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THE VALUATION OF THE FAMILY POLICY

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THE purpose of this paper is to describe the methods upon which the computations of policy reserves and related Annual Statement items for the Family Policy issued by the company with which I am connected are based. Because this policy provides life insurance coverage for all members of a family (each of the parents and any children) in a single contract for which the premium depends only on the age at issue of the father, these computations raise several new points of professional interest. This interest is, of course, heightened by the fact that very substantial amounts of insurance are being placed in force on this plan by my company and many other companies.

DESCRIPTION OF POLICY PROVISIONS PERTINENT TO VALUATION

While my company now offers several policies providing life insurance coverage on dependent children in connection with coverage on parents, all of the points of interest are present in the valuation of the Regular Ordinary Family Policy we now offer, so that the descriptions in the paper will be primarily in terms of this contract. The benefits of the other policies are briefly described at the end of this section.

The insurance provided by one unit of the Regular Ordinary Family Policy is as follows:

$5,000 of whole life insurance (referred to as the Basic Amount of Insurance) is provided on the life of the insured.[1] $5,000 of additional accidental death benefit is also provided on the life of the insured.

If the insured's and his wife's ages are the same, $1,000 of yearly term insurance is provided on her life. If the ages are different, the amount of the wife's yearly term insurance is determined by a scale designed, on the basis of my own company's recent mortality experience, to produce approximately the same mortality cost as if the ages were the same. The wife's yearly term insurance expires on the policy anniversary nearest the insured's 65th birthday. If the insured dies before then, it becomes paid-up term insurance expiring on that date. If the wife dies during the period for which yearly term insurance on her life is provided, an additional $1,000 of yearly term insurance is provided on the life of the insured during the period between the wife's death and the policy anniversary nearest his 65th birthday.

$1,000 ($250 before the first birthday) of yearly term insurance is provided on the life of each dependent child, ending at the earlier of

[1] In this policy, "the insured" refers to the husband.

a) the first policy anniversary subsequent to the child's 21st birthday; and
b) the policy anniversary nearest the insured's 65th birthday.

This benefit also becomes paid-up term insurance upon the death of the insured. Family Policies issued in Canada must provide death benefits for dependent children somewhat different from those just described, in the event of death before the fifth birthday.

The premiums, which are payable during the insured's lifetime, reduce on the policy anniversary nearest his 65th birthday. Other policies now offered by my company, which provide life insurance coverage on dependent children in connection with coverage on parents, include the Monthly Debit Ordinary Family Policy, which provides that premiums may be collected at the home of the insured and which has a smaller death benefit on the insured than does the Regular Ordinary Family Policy; the Regular Ordinary Parent and Children Policy providing modified endowment at 65 coverage on the life of one parent and term insurance on his or her children; and the Monthly Debit Ordinary Parent and Children Policy providing coverage similar to the latter on the level premium Monthly Debit Ordinary basis.

METHOD OF IN-FORCE ACCOUNTING

On the issue of a one-unit Family Policy, we add $8,000 ($5,000 basic amount, an average of $1,000 for the wife's term insurance, and an average of $2,000 for the children's term insurance) to the Whole Life and Endowment column of the Policy Exhibit. Of this amount, $2,000 will be terminated by decrease at whatever is found, by sampling methods, to be the average duration at expiry of the children's yearly term insurance. ($2,000 is slightly less than the average amount of coverage provided with respect to children, as determined from several samples of recent issues; and 15 years is the average duration of coverage similarly determined.) The additional $1,000 will be terminated by decrease when the insured attains age 65. On the death of the wife or a child, if their insurance has not yet been terminated by decrease, $1,000 will be shown as decreased by death. The Exhibit is restored to balance by the inclusion of $1,000 as a negative lapse in line 15.

On the death of the insured prior to age 65, there will be an entry in line 10, ceased by death. The exact amount of paid-up term insurance available will become a transfer deduction from the Whole Life and Endowment column as well as a transfer addition to the Term column, and in this case also the Whole Life and Endowment column of the Exhibit is restored to balance by a lapse entry.

In the absence of loans and dividend additions, when a policy is con-

tinued as extended insurance after a premium default, the amount of insurance carried in force is reduced (if this transaction has not already been put through on account of the insured's having attained age 65) to $5,000 per unit to agree with the death benefit then available on the life of the insured. (This reduced amount is subject to the usual further adjustments if there are loans or dividend additions on the policy.) The additional insurance (a maximum of $3,000 per unit) carried prior to the premium default is terminated by an entry in Line 15 (lapse) of the Policy Exhibit.

LIFE INSURANCE RESERVE

A. *Policy Reserve (Premium-Paying)*

A theoretically complete analysis of the reserve for the death benefits would take into account the amounts of benefits, the ages of those currently insured, the various contingencies as to order of deaths among those insured, and the probabilities of future births of children to become insured. The resulting refinement in calculation procedures does not seem warranted in view of its expense, and it is questionable whether an extensive system of record-keeping of dependents' in-force would actually result in a significantly more accurate reserve. Thus the following theory was formulated which, many tests reveal, closely approximates the theoretical ideal.

The valuation net annual premium per unit with respect to the basic amount of insurance during the tth policy year for a policy issued at age x of the insured is defined (for the Regular Ordinary plan) as

$$\pi_t = 5{,}000\, P_x + 1{,}000\, P^1_{x:\overline{65-x}|} - 1{,}000\, A^{\ 1}_{x+t-1:\overline{1}|} \quad \text{where} \quad x+t \leqslant 65$$

$$= 5\,000\, P_x \text{ where } x+t > 65\,.$$

It may easily be verified that the present value at issue of these net premiums

$$\frac{\sum_{t=1}^{\omega-x} D_{x+t-1}\pi_t}{D_x}$$

is actually $5{,}000\, A_x$. The amount deducted as the third term of the formula for π_t before attained age 65 of the insured is applied to purchase the term insurance provided on the wife if she is living, or on the insured if she is not living.

The valuation net annual premium per unit with respect to the con-

tingent term insurance on the wife which becomes paid-up at the death of the insured is defined as

$$\rho = 1{,}000\ \frac{{}_{65-x}A^{1}_{xx}}{\ddot{a}_{x:\overline{65-x}|}}.$$

The terminal reserves generated by ρ take the form

$$1{,}000\,{}_{65-x-t}A^{\ 1}_{\overline{x+t:x+t}} - \rho\,\ddot{a}_{x+t:\overline{65-x-t}|}$$

while the insured is living.

These formulas will yield exact reserves when the age of the insured equals the age of the wife. When the ages are unequal, the fact that the amount of insurance on the life of the wife is based on the difference in their ages produces theoretically ideal reserves approximately equal to the above.

The valuation net premium computed by my company to cover the children's yearly term insurance is based on the assumption of a stationary population of insured children and premium-paying fathers. The total mortality cost for the year is divided among the premium-paying fathers. A similar calculation, assuming a child's term insurance expires at the earliest of the father's death, the anniversary nearest the father's 65th birthday, and the anniversary following the child's 21st birthday produces a lower net premium which we will denote by τ. The difference between the two, which we will denote as κ, represents the valuation net premium payable during the insured's lifetime and prior to the policy anniversary nearest his 65th birthday for the paid-up term insurance provided for the children after his death. While the insured is living, the terminal reserves for this benefit are taken as zero. This appears conservative since premiums are payable to age 65 of the insured, but the coverage will generally cease at an earlier duration, namely when the youngest child attains age 21.

Therefore, if we let V_t equal the terminal reserve on the basic amount plus the terminal reserve generated by the premium ρ, the portion of the premium-paying mean reserve per unit on the tth December 31 following issue, with respect to the entire death benefit on the life of the insured, equals

$$\frac{V_{t-1} + V_t + \pi_t + \rho + \kappa}{2}.$$

The remaining benefits are yearly term benefits and are valued in the customary manner, using reserve factors covering the period extending to the next policy anniversary. The portion of the premium-paying mean reserve per unit of Family Policy with respect to the term insurance

provided on the wife before the insured attains age 65, or, if the wife has died, provided on the insured while he is under age 65, is

$$C_t = \frac{1{,}000}{2} A_{\overline{x+t-1:\overline{1}|}}^{\ 1}.$$

This is the regular expression for valuing term insurance expiring in the following year. C_t is an approximation to the theoretical expression (assuming the wife is living)

$$\frac{F_d}{2} \cdot {}_1A_{\overline{x+t-d-1:x+t-1}}^{\ \ 1} + \frac{1{,}000}{2} {}_1A_{\overline{x+t-1:x+t-d-1}}^{\ \ 1},$$

where d is the insured's age less that of the wife ($d \gtrless 0$) and F_d is the amount of insurance provided by the scale of benefits for a given value of d. If the wife is not living, or if she is living but d equals zero, then in each case the single-life formula for C_t is exact.

The portion of the premium-paying mean reserve per unit of Family Policy with respect to the term insurance on the dependent children is $\tau/2$.

In summary, the mean premium-paying reserve per unit on the tth December 31 subsequent to issue is

$$\frac{V_{t-1} + V_t + \pi_t + \rho + \kappa}{2} + C_t + \frac{\tau}{2}.$$

B. *Reserve for Nondeduction of Deferred Fractional Premiums*

A deferred premium asset is held with respect to the total amount of valuation annual premium (including the associated accidental death benefits and premium waiver disability benefits). The asset takes into account the distribution of premiums by frequency of premium payment. The reserve for nondeduction, on the full annual premium basis, is computed by applying whole life reserve factors to the premium 5,000 P_x (which can be shown to be the average equivalent of π_t, using the relationship given in subsection A) and (for attained ages under 65) term to age 65 reserve factors to the sum $\rho + \kappa$.

