

Pension Plan Life-Cycle Funding Approach

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Abstract

During its life cycle, a pension plan experiences fluctuation much like the ups and downs in the life of a human being. This paper reviews the life cycle of a typical pension plan, examines the issues relating to its funding, and proposes a new funding approach that tries to balance the various concerns. This funding approach should be flexible enough so that some of its key components can be adjusted automatically along with the maturation of a pension plan to reflect the changing concerns in funding at various stages. This funding approach is called a Pension Plan Life-Cycle Funding Approach. It is built around two key risks related to benefit security in the life cycle of a pension plan:

1. A “sudden death” situation (i.e., a plan sponsor becoming insolvent and the plan is forced to wind up), and
2. A “horrible lengthy life” situation: for example, a plan is kept ongoing but faces continual bad luck—it has experienced more losses than gains, and that makes the cost of funding the pension plan higher than anticipated.

Although legislation and regulations differ by geographic location, the fundamentals around pension plan funding are very similar. The approach presented in this paper is created with the expectation that rules and regulatory constraints can be amended in the future, if needed, to fit a sound pension plan funding approach accepted worldwide. This paper provides a high-level view of the suggested funding approach so that readers may focus on the reasoning behind this concept. Details about how this approach can be adopted in practice have been reserved for subsequent papers.

1. Introduction

There have been many concerns in recent years about pension plan funding after dramatic deterioration of the funding status for many defined benefit pension plans. A few pension actuaries have commented that the existing funding approach is broken. Others have suggested modifying our existing funding approach or introducing financial economics into pension plan funding. We have not experienced such a high volume of discussions for many years. Our funding approach has not received that much public attention in the past. I believe this is a good time to revisit the approach for funding a pension plan in this changing world. It is necessary that we change when the times change. Rules may need to be rewritten to incorporate new theories. With better research and technology, we should be able to improve our existing funding approach to better fulfill the needs of a now more complicated society.

We can be very creative and come up with many different solutions to solve a problem. But we should remember that *the right solution must reflect reality*. I will share my view of the current funding approach in Section 2: The Deal with the Devil. In Section 3, Value at Risk, I will explain the fundamental concept behind the funding approach that I propose. Then we will go through the life cycle of a pension plan in Section 4. It is important to understand fully the nature of the financial product that we are valuing. To close, I will introduce the Pension Plan Life-Cycle Funding Approach in Section 5.

Sections 2 and 3 provide basic information. Some readers may wish to skip these sections and focus on Sections 4 and 5.

2. The Deal with the Devil

2.1 What Is the Story?

Before we can develop a right solution, we must first go back to the fundamentals. Let's begin with a story. Once upon a time, there was a conversation between Adam and the Devil. They are playing a game about life. The sole tool used in the game is a coin that shows a picture of heaven on one side and a picture of hell on the other. The Devil flips the coin. If the heaven side lands up, the gates of heaven open and Adam can go in. If it's hell, the gates of hell open, and the Devil takes Adam in with him. But there is an option. If the coin comes up "hell," the Devil would allow Adam to purchase his life back for the sum of one dollar (\$1). This substitution option requires Adam to put money on the table before the coin is flipped.

Before the game starts, Adam thinks he needs advice because he is betting on his life. Adam brings his entire savings of eight quarters (\$2) and seeks out his friend who works as a pension consultant. The consultant reviews the case and argues that the substitution option is the key component of the game. To find out the amount that Adam should put on the betting table, the consultant applies his pension funding knowledge to value the substitution option. Based on statistical principles, the probability for each side to come up a fair coin is 0.5. If heaven comes up, the cost to buy back is \$0 because Adam does not need to utilize the substitution option. On the other hand, if hell comes up, the cost to buy back is \$1. After considering future contingencies, the expected value for the substitution option is determined to be \$0.50 ($\$0 \times 0.5 + \1×0.5). To be conservative, the consultant adds a provision for adverse deviations to the result calculated. At the end, he suggests that Adam put three quarters (\$0.75) on the table. Adam follows the advice, perhaps not too wisely, and enters the game.

