

The increase in the premium rate due to reduced interest earnings is even smaller than the effect of higher mortality. The distribution of the changes in premium by amount is uniform, except as it is dampened by deficiency reserve requirements.

The impact of increased lapse rates is highly sensitive to policy size. The pattern of rate decreases by size becomes $2.54, $.93 and $.31 compared to "standard" differences of $2, $.63 and $.24. An astonishing

property of these premiums can be observed at policy size $30,000: the premium rate per $1,000 is *lower* than the comparable standard premium. This is explained by the more rapid recovery of the deficiency reserve.

The distribution of the premium rate increases attributable to increased nonforfeiture values and reserves is opposite to that due to any of the other variations. Premium rates for larger amounts increase more than for smaller amounts because of the increase in the valuation net premium and deficiency reserve requirements.

For a policy size of $3,000, the annualized monthly premium rate is 15% above the corresponding standard premium; this increase reduces to 3% for a policy size of $30,000. The relationship between the monthly and annual premium suggested by this illustration is:

$$\tfrac{1}{12}\,P^{(12)} = .0846P + \frac{.75}{A},$$

where A represents policy size in thousands. This relationship is quite different from that traditionally used for fractional premiums. For a company using a policy fee system, it is a simple matter to include a "collection charge" and closely approach the indicated relationship.

CONCLUSIONS

The following conclusions are suggested by this analysis of gross premiums and profit objectives:

(1) recent developments in the industry have made it possible, and perhaps necessary, to refine rate structures for nonparticipating insurance;
(2) the suggested method of handling contingency and profit margins is one which makes sense to stockholders;
(3) the proposed calculation technique allows complete flexibility in selecting assumptions without great attendant increase in the work required to determine premium rates;
(4) a related technique offers a suitable basis for assigning value to a block of business or an agency organization;
(5) premium rates determined by the proposed method and technique are not unrealistic competitively, given typical assumptions; there are, however, significant and systematic departures from the usual pattern;
(6) direct recognition of such factors as mode of payment, retention limits, deficiency reserve requirements and other variables may suggest certain modifications or refinements of the customary rate pattern to recognize the effects of such factors upon premium rates.

APPENDIX A

SUMMARY OF STANDARD ASSUMPTIONS

Mortality

A select modification of Mortality Table X_{18} (Appendix B) with each mortality rate increased .00025 for a contingency margin of 25 extra deaths per 1,000 per century.

Interest

3.75% for policy years 1 through 5, decreasing .05% each year thereafter, reaching 3.00% in policy year 20 and remaining constant thereafter.

Dollar Expenses (per policy)

Item	Age	Policy Size			
		$3,000	$6,000	$12,000	$30,000
(*a*) First Year	15 & 25	$35	$40	$45	$60
	35	40	45	55	65
	45	50	55	65	70
	55	60	65	70	75
(*b*) Renewal Years	all	4	5	7	13
(*c*) Claim Expense	all	25	30	40	70
(*d*) Other Termination Expenses	all	5	6	8	14

Percentage Expenses

(*a*) Commissions (including expense reimbursement allowance of 30% of first year commission)

Plan	Age	Commission Rate for Policy Year			
		1	2–10	11–15	Subs.
Whole Life	all	78%	7½%	5%	2%

(*b*) Other Percentage Expenses: 2% for premium taxes; all ages, policy years and policy sizes.

Indirect Expenses

First year expense equal to 15% of the present value of commissions.

Persistency

Linton A termination rates used as probabilities of voluntary withdrawal; all policies are assumed to surrender after 30 years.

Nonforfeiture Values

1941 CSO 3% minimum values.

Reserves

1941 CSO 3% CRVM reserves, including deficiency reserves.

Mode of Payment

Annual.

Profit Objectives

(*a*) Yield on surplus invested in new business: 15%.

(*b*) Present value of profit as a multiple of present value of commissions: 10%.

APPENDIX B

MORTALITY TABLE X_{18} (WITH SELECT MODIFICATION)

Mortality Table X_{18} represents the experience of fifteen large companies between policy anniversaries in 1950 and 1954, excluding policy years 1 through 5. The following select modification was prepared for use in calculating premiums for nonparticipating insurance and is based on the published intercompany experience for the same period.

Mr. Norman F. Buck has previously published a select modification of this table together with a lucid discussion of three techniques for determining select rates.[4] Because Mortality Table X_{18} was derived from a pool of experience weighted heavily toward early durations, a conservative estimate of select mortality is indicated as most appropriate for calculating gross premiums. The following table was developed in a manner similar to the second of Mr. Buck's three suggested methods: coefficients of selection were determined for policy year 1, related to the ultimate mortality rate at the same attained age, and graduated with the criterion that these coefficients increase by issue age. Coefficients for policy years 2 through 5 were determined by grading smoothly to zero at duration 6. Select mortality rates for issue ages 71 through 75 were determined by holding constant the coefficients of selection for age 70. The mortality rates for policy year 1 are very close to those shown in Mr. Buck's modification; for policy years 2 through 5, the mortality rates are deliberately overstated. It is submitted that the margins in select years 2 through 5 and in the early ultimate years are sufficient to offset any understatement of true ultimate mortality.

[4] *TSA* IX, 28.

MORTALITY TABLE X_{18}
(WITH SELECT MODIFICATION)
MORTALITY RATE PER 1,000

Age at Issue	Policy Year 1	Policy Year 2	Policy Year 3	Policy Year 4	Policy Year 5	Att. Age	Ultimate
0	6.33	1.00	.78	.66	.58	5	.52
1	1.00	.78	.66	.58	.52	6	.47
2	.77	.66	.58	.52	.47	7	.43
3	.65	.58	.52	.47	.43	8	.40
4	.57	.52	.46	.43	.40	9	.38
5	.51	.46	.42	.40	.38	10	.37
6	.46	.42	.39	.38	.37	11	.39
7	.41	.39	.37	.36	.39	12	.43
8	.38	.37	.36	.38	.43	13	.47
9	.36	.36	.38	.42	.47	14	.51
10	.35	.37	.42	.46	.50	15	.55
11	.37	.41	.45	.50	.54	16	.61
12	.40	.45	.49	.54	.60	17	.67
13	.44	.49	.53	.59	.66	18	.75
14	.47	.52	.58	.65	.74	19	.81
15	.51	.57	.64	.73	.80	20	.85
16	.56	.62	.71	.78	.84	21	.87
17	.61	.69	.76	.82	.85	22	.89
18	.67	.74	.80	.83	.87	23	.90
19	.72	.77	.81	.85	.88	24	.92
20	.74	.78	.82	.86	.90	25	.93
21	.75	.79	.82	.87	.90	26	.95
22	.75	.79	.83	.87	.92	27	.98
23	.75	.79	.84	.89	.95	28	1.00
24	.75	.79	.84	.91	.96	29	1.04
25	.74	.80	.86	.92	1.00	30	1.08
26	.75	.81	.87	.95	1.03	31	1.13
27	.75	.82	.90	.98	1.08	32	1.18
28	.76	.84	.92	1.02	1.12	33	1.24
29	.77	.86	.95	1.06	1.18	34	1.32
30	.78	.88	.99	1.10	1.25	35	1.41
31	.80	.91	1.02	1.17	1.33	36	1.53
32	.82	.94	1.08	1.24	1.44	37	1.68
33	.84	.98	1.14	1.33	1.57	38	1.87
34	.88	1.03	1.22	1.45	1.74	39	2.10
35	.92	1.10	1.33	1.61	1.95	40	2.36
36	.97	1.19	1.46	1.79	2.19	41	2.64
37	1.04	1.30	1.62	2.00	2.44	42	2.95
38	1.13	1.44	1.80	2.22	2.72	43	3.28
39	1.24	1.59	1.99	2.47	3.01	44	3.63
40	1.36	1.74	2.20	2.72	3.32	45	4.02
41	1.48	1.91	2.41	2.99	3.67	46	4.45
42	1.61	2.09	2.64	3.29	4.05	47	4.92
43	1.74	2.27	2.89	3.61	4.46	48	5.46
44	1.87	2.46	3.16	3.97	4.93	49	6.06
45	2.01	2.67	3.44	4.37	5.45	50	6.72
46	2.16	2.89	3.77	4.81	6.03	51	7.45
47	2.31	3.14	4.13	5.30	6.66	52	8.21
48	2.48	3.42	4.52	5.83	7.32	53	9.02
49	2.67	3.71	4.95	6.37	8.01	54	9.92