The results, combined with the results of the comparable calculations for all other plans are then adjusted from an annual premium basis to the level of the deferred premium asset by applying the ratio of deferred to annual premiums in total.

C. *Reserve for Dependents' Paid-Up Term Insurance (after Insured Has Died)*

When a report of the death of the insured under a premium-paying Family Policy is received, the in-force card is removed from the premium-

paying in-force and set aside in a suspense (died) file. As soon as the necessary particulars are received (that is, ages of dependents, amounts of insurance, year of expiry, etc.) the card in suspense is removed and the necessary paid-up term cards are set up. The valuation of these paid-up term cards is performed by applying the usual single-life reserve factors, which vary by year of birth and year of expiry, to the appropriate amounts in force.

The valuation of the suspense (died) file is necessarily based on approximate assumptions since all the needed particulars have not yet been finally ascertained on the statement date. The age at death of the insured is, of course, available. The reserve factors are based on average ages and durations consistent with those used in determining the children's valuation net premiums.

WAIVER OF PREMIUM DISABILITY BENEFIT

The waiver of premium benefit on these policies consists of waiver of the contract premium in the event of the insured's qualifying for disability benefits. The contract premium decreases at the insured's age 65, and the reserve factors take this into account by considering the benefit a waiver for life of the contract premium effective at the insured's age 65 and later, plus a waiver, for the period from disability to age 65 of the insured, of the difference in annual contract premiums before and after age 65. Since this benefit is provided without specific extra premium charge, valuation premiums are assumed to be payable during the lifetime of the insured.

Active life reserves are computed according to our usual assumption that the amount waived is 90% of the respective contract premiums. Disabled life reserves are also computed according to that assumption.

ACCIDENTAL DEATH BENEFIT

The actual amount of accidental death benefit (that is, $5,000 per unit of Family Policy) is carried in the valuation detail records. Standard whole life accidental death benefit reserve factors (premiums and benefits for life) are applied to the amount of accidental death benefit in force.

ACCOUNTING AT DEATH OF INSURED

If the death of the insured occurs during the period for which dependents' coverage is provided, it is necessary to make an accounting entry on the books in addition to the normal death claim charge which arises when the basic amount is paid. This additional accounting entry consists of a charge to the death claim account and an offsetting credit to the premiums paid by death of insured account. The amount of this account-

ing entry is determined by multiplying the amount of insurance with respect to each dependent insured by the appropriate net single premium term insurance factor, considering the year of birth and period of coverage.

As mentioned in subsection C above, there are a number of claims outstanding at year-end where all the necessary particulars have not yet been finally ascertained, but where an approximate reserve has been set up as a result of the insured's death being reported. In addition, there are some claims where the exact reserve has been set up by year-end but where there was not sufficient time to enter the death claim–premium accounting transaction on the company's books. For both of these situations, a death claim liability and an offsetting negative premium liability are set up to adjust the revenue accounts. In the former case where complete information is not available, approximate methods are used to determine the amounts. In the latter case where the necessary detailed information is available, the amounts are determined using all known information.

In addition, there are some claims which have not yet been reported by December 31. Since no fully paid-up reserve for paid-up term benefits has been set up for these claims, it is necessary to make provision for them in the liability for death claims incurred but not reported. The usual type of reconciliation is employed using average factors to determine the amount of liability.

CONCLUSION

The methods presented have been designed to provide an accurate and efficient Annual Statement treatment for Family Policies as applicable to the preparation of the Association Blank. They have necessarily been strongly influenced by the specific benefits and provisions of our policies, as well as by our procedural and organizational pattern. Thus, it is unlikely that another company would adopt the system we have developed without modification. Nevertheless, it is hoped that this paper, by illustrating how one company has developed its procedures, will make easier the task of other actuaries engaged in developing their version of this interesting new plan of insurance, the Family Policy.

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DISCUSSION OF PRECEDING PAPER

H. D. GARBER:

We are indebted to Mr. Sarnoff for his comprehensive report of the way in which his company is handling the complex reserve and annual statement problems arising from the introduction of family policies. The information presented in this paper will undoubtedly be of assistance to companies planning to issue a family policy and should provide some new ideas for those companies which already have a family policy on the market. The procedures which we have developed at the Equitable differ in some important respects from those adopted by Mr. Sarnoff's company. In this discussion I shall describe our methods of calculating reserve liabilities and of preparing annual statement Policy Exhibit information for our family policies.

The Equitable issues two family policies. Each unit of our Family Protection policy, which is the more popular of the two, provides the following insurance benefits: (*a*) $5,000 of whole life insurance on the husband, (*b*) term insurance on the wife amounting to $3,000 before age 31, then decreasing by $100 per year to $500 for ages 55 through 64 and expiring at age 65, and (*c*) $1,000 of level term insurance on each insured child to age 25 or the policy anniversary nearest the wife's 65th birthday, if earlier. Premiums on this policy are payable during the husband's lifetime. The initial premium, which depends only on the age of the husband, is reduced on the policy anniversary nearest the wife's 65th birthday. If the wife dies before that date there is a partial reduction in the premium. When the husband dies, the remaining benefits, if any, on the surviving family members become paid-up.

Our other policy, the Family Security policy, provides endowment insurance maturing at the husband's age 65 on both the husband and wife and term insurance on the children. The procedures which we have developed for handling these two policies are, for the most part, analogous and I shall limit my discussion of specific procedures to those applicable to the Family Protection policy.

Reserves

From an administrative point of view it is desirable to limit the number of subdivisions used in calculating reserve liabilities. With this in mind we developed valuation net premiums and reserve rates which vary only according to the husband's age at issue, the policy duration and

whether the wife is alive or not. Under this approach the reserve rates for each issue age of the husband reflect appropriate assumptions as to the average age of the wife, if alive, the average number and age distribution of the insured children and the likelihood of future births into the family. For example, we assumed that at issue the family of a husband aged 20 would consist of a wife aged 19 and a child aged 0, and that two additional children would be born two and four years after issue, respectively. Similarly, for an insured age 40 we assumed at issue a wife aged 37 and children aged 10, 8 and 6. Various tests we carried out indicated the use of average family composition assumptions for each age of the husband would produce approximately the same aggregate reserve as a calculation which recognized actual composition of each insured family.

In order to determine valuation net premiums and mean reserves for our Family Protection policy, we subdivided the insurance benefits provided into three groups: (1) the whole life insurance on the husband's life; (2) the decreasing term insurance on the wife, including the paid-up insurance provided after the death of the husband; and (3) the level term insurance on the insured children, including the paid-up insurance provided after the death of the husband. Separate valuation net premium and mean reserve rates were calculated for each of these three groups. Before the policy anniversary nearest the wife's 65th birthday, the net premiums and reserves applicable to the entire policy are computed as the sum of (1), (2) and (3) if the wife is alive or the sum of (1) and (3) if the wife is dead. After this anniversary only the husband's benefits are involved.

The net premium and reserve factors for the benefits payable under (1) above were calculated in the usual manner. In determining these factors for the wife's benefits, identified by (2) above, we assumed that premiums are payable during the joint lifetime of the husband and wife, but not beyond the termination of the wife's benefit. The net premiums and reserves for the children's benefits, identified by (3) above, were computed on the assumption that such premiums are payable during the husband's lifetime until the coverage on the youngest child ceases. In the latter case, the present value of the benefits includes the value of benefits payable with respect to any future births anticipated under our family composition assumptions. (It might be mentioned that in these calculations we used a q_0 of .00750, instead of the published CSO value of .02258.) The terminal reserves for the children's benefits are negative at most points and those for the wife's benefits are negative at some points, but the minimum reserve which we plan to hold with respect to each benefit is one-half of the respective valuation net premium.

Since the gross premium for children's benefit is payable to the policy anniversary nearest the wife's 65th birthday, it may appear inconsistent to assume that valuation premiums are payable only to the point where the coverage on the youngest child expires. In our dividends, however, we expect to recognize, on an average basis, the termination of the coverage on the children, and the liability for the excess premiums will be reflected in the dividend liability rather than in the reserve liability.

When the husband dies, individual valuation cards are set up for the paid-up benefits, if any, remaining on the lives of the wife and children. Under our procedures, however, the aggregate net single premium for these benefits is not charged as a death claim but rather is deducted from reserves released by death. The purpose of this deduction is to keep the tabular cost in balance.

Policy Exhibit

When a Family Protection policy is paid for we include $8,000 per unit as a new issue on line 2 of the Whole Life and Endowment column of the Policy Exhibit. Of this amount, $5,000 is for the whole life insurance on the husband, $1,500 for the decreasing term insurance on the wife, and the remaining $1,500 for the level term insurance on the children. The amounts included with respect to the wife's and the children's coverages are approximately equal to the average insurance benefits provided for those dependents between the issue of the policy and the policy anniversary nearest the wife's 65th birthday.

No entry is made in the Policy Exhibit on the death of an insured child, but when a wife dies $1,500 is terminated by death (line 10). On the policy anniversary nearest the wife's 65th birthday, there is a termination by decrease (line 16) of $3,000 if the wife is alive or $1,500 if the wife is dead.

When the husband dies, there is a transfer deduction in the Whole Life and Endowment column and a transfer addition in the Term column for the paid-up dependents' benefits, if any. In determining the amount of the transfer we include the ultimate amount of the wife's benefit (*i.e.*, $500) and $1,000 for each insured child. There is also a termination by death (line 10) of an amount equal to the insurance in force before the death of the husband (*i.e.*, $8,000 if the wife is alive or $6,500 if the wife is dead) less the amount transferred to the Term column. On the average, the amount terminated by death when the husband dies will be about $5,000, or the husband's basic insurance benefit.

On surrender or lapse, the total amount of insurance in force just before the termination is included on line 14 (for surrenders) or on line 15 (for lapses).