The Devil then flips the coin, and it lands hell side up. The Devil is more interested in Adam's money than his life, so he asks Adam whether he would like to use the substitution option. Adam looks at his money on the betting table and sees he is short by one quarter (\$0.25). It is too late now because the rules require Adam to put money on the table in advance if he would like to use the substitution option after the flip. Sadly, Adam needs to hand over his life.

I tried to create this parable as a simplified illustration about the existing pension plan funding situation. The characters in the story are described below:

Name/Item in Parable	Represents
Adam	Pension plan sponsors
Consultant	Actuaries who provide funding advice to plan sponsors
Fair coin	The risks affecting the true cost for providing pension benefits. Its outcome is beyond anyone's control.
Quarters in pocket	Current financial strength of the pension plan sponsor
Quarters on betting table	Assets allocated to pension plan to provide benefit security at the valuation date

2.2 What Is Deficient With Pricing?

I believe one of the deficiencies of our existing pension plan funding approach is that we try to price the future contingencies as if the risks can be transferred easily. We try to come up with a single liability figure to represent a large range of potential outcomes. The pension plan liabilities are determined based on expected value calculated using best-estimate assumptions about future contingencies to measure the known risks then advanced by adding provision for adverse deviations to protect against small deviations from the measurement. This is the approach used in the above story, and it turns out to be deficient.

The expected value approach is commonly used in pricing financial products so that such products can be traded in the market and the risks can be transferred to other parties in a very short time. Pricing is a financially sound approach in valuing a product when the product has high liquidity. However, pension plan benefits are generally illiquid liabilities guaranteed by the plan sponsor. The sponsor can wind up the plan to settle its obligations. A division or the whole corporation can be bought or sold as part of mergers and acquisitions, with the plan being transferred to another plan sponsor. But other than cessation of plan sponsorship, the sponsor cannot transfer the risks away easily through trading a portion or the whole plan with another corporation, the banks, or retail investors. In most cases the sponsor is assuming both the liability and the commitment of maintaining the plan. We do not yet have a developed financial market that can be used to trade pension plan liabilities. Pricing is financially sound only if the product has a liquid trading market. Unless we can develop sophisticated derivatives to

hedge pension plan funding risks, *the use of pricing is insufficient for funding when there is no market.*

What about purchasing group annuities for the members? Aren't risks being transferred out? In theory, purchasing group annuities can transfer some funding risks of a pension plan. A major concern is that most of the active members' actual retirement benefit is subject to future salary changes and future decrements, which make it very difficult to determine the cost precisely. The form of benefit payout is also not determinable in advance because members may elect to receive a lump-sum payment or a monthly pension. Even if insurance companies can offer such a complicated product, the premium to be charged for getting such protection is likely to be expensive. These constraints combined with the current low interest rate environment would lead to high costs for complete risk transfers, which will likely discourage a plan sponsor from obtaining such insurance protection. Also, if insurance companies are unable to provide (nonlevel) group annuities with cash payments that can exactly match those provided by some plans with sophisticated plan provisions, it is inappropriate to fund the pension plan as if such coverage was available for purchase. The risks involved in pension plan funding simply cannot be easily transferred away in reality for various reasons. On the other hand, advantages for self-insuring remain that can provide the plan sponsor some degree of control on contributions and allow the sponsor to fund the benefits at a lower cost through investing in riskier asset types that have higher long-term expected returns.