MORTALITY TABLE X_{18}—*Continued*

Age at Issue	Policy Year 1	Policy Year 2	Policy Year 3	Policy Year 4	Policy Year 5	Att. Age	Ultimate
50	2.86	4.02	5.38	6.95	8.78	55	10.91
51	3.05	4.33	5.83	7.58	9.62	56	12.01
52	3.24	4.65	6.32	8.27	10.56	57	13.22
53	3.43	5.00	6.85	9.03	11.58	58	14.55
54	3.62	5.37	7.43	9.86	12.70	59	15.99
55	3.82	5.76	8.06	10.77	13.91	60	17.57
56	4.02	6.19	8.74	11.74	15.23	61	19.28
57	4.23	6.63	9.47	12.79	16.66	62	21.12
58	4.44	7.10	10.24	13.92	18.18	63	23.10
59	4.64	7.59	11.07	15.12	19.82	64	25.25
60	4.83	8.10	11.93	16.40	21.59	65	27.61
61	5.11	8.70	12.91	17.83	23.55	66	30.21
62	5.39	9.33	13.96	19.38	25.71	67	33.08
63	5.66	10.00	15.10	21.09	28.08	68	36.24
64	5.93	10.71	16.34	22.96	30.70	69	39.66
65	6.21	11.48	17.70	25.01	33.51	70	43.30
66	6.65	12.44	19.28	27.29	36.55	71	47.09
67	7.11	13.48	20.98	29.70	39.70	72	51.00
68	7.61	14.59	22.78	32.21	42.94	73	55.01
69	8.13	15.76	24.63	34.78	46.26	74	59.23
70	8.66	16.95	26.52	37.41	49.75	75	63.80
71	9.42	18.36	28.60	40.28	53.59	76	68.85
72	10.20	19.80	30.80	43.38	57.83	77	74.52
73	11.00	21.32	33.18	46.82	62.60	78	80.92
74	11.85	22.97	35.80	50.67	67.97	79	87.99
75	12.76	24.79	38.75	55.03	73.91	80	95.64
						81	103.78
						82	112.32
						83	121.20
						84	130.45
						85	140.12
						86	150.27
						87	160.98
						88	172.39
						89	184.75
						90	198.38
						91	213.71
						92	231.24
						93	251.47
						94	274.90
						95	303.03
						96	343.36
						97	409.79
						98	522.62
						99	708.55
						100	1,000.00

DISCUSSION OF PRECEDING PAPER

JAMES E. HOSKINS:

The outstanding feature of Mr. Anderson's paper is his systematic treatment of the profit loading. As he indicates, the premium computation methods in actuarial literature have allowed for profit either by a constant per $1,000 or by assigning values to the mortality and other factors which are more conservative than those which are considered "most probable." The previous papers, however, have not suggested any theoretical basis for the amount of such loadings. Mr. Anderson now proposes that the profit loading in test rates be based on the same consideration that governs investment in other lines of business, namely, the return desired on the amount invested, taking into account the degree of risk in the enterprise.

In practice a rough estimate of the profit level produced by this consideration has sometimes been used to validate the loadings used in previous methods of computation. Mr. Anderson, however, introduces the concept into the computation directly and exactly, subject, of course, to the actuary's judgment as to the "most probable" values of the various factors and as to the rate of return which stockholders may reasonably expect.

In an extreme hypothetical case where the valuation reserve is merely the reserve arising from the premium assumptions, then since no surplus would be invested in a policy, Mr. Anderson's proposal would allow for no profit. The stockholders, however, would assume the risk of adverse statistical fluctuations from the "most probable" assumptions, and would therefore expect some return beyond the interest earned on the securities in which the capital and free surplus is invested.

The common method of providing for profit by choosing slightly conservative values of the mortality and interest factors has the property of permitting the profit loading to be proportioned to the risk assumed, if it is felt that the chance of a given degree of adverse fluctuation in one factor is materially greater than in the other. The various possible values of the mortality rate and of the interest rate may be regarded as frequency distributions, and their standard deviations may differ considerably. Specifically, the stockholder might feel that he ought to get a higher rate of "most probable" return on that part of his money invested in term insurance than on the part invested in endowments, or vice versa,

with intermediate rates for intermediate plans. This is analogous to Mr. Anderson's suggestion that different values might be assigned to the risk of investing in new business and to that of investing in new agency organization, although in his illustration he assigns practically the same values to each. Perhaps Mr. Anderson's method can be modified so as to accomplish this result with more accuracy than the method of conservative mortality and interest factors.

Mr. E. A. Rieder (*RAIA* XXIX) suggested that ideal commissions should be related to the expected profit as it varies with plan and age, and expressed the profit as a percentage of the amount at risk plus another percentage of the initial reserve. He merely implied that profit does in fact tend to arise largely in this way, not that it ought to arise in this way because of the respective mortality and investment risks assumed. If, however, we combine Mr. Anderson's suggestion that certain overhead expenses should be related to commissions (which incidentally is a sort of compromise between relating them to premiums and to amount of insurance) with Mr. Rieder's suggestion that commissions should be related to expected profit, the result is to distribute overhead among the various policies in proportion to the respective amounts of profit which they are expected to yield, which virtually charges each policy with that part of the overhead which it can afford to carry.

At one point Mr. Anderson says, "If there is a uniform distribution of profit throughout the rate structure, the realization of the profit objective cannot be thwarted by a shift in the distribution of sales." The paper recognizes that theoretical premiums must often be modified by competition. If a company's scale of rates achieves its profit objective on the average, on its previous distribution of sales, but departs from the objective at individual points to meet competition, it is unlikely that this procedure will itself cause a shift in distribution. At the points where the profit is below the objective, there is no more incentive for a prospect to buy from this company than from one of the competing companies. It is unlikely that either agent or prospect will recognize that that particular rate is more of a bargain than the rates for other plans or ages. In bending his curves of theoretical rates to meet competition, the actuary may merely be giving credence to the judgment of other actuaries as to the "most probable" values of the various factors.

It is sometimes said that the ideal cash value is the natural reserve, subject to modification for asset depreciation. The natural reserve as defined by Mr. B. E. Shepherd in *TASA* XLI is in effect the accumulation of the no-profit premium. Mr. Shepherd points out that if the valuation reserve were the natural reserve, the annual emerging profit would be

the excess of the gross premium over the natural premium, less percentage expenses. This amount would be nearly independent of duration, except in the first policy year. Under Mr. Anderson's theory, however, that the profit should be proportionate to the surplus invested in the policy, the emerging profit should start high and decrease with duration. If cash values are made equal to the natural reserve as defined by Mr. Shepherd, then on a surrendered policy the company will not have received as much profit as it should have received under Mr. Anderson's theory. If, however, the expected profit is treated as an expense, *i.e.*, as compensation to the stockholders for the service which they perform, then the natural reserve as modified to allow for this additional expense item is a proper basis for cash values.

In listing the premium factors to be considered, Mr. Anderson omits one which some actuaries have considered important, namely, allowance for the likelihood that, on the average, the payment of $100 of cash value may require the sale of assets having a book value greater than $100, or, if paid out of current income, may prevent investment at a favorable rate. In other words, there is reason to believe that more cash surrenders occur when the market is down than when it is up. This is elaborated in Mr. Richardson's paper, "Guaranteed Cash Surrender Values under Modern Conditions," in *TASA* XXXIX at page 261, and in the discussion of the paper in *TASA* XL, especially that by Mr. Gray on page 142. However, Mr. Anderson's method permits making this allowance by substituting for the actual cash value an appropriately increased amount.

LEON D. FORBES:

Mr. Anderson has presented a most thorough and interesting paper. To many of us the most intriguing aspect of the paper is his handling of the profit objective as a yield rate on the amount of surplus invested to acquire that profit. Those few of us in the reinsurance business were also quite interested in the several statements regarding reinsurance.

The first statement to jar my enjoyment of the paper appears on page 358 where, in the list of parameters which uniquely define the theoretical premium rate, is included as parameter number 4, "the expenses incurred, including taxes and reinsurance costs." Those of us in the reinsurance business who like to keep things in their proper perspective might prefer to say perhaps the same thing in a little different way. I should like to suggest that parameter number 4 be changed to read, "the expenses incurred, including consulting actuarial fees, and properly reflecting the reduction in general overhead expense which may be achieved through a reinsurance relationship."