W. J. D. LEWIS:

Mr. Sarnoff is to be congratulated on his lucid and elegant presentation of the Prudential's solutions to the various valuation, accounting and exhibit problems which arise in the administration of the Family Policy.

Particularly, I was impressed by Mr. Sarnoff's outline of the Prudential's valuation procedures. These procedures provide the solutions to problems which must have vexed at least one actuary in every company which claims a Family Policy in its portfolio. The techniques in the paper are ingenious and actuarially precise. I think a brief description of my own company's valuation methods, which, although less precise than Mr. Sarnoff's, are in many respects simpler, will provide a useful supplement to his paper.

Confederation Life Association's Family Policy is identical in most respects with the Prudential's. The two main points of differences are, first, that under the Association's contracts all premiums cease at age 65 and the policy becomes paid-up and, secondly, that the total children's insurance per unit policy ($5,000 on insured) is limited to $5,000. If there are more than five children insured at any time under the contract, the insurance on each child is $5,000 divided by the number of children.

The mean reserve held by the Association at duration t on a premium paying policy is of the form

$$M_{(x,\ t)} + K\ . \tag{1}$$

The first term in this expression, $M_{(x,t)}$, depends on the insured's age at issue x and the duration t. It provides the reserve for:

a) The basic insurance, $5,000, Life to 65 on the insured;
b) the term insurance on the life of the wife prior to the insured's attaining age 65;
c) the contingent term insurance of $1,000 on the insured's life after the death of the wife prior to age 65.

The second term, K, is a constant independent of age or duration and provides, during the premium paying period, the reserve for:

d) the term insurance on the children during the insured's life;
e) the paid-up term insurance benefit on the wife's life, commencing on the insured's death;
f) the paid-up term insurance benefit on the children's lives, commencing on the insured's death.

The Association employs a Karup attained age valuation method. The first term in formula (1) fits exactly into this method and the second term is provided for by adjusting the Karup reserve by K times the number of units of Family Policy in force on any year-end.

The development of the reserve $M_{(x,t)}$ is of some interest since it sheds some additional light on the reserve error introduced if the wife's age at issue, y, is different from that of the insured.

The net annual premium required to provide benefits (*a*), (*b*) and (*c*) above is given by the following expression:

$$\pi = \frac{5{,}000\ \mathrm{A}_x + S_{x:y}\,\mathrm{A}^{1}_{xy:\overline{65-x}|} + 1{,}000\ \mathrm{A}^{2}_{\underset{1}{xy:\overline{65-x}|}}}{\ddot{a}_{x:\overline{65-x}|}}, \tag{2}$$

where $S_{x:y}$ is the amount of term insurance on the wife's life.

Now, recognizing that

$$\mathrm{A}^{2}_{\underset{1}{xy:\overline{65-x}|}} = \mathrm{A}^{1}_{x:\overline{65-x}|} - \mathrm{A}^{1}_{xy:\overline{65-x}|}, \tag{3}$$

the expression (2) for π reduces to

$$\pi = \frac{5{,}000\ \mathrm{A}_x + 1{,}000\ \mathrm{A}^{1}_{x:\overline{65-x}|}}{\ddot{a}_{x:\overline{65-x}|}}, \tag{4}$$

provided that

$$S_{x:y} = \frac{1{,}000\ \mathrm{A}^{1}_{xy:\overline{65-x}|}}{\mathrm{A}^{1}_{xy:\overline{65-x}|}}. \tag{5}$$

The terminal reserve provided by π at duration t is then

$$\begin{aligned} \mathrm{V}_{(x,\,t)} &= 5{,}000\ \mathrm{A}_{x+t} + S_{x:y}\,\mathrm{A}^{\overset{1}{}}_{x+t:\overline{y+t}:\overline{65-x-t}|} \\ &\qquad + 1{,}000\ \mathrm{A}^{\overset{1}{}}_{\underset{1}{x+t:\overline{y+t}:\overline{65-x-t}|}} - \pi\,\ddot{a}_{x+t:\overline{65-x-t}|} \end{aligned} \tag{6}$$

$$= 5{,}000\ \mathrm{A}_{x+t} + 1{,}000\ \mathrm{A}^{\overset{1}{}}_{x+t:\overline{65-x-t}|} - \pi\,\ddot{a}_{x+t:\overline{65-x-t}|}, \tag{7}$$

provided

$$S_{x:y}\mathrm{A}^{\overset{1}{}}_{x+t:\overline{y+t}:\overline{65-x-t}|} = 1{,}000\ \mathrm{A}^{\overset{1}{}}_{x+t:y+t:\overline{65-x-t}|}, \tag{8}$$

which is true if

$$S_{x:y} = S_{x+t:y+t}, \tag{9}$$

or if $S_{x:y}$ depends only on the difference in age $x - y$.

Values of $S_{x:y}$ were calculated on an IBM 705 computer from first

principles in order to determine the extent to which equation (9) is true.

It was found that for values of x ranging up to age 50 the percentage difference of $S_{x:y}$ from $S_{40:40-x+y}$ was, on the valuation basis, CSO $2\frac{3}{4}\%$, as follows:

Value of $x - y$	Percentage Error in $S_{x:y}$ from $S_{40:40-x+y}$
10	5 %
5	$2\frac{1}{2}\%$
0	0 %
− 5	$2\frac{1}{2}\%$

These errors in assuming that equation (9) is true are small and produce negligible errors in the reserve. Accordingly the terminal reserve $V_{(x,\ t)}$ is calculated from equation (7) and

$$M_{(x,\ t)} = \frac{V_{(x,\ t-1)} + V_{(x,\ t)} + \pi}{2}. \qquad (10)$$

One other interesting point arising out of this analysis is that, on the death of the wife (y) at duration t, a second wife at age $z + t$ can be substituted without altering the valuation premium or reserve provided the amount of insurance on the second wife's life is $S_{x:z}$.

The Association decided, as far as benefits (d), (e) and (f) are concerned, to make charges against the gross premium on the plan at the beginning of each policy year for the term costs of the benefits during that policy year. With sufficient theoretical accuracy the charge, for the tth policy year, would be of the following form:

$$B\sum_{a} A^{\ 1}_{\overline{a+t-1:1}\rceil} + S_{x:y} A^{\ 1}_{\overline{x+t-1:1}\rceil} A^{\ 1}_{\overline{y+t:65-x-t}\rceil} + BA^{\ 1}_{\overline{x+t-1:1}\rceil} \sum_{a} A^{\ 1}_{\overline{a+t:m}\rceil}, \qquad (11)$$

where:

a is the age at issue (possibly negative) of a child insured in the tth policy year;

$B = \$1{,}000$, or $\dfrac{\$5{,}000}{n}$ if the number of children, n, insured in the tth policy year is more than 5.

$m =$ lesser of $21 - a - t$ and $65 - x - t$

and the summation $\sum_{a}$ is taken over all children insured in the tth year.

The first term in (11) depends primarily on the number of children insured. For any one child it has the tendency to decrease until about attained age 11 or 12 and then increase. It increases sharply as new children

are insured. The second term increases with duration but is fairly small (less than $2.00 per unit policy). The final term, like the first term, depends on the number of children insured but tends to decrease with duration and is small (less than $0.25 per child).

These considerations, combined with a series of tests on assumed and average family age distributions led to the conclusion that a flat charge of $2K$ per unit policy could be used to approximate with adequate accuracy the necessary annual cost of the three benefits. The unexpired portion of this charge, at any year-end, is K and this amount is reserved as shown in equation (1). The value of K was checked by means of an accurate calculation based on the first 500 contracts written.

GEORGE C. CAMPBELL:

Mr. Sarnoff has been of much service to the profession by contributing a paper on the valuation of the Family Policy. As far as I know, his paper is the first in this field which is of great practical importance.

I want first to comment briefly on Mr. Sarnoff's paper and then to give some corresponding information about the Metropolitan Family Policy.

Mr. Sarnoff indicates that the in-force is adjusted through the lapse line for some transactions where I should prefer the decrease line. I believe that the lapse line should be used to reflect terminations of whole policies before they have nonforfeiture values. My comments later about Metropolitan practices will indicate our thinking on this point.

Mr. Sarnoff states in connection with accounting at death of the insured that he intends to put cross entries through the books of account for an amount in addition to the normal death claim charge to take care of the single premiums for the term insurance on the wife and children. We do not make such entries. When the father dies, the reserve released at death is the premium paying reserve on his policy less the reserves required for the term insurance on his dependents. Although the net reserve released could be negative in some individual cases, I prefer this nonledger recognition of the realignment of reserves for gain and loss purposes rather than bookkeeping cross-entries in the premium and death claim accounts.

Moving along to premiums and reserves, I should prefer to consider together rather than separately Mr. Sarnoff's first formula for the premium on the husband and the formula given later designated by C_t. If these two varying premiums were brought together, we would come immediately to a level premium to age 65 and a lower level premium thereafter. These premiums might then be considered as covering the basic $5,000 on the husband, plus $1,000 term to 65 on the wife payable at the death of

the wife before 65 regardless of the survival of the husband. The term to 65 symbol would be quite unusual because the husband would pay the premium during his life to age 65 to provide term insurance for his wife to age 65, whether or not he is alive at the time of her death. The reserve for this part of the benefit would follow, using the normal expressions. If this rearrangement of the formulas were made with the same symbols defining somewhat different benefits as between husband and wife, the expression the author gives for ρ would be defined differently in words as term insurance on the husband after the death of the wife, with premiums payable during the life of the husband limited to age 65. I believe this approach, which differs only slightly from the author's except for verbal interpretation, more clearly conforms with net level premium valuation.