2.3 When Can We Use Pricing?

The illiquid nature of a pension plan makes our existing pension plan funding approach, which is similar to pricing, an insufficient approach when used to determine the funding requirements for a pension plan. The pricing approach remains valid in determining the accounting position of a pension plan for the plan sponsor so that all amounts in the sponsor's financial statement can be marked-to-market. Marking-to-market is important because a mark-to-market financial statement helps investors to determine the fair value of a company so that they can decide whether they should buy, hold, or sell its stocks. As stock trading can transfer risks between investors, having mark-to-market financial statements is important for financial reporting. In fact, the worldwide accounting profession's view is converging that market value is the one true and relevant measure of a company's worth.¹

¹ Mercer Human Resource Consulting 2003.

However, the objective for funding a pension plan is to provide benefit security, which is not only different from pricing (i.e., finding a tradable value or market value for a product) but requires complicated analysis (i.e., requires full understanding of all contingencies) before a well-informed decision can be made about the contribution and desired surplus levels. *It is inappropriate to use the mark-to-market approach to value liability if the product itself has no liquid trading market and the owner is required to assume all positive and negative outcomes.* It is important to use different approaches for funding and accounting valuations. If pricing or finding the market value of pension plan liabilities provides insufficient information for funding pension plans, what's next?

2.4 Deal With The Devil Again

Let's revisit this story. This time the game remains the same except that Adam seeks advice from his wife, Eve. Eve learned the rules and came up with the following analysis:

Picture on Coin	Number of Quarters on Table	Outcome
Hell	0 (\$0.00)	Death
Hell	1 (\$0.25)	Death
Hell	2 (\$0.50)	Death
Hell	3 (\$0.75)	Death
Hell	4 (\$1.00)	Life
Hell	5 (\$1.25)	Life
Hell	6 (\$1.50)	Life
Hell	7 (\$1.75)	Life
Hell	8 (\$2.00)	Life
Heaven	Any	Life

Eve's conclusion is that Adam will lose his life only if both a picture of hell shows up and fewer than four quarters (\$1) are placed on the betting table in advance. Based on the results of the various scenarios, a safe one-time bet is to put four to eight quarters (\$1 to \$2) on the table before entering the game.

Adam places four quarters (\$1) on the betting table. The Devil flips the coin, and a picture of heaven shows up. Adam takes his money back and leaves knowing he is assured a trip to heaven.

What is different between the first and second game? A quick answer is the amount of information provided. In the first game, the consultant summarizes all the

risks into a single value. That helps to simplify the situation, but it fails to show the complete picture to the users of the information. If lesser means better, then there is nothing wrong with that. However, the single number provided rarely turns out to be the exact right number. It is sure to be either excessive or insufficient. In the second game, Eve described the consequences of various potential outcomes. Adam then made an informed decision after considering his situation and reviewing the consequences.

Instead of finding the expected value or long-term average for pension plan liabilities, actuaries should illustrate the risks of funding a defined benefit pension plan to the plan sponsor. A decision should then be made by the sponsor about the level of benefit security it desires. Instead of pricing the liabilities of a pension plan and valuing the plan as if it were a tradable financial product, *plan sponsors need to know the incremental costs to increase the probability that there are sufficient assets in the pension plan to cover future benefit obligations.*

3. Value at Risk

3.1 Fully Funded Is Not Enough

Our current funding approach fails to properly illustrate the risks involved in providing pension benefits. Let's look at the following dialogue between a plan sponsor and its plan actuary:

Party in Dialogue	Conversation
<i>Plan sponsor:</i>	What is the funded status of the pension plan?
<i>Actuary:</i>	The plan is fully funded.
<i>Plan sponsor:</i>	What do you mean by fully funded?
<i>Actuary:</i>	Using best-estimate assumptions to determine the benefits costs, the value of assets is sufficient to pay for the future benefits. However, future experience may deviate from expected, which will result in gains and losses. The plan experience will be reflected in future valuations. Since our valuation starts with best-estimate assumptions, about 50 percent of the time the actual cost will be higher and about 50 percent of the time the actual cost will be lower than the best-estimate liability calculated. The good news is that we have included a provision for adverse deviations in the calculations as required by the actuarial valuation standards. As such, the probability of having insufficient assets may equal 40 percent instead of 50 percent.
<i>Plan sponsor:</i>	Are you saying we have enough money to cover ourselves for 60 percent of the time only? What if things go wrong in the future? How much additional contribution should I make now to enhance the benefit security level?
<i>Actuary:</i>	Sorry, your plan is fully funded based on the most recent actuarial valuation. Additional contributions are not permitted. Also, the funding approach used doesn't provide estimates about the additional cost to maintain current benefit security levels if actual experience is worse than my guess. So, I don't have an answer for you.
<i>Plan sponsor:</i>	Are you telling me that I am living on your guess, and you have no idea about how bad the situation can turn out to be? What if your guess is wrong? Will you pick up the shortfall for me?