The second statement which I found somewhat disturbing reads, in part, "The calculations suggest that the additional premium on account of reinsurance is $2.46 per $1,000, plus $2.34 per policy." Mr. Anderson mentions that the illustration is unrealistic because it is based on an assumption of 100% reinsurance. This additional premium, of course, covers commissions and premium taxes on the additional premium itself, in addition to the reinsurance costs. The reinsurance costs include the expense of administering reinsurance in the home office of the ceding company, as well as the expenses and profit of the reinsurer.

Many people would consider reinsurance costs to be either the reduction in profit or losses incurred on reinsured business. To determine this reinsurance cost, it is necessary to make several decisions. Decisions must be made as to how much of the general overhead expense of the company shall be allocated to the reinsured portion of the business, how the expense of administering reinsurance shall be allocated, and how allowance is to be made for the reduction in overhead which the ceding company attains by relying on the underwriting assistance and underwriting research program of the reinsurer. Some value should also be placed on the supplemental services of the reinsurer, such as training of personnel in various phases of the insurance business and assistance in entering new lines of business.

With regard to determining how much of the general overhead expense of the ceding company should be allocated to reinsured business, two extreme examples might be cited. One might assume that if it were not for the reinsurance relationship, the ceding company could not issue more than its limit of retention and that, therefore, all general overhead expense should be allocated to retained business and none to reinsured business. At the other extreme it could be assumed that reinsured business should bear its full pro-rata share of the general overhead expense of the ceding company. The most reasonable position probably lies somewhere between these two extremes.

With regard to the expense of administration of reinsurance in the office of the ceding company, I believe a rather good argument can be made for assessing this expense to all policyholders when determining reinsurance costs. This seems reasonable if one considers reinsurance to be mainly a service to one's field force. In any event, this expense is not large.

In his calculation of additional premiums necessitated by reinsurance, Mr. Anderson took what we would consider a rather severe view in each instance:

a) The reinsured business was required to bear its pro-rata share of the general overhead expense of the ceding company.

b) The expense of administering reinsured business was allocated to the reinsured business only.

c) No credit was given for the reduction in overhead attained by the ceding company through utilizing the underwriting assistance and the underwriting research program of the reinsurer.

d) No credit was given for the value of the reinsurer's supplemental services.

We thought it would be interesting to compare the reinsurance cost with the profit margin contained in our own premium scale for the same age and plan as was used by Mr. Anderson in his example. Our company is a competitive non–New York company. Our rates are the same as those of Company G shown in the paper, and at age 35, the age used in the example, five of the ten companies included in the table had rates higher than the Lincoln, and four had lower rates in the larger amount band where our reinsurance would arise. We followed his severe assumption regarding allocation of general overhead expense and ignored the value of all reinsurance services. We found that the reinsurance cost fell within the profit margin contained in our scale. I should not want to imply that on the basis of this one calculation this relationship would hold for other ages at issue and mortality classifications—a broad study would be necessary to investigate that point.

Neither we nor Mr. Anderson took into account the effect of the recapture privilege on reinsurance costs or of the negative carry-over limit on the experience refund account.

ROBERT MERRITT:

The author is to be congratulated on a fine piece of work. The value of this paper is by no means confined to nonparticipating insurance, for its analysis is so general and yet so detailed as to make the extension to participating business obvious and straightforward. It is also a striking illustration of the extreme power and usefulness of a medium-scale computer in asset share work.

One of the most stimulating parts of the paper was the projection of the development of a model agency. Although one may suspect that new agencies are sometimes opened from motives that do not include the immediate prospect of profit, Mr. Anderson has provided the tools for objective financial analysis when that is the aim.

I do not follow the author in his discussion of the recouping of investment in agency development via loading in gross premiums. He states that "the yield on surplus invested in agency development" is independent of "the yield on surplus invested in new business"; while this seems reason-

able enough on the surface, it would appear that the yield on new business is subject to a practical upper limit dictated by a competitive ceiling on gross premiums. This in turn places a ceiling on the rate of agency development, and the yield obtainable on investment in such development. In short, I think the two yield rates cannot in fact be independent.

By using present value of commissions as a sort of yardstick for other financial influences, the author is tacitly assuming continuation of something like the present level and incidence of commissions on all policies. If there existed an aggressive direct-writing company, paying no commissions, this yardstick would fail; or if a company with an already favorable net cost were to cut premiums and commissions sharply on a few special policies in an effort to improve its competitive position still further, the yardstick would be bent out of shape. Present value of commissions, in other words, is not under all circumstances the whole measure of a company's desire for new business.

It does seem to me that Mr. Anderson tosses off rather lightly the potential effect of inflation on expenses when he calls it "probably of minor consequence because the major portion of expenses are either incurred at issue or contractual." In so saying, he is doubtless referring to underwriting costs, commissions and premium taxes. Nonetheless, the other renewal expenses are scarcely negligible in relation to the "present value of book profit at issue." If we were to assume an inflationary increase in such expenses equivalent to a compound interest rate of 2%, this would replace the term E_t/A in the expression for ${}_tB$ by

$$\frac{E_t}{A}(1.02)^{t-1}.$$

Hence ${}_tB$ would be decreased by

$$\frac{E_t}{A}[(1.02)^{t-1}-1] = \frac{.02E_t}{A}\, s\overline{t-1|}^{2\%};$$

and Z would be decreased by

$$\frac{.02E_t}{A}\sum_{t=2}^{30} F_t\, s\overline{t-1|}^{2\%}.$$

Using the author's "standard" assumption, the expression at issue age 35 turns out to be $.623E_t/A$. The function E_t/A is, of course, largest in the \$3,000 size class; and here the decrease in Z is \$.47. This is 19% of the book profit of \$2.42. In the largest size class the reduction is \$.27 or 13% of the book profit of \$2.09.

Claim expenses and termination expenses are also affected, but the ef-

fect on book profit is, of course, much smaller because these expenses are incurred only at the time of the event.

Mr. Anderson's suggested treatment of indirect expenses is of considerable interest; I have often thought that so large a part of a company's expenses are relatively fixed that there are great difficulties in assessing all expenses according to the usual indexes—per policy, per thousand and per dollar of premium. However, if asset shares are regarded as a mathematical model of part of a company, it would seem that apportionment of indirect expense is not necessarily better when related to present value of future commissions than when apportioned per M. It would be better only if the over-all level of indirect expenses were more nearly proportional to the total present value of commissions on new business. By definition there is no direct relationship, and the willingness of management to spend money is the real determinant. This willingness of management to spend money is probably going to be more closely related to volume of new business than to value of commissions.

LALANDER S. NORMAN:

Mr. Anderson has performed a great service in presenting his masterful review of the factors involved in gross premiums and profit margins, and in setting out so concisely a technique for handling them.

One of the factors dealt with is the effect of reinsurance. A technique is indicated for bringing yearly renewable term reinsurance premiums less experience refunds into the picture on any given assumption as to retention by the issuing company. Mr. Anderson includes an illustration and the warning that "because the illustration is quite artificial and because the assumption of 100% Y.R.T. reinsurance is unrealistic, the results require careful scrutiny if false conclusions are to be avoided." Without this warning a false impression as to the true cost of reinsurance might have been left. Some additional points which may be given consideration in this connection include:

1. Typical gross premiums in use today for reinsurance with provision for experience refund are materially below the 1941 CSO values of q for the principal insurance ages.
2. The first year Y.R.T. reinsurance rate is typically one-half the renewal rate for the same attained age. In terms of the author's formula, this would reduce the modified mortality rate for the first policy year by $.275\ q^{CSO}$.
3. Typical refund calculations include devices for limiting the amount of claims charged against the issuing company when chance fluctuation causes high peaks in actual mortality. A portion of the actual mortal-

ity is thus removed from the claims charge and absorbed, instead, in the reinsurer's retained share of the reinsurance premiums. The total claim charges that are made against the issuing company when computing experience refunds will, therefore, average out to something less than the total actual mortality represented by the q factor of the formula suggested in the paper.