This completes my comments on the author's paper. The remainder of my remarks will concern our Metropolitan policy. These comments will apply specifically to our $5,000 basic policy, although with suitable variations, the same principles apply to the $3,000 policy.

The principal Metropolitan Family Policy provides $5,000 Endowment at 85 on the husband, plus $1,250 term to 85 for wife of equal age and $500 pure endowment, plus $1,000 term to age 25 on each child under 18 at issue or born after issue. The insurance on wife and children becomes paid-up if the father dies. The gross premium during the first five years is 85% of the ultimate premium. If the wife dies before the husband, the premium will be reduced to the "one-parent" basis. Our policy does not increase the insurance on the husband at the death of the wife. This brief description naturally covers only the major policy provisions.

We carry the $5,000 policy into the issue and into the in-force for $8,250, which includes $6,250 for husband and wife of equal ages, plus $1,000 on each of two children existing, on the average, at issue.

In our policy exhibit, the amount paid at the death of the husband, the wife or a child goes into the death claim line. If a child dies, the in-force remains unchanged, and the amount in the death claim line is offset by a negative decrease. If the wife dies, the in-force is reduced by the amount of the claim, and no further adjustment is necessary for equal ages. Any adjustment required because the wife is of unequal age with the husband is made through the decrease line. If the husband dies, the remaining in-force is transferred from the life and endowment column to the term column where the net additional amounts are set up in the increase line.

At default of premium payments before the policy has a value, the whole amount would terminate through the lapse line. At default after the policy has a nonforfeiture value, the premium paying amount carried

in the policy exhibit would be decreased in the life and endowment column to the nonforfeiture term amount which would be transferred to the term column.

At some later date we shall determine from appropriate samples the duration at which the insurance on the children should be terminated because of the age limitation.

We start our reserve factor calculation with the basic Endowment at Age 85 premium on the husband adjusted for the fact that gross premiums and the corresponding net premiums during the first five years are only 85% of the ultimate.

We compute another ultimate premium, taking 85% of it during the first five years, valued by appropriate joint life annuities on the lives of both husband and wife, limited to age 85, to provide the term insurance and pure endowment insurance on the life of the wife. The use of the joint life annuities on the premium side increases the premium to provide for paid-up insurance on the wife after the death of the husband, since on the benefit side we have the single premium for the complete benefit to the wife.

The terminal reserve factors for benefits to husband and wife follow from these premiums. We computed them by the u and k method, and used continuous functions throughout to provide for immediate payment of claims and the nonpayment of premiums beyond the month of death.

The ultimate net premium for children included in the gross premium is the same for a given age of husband and wife regardless of the number of children or whether there are any children. This same premium continues after the children reach age 25 and even after the death of any or all the children, so long as the father is alive and under age 85. This premium is valued by a life annuity to age 85 on the life of the father. On the benefit side for existing children at issue, we have the single premium for the term insurance, limited at age 25, weighted by the age distribution of the children being insured and by the average number of children per unit. We have also on the benefit side a single premium for unborn children, reflecting the discounted value of yearly probabilities of birth to a wife of given age and the cost at birth of the insurance to be provided.

The reserve both for existing and for unborn children at any point in time is the present value of the future benefits less the present value of the future premiums. We found for our distribution of children that the mean reserve for the children's portion of the benefit was positive at the end of the first calendar year. Although our computations produced a smaller factor, we used the conservative round amount of $1 per unit for the children's benefit in our 1957 valuation.

The mean reserve for the children becomes negative by the end of the second calendar year and remains negative thereafter. I believe that it would be proper theoretically to take credit on the policy as a whole for the negative reserve on the children's benefits, since the premium for these benefits cannot be terminated separately. As a practical matter, however, it might be more satisfactory to take no credit for the negative reserves which develop after the first calendar year.

LOWELL DORN:

Mr. Sarnoff's interesting and informative paper outlines his company's approach to the problems posed by this unique development in life insurance, family policies. Each company's approach must depend, of course, on the benefits provided by its policy and be tailored to its own special problems for handling its valuation and other year-end work.

Our family policy is very similar to the one described in Mr. Sarnoff's paper, but there are some differences. For each unit, it provides (*a*) a basic face amount of $5,000 whole life insurance on the husband; (*b*) where the wife is the same age as the husband, $1,250 of term to his age 65 on the wife; and (*c*) the insurance on the children. If the wife is older or younger, naturally the amount of insurance on her life is adjusted; however, it varies only with the difference between the ages of the husband and the wife.

If the wife predeceases her husband before he reaches age 65, then her term insurance is paid and is automatically replaced by $1,250 of term to age 65 on his life. This means that we have only one table of nonforfeiture values in the policy. Upon the husband's death prior to age 65, the wife's and children's insurance is continued as paid-up insurance.

Now, for the purpose of nonforfeiture calculations and minimum cash value tests and valuation, we found it convenient to consider the policy as consisting of four separate elements. They are (1) the basic $5,000 of whole life on the husband; (2) the $1,250 of term insurance to age 65 on the wife, replaced by a similar amount on the husband if she predeceases him; (3) the provision for making the wife's insurance paid-up if the husband dies first; and (4) the provision for the children's term insurance, both before and after the father's death.

We found that under any reasonable assumptions regarding the average ages and average number of children covered by the policy, the combined reserves for these last two elements were negative, almost without exception. As a practical matter, they could be ignored in computing terminal reserves and nonforfeiture values. Hence, for valuation purposes, during the husband's lifetime, we hold a mean reserve for the $5,000 whole

life and the $1,250 term to age 65. We add an extra $3.50 per unit to cover the children's benefit and the provision for making the wife's insurance paid-up if the husband predeceases her; this represents one-half of the average net premium for these two coverages combined, without taking credit for any negative terminal reserves on these extra coverages.

After the husband's death, we hold the exact theoretical reserves for the wife's and children's paid-up term insurance since, under the law, it would be impracticable, if not impossible, to do otherwise.

We have generally taken a practical approach in nonforfeiture computations and valuation procedures. In issuing this policy, we contemplated simplified, streamlined operations, and we are adhering to that concept throughout. Certain aspects of the policy, especially the children's insurance, could lead into a theoretical jungle, but the policy is built on averages for the purpose of providing expense savings. We feel that this should be reflected in our valuation and annual statement procedures.

In the policy exhibit we count each unit as $8,000 during the husband's lifetime, reducing this to $5,000 when he attains age 65. In the case of the wife's and the children's paid-up term insurance after the husband's death, we show the exact amount.

I shall not try to cover other areas in which problems have arisen and our solutions to them. We do not plan to use the lapse line in the policy exhibit for simply reflecting adjustments. We plan to use the decrease or increase line.

We are issuing a great deal of business on this plan, and the cost to policyholders is low. However, it can be kept low only if we hold down the expenses, and that is our basic objective.

GERARD A. VICINO:

Mr. Sarnoff has presented a highly informative paper on the Family Policy. This is the first paper in the literature dealing with certain actuarial phases of this new and popular plan of marketing life insurance on a wholesale basis. In my discussion I should like to present my thoughts relative to net premiums, reserves, nonforfeiture values, and substandard premiums.

Regarding reserves for the death benefit coverages on the insured and his wife, Mr. Sarnoff plunges immediately to the end results by defining the valuation net premiums. It may prove instructive and helpful to show the direct development and rationale of the various net premium elements. My discussion of this phase will be confined to the life insurance

benefits on the insured and his wife, exclusive of the benefits on the lives of the dependent children, the disability waiver and additional accidental death benefits on the insured, and the conversion benefit on the wife, that are provided in a Family Policy of the type described by Mr. Sarnoff as the Regular Ordinary Family Policy issued by his company. While Mr. Sarnoff refers to the insurance provided on the life of the wife as being on the "yearly term" basis, I do not know whether this coverage is actually so described in the policy itself or if it is defined, as in my company's policy, simply as "term insurance" without qualification. In any event, it is my feeling that even if the wife's coverage is called yearly renewable term insurance in the policy, the definition is of no consequence since there is no provision in the policy for subsequent discontinuance of the wife's coverage with a corresponding premium reduction. I shall therefore assume that the policy refers to the wife's coverage simply as being term insurance.

The premium for this policy is payable only so long as the insured may live and the benefits, per unit, consist of:

a) \$5,000 of whole life insurance on the life of the insured;
b) \$1,000 of term insurance on the life of his wife expiring on the policy anniversary nearest the 65th anniversary of the insured's date of birth if the wife's age is the same as her husband's, more if she is younger, and less if older; and
c) \$1,000 of contingent term insurance on the life of the insured, commencing at his wife's death and terminating at the policy anniversary nearest his 65th birthday.

Using the above particulars of the policy, we may directly develop the total net premium and its distribution into three parts corresponding to the various elements of coverage.

Development of Composite Pre-65 Net Premium

If x denotes the issue ages of the husband and wife, the present value at issue of these three elements of coverage are 5,000 A_x, 1,000 $A^{1}_{x:\overline{65-x}|}$, and 1,000 ${}_{65-x|}A^{2}_{xx}$, respectively, where the last symbol refers to a benefit payable on death of the *insured* if he dies subsequent to his wife rather than, as is implied in Mr. Sarnoff's formula for ρ, to the wife's dying after her husband. Since the coverage remaining after age 65 is simply whole life insurance on the insured and since the gross premium payable after 65 is either exactly or essentially the whole life premium, it is logical to define the post-65 level net premium as the regular whole life premium,

i.e., 5,000 P_x. Then if Z denotes the composite pre-65 level net premium, we have

$$(Z - 5{,}000\ P_x)\ \ddot{a}_{x:\overline{65-x}|} + 5{,}000\ P_x \ddot{a}_x = 5\,000\ A_x + 1{,}000\ A^1_{x:\overline{65-x}|} + 1{,}000\,|_{65-x} A^1_{xx},$$

whence

$$Z = 5{,}000\ P_x + 1{,}000\ P^1_{x:\overline{65-x}|} + 1{,}000\ \frac{|_{65-x} A^1_{xx}}{\ddot{a}_{x:\overline{65-x}|}}.$$

The first term of this expression is the net premium for the husband's permanent insurance and the sum of the second and third terms represents the net premium for the wife's term and husband's contingent term coverages combined. The second term is too small to represent the premium for the wife's coverage because it assumes the premium is payable to the wife's death even if she dies after her husband. The third term, on the other hand, is too large to represent the premium for the husband's contingent term insurance because it ignores the fact that the premiums which would have been payable for the wife's coverage had she not died are not eliminated but instead are transferred towards the husband's contingent term insurance.