Assuming the projected actual cost is symmetrically distributed, then the best-estimate liability equals the average cost of providing the benefits. This implies that 50 percent of the time the actual cost is lower, and 50 percent of the time the actual cost is higher than the best-estimate liability calculated. In other words, if a pension plan's funding ratio is at 100 percent exactly, the implication is not that the plan is fully funded. The true message to the users is that the plan has sufficient assets to cover the liabilities for 50 percent of the time. Half of the time there will be excess assets, and half of the time there will be shortfalls in the future.

A weakness of our existing funding approach is that we give out one liability number, which is almost absolutely going to be a wrong number. Then we ask the plan sponsors to fund their pension plans as if this number, although justifiable in theory, will turn out to be the correct number. When experience deviates from what is assumed, the funding ratio has to be continuously revised to reflect new information. This is a reactive funding approach. A reactive approach can explain problems after they occurred, but it fails to quantify the impact of potential problems in advance. Time was spent on calculating experience gains and losses in the past intervaluation period and not enough *on anticipating potential gains and losses after the current valuation date. Whatever happened in the past stays with the past. It is the future that plan sponsors should be concerned about.*

Given that we already know that deviations between assumptions and future experience are the nature of pension plan funding, we need a proactive funding approach. A proactive approach provides a clear map for the users so that they can see its current location, the roads upfront, and the intersections that they will be crossing later. If benefit security is defined to be the complete promise of the plan sponsor unlimited by the degree funded, one way to design a proactive funding approach is to include the concept of value at risk (VaR) in funding pension plans.

3.2 Economic Capital

Value at risk gives a broad idea of the worst loss that can be incurred. More formally, VaR describes the percentile of the projected distribution of the outcome over the target horizon. If c is the selected confidence level, VaR corresponds to the $1 - c$ lower-tail level. For example, with a 95 percent confidence level, VaR should be such that it exceeds 5 percent of the total number of observations in the distribution.² The choice of the confidence level is relatively arbitrary. Users should recognize that VaR

² Jorion 2001.

does not describe the worst-ever loss but is rather a probabilistic measure that should be exceeded with some frequency.³

After the VaR has been calculated, the amount of economic capital then can be determined based on the risk tolerance and to reflect the level of security desired. Economic capital represents the amount of capital that needs to be set aside to cover most of the potential losses. VaR can be viewed as a measure of potential losses in a financial activity that requires economic capital to support.

VaR measures in funding pension plans should be determined based on the probability of ruin, which is the probability that plan assets are not sufficient to cover liabilities, resulting in insolvency at any future date within a selected time horizon. Economic capital in funding pension plans (which will be redefined in Section 5) is defined to be the amount of assets in excess of the wind-up liabilities at the valuation date. The plan sponsor is responsible to ensure that there is sufficient capital (or assets) in the pension plan to provide benefits and protect against future contingencies. Sufficient capital (or assets) provides a cushion against experience losses. Actual capital can be set to equal the VaR measure or below it for a relatively less secure funding decision. The greater the amount of capital (or assets) in the pension plan, the greater the level of protection provided to the members, and vice versa.