4. Business written and reinsured, that might not be issued in the absence of reinsurance, results, under the author's approach, in contributions to profit in accordance with assumed objectives, commissions to the issuing company's agents, and margins to cover a share of other agency expenses, issue costs, servicing costs and home office overhead.
5. In addition, the use of reinsurance facilities will normally enable the issuing company to write a substantially larger volume of business within its own retention limit. Whether introduced specifically into the formula or not, an appraisal of the effect of reinsurance may properly give consideration not only to the reinsured business itself, but to the aggregate increase in the volume of retained business, and profits therefrom, that may result from being able to go into the marketplace with a full and adequate line of products and service.
6. Dealing with the reinsurance costs as a part of the mortality cost, in the author's suggested manner, is natural and proper as a means of handling the mathematics, but this should not be permitted to obscure the fact that the cost of reinsurance in most cases covers the purchase of selection know-how and incidental services in addition to the transfer of mortality risks.

HARWOOD ROSSER:

Mr. Anderson has presented an admirable treatment of the subject, delving into background material and underlying philosophies, and emphasizing the impact of competition. But he has assumed a fairly sophisticated audience, at least if they are to obtain results as refined as his.

His innovations, such as using different interest or yield rates for different purposes, are most intriguing; but I shall leave the discussion of them to others. His seeking to take into account many additional factors is laudable—up to a point. Now I have the utmost respect for, and some acquaintance with, electronic computers. However, when he finds himself, by his own admission, astride an IBM 650, and able to dismount only with great difficulty, one suspects that, for many of us, he may have gone too far.

Fortunately, we can reach essentially the same destination, but on less expensive steeds. In his Table 3, Mr. Anderson illustrates a plan with

annual premiums, and without survival or conversion benefits. As he remarks, "numerous simplifications develop." Then his formulas, and hence his results, become quite similar to those implicit in my 1951 paper,[1] "A Present Value Approach to Profit Margins and Dividends."

This approach allows equal flexibility with his as to the choice of mortality, persistency and interest assumptions, including the discounting of margins at a varying rate. The major difference is that my method provides for having the cash value in hand each year, rather than the reserve. As the author implies, this does not matter as long as a single rate of interest throughout each year is used. A minor difference in our treatments lies in the handling of indirect and of termination expenses. Also, Mr. Anderson is able to distinguish between annual premiums and those payable more frequently. But he concedes, by his formula following Table 6, that this point will not be reexamined from first principles by every company, for each plan, age and size of policy.

As to simplicity, the technique of my paper requires very little actuarial background for its actual application. Moreover, it can be fairly readily mechanized on a lesser machine than the 650 computer.

To facilitate comparison of the similarities and divergences between Mr. Anderson's method and mine, I have tried to build a bridge between the two, in the form of Table 3 $(a)'$. This is a modification of my approach in order to produce Mr. Anderson's results. Column 21 in this table is, for each duration, within two cents of the figure in the column headed "Book Profit per $1,000 in Force" in his Table 3, Part (a). That is, the figures are identical except for accumulated rounding differences, of which he has already spoken. (Dollar calculations in my table were carried out to four decimals. To save space, however, they are shown only to two.) Likewise, column 24 of my table corresponds to his final column, with a little wider numerical spread.

Through column 16, Table 3 $(a)'$ has been modeled as closely as possible after Table 1 of my 1951 paper. This column 16, which totals to $9.05, is the counterpart of column 11 in the earlier Table 1, and gives the discounted margins on the basis of holding cash values only. Because of the way Mr. Anderson assessed "Other Termination Expenses," it was necessary to include two columns just to provide for these. Including these expenses elsewhere would eliminate this. (Ignoring them completely would affect the premium by only $.08.)

Beyond column 16, Table 3 $(a)'$ is taken up mainly with adjusting the margins to the basis of the reserves that one intends to hold. When, as here, the valuation rate is under 4%, and the yield required on borrowed

[1] *TSA* III, 187.

TABLE 3(a)′

PROFIT MARGINS ON TWO BASES—A REARRANGEMENT OF TABLE 3(a) WHOLE LIFE—AGE 35—POLICY AMOUNT $3,000—GROSS PREMIUM $22.35

POLICY YEAR t	EARNED INTEREST RATE i_t	$1{,}000\times(1+i_t/2)\times(1+Q/A)$	TERMINAL RESERVE ${}_tV$	CASH VALUE ${}_tCV$	NET RISK (INC. D. C. EXPENSE) (2)-(4)	MORT. RATE $1{,}000q_t$	WITHDRAWAL RATE w_t	MORTALITY COST (INCL. EXPENSE) (5)×(6)/1,000	COMM. RATE C_t	WITHDRAWAL EXPENSE (EXCL. CASH VALUE PAID) $1{,}000\times(7)\times W/A$	DEDUCTIVE ITEMS BEGINNING OF YR. $\frac{(4)+(8)+(10)}{1+i_t}$	EXPENSES OTHER THAN TERMINATION	
												Percentage $P'\times[.02+(9)]$	Dollar* $1{,}000\times E_t/A$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1.....	.0375	1027.24	0	0	1027.24	1.17	.100	1.20	.780	.17	1.32	17.88	16.99
2.....	"	"	15.69	0	1027.24	1.35	.060	1.39	.075	.10	1.43	2.12	1.33
3.....	"	"	31.65	14.42	1012.82	1.58	.050	1.60	"	.08	15.52	"	"
4.....	"	"	47.88	30.94	996.30	1.86	.044	1.85	"	.07	31.68	"	"
5.....	"	"	64.35	47.71	979.53	2.20	.040	2.16	"	.07	48.13	"	"
6.....	.0370	1026.99	81.08	64.73	962.26	2.61	.036	2.51	"	.06	64.90	"	"
7.....	.0365	1026.74	98.04	82.00	944.74	2.89	.032	2.73	"	.05	81.80	"	"
8.....	.0360	1026.48	115.24	99.50	926.98	3.20	.029	2.97	"	.05	98.95	"	"
9.....	.0355	1026.23	132.66	117.23	909.00	3.53	.027	3.21	"	.05	116.35	"	"
10.....	.0350	1025.98	150.28	135.17	890.81	3.88	.025	3.46	"	.04	133.98	"	"
11.....	.0345	1025.73	168.11	153.31	872.42	4.27	.024	3.73	.050	.04	151.84	1.56	"
12.....	.0340	1025.48	186.11	171.63	853.85	4.70	.023	4.01	"	.04	169.90	"	"
13.....	.0335	1025.22	204.29	190.13	835.09	5.17	.022	4.32	"	.04	188.18	"	"
14.....	.0330	1024.97	222.62	208.79	816.18	5.71	.021	4.66	"	.04	206.67	"	"
15.....	.0325	1024.72	241.08	227.58	797.14	6.31	.020	5.03	"	.03	225.32	"	"
16.....	.0320	1024.47	259.67	246.50	777.97	6.97	"	5.42	.020	"	244.14	.89	"
17.....	.0315	1024.21	278.36	265.52	758.69	7.70	"	5.84	"	"	263.11	"	"
18.....	.0310	1023.96	297.14	284.63	739.33	8.46	"	6.25	"	"	282.17	"	"
19.....	.0305	1023.71	315.97	303.80	719.91	9.27	"	6.67	"	"	301.32	"	"
20.....	.0300	1023.46	334.85	323.02	700.44	10.17	"	7.12	"	"	320.56	"	"
21.....	"	"	353.75	342.25	681.21	11.16	"	7.60	"	"	339.69	"	"
22.....	"	"	372.65	361.49	661.97	12.26	"	8.12	"	"	358.87	"	"
23.....	"	"	391.53	380.70	642.76	13.47	"	8.66	"	"	378.05	"	"
24.....	"	"	410.36	399.87	623.59	14.80	"	9.23	"	"	397.22	"	"
25.....	"	"	429.12	418.96	604.50	16.24	"	9.82	"	"	416.32	"	"
26.....	"	"	447.78	437.96	585.50	17.82	"	10.43	"	"	435.37	"	"
27.....	"	"	466.33	456.84	566.62	19.53	"	11.07	"	"	454.31	"	"
28.....	"	"	484.73	475.56	547.90	21.37	"	11.71	"	"	473.11	"	"
29.....	"	"	502.97	494.13	529.33	23.35	"	12.36	"	"	491.77	"	"
30.....	"	"	521.01	512.49	510.97	25.50	.975	13.03	"	0†	510.21	"	"

* Including first year indirect expenses of $3.66.