Development of Pre-65 Level Basis Breakdown

The complete level net premium for the wife's coverage is obviously

$$\alpha = 1{,}000\ \frac{A^1_{x:\overline{65-x}|}}{\ddot{a}_{xx:\overline{65-x}|}}.$$

Similarly, the additional level net premium, β, for the husband's contingent term insurance, being in addition to the α premiums transferred to this reversionary coverage following his wife's death, is determined from the formula

$$1{,}000\,|_{65-x} A^1_{xx} = \alpha\,(\ddot{a}_{x:\overline{65-x}|} - \ddot{a}_{xx:\overline{65-x}|}) + \beta \ddot{a}_{x:\overline{65-x}|}.$$

Thus the pre-65 level premiums for the three elements of coverage outlined previously are as follows:

$$\phi = 5{,}000\ P_x \qquad \text{for element } (a)$$

$$\alpha = 1{,}000\ \frac{A^1_{x:\overline{65-x}|}}{\ddot{a}_{xx:\overline{65-x}|}} \qquad \text{for element } (b)$$

$$\beta = \frac{1{,}000\,|_{65-x} A^1_{xx} - \alpha\,(\ddot{a}_{x:\overline{65-x}|} - \ddot{a}_{xx:\overline{65-x}|})}{\ddot{a}_{x:\overline{65-x}|}},$$

as the additional premium for element (*c*). As before, the symbol ${}_{65-x|}A^{1}_{xx}$ refers to a benefit payable if the husband dies after his wife.

The sum of these three level premium elements will be found, by way of verification, to be equal to Z, the composite pre-65 level premium. This level basis breakdown of the pre-65 composite premium, calculated on the basis of realistic mortality and interest, is useful for computing not only standard gross premiums but also substandard premiums.

Development of Pre-65 Unlevel Basis Breakdown

While normally it is not necessary to proceed beyond the composite basis or the level basis breakdown, in the case of the Family Policy we need to convert the level premium elements to an unlevel basis in order to meet two practical requirements of the policy, *viz.*:

(1) The variation in the amount of the death benefit on the wife whenever her age differs from her husband's should depend only on the differential in their ages and not on their actual ages, and
(2) The total pre-65 nonforfeiture equity of the policy should be determined and applied solely with reference to the policy duration, the insured's age, and the amount of whole life insurance on his life.

The first requirement, in turn, implies that the implicit gross premiums for the wife's coverage included in the total pre-65 gross premium should be determined according to a scale that not only varies with the differential in the husband's and wife's ages, but also varies in the same manner as the scale for the amount of the wife's death benefit. The assumption that the wife's coverage is largely yearly term insurance obviously meets both requirements.

One approach towards achieving an unlevel premium breakdown is to keep β (the pre-65 additional level net premium for the husband's contingent term insurance) level at the same value as under the level basis. Then adopting the same definition for the tth year premium, ϕ_t, for the husband's permanent coverage as used by Mr. Sarnoff, and denoting the tth year premium for the wife's term insurance by α_t, we have

$$\phi_t = 5{,}000\ \mathrm{P}_x + 1{,}000\ \mathrm{P}^{1}_{x:\overline{65-x|}} - 1{,}000\ c_{x+t-1}$$

$$\alpha_t = 1{,}000\ c_{x+t-1} + 1{,}000\ \mathrm{P}^{1}_{x:\overline{65-x|}} \left(\frac{\ddot{a}_{x:\overline{65-x|}}}{\ddot{a}_{xx:\overline{65-x|}}} - 1 \right)$$

$$\beta_t = \beta\ .$$

Thus, on this basis the unlevel net premiums for the wife's coverage are the one-year term premiums at the attained age each increased by a level

positive quantity that is independent of the duration. The present value at issue of each of these three premium elements agrees with the present value of the corresponding level basis counterpart (when, in connection with α_t, it is remembered that following the wife's death the remaining α_t premiums are applied towards the husband's term coverage).

A second approach is to allow all three premium elements to vary by duration. One solution is the following:

$$\phi_t = \text{same value as in the preceding paragraph}$$

$$\alpha_t = 1{,}000\, c_{x+t-1} \frac{\ddot{a}_{x:\overline{65-x}|}}{\ddot{a}_{xx:\overline{65-x}|}}$$

$$\beta_t = \beta + 1{,}000\, (\mathrm{P}^1_{x:\overline{65-x}|} - c_{x+t-1}) \left(\frac{\ddot{a}_{x:\overline{65-x}|}}{\ddot{a}_{xx:\overline{65-x}|}} - 1 \right),$$

where these unlevel tth year premium elements apply to the husband's permanent, the wife's term, and the husband's contingent term coverages, respectively. Under this method the premiums for the wife's coverage are a constant multiple of the attained age one-year term premiums.

Pre-65 Reserves

i) Husband and wife both alive:

Let

$${}_tV'_x = {}_{|65-x-t}\mathrm{A}^{\frac{2}{1}}_{x+t:x+t} - \frac{{}_{|65-x}\mathrm{A}^2_{xx}}{\ddot{a}_{x:\overline{65-x}|}} \ddot{a}_{x+t:\overline{65-x-t}|}$$

and ${}_t\mathrm{MV}'_x$ = corresponding tth year mean reserve.

If ${}_t\mathrm{MV}_x$ and ${}_t\mathrm{MV}^1_{x:\overline{65-x}|}$ denote the tth year mean reserves per dollar for the ordinary life and term to age 65 plans, then the tth year mean reserve, ${}_t\mathrm{MV}^{\mathrm{F}}_x$, per unit of the Family Policy exclusive of coverage on dependent children is given by the formula

$${}_t\mathrm{MV}^{\mathrm{F}}_x = 5{,}000\ {}_t\mathrm{MV}_x + 1{,}000\, {}_t\mathrm{MV}^1_{x:\overline{65-x}|} + 1{,}000\ {}_t\mathrm{MV}'_x .$$

This simple expression results irrespective of which of the four previously-derived sets of net premiums (*i.e.*, the composite, level split, or either of the two unlevel split bases) is employed, and is equivalent to Mr. Sarnoff's formula. This result may be easily verified by general reasoning. It will be noticed that the total reserve is largely made up of published values.

ii) Wife dead, husband alive:

Each of the four sets of net premiums derived previously produces

$$MV_x^F = 5{,}000\ {}_tMV_x + 1{,}000\ {}_tMV_{x:\overline{65-x}|}^1 - 1{,}000\ \frac{{}_{65-x}|A_{xx}^2}{\ddot{a}_{x:\overline{65-x}|}}\left(\frac{\ddot{a}_{x+t-1:\overline{66-x-t}|} + \ddot{a}_{x+t:\overline{65-x-t}|} - 1}{2}\right),$$

which also corresponds to Mr. Sarnoff's result. This formula can also be verified by general reasoning.

I might point out a possibly misleading feature of Mr. Sarnoff's formula for the total mean premium-paying reserve. While the form of his formula is the same regardless of whether the wife is living or dead, the *numerical* value produced by his formula changes when the wife dies, since the element ${}_{65-x}|A^2_{\overline{x+t}:x+t}$ of the terminal reserves generated by the premium he defines as ρ vanishes when the wife dies.

iii) Husband dead, wife alive:

Each of the four sets of net premiums previously derived produces

$${}_tMV_x^F = 1{,}000\ \frac{A^1_{\overline{x+t-1:\overline{66-x-t}|}} + A^1_{\overline{x+t:\overline{65-x-t}|}}}{2}.$$

However, as Mr. Sarnoff suggests, it would be preferable to value the actual amount of insurance on the wife with the paid-up reserve factor applicable to her actual age and the duration to expiry.

The accompanying table outlines the pre-65 annual premium premium-paying tth year reserves for the coverages on the husband and wife. The symbol δ refers to the premium ${}_{65-x}|A^2_{xx} \div \ddot{a}_{x:\overline{65-x}|}$. It will be noticed that the reserves for the husband's permanent coverage, on the one hand, and those for the husband's term insurance and the wife's term insurance (if living), on the other hand, depend upon the assumed premium basis. In certain situations knowledge of the applicable split of the total pre-65 reserve is needed.

Where punched cards are used for valuation, the above reserve formulas suggest, as one possibility, the creation of three cards: one for \$5,000 of regular ordinary life insurance, one for \$1,000 of term to age 65 insurance, and one for \$1,000 ${}_tMV'_x$, the special reserve defined above in section (i). If the husband dies before 65, being survived by his wife, all three cards would be terminated and a new card created for the paid-up benefit on the wife. If the husband survives to 65, the second two cards would be terminated. If the wife predeceases her husband before his age 65, the third card can be terminated and either no new card created (for

economy and conservatism) or, alternatively, a negative reserve card can be created.