† Withdrawal expenses not assessed since the assumption of 100% terminations is for convenience only.

TABLE 3(*a*)′—*Continued*

Policy Year t	Persistency & Discount Factor: F_t	Margin: Based on Cash Values		T. R. Less C. V. (3)−(4)	Annual Persistency Rate $1-q_t-w_t$	Aux. Function $\frac{(17)\times(18)}{1+i_t}$	Margin Released as Cash Value Approaches Res. $(17)_{t-1}-(19)$	Margin: Adjusted for Reserves Held			Value of Future Margins (23) ÷ (14)
		Beg. of Year $P'+(4)_{t-1}-(11)-(12)-(13)$	Value at Issue (14)×(15)					Beg. of Year (15)+(20)	Value at Issue (14)+(21)	(22) Summed Upward	
	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
1.....	1.0000000	−13.84	−13.84	0.	.89883	0	0	−13.84	−13.84	2.39	2.39
2.....	.7815913	17.46	13.65	15.69	.93865	14.20	−14.20	3.27	2.55	16.23	20.76
3.....	.6379484	3.37	2.15	17.23	.94842	15.75	— .05	3.31	2.11	13.68	21.44
4.....	.5261243	1.64	.86	16.94	.95414	15.58	1.65	3.29	1.73	11.56	21.98
5.....	.4365185	1.71	.74	16.64	.95780	15.36	1.58	3.28	1.43	9.84	22.53
6.....	.3635630	1.70	.62	16.35	.96139	15.16	1.48	3.19	1.16	8.40	23.11
7.....	.3039355	1.83	.55	16.04	.96511	14.94	1.41	3.24	.98	7.24	23.83
8.....	.2550706	1.94	.50	15.74	.96780	14.70	1.34	3.28	.84	6.26	24.54
9.....	.2146585	2.04	.44	15.43	.96947	14.45	1.29	3.33	.72	5.42	25.26
10.....	.1809608	2.14	.39	15.11	.97112	14.18	1.25	3.40	.61	4.71	26.01
11.....	.1528128	2.79	.43	14.80	.97173	13.90	1.21	3.99	.61	4.09	26.78
12.....	.1291241	2.86	.37	14.48	.97230	13.62	1.18	4.04	.52	3.48	26.96
13.....	.1091716	2.90	.32	14.16	.97283	13.33	1.15	4.05	.44	2.96	27.11
14.....	.0923525	2.92	.27	13.83	.973 9	13.03	1.13	4.05	.37	2.52	27.26
15.....	.0781616	2.92	.23	13.50	.97369	12.73	1.10	4.02	.31	2.14	27.42
16.....	.0661784	3.56	.24	13.17	.97303	12.42	1.08	4.64	.31	1.83	27.64
17.....	.0559944	3.52	.20	12.84	.97230	12.10	1.07	4.58	.26	1.52	27.18
18.....	.0473420	3.47	.16	12.51	.97154	11.79	1.05	4.52	.21	1.27	26.73
19.....	.0399954	3.44	.14	12.17	.97073	11.46	1.05	4.48	.18	1.05	26.29
20.....	.0337606	3.36	.11	11.83	.96983	11.14	1.03	4.39	.15	.87	25.83
21.....	.0284713	3.45	.10	11.50	.96884	10.82	1.01	4.46	.13	.72	25.42
22.....	.0239862	3.50	.08	11.16	.96774	10.49	1.01	4.51	.11	.60	24.88
23.....	.0201847	3.56	.07	10.83	.96653	10.16	1.00	4.56	.09	.49	24.20
24.....	.0169644	3.61	.06	10.49	.96520	9.83	1.00	4.61	.08	.40	23.36
25.....	.0142383	3.67	.05	10.16	.96376	9.51	.98	4.66	.07	.32	22.35
26.....	.0119325	3.72	.04	9.82	.96218	9.17	.99	4.70	.06	.25	21.11
27.....	.0099836	3.77	.04	9.49	.96047	8.85	.97	4.74	.05	.20	19.61
28.....	.0083382	3.85	.03	9.17	.95863	8.53	.96	4.81	.04	.15	17.80
29.....	.0069507	3.91	.03	8.84	.95665	8.21	.96	4.87	.03	.11	15.59
30.....	.0057820	4.04	.02	8.52	.00000	0	8.84	12.88	.07	.07	12.88

surplus is 15%, this adjustment is substantial: $1.58 of gross premium. (If the persistency and discount factors in column 14 are figured at the valuation rates in column 1 instead, and then applied to column 20, the sum of the resulting present values will be found to be zero, as implied above. That is, the need for this adjustment disappears if the two interest rates coincide at each duration, whether constant or not. On this basis, columns 16 and 22 would have the same total, $28.33—quite a change from Mr. Anderson's $2.42!)

One note of caution should be sounded for those who might wish to follow along the lines of Table 3 $(a)'$, including the adjustment to the basis of reserves. This cannot be accomplished by simply substituting reserves for cash values throughout, as one might suppose. Instead, a modification of Table 3 from my paper would produce a more logical worksheet than the rather ungainly Table 3 $(a)'$, whose format was not dictated by simplicity alone.

In practice, either method (his or mine) would start with an experimental premium (Mr. Anderson recommends the net valuation premium), as of course the appropriate gross premium is not yet known. For space reasons, and to avoid considerable repetition, he omitted showing this calculation; and I have followed suit. However, his examples would be easier to follow if he had given us the numerical values of certain unprimed functions, such as X and Z—*i.e.*, values in the bottom line of Table 3, but based on the net premium. Perhaps he will furnish these in his reply.

On a rather personal point, I wish to commend Mr. Anderson most highly. I do not believe that he even mentioned the naughty words "asset shares." My sentiments on this score are recorded in the discussion of my 1951 paper.

In summary, one segment of the profession will hail the greater generality of Mr. Anderson's approach. Another segment will find some difficulty in following his formulas, or in preparing computer programs therefrom; and will turn to certain of the older papers as not being entirely outdated. Moreover, these two segments will not be mutually exclusive. It is hoped that this discussion may assist some in both groups.

STUART A. ROBERTSON:

The spectacular development of processing and computation facilities has made feasible the use of methods that would have been wholly impractical just a decade or two ago. A paper that describes modern techniques for pricing nonparticipating insurance is timely, if not overdue, and we are indebted to Mr. Anderson for his very able treatment of the

subject. Discussion of his paper is certain to reveal that there will be no universal acceptance of his suggested profit objectives, and there may not be complete agreement on some other details. Such differences should not be permitted to obscure the very real value and great potential of the techniques described in the paper, nor to detract from our appreciation of this fine piece of work.

The expression of profit, or a substantial part of it, in terms of an investor's return on surplus expended to acquire new business is an appealing concept. We are quite accustomed to viewing the surplus that is so consumed as an investment, and it seems only logical for a company to seek to fix the return on that investment at some uniform rate for all segments of its ordinary business. Closer scrutiny, however, reveals certain features that will cause some of us to consider this approach to be not entirely suitable unless modified. Two of those features on which I will comment briefly are:

1. The method fails to satisfy the desirable condition that the amount of profit be related in some manner to the risk that is attached to production of that profit.
2. Because the amounts of both the investment and the profit are on the "book" basis, there exists a certain artificiality, the extent of which is not uniform for all segments of the company's business.

Normally, an investor will expect a greater or lesser rate of return on his funds according to the degree of assurance that the expected return will actually be realized. In nonparticipating life insurance, the risk attached to realization of the expected profit must be measured on the basis of judgment of the probability of future experience departing from our best estimates of mortality, investment return, expenses and termination rates. The effect of such departures will vary for different segments of our business. Individually, our best estimates of future experience, and our judgments of the probability of departure from those estimates, will doubtless differ, but I should expect to find general agreement among us that each dollar of profit that is expected to be derived at a particular time in the future, as a result of realization of our best current estimates of future interest and mortality rates, can be viewed with much less confidence in policies that stress the savings element of life insurance than in policies that do not. This suggests that it is appropriate to seek profits on policies with a high investment element that, when measured by our best current estimates of future experience, will be higher than those of policies where that element is less important.

Applied to a specific example, we might compare income endowment

insurance with ordinary life. An investment earnings rate such as to be equivalent to a rate, say, ½% lower than whatever rate is judged now to be most probable, clearly falls within a reasonable range of possible experience. The effect on profits, in the case of income endowment insurance, would be very marked compared with the effect on the profits of ordinary life. It is true that an unfavorable departure from our estimate of mortality experience would have somewhat greater effect on ordinary life than on income endowment; but the probability of a departure from the mortality assumption that is so extreme as to affect the profits of income endowment insurance by an amount comparable to a reduction of ½% in interest earnings is almost negligible. The surplus investment required to produce the income endowment business may be just about the same—perhaps appreciably less—than that for ordinary life policies. Thus, the profit objective as a fixed return on invested surplus produces profits that are relatively low on a segment of the business for which there is justification in seeking profits that are relatively high. An extreme illustration of the point would be the case of single premium life or endowment insurance. Acquisition of such business customarily results in an immediate contribution to surplus, rather than a diminution of surplus; yet, if based upon our best estimates of future experience, there is a substantial risk arising out of the possibility of experience—especially with respect to interest rates—failing to conform to assumptions. The stockholders are entitled to a profit to compensate them for the risk assumed, irrespective of the fact that on the basis of "book" values that segment of the business does not call for use of their money.

Mr. Anderson apparently is not in disagreement with the principle expressed, for in his illustration the assumed investment return is a reducing rate that eventually levels off at a rate ¾% lower than the initial rate. I suspect that his best estimate of future experience would not follow this reducing trend, and that he recognizes the reducing rate as the probable source of an additional profit margin not included in the expressed profit objectives.

Pursuit of this line of thought suggests, as one approach, that nonparticipating premiums may be calculated on the basis of moderately conservative assumptions such as to produce no profit if the assumptions are borne out by experience. Funds of the stockholders are pledged to absorb the loss if experience is less favorable than expressed by those assumptions, and in return for the use of their money and for the risk of loss the stockholders are rewarded to the extent that experience as it develops is more favorable than expressed by those conservative assumptions.

Failure of the proposed profit objective to relate profits to the risk

undertaken is not a valid objection if it is intended that the yield rate on surplus invested in new business be varied according to policy plan, age at issue and other factors that affect the degree of uncertainty with respect to profits. However, establishment of reasonable rules for the pattern of that variation appears to be subject to just as many limitations and arbitrary aspects as the traditional methods of computing theoretical premiums.

Whatever method is employed to determine premium rates, a very interesting result may then be derived by reversing Mr. Anderson's approach, and thus finding what effective yield is expected to be earned on the surplus invested in a block of new business. With suitable weighting factors introduced on the basis of expected distribution by age and amount, the result is available for each policy plan and, with another weighting operation, for new business as a whole. The result is still subject to the limitation that exists due to the artificiality of book profits, but the figures, with that limitation recognized, will be of real interest to the company.

The second questionable feature of the proposed profit objective relates to the somewhat artificial character of book profits. The profits of a life insurance operation must always be viewed through the fog of valuation practices, and it is possible to lose sight of the fact that the valuation standard has no effect whatever on the amount of profit; it merely governs the timing of the release of that profit on the company's books. However, the fact remains that money belonging to stockholders will be tied up while profits are being earned. The question is whether that amount of money is reasonably related either to the statutory reserve or to the more stringent reserve standard arbitrarily adopted by the company.

When issuing a block of life insurance business, the company absolutely guarantees performance of the policy contracts. Throughout the life of that business, the company will hold funds, somewhere in its accounts, that back up that guarantee, being of such amount as to give positive assurance of payment of the promised benefits under the most adverse conditions that, within reason, may be conceived. For the purpose of profit analysis, it is unimportant whether those funds are in the regular reserve account, a deficiency reserve, securities valuation reserve, contingency fund, unassigned surplus, or capital. Transfers from one of those accounts to another are in the nature of bookkeeping transactions that do not affect benefits, expenses or real profits. The excess of the total assets so required to be held, over the fund actually accumulated from income less expenditure, constitutes the stockholders' investment in that block of business. Should the company, through choice, hold funds in excess of the amounts

required to guarantee performance of its contracts, the excess is available for distribution to stockholders, and, until so distributed, earns for the stockholders a return equivalent to the rate experienced by the company on its invested assets.

The artificiality of book profits is especially clear in the case of business for which deficiency reserves are required. In a well-managed company, with adequate premium rates, the total fund needed for absolute guarantee of performance of the contracts is unaffected by this peculiarity of the law, and, for every dollar carried in the deficiency reserve account, a company is justified in carrying in the surplus account one dollar less than the amount that would be regarded as necessary in the absence of the deficiency reserve.

Turning now to the second component of Mr. Anderson's profit objectives, this has the effect of making a charge to all policies for the use of the facilities that the company has made available for the purpose. Certain funds of the stockholders have been devoted to building those facilities, as distinguished from funds applied to the acquisition of business, and it seems perfectly logical to make some reasonable charge to all who will benefit from the use of those facilities. While there may well be other approaches to determining some consistent charge to be made on this account, the author's method seems to be particularly suitable because, usually, the most important element by far in the cost of building the organization is that attached to development of the agency department. The value attached to the use of services of that department is theoretically related to the present value of the commission expense and it seems appropriate for the profit charge to be a function of that present value.

Whether or not the profit objectives suggested by Mr. Anderson are universally accepted, his formulas and various modified versions of them will be recognized as invaluable tools for the study of profits, the determination of theoretical premiums and the valuation of business in force. He has given us the basis for an entire family of formulas that can be made to suit the individual requirements of any specific task. One adaptation of the method, somewhat simpler in some respects and perhaps more flexible, will be described. The simplicity and flexibility are not necessarily desirable features where an electronic computer is employed, but may be of value in other situations, and these versions of the formulas will serve to illustrate the versatility of the method.

If the discount factors are computed on the basis of expected investment yield, the present value of book profits over the entire life of the policy, computed as of the date of issue, is independent of the valuation standard. Some will consider it a suitable index of profitability. One by-

product of the calculation can be the number of years of profits which must be included to produce a positive profit result—in other words, the time required to restore invested surplus on the basis of the valuation standard employed. Whatever its profit objective, this will be a matter of interest to the company. Another product is the asset share, which might be especially interesting to companies accustomed to testing premiums by the asset share method.

Mr. Anderson's notation is employed, with the following additions:

${}_tM$ = Profit, as of the beginning of year t, on the basis of the gross premium, after making provision for accumulating a fund equal to just the policy's cash value.

f_t = Discount factor, comparable to the author's F_t, but computed on the basis of the assumed investment earnings rate, i_t, rather than the investor's rate of return, j_t.

a = Factor to approximate the additional premium income anticipated on account of mode extras, reduced for loss of interest on fractional payment of premiums and loss on account of nondeduction of deferred installments on death.

Separate studies for each mode of payment, as undertaken by Mr. Anderson, will be useful when determining the additional charge required for the privilege of making fractional payments, but where that has been determined, it will be appropriate to introduce the "a" factor described above, and adjust all assumptions to a composite basis that rests on the expected distribution of business according to mode of payment. There is implicit in that step the introduction of a withdrawal rate for duration 0, thus reducing the first year exposure to a level amount equivalent to the reducing amount actually expected after accounting for first year lapses among the policies paid on a fractional basis. Withdrawal rates at other durations are to be similarly adjusted. Expense constants, likewise, are to be appropriate for the assumed distribution of business according to mode of payment.

Calculations may take the following form, with tabulations provided for the results of the eight expressions listed:

$$(1)\quad {}_tM = (1+a)\,\mathrm{P}'\,(1-C_t-P_t) - \frac{E_t}{A} - q_t\left[v^{1/2}\left({}_tD+\frac{Q}{A}\right) - v\cdot {}_tCV\right] - (v\cdot {}_tCV - {}_{t-1}CV)$$

$$(2)\quad f_t M_t$$

$$(3)\quad \sum_0^n f_t M_t$$

$$(4) \quad \sum_{0}^{n} f_t M_t - f_{n+1}({}_nV - {}_nCV)$$

$$(5) \quad {}_nCV + \frac{1}{f_{n+1}} \sum_{0}^{n} f_t M_t$$

$$(6) \quad f_t(1 - C_t - P_t)$$

$$(7) \quad \sum_{0}^{n} f_t(1 - C_t - P_t)$$

$$(8) \quad \sum_{0}^{n} f_t .$$

The f_0 discount factor will be 1, and f_1 will be $(1 - w_0)$; otherwise

$$f_{t+1} = \frac{1 - q_t - w_t}{1 + i_t} \cdot f_t .$$

The initial expenses that are independent of premium will be designated E_0, and ${}_0M$ is simply $-B_0/A$, the general formula being applicable for values of t equal to or greater than 1.