In addition to the various reserves mentioned by Mr. Sarnoff, I feel that a reserve should be accumulated during the premium-paying period to meet the cost of the conversion benefit available to the insured's wife and dependent children. The desirability of developing specific reserves for this contingent liability is evident in view of the fact that not only may the wife convert her actual amount of insurance but each of several covered children may convert up to five times the amount of term insurance each held before conversion and that up to three units of insurance may be issued.

Premium Basis	Terminal Reserves		Mean Reserves	
	Husband and Wife Both Living			
	Husband's Permanent Coverage	Husband's and Wife's Term Coverages	Husband's Permanent Coverage	Husband's and Wife's Term Coverages
Composite and Level Split Bases	$5{,}000\ {}_tV_x$	$1{,}000\ {}_tV^1_{x:\overline{65-x}|} + 1{,}000\ {}_tV'_x$	$5{,}000\ {}_tMV_x$	$1{,}000\ {}_tMV^1_{x:\overline{65-x}|} + 1{,}000\ {}_tMV'_x$
Unlevel Split Bases	$5{,}000\ {}_tV_x + 1{,}000\ {}_tV^1_{x:\overline{65-x}|}$	$1{,}000\ {}_tV'_x$	$5{,}000\ {}_tMV_x + 1{,}000\ {}_tMV^1_{x:\overline{65-x}|} - 1{,}000\ (c_{x+t-1}/2)$	$1{,}000\ {}_tMV' + 1{,}000\ (c_{x+t-1}/2)$
	Husband Living, Wife Dead			
	Husband's Permanent Coverage	Husband's Term Coverage	Husband's Permanent Coverage	Husband's Term Coverage
Composite and Level Split Bases	$5{,}000\ {}_tV_\lambda$	$1{,}000\ {}_tV^1_{x:\overline{65-x}|} - 1{,}000\delta \cdot R_t$	$5{,}000\ {}_tMV_x$	$1{,}000\ {}_tMV^1_{x:\overline{65-x}|} - 1{,}000\delta \dfrac{R_{t-1}+R_t-1}{2}$
Unlevel Split Bases	$5{,}000\ {}_tV_\lambda + 1{,}000\ {}_tV^1_{x:\overline{65-x}|}$	$-1{,}000\delta \cdot R_t$	$5{,}000\ {}_tMV_x + 1{,}000\ {}_tMV^1_{x:\overline{65-x}|} - 1{,}000\ (c_{x+t-1}/2)$	$1{,}000\ (c_{x+t-1}/2) - 1{,}000\delta \dfrac{R_{t-1}+R_t-1}{2}$

LEGEND

λ = husband's issue age
${}_tV_x$ = regular Ordinary Life terminal reserve per dollar
${}_tMV_x$ = regular Ordinary Life mean reserve per dollar
${}_tV^1_{x:\overline{65-x}|}$ = regular Term to Age 65 terminal reserve per dollar
${}_tMV^1_{x:\overline{65-x}|}$ = regular Term to Age 65 mean reserve per dollar
$R_t = \bar{a}_{x+t:\overline{65-x-t}|}$
$\delta = ({}_{65-x}A^2_{xx})/(\bar{a}_{x:\overline{65-x}|})$
${}_tV'_x$ and ${}_tMV'_x$ = terminal and mean reserves generated by δ

Determination of Wife's Premium and Death Benefit Scales

If x and y are the issue ages of the husband and wife, respectively, and d denotes the algebraic excess of x over y, then the precise amount of death benefit, F_d, on the wife per unit of the policy is given by the expression

$$F_d = 1{,}000\,\frac{A^{1}_{x:\overline{65-x}|} + {}_{65-x|}A^{2}_{xx} - {}_{65-x|}A^{2}_{x:x-d}}{A^{\frac{1}{}}_{x-d:\overline{65-x}|}}.$$

Values of F_d can be calculated (on the basis of realistic mortality and interest) for a simple network of values of x and d. The resulting pattern of values of F_d coupled with the assumed approximately one-year term nature of the coverage on the wife can be utilized to arrive at reasonable scales of death benefit and implicit gross premiums relative to the insurance on the wife.

Cash Values

While for practical reasons it is desired to determine the pre-65 total nonforfeiture equity of the policy solely with reference to the permanent whole life insurance on the insured, it seems to me that any test of a proposed scale of cash values as to whether it meets the statutory minimum values should, under the present wording of the law, be based on the policy as a whole and not on only that portion of the policy remaining after eliminating the term coverage on the wife. It seems justifiable, however, to exclude the coverage and premiums on the children from any minimum values calculations in view of the indefiniteness, as far as any particular policy is concerned, of the number of children covered, their ages, and durations of insurance. With the children eliminated, the policy is simply one which promises to pay, per unit, $5,000 on death of the insured whenever it may occur, $1,000 on death of the wife if she dies prior to a specified date, and $1,000 additional on the insured if he dies after his wife but before the specified date, and the premiums are payable so long as the insured may live. Even if the policy describes the wife's insurance as being on the yearly renewable term plan, I feel this same viewpoint is applicable because there is no provision in the policy for discontinuance of the wife's coverage with a corresponding reduction in the premium payable.

The minimum cash values under this policy-as-a-whole interpretation of the standard nonforfeiture law can then be determined quite easily, except for one difficulty. When the insured's and his wife's ages are the

same, the present value at issue of the adjusted premiums, per unit, is given by the expression

$$5{,}000\ A_x + 1{,}000\ A^1_{x:\overline{65-x}|} + 1{,}000\ |_{65-x}A^2_{xx} + .02S$$

$+.40$ (first year plan adjusted premium but $\geq .04S$)

$+.25$ (smaller of first year plan adjusted premium and Ordinary Life adjusted premium for S dollars, but $\geq .04S$)

Some question arises as to how S, the equivalent level amount, should be determined. A number of reasonable methods of calculating S suggest themselves, and in any event the problem is not great, since the equivalent level amount cannot be much larger than \$5,000.

If the value of S is determined (for example, it might for simplicity and liberality be taken as \$5,000), then the uniform ratio r required by the standard nonforfeiture law is obtained by equating the product of r and the present value at issue of the gross premiums for the wife's and husband's coverages (exclusive of the disability and accidental death benefits) to the expression given in the preceding paragraph. With r determined, it can then be shown that the minimum cash value at the end of the tth year is as follows, where G represents the gross premium for the husband's and wife's coverages before the policy anniversary nearest the insured's 65th birthday, and g the corresponding premium payable thereafter, and ${}_tV^F$ denotes the total terminal reserve for the wife's and husband's coverages:

(i) Husband and wife both alive (pre-65)

$${}_tV^F_x - (rG - Z)\ \ddot{a}_{x+t:\overline{65-x-t}|} - (rg - 5{,}000\ P_x)\ {}_{65-x-t}|\ \ddot{a}_{x+}$$

(ii) Husband alive, wife dead (pre-65)

$$\text{Value in (i)} - 1{,}000\ |_{65-x-t}A^2_{\overline{x+t:x+t}}$$

(iii) Wife alive, husband dead (pre-65)

$$1{,}000\ A^1_{\overline{x+t:65-x-t}|}$$

(iv) Husband alive (post-65)

$${}_tV^F_x - (rg - 5{,}000\ P_x)\ \ddot{a}_{x+t}\ .$$

The symbol Z in (i) refers to the composite net premium derived in an earlier section. These expressions indicate that the minimum cash value at any duration depends upon whether the husband or wife or both are living at that time. In the case of item (iii), it might well be argued that

the minimum value should be based on the wife's actual age and actual amount of insurance.

The pre-65 cash values generally allowed during the insured's lifetime for a policy of the type under discussion are, per unit, equal to the net level reserves for $5,000 of whole life insurance on the insured which is assumed to be financed by the unlevel premiums

$$5{,}000\ P_x + 1{,}000\ P^1_{x:\overline{65-x}|} - 1{,}000\ c_{x+t-1}$$

less a surrender charge that grades to zero at the end of a certain period or at the end of the mortality table. Thus the values allowed in practice are equal to

$$5{,}000\ {}_tV_x + 1{,}000\ {}_tV^1_{x:\overline{65-x}|}$$

less the surrender charge applicable to the tth year. If the surrender charges are properly chosen, the minimum values given above will be met.

While some companies allow a cash value with respect to the wife's paid-up term insurance, others do not. Under the approach discussed above, cash values for this paid-up benefit (following the insured's death) would be indicated.

If the scale of death benefit on the wife is determined so as to produce approximately the same mortality cost as when the husband's and wife's ages are the same and if the proposed scale of cash values is tested against the minimum values computed on the basis of the policy as a whole, there would be no need for constructing an implicit gross premium scale applicable to the wife's term insurance. However, some insurance departments have already been exposed to the concept of such an implicit scale. It would, therefore, be preferable to prepare and submit such a scale to forestall any possible delay in securing approval of the policy.

Substandard Extra Gross Premiums

The method used in my company to develop the substandard extra gross premiums for table ratings may be of interest to other actuaries. My company's Family Policy (not yet introduced) is essentially the same one discussed above, except that the reversionary death benefit on the insured and the wife's term coverage (when her age is the same as her husband's) are $1,250 per unit. We decided that the policy would be made available for substandard husbands up to Table D but would be declined if the wife were substandard. Substandard children at issue are to be handled by excluding them. Since the disability waiver and additional accidental death benefits are automatic, an assumption had to be

made as to the average rating for these benefits applicable to each life rating. The assumption adopted for the supplemental benefits, together with the acceptable underwriting limits, is shown in the following table.