The voluntary withdrawal rate does not enter into calculation of the basic function, ${}_tM$. This facilitates the making of the study on the basis of more than one set of total termination rates. Either expression (3) or (4), for the value of n that corresponds to assumed date of termination of the last policy, presents the present value of all future profits discounted to the date of issue on the basis of the rate of interest assumed to be earned by the company on its investments. That value of n for which expression (4) is first positive indicates the number of years required for the emerging book profits to restore the surplus investment.

Expression (5) will be recognized as the asset share.

Expressions (6) and (7) will be useful in readily making a determination of the effect on profit margins resulting from a change in the gross premium. They can be used also to determine the effect of any change in the expenses that are expressed in the form of a percentage of premium income.

Expression (8) is used to examine the effect of any change in the assumption as to expenses other than those related to premium income, including such changes as are required by a change in policy size. The basic calculations may be made for just one policy size, and results easily modified for change in size.

A particularly informative set of calculations is provided by producing

four sets of ${}_tM$ results. One is based on the actuary's best estimate as to future experience; the others are the results of altering the interest and the mortality assumptions, singly and combined, to a moderately severe basis. Steps (2) to (5) are performed with the discount factor the interest component of which is consistent with the applicable assumptions for the calculation of ${}_tM$ and with the withdrawal rate on the basis of the "most probable" estimate. Then they may be recomputed with the withdrawal rate changed to a moderately conservative assumption. Factors (6) to (8) will also be computed on the basis of the various interest and termination assumptions. This array of results will be adequate to show the effect on profits of a change in any one of the major assumptions, and also the combined effect of altering all of the major assumptions. Probably the most pertinent products are the set of calculations based on the best estimates of future experience and the set based upon moderately conservative assumptions as to all of the factors. A company may then establish whatever requirements it considers to be appropriate for profits according to expected conditions, and for profits based upon moderately poor experience with respect to all factors, subject always to requirements imposed by competitive considerations.

Before concluding, I should like to make just one comment concerning the author's observation that the quantity discounts theoretically justified do not closely approximate discounts effected by use of the policy fee method of grading. It should be emphasized that that conclusion relates to the specific illustration and assumptions that were employed. Some companies will find that expense factors applicable for their calculations support the policy fee method of grading very well.

I congratulate Mr. Anderson on his excellent paper and express again my admiration for his well-organized presentation and description of some powerful actuarial tools.

HILLARY J. FISHER:

In his paper, Mr. Anderson has made a valuable contribution to actuarial thought, particularly in emphasizing the importance of profit analysis and in presenting a logical method of determining and measuring profit. The several concepts expressed in this paper could be developed into a number of new papers.

Mr. Anderson's suggestion that profits be expressed as the rate of yield on the amount of surplus invested is one that is meaningful to stockholders, since they generally have an investor's viewpoint.

The author further indicates that the amount invested is not a certain sum determined at the time new business is placed on the books, or at the time a new agency is launched. The sum invested may, in fact, increase

for several years, after which it should decrease steadily. Book profits on the new business or new agency are partly return on investment and partly return of "capital" to surplus. The rate of yield, therefore, can be determined only by commuting the amounts involved—that is, by converting the time factor to a yield factor.

The various "levels" of profit which Mr. Anderson suggests—profit on new business, on a new agent, on a new agency, on a new line, or even on a new company or a going company as a whole—point up the inadequacies of the Annual Statement in aiding management in assessing the profitability of each level of operation, for an investment at any of these levels is treated in the Statement as a current operating cost. But it usually is several years, at least, before there is any return on such investment, and it is many years before the invested sum is returned. In later years, the sums returned and the yields are both treated as current operating gains for those years. As long as the money coming back exceeds the money going out, we say we are making money. But we really do not know how profitable, relative to other forms of investment, each undertaking has been.

HARRY M. SARASON:

It is commonplace in human thought that the things that we think are perfectly satisfactory are the things that may be most dangerous. It is not a universal rule, but it is commonplace.

Now here we are talking about the dangers of nonpersistency and discussing the mortality, but there has not been a word said about renewal expenses in spite of the fact that we recently had a terrific increase in federal income tax.

E. FORREST ESTES:

While Mr. Anderson's paper presumably concerns itself with nonparticipating insurance only, many aspects of the paper are equally pertinent to the field of participating insurance. In particular his analysis of the various factors inherent in premium construction bears rereading by the actuary of a participating, as well as a nonparticipating, company; his meaningful discussion of contingency margins is particularly searching.

In the construction of gross premium schedules for participating insurance it is instructive to analyze realistically and in detail the component parts of a gross premium with respect to major plans and at strategic ages (including the required addition for a unit annual dividend margin). By varying assumptions it is possible to gain perspective as to the relative effect of such variations—this being helpful in evaluating the gross premium schedule finally to become adopted, as well as adding to background in the final determination of the initial dividend schedule.

Mr. Anderson's discussion of "contingency margins" points up an area in which there is need for more precise actuarial thinking. Management habitually thinks of "free surplus" (or its equivalent) as a contingency catch-all, also leaning heavily on margins contained in redundant statutory valuation mortality tables. There can be some question, however, as to whether management may be aware of the real size of contingent liabilities that appear as a part of "surplus." To illustrate: The income endowment type of insurance policy, as originally issued by most companies, contained the large mortality margins inherent in the American Experience Table; the guaranteed maturity value was, however, based on inadequate annuity assumptions. A priori thinking provided that any annuity losses would be offset by mortality and interest gains—which was not unsound if the thinking also included consistent plans for accumulating (as earmarked surplus or as a contingency reserve) a required portion of such gains. It would be interesting to know whether companies having such business in force are aware of its probable ultimate drain on "free surplus."

EDWARD A. GREEN:

I have read Mr. Anderson's excellent paper with great interest. Although I have not prepared a formal discussion I should like to comment briefly on his observation that risk classes for investment purposes deserve attention.

When I first came into the insurance industry we were concerned with the problem of declining yields on new investments. In that period we attempted to discourage policyholders from taking policies with substantial investment elements by imposing limits on the amount of single premium business and advance premiums we would accept. In order to encourage the taking of policies with substantial investment elements during periods of higher yields on new investments our only device to date has been lower premium scales—scales whose investment earnings assumption can be no more than a compromise between yields available at the time the policy is taken and expected long-term ones.

I think we need a better method of drawing funds to us when investment opportunities are favorable and discouraging excessive emphasis on the investment element when yield rates are low on new investments. This objective may be attained by rate and dividend formulas developed from asset share computations utilizing a "generation" approach to investment income allocation. Under such an approach greater weight would be given to yields available at the time funds are received than is now accorded. With modern mechanical equipment, I think we can do a great deal more along these lines than we have in the past.

Rate and dividend formulas developed through such methods might be expected to give better recognition to the differing effects on over-all company experience of such policy lines as group annuities, single premium life insurance and limited payment life insurance policies, and reduce the need for artificial underwriting rules designed to minimize investment antiselection.

As actuaries we all spend a good deal of time in refining our analyses of mortality and expenses and we carry these refinements into our theoretical rate structures. We have not given comparable attention to developing earnings assumptions which give weight to investment conditions expected to prevail at the time premiums are received. For high reserve contracts refinements of earnings assumptions may be found to have greater financial importance than those now being made in other assumptions affecting premium rates.

(AUTHOR'S REVIEW OF DISCUSSION)

JAMES C. H. ANDERSON:

One of the most gratifying rewards an author of a paper for the *Transactions* receives is the discussion evoked by his efforts. In this case, the reward has been ample, and I thank the participants for their generosity. It is also appropriate for me to express at this time my appreciation for the efforts of a number of individuals who contributed much to the development of the ideas embodied in this paper. My many friends and former associates with Bowles, Andrews & Towne, Inc., participated in this work from the very beginning. In particular, I thank Mr. William A. Dreher, for his constant encouragement and for his tireless review and criticism of this paper. My thanks, too, to Mr. David N. Wilson, of Actuarial Computing Service, Inc., for producing in expert fashion the results exhibited in the paper.

Mr. Hoskins observed that the use of "most probable" values for the various factors affecting gross premiums produces, in cases where no surplus investment is required, no profit to stockholders; in fact, he might have gone a step further and pointed out that if there is surplus release at issue in excess of the required present value of profit, the stockholders may have a "profit" which is actually negative. This criticism is altogether valid, and were the paper to be rewritten today, I should be inclined to find substitute wording for "most probable." The probabilities of statistical fluctuations (as distinguished from statistical departures from the mean, for which contingency margins are required) makes the estimation of future mortality, interest, persistency, etc., similar to a

gaming operation in which the insurance company represents the "house." Some margin, or "house percentage," is required if the insurance company is to have a reasonable probability of remaining solvent. Therefore, a satisfactory variation of the method would be to replace "most probable level" with "the level at which a gamble is worth while." This possibility may also have been in Mr. Hoskins' mind when he stated, "Perhaps Mr. Anderson's method can be modified so as to accomplish this result. . . ." I am indebted to Mr. Hoskins for contributing a very concise explanation of the proposed method of allocating indirect expenses; he states, quite properly, that the method "virtually charges each policy with that part of the overhead which it can afford to carry." His comments regarding the adjustment of calculated premium rates on account of competitive considerations are also well taken; I should, however, like to add one qualification to Mr. Hoskins' remarks on this subject; it is unlikely that reducing calculated premium rates will in itself cause a shift in the distribution of business, *unless the reduction results in a rate below the average of rates of competing companies.*

Mr. Hoskins' remarks regarding natural reserves are most interesting, because it is necessary to calculate a natural reserve in a different manner if the theory proposed in this paper is adopted. The natural reserve is equal to the valuation reserve less a deduction, which is precisely equal to the excess of the present value of the remaining profits under the policy, were it to remain in force, over a percentage of the present value of the remaining commissions under the policy, were it to remain in force. The calculation can be made in accordance with the techniques described, assuming a zero lapse rate throughout the period of the policy. This, in effect, gives the policyholder a "repayment right"—that is, the right to discharge at any time the accumulated "indebtedness" to stockholders on account of the initial surplus drain. Unfortunately, the cash values suggested by such an approach are often materially less than the minimum values permitted by law.

Mr. Robertson joins with Mr. Hoskins in pointing out the weakness of most probable estimates. As stated above, I now agree with that viewpoint. Mr. Robertson's second objection, that the use of book profit introduces a certain artificiality into the calculation, is one that I am not nearly so inclined to agree with. It is generally acknowledged that life insurance accounting is quite artificial. If a company were able to choose between this artificial method and a more realistic one, it could reasonably be argued that the results of this voluntarily adopted accounting system are also artificial; because no choice is available, these results are very real indeed. Whether or not the book profit approach is artificial de-

pends upon the ultimate disposition of the profits released. If stockholders plan to disperse these profits in dividends, if they plan to reinvest these profits in the acquisition of further amounts of new business or additional agency plant, the book basis is not at all unrealistic since it measures *available* surplus. Mr. Robertson's remarks on this subject are perhaps more applicable to a mutual company than they are to a stock company.

An important observation included in Mr. Robertson's remarks is that the proposed method can be reversed to solve for a yield rate, given a fixed gross premium. This approach has, in fact, been used with some success. Experience has indicated that the most satisfactory technique is to calculate the value of profit at two yield rates, then to interpolate to determine an approximate yield rate for a given gross premium.

The interest in measuring the relative importance of various of the factors entering into the determination of gross premiums, which was expressed by Mr. Estes, corresponds quite closely to that voiced by Mr. Robertson. The method outlined by Mr. Robertson for measuring certain of the factors is certainly more appropriate for participating insurance than the method outlined in the paper; it is difficult to justify a return on invested surplus at rates suitable for venture capital when the investment is being made by other policyholders in a mutual company.

Mr. Green presents compelling reasons for the development of yield rates by generation of policyholders. Although such a concept was alluded to in the paper, I deliberately refrained from expressing any opinion as to the propriety of such a scheme since the idea is not essential to the techniques described. It is, at this point, appropriate for me to express my total agreement with Mr. Green's remarks.

Two of Mr. Fisher's observations appeal to me very much. I was particularly attracted by his statement that profits are partly a return on investment and partly a return of "capital" to surplus, and by his proposition that the time factor could be converted into a yield factor.

In Mr. Forbes, the reinsurance companies have an able, spirited and good-humored advocate. His case for the intangible services offered by reinsurance is a strong one. The most persuasive of his arguments for reducing the calculated cost of reinsurance concerns the allocation of general overhead expenses. He states that the extremes are the allocation of a full pro-rata share of general overhead against all business, or the allocation of such expenses against fully retained business only, and suggests that the most reasonable position probably lies between such extremes. Being charitably inclined, I would acknowledge that charging all indirect expenses to fully retained business is reasonable. With such an

assumption, and further assuming that the factor for charging indirect expenses to fully retained business is unaffected by the allocation of the same expense to less business, the revised premium rates calculated in accordance with the assumptions underlying Table 6 in the case of 100% Y. R. T. reinsurance become as follows:

$ 3,000	$24.37
6,000	22.45
12,000	21.31
30,000	20.45

An equivalent statement to that originally made in the paper would now be that the calculations suggest that the additional premium on account of reinsurance is $1.76 per $1,000 plus $2.04 per policy. Having retreated this far, there is one point on which I am prepared to stand: simply that reinsurance is one—and a significant one—of the factors which affect the over-all level of company profit. Certainly, for a company which intends to grade premiums by policy size, it would be out of the question to raise premium rates at amounts in excess of the company's retention limit. It is of significance to recognize that on a great deal of business which involves reinsurance the ceding company is probably marketing this product at a very small profit, or perhaps even at a loss. Although there is no practical way to avoid this—other than introducing an issue limit, if this is deemed practical—the total profit must nevertheless be sufficient to meet the objectives of the stockholders. This means that additional profit is required on nonreinsured business in order to offset the profit reduction on reinsured business.

Mr. Rosser is due an apology for my failure to acknowledge the numerous similarities between his 1951 paper and mine. He has summarized, very aptly, the differences between his proposal and that under discussion here. His remarks regarding the difficulty of calculating premium rates according to what is a complex series of assumptions and techniques leave me unimpressed. This criticism is certainly a devastating one in cases where no medium to large scale computer is available. Today such machines are universally available, at least on a consulting or part-time basis, and one firm has already offered to make calculations according to this proposal to all comers; I should be happy to compare the cost of such an outside purchase to the cost of performing the job according to Mr. Rosser's techniques behind a company's own doors. Mr. Rosser has built a most interesting bridge from his method to mine. This bridge is a masterpiece of engineering, but one questions whether or not it will ever carry much traffic. I have a little difficulty in reconciling two statements he

offers in his discussion. The first statement is ". . . and hence his results become quite similar to those implicit in my 1951 paper . . ."; and the second statement is, ". . . when, as here, the valuation rate is under 4%, and the yield rate required on borrowed surplus is 15%, this adjustment is substantial: $1.58 of gross premium." My thinking is not sufficiently advanced to admit that these results, which differ by $1.58, are "similar."

Fortunately, on one point Mr. Rosser and I are in complete agreement. It is comforting to find a colleague occupying a similar, radical position. Asset shares are interesting animals to calculate and to contemplate; my difficulty—and I am sure it is his, too—is interpreting these animals once constructed. Perhaps no other term, unless it is "profit," is as much misused both by actuaries and by other persons interested in the financial mechanics of the life insurance business.

The procedural rules of the Society are such that the author of a paper for the *Transactions* is armed with the ultimate weapon, that of having the last word. My present inclination is to regard this rule as a good one, reserving the right to change that opinion if in the future this weapon should be turned upon me. I hope that in this case the weapon has been used sparingly, because these discussions have pointed out a number of differences in viewpoint, and contributed several important corrections and qualifications to the thesis outlined in the paper. I repeat my thanks to those who participated. For the record it should be noted that all of this, excepting the author's reply, was written before the 1959 Federal income tax law for life insurance companies was finally enacted; this law will have an important impact on future thinking with respect to premium rates and profit margins.