Husband's Life Classification	Assumed Average Disability and Additional Accidental Death Rating	Maximum Underwriting Acceptance for Disability and Additional Accidental Death Rating
Table A.....	1½×	2½×
Table B.....	2×	3×
Table C.....	2½×	3½×
Table D.....	3×	4×

The excess, per unit, of the pre-65 multiple-table net premium for both the husband's and wife's term coverages combined over the corresponding standard premium can be shown to be less than

$$1{,}250\,\frac{A^{1}_{x:\overline{65-x}\rceil} + {}_{65-x|}A^{2}_{xx}}{\ddot{a}_{x:\overline{65-x}\rceil}}\left(\frac{\ddot{a}_{x:\overline{65-x}\rceil}}{\ddot{a}'_{x:\overline{65-x}\rceil}} - 1\right),$$

where the unprimed symbols are based on standard mortality and the primed function on multiple-table mortality. This inequality results from the fact that the value of ${}_{65-x|}A^{2}_{xx}$ is reduced when the life insured (*i.e.*, the husband) is substandard and the counter life (*i.e.*, the wife) is standard, as compared to the value when both lives are standard.

Pre-65 substandard extra term net premiums were calculated by the above expression for each of the four life table ratings and quinquennial ages on the basis of realistic mortality and interest. Then the pre-65 substandard extra gross premiums for quinquennial ages within each table classification were calculated as the sum of the following elements:

a) the company's regular ordinary life substandard extra gross premium for $5,000 of insurance;
b) the substandard extra net premium for the term coverages, calculated by the foregoing formula, plus a suitable loading;
c) the substandard extra gross premium for the additional accidental death benefit, using the assumed ratings in the above table; and
d) the disability extra gross premium, including waiver of (*a*), (*b*), and (*c*) as well as the total standard premium.

The substandard premiums for individual ages were then secured by interpolation within each life rating.

The post-65 substandard extra gross premiums are, per unit, the same as those applying to our regular ordinary life policy for $5,000 of insurance.

HERBERT L. FEAY:

The reserve that Mr. Sarnoff develops for his basic amount of insurance is equivalent to the reserve for $5,000 of ordinary whole life insurance plus $1,000 of term insurance to age 65, both on the life of the husband. The use of the one-year term factors in defining the net premiums and reserves for the insurance on the husband and wife, it seems to me, introduces an unnecessary actuarial complication. That complication is not needed for either the reserves or cash values but may be useful in securing the proper answer to the cash value problem. This answer is also secured by others by use of another actuarial complication involving the classification of some of the benefits as "Payor Benefits" not needing reserves and cash values.

Analysis of Benefits

My analysis of the insurance (other than disability and accidental death benefits) provided by one unit of the Family Policy of the Prudential type is as follows:

(1) $5,000 of ordinary whole life insurance on the husband.
(2) $1,000 of term insurance on each child to age 25 but not beyond age 65 for the husband.
(3) $1,000 of survivor term insurance on husband to his age 65 after death of wife if wife dies before husband attains age 65.
(4) $\$K$ of contingent term insurance on wife if she dies before husband and before he attains age 65.
(5) $\$K$ of survivor term insurance on wife if she dies after husband and before he would have attained age 65.

The amount of insurance, $\$K$, is selected so that benefits (3) and (4) above will be the equivalent of $1,000 of term insurance to age 65 on the life of the husband. Assuming the same mortality rates for both lives, K equals 1,000 if the wife and husband are of equal age, K is greater than 1,000 if the wife is younger, and K is less than 1,000 if the wife is older than her husband.

Presumably a company which has a differential in premium rates based on sex should have K equal to 1,000 at some age from 3 to 5 years higher than the age of the husband.

For a specified difference in the ages of husband and wife the spread of the values of K from 1,000 increases with age at issue. These variations in K are small and usually are ignored, and the values are determined by use of an average age for the husband.

Net Premiums and Reserves

All benefits involved can be valued on the full preliminary term method, so that the discussion of net premiums and reserves can be confined to those for the net level premium method.

The single premium value of benefit (1) is obvious. The single premium value of benefits (3) and (4), because of the method of determining the value of K, is single premium term insurance to age 65 on the husband.

The single premium value of benefit (5) is $A^{1}_{xy:\overline{65-x}|}$ with x equal to the husband's age, and y equal to the wife's age.

The single premium value of the insurance on the children will depend on the assumptions regarding the children in the family. Mr. Sarnoff assumes a stationary number and stationary ages for the children in a family for all ages of the husband and all policy durations. Another assumption that has been used is that only the term insurance on the children who are alive and actually insured should be considered. I believe a more exact procedure than either of these assumptions is the use of a Family Index Table similar to the one developed by Mr. Elder A. Porter of the Manhattan Life Insurance Company. Incidentally, I understand that the Manhattan was the first company to bring out a Family Policy in the current development of this plan. Later in this discussion I give a Family Index Table and suggest a method of determining net premiums and reserves for the term insurance for the children.

Only the life contingencies of the father are involved in the annuities used to obtain net level annual premiums from the net single premiums for the benefits.

Under the Commissioners Standard Valuation Law, a company is not free to select the net premiums to correspond with the benefits granted. This law provides that, for a policy with a uniform amount of insurance and requiring the payment of uniform premiums, the modified net premiums shall be a uniform percentage of the respective contract premiums. The law further provides that the reserves for policies with varying amounts of insurance or with varying premiums shall be calculated by a method consistent with this principle. The principle as applied in practice has required that the net premiums for all benefits must be level when the gross premiums are level and must vary directly with the gross premiums when the gross premiums vary by policy duration. Examples are level premium policies with disability benefits terminating at age 65 (without a reduction in gross premiums), level gross premium Family Income insurance policies providing decreasing insurance benefits, and Modified Life policies with level death benefits and with gross premiums that double at the end of five years. On this basis for a consistent method,

the total net annual valuation premiums for all benefits provided by a policy must be a constant percentage of the contract gross premiums. This rule should be followed for the Family Plan unless special approval is secured from supervisory officials for a different loading procedure. This different procedure must be consistent with the constant percentage factor for level premiums. This could be a cause of difficulty for a company with a constant policy charge or with gross premiums graded by size.

Because of my assumption that all benefits of the Family Plan of the Prudential type can be valued on the full preliminary term basis, I am not discussing the effect on annual valuation premiums and on reserves of the varying benefits of this Family Plan. A company which provides the benefits listed but with no premiums after age 65 or which uses endowment insurance for benefit (1) will have this problem for the older ages at issue.

A company could determine the valuation net premiums that it desires, using the proper temporary life and whole life annuity values for the husband, and then load those premiums by a constant percentage. This can produce problems for the cash value calculations. The annual adjusted premiums for the Standard Nonforfeiture Method must also be a uniform percentage of the respective annual gross premiums and there is no provision for a consistent method as is the case for reserves. If you desire the Family Plan to have the same cash values and reserves after age 65 as a $5,000 ordinary whole life policy on the life of the husband, you will have a nice problem of adjustment in trying to determine the minimum values of the law.

From the discussion of Mr. Sarnoff, I understand that he has level, net valuation premiums for the life of the husband for benefit (1) and level net premiums during the life of the husband prior to his attained age 65 for the other four benefits in my list. He determines separate premiums and reserves for the benefits, except that benefits (3) and (4) are combined and valued as the equivalent of $1,000 of term insurance to age 65 on the husband.

Following his assumptions for premiums, the level net premiums and reserves for benefits (1), (3) and (4) equal the sum of the level net premiums and reserves for two standard plans on the husband, namely $5,000 of ordinary whole life and $1,000 of term insurance to age 65.

Survivor Insurance on Wife

As indicated by Mr. Sarnoff, reserves in excess of half the annual net premiums are needed for benefit (5). The reserve liability for this benefit is not negative as is sometimes indicated in arguments that this benefit

can be ignored in determining reserves and cash values. The negative reserve argument would apply only if premiums are continued for a considerable period after age 65, the terminating age for the benefit. Such a continuance of premiums for valuation purposes must be reflected in the gross premiums.

When the husband and wife are of equal age, the amount of K for the wife's survivor insurance is 1,000, and the net level annual premiums on the basis of the CSO Table with 3% interest are as follows:

WIFE'S AGE EQUALS HUSBAND'S AGE

Age of Husband	Annual Level Net Premium
20	$1.11
25	1.30
30	1.53
35	1.80
40	2.10
45	2.42
50	2.68

The net level annual premiums are payable during the lifetime of the husband prior to age 65 and do not terminate on the death of the wife prior to the death of the husband.

If the wife is 5 years younger than the husband, the average value of K is taken as 1,482; and if the wife is 5 years older than the husband, this average value is assumed to be 669.

The net level annual premiums determined on the same standards and procedures as for the above but with differences in ages for husband and wife are as follows:

ANNUAL LEVEL NET PREMIUMS

Age of Husband	Wife 5 Years Younger	Wife 5 Years Older
25	$1.64	$1.06
35	2.20	1.48
45	2.91	2.02

It is obvious from these premiums that reserves in excess of half the net premiums are required for this survivor insurance.

Insurance Departments will undoubtedly approve approximate methods for determining such reserves. Probably an average age for the wife three or four years younger than the husband's age can be used. If the calculations must be by hand-operated machines, values can probably be determined for 5-age and 5-year-duration intervals with intervening values interpolated.

Insurance on the Children

It is not correct to assume that the term insurance on the children is the insurance for 2.2 children from age 5 to age 25. The actual number of children and the average age of the children vary by attained age for the husband and wife, and children can be added to existing families. These conditions can be incorporated into a Family Index Table.

Using the Family Index Table developed by Mr. Porter, together with additional information from U.S. Census, Social Security, and Railroad Retirement statistics, I developed a Family Index Table for families insured under Family Policies. The following figures are from my table:

FAMILIES INSURED UNDER FAMILY POLICIES

Age of Living Husband	Average Number of Children	Average Attained Age of Children
22	1.31	2.00
27	1.74	4.00
32	2.53	6.00
37	3.05	10.00
42	2.95	13.00
47	2.72	15.00
52	2.46	16.00
57	2.16	17.00
62	1.48	18.00

This is an aggregate table for all policy durations for each attained age of the husbands who are alive and whose families are insured under Family Insurance policies. The children are included until age 25 but not beyond attained age 65 for the husband.

If the husband is alive at age y, the number of children for his attained age y is given in the Family Index Table, and one-year term insurance must be provided for those children. If the husband dies between attained age y and $y + 1$, single premium paid-up term insurance must be provided for the children at the end of the one-year term period. The number of children granted this paid-up term insurance has been taken to be the number of children at attained age $y + 1$ as given in the Family Index Table. Only those children are provided with paid-up insurance, as no new children are to be added to the policy after the death of the husband.

It is a comparatively simple actuarial exercise to devise commutation columns to secure the single premium value of the term insurance for the children included in the Family Index Table. This single premium is divided by a temporary life annuity to age 65 on the life of the husband to secure the annual level net premium.

Net premiums that I calculated for the children's insurance using the Family Index Table and CSO mortality rates with 3% interest are as follows:

PREMIUMS FOR CHILDREN'S INSURANCE

Husband's Age at Issue	Annual Net Premium
20	$5.83
25	5.89
30	6.03
35	5.88
40	5.68
45	5.49
50	5.15

These net premiums indicate that reserves will accumulate only for the younger ages at issue and that the accumulation will be small. Probably an average reserve of $4.00 per policy unit will be entirely adequate for the children's term insurance in the Family Insurance plan.

Cash Values

The paper does not mention cash values but, as the song says, "you can't have one without the other." In general, the same problems are involved in the determination of cash values as in the determination of reserves, but there are differences in the legal requirements for determining cash values and reserves that will not permit using the same procedures for both.

The Nonforfeiture Law does not contain a provision that cash values for policies with varying premiums and benefits can "be calculated by a method consistent with the principles" outlined for level insurance with level premiums. Instead the Nonforfeiture Law requires that the adjusted premiums for cash values must be a constant percentage of the gross premiums and that the present value of all benefits must be included in the calculation except for benefits that can be eliminated under a special subsection. The Nonforfeiture Law also provides for using an equivalent level amount of insurance for varying insurance benefits under a policy with benefits that vary by duration.

The insurance on the children can be eliminated from consideration for the cash values, either because the inclusion of those benefits will reduce the cash values or because these benefits would not have cash values if they were granted by a separate policy.

Benefit (5), the survivor term insurance on the wife, can be called a reversionary annuity for the premiums for term insurance granted after the death of the husband. The premiums to be waived must be the

premiums for the attained age of the wife at the death of the husband. Apparently this position has been accepted by supervisory officials.

With benefits (2) and (5) eliminated, the remaining three benefits are combined into whole life insurance and term insurance on the life of the husband as is done for the reserves. Since these two benefits are combined into one policy, the equivalent level amount of insurance for determining expense allowances must be on the whole life insurance plan.

Some companies include benefit (5) in the cash value calculation. When this is done the value of that benefit is included in the calculation for the equivalent amount of whole life insurance.

The table that follows illustrates the increases in minimum cash values that result when benefit (5) is included in the cash value calculations on the CSO Table with 3% interest.

HUSBAND AND WIFE OF EQUAL AGE

HUSBAND'S AGE AT ISSUE	POLICY DURATIONS			
	5	10	15	20
20	$1.62	$6.03	$10.29	$13.94
30	2.28	6.86	7.19	11.16
40	−1.22	4.39	3.62	−0.51
50	−1.75	−3.50	0.00	.....

RALPH E. EDWARDS:

The practice of my Company, Baltimore Life, differs in one respect from that described by Mr. Sarnoff's excellent paper and the discussions here. In the Policy Exhibit of the Annual Statement, while the husband is alive, we show the amount of term insurance on the wife and children in the Term Policies Amount column, with zero in the Term Policies Number column.

(AUTHOR'S REVIEW OF DISCUSSION)

PAUL E. SARNOFF:

I am grateful for the interest shown in the valuation aspects of the Family Policy.

The discussion indicates that it is the practice of some companies to use the Decrease line of the Policy Exhibit rather than the Lapse line under certain circumstances when the coverage of the dependents is terminated. My Company has processed terminations of integrated policies, such as those containing decreasing term benefits in conjunction

with a level benefit for life, in the following way. When the balance of the policy is to remain in force, the Lapse line is used to terminate that portion of such an integrated policy which lapses prior to the date on which that benefit would have automatically decreased. Thus in the similar case of Family Policies, we find it consistent to use the Lapse line in other instances than those where an entire policy terminates without value.

We learn, also, from the discussion that some companies reflect the establishment of the dependents' paid-up term insurance upon the death of the insured by a negative entry in the Reserves Released by Death line of the Analysis of Increase in Reserve. It is the practice of my Company to enter a death claim–premium accounting transaction upon the books to reflect the establishment of dependents' paid-up term insurance. We hold death claim liabilities with respect to dependents' paid-up term insurance for pending and incurred but unreported death claims, since under our method of operation such claims are not reported for valuation in-force purposes as death claim terminations prior to the statement date. We reflect the subsequent establishment of dependents' paid-up term coverage as an accounting transaction involving the death claim account because the death claim liability has previously been used to recognize the liability on these claims.

Mr. Campbell regroups two elements of the valuation net premium and arrives at the following valuation net premiums exclusive of the premiums for the children's term insurance:

a) A $5,000 whole life premium for the basic amount;
b) A $1,000 term to 65 premium which reflects (assuming the husband and wife have equal ages) a $1,000 death benefit on the life of the wife irrespective of the order of death. The premium for this benefit is assumed payable during the lifetime of the insured prior to his age 65;
c) A premium for $1,000 of contingent term insurance on the life of the insured if he is second to die, prior to age 65. Premiums for this benefit are assumed to be payable during the lifetime of the insured prior to his age 65.

In Mr. Campbell's view, his rearrangement more clearly conforms to the principles of net level premium valuation since each of the three elements is level during the period prior to the insured's age 65 and, of course, the first element is level thereafter. However, the methods described in my paper also result in a net level premium valuation for the policy as a whole in that the valuation net premiums in total agree with the total of the premiums described by Mr. Campbell during the period prior to age

65 of the insured, as well as with the $5,000 whole life premium payable thereafter.

Section ii of Mr. Vicino's discussion of pre-65 reserves suggests that the net single premium contingent term insurance element of the terminal reserves vanishes when the wife is dead and the insured alive. The reserve factors used to value Family Policies issued by my Company are not affected by such an adjustment. Our policies are administered so that a new wife may be insured upon the remarriage of the insured (subject to some underwriting requirements). Therefore, the same reserves are computed with respect to all Family Policies where the insured is alive, whether or not his wife is living. This practice results in a slight overstatement of the reserve.

Mr. Vicino considers there is a need for reserves to be built up in order to provide for the cost of the conversion benefit. Since the insurance on any child may be converted without evidence of insurability at expiry, in the ratio of five thousand dollars of insurance for each thousand in effect prior to expiry, he points out that there is an opportunity for antiselection. One of the benefits considered in the determination of the net premium τ was the cost of this conversion benefit. This cost is thus reserved for during the lifetime of the insured in the same manner as the death benefit itself. Since the premium for this benefit is payable until age 65 of the insured, but coverage on the children generally ceases at an earlier date, namely at age 21 of the children, we know that negative reserves will result at most durations. Even though antiselection is expected in comparison with experienced standard ordinary mortality, the total value of the benefit under a converted policy is expected to be less than that provided by the valuation table we employ. There is, therefore, no necessity for my Company to hold an extra mortality reserve for policies which have been converted from Family Policies.

Mr. Feay's discussion includes a statement that some companies treat the dependents' paid-up term insurance as a payor benefit which does not need reserves. I should imagine, however, that even with respect to payor benefits, reserves of one-half the net extra premium are held while the insured is alive and a temporary annuity of the net annual premium is held after the insured has died. He states that all benefits can be valued on a full preliminary term method. Since the valuation net premium per thousand is less than that for a 20-Payment Life policy, this statement is true in those states which have adopted the Standard Valuation Law, but it is not true in any jurisdiction which requires full net level premium reserves. Mr. Feay's statement of the assumptions regarding the stationary population used to compute the cost of the children's insurance

is somewhat oversimplified. I should have made it clearer in my paper that our stationary population includes increments for births and expiries due to children attaining age 21 or to the insured's attaining age 65. The total cost of the benefits provided for the stationary population is distributed equally among the premium payers below attained age 65.

Mr. Feay comments that our valuation premiums for benefit (1) are level and that the premiums for the other four benefits are also level. In the aggregate it is correct that the total valuation net premium for the policy is level until age 65 and at that age changes to a new level that is maintained during the remaining lifetime of the insured. I should point out, however, that in our analysis of the benefits, the valuation net premiums for some of the components, namely benefit one and the sum of benefits three and four, is distinctly unlevel. The use of level premiums for each benefit simplifies the arithmetic and might be regarded as more sophisticated than the yearly term approach we have developed. However, the yearly term approach was chosen by my Company because it was adaptable both to a verbal presentation, and to a symbolic presentation where a more rigorous proof was desired. In fact, on several occasions we demonstrated the equivalence of the methods.

I should like to thank Mr. Feay for his interesting comments on cash values, and Mr. Vicino for his lucid presentation of his analysis of Family Policy reserves. Finally, I should like to thank Mr. Garber, Mr. Lewis, Mr. Campbell, Mr. Dorn, and Mr. Edwards for their descriptions of their companies' practices. This additional information should be of real assistance to actuaries whose companies are planning to enter the Family Policy market but have not yet done so.