To illustrate the VaR concept, we will look at the cost to provide a \$10,000 nonindexed pension payable annually in advance to one retiree aged 65. To simplify the calculations, this example is structured so that mortality is the only risk involved. The detailed calculations and assumptions used can be found in the Appendix. The key figures are summarized below:

Confidence Level	Present Value of Benefits	Increases to Next Level
0%	\$ 10,000	n.a.
25	103,000	\$93,000
50	127,600	24,600
75	142,100	14,500
100	169,900	27,800

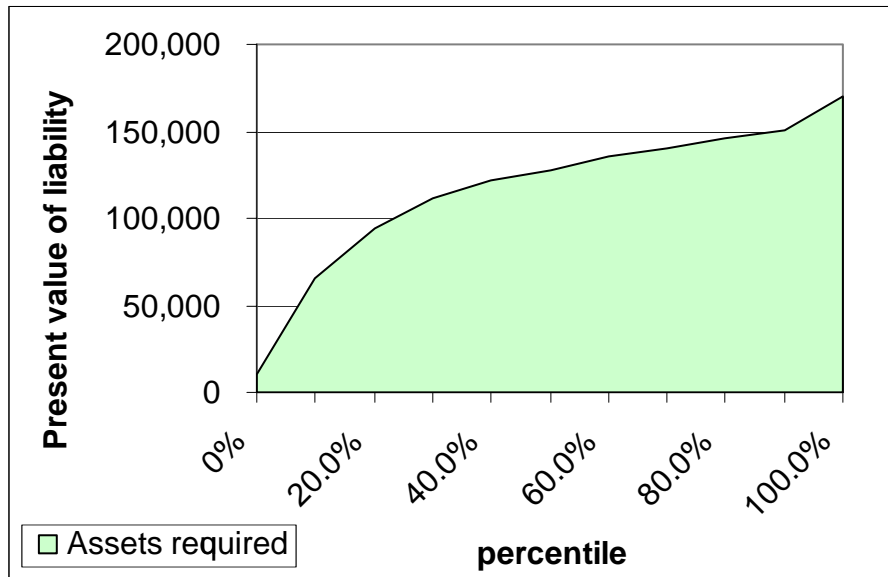
³ Jorion 2001.

The actual cost to provide the benefits ranges from a present value of \$10,000 to \$169,900. The width of the range is \$159,900 ($\$169,900 - 10,000$), which is greater than the actuarial liability calculated based on the expected value method, \$117,900 ($\$10,000 \times 11.7902$ annuity factor). It is also interesting to find that the amount of assets calculated based on the expected value method (\$117,900) is only sufficient to cover, without further contributions, all benefit payments for 39 percent of the time (i.e., the member dies before the assets remaining in the plan with investment return become less than the next payment amount). It should be clear from this example that the use of the expected value method, which is similar to pricing, is insufficient to ensure benefit security.

3.3 Is the VaR Method Going to Cost More?

Let's assume that the member actually dies at the age that exactly matches those at a 75 percent confidence level. If valuations are performed annually and any funding shortfalls are funded by additional contributions immediately, and assuming further that the surplus will be refunded to the employer, then the present value of future benefit payments less the present value of future surplus refunded, calculated using the existing funding method, equals the assets (economic capital plus wind-up liability at valuation date) required under the VaR method at the 75 percent confidence level. In other words, both methods give a present value of \$142,100. This should not come as a surprise, because the actual cost of the benefits is always the same, no matter which funding methods are used. Under our existing funding method, the future experience loss, which has a present value of \$24,200 ($\$142,100$ less $\$117,900$), is funded through future contributions. Under the VaR method, the plan sponsor has the option to prefund this \$24,200 cost in advance. The funding methods selected affect only the allocation of cost (adjusted with interest) into different periods, but they do not affect the actual cost incurred. Mathematically, the present values of the actual benefit payments are the same in either method. Although the cost does not change, the reactive approach failed to advise the potential variations of the true cost in advance.

The following graph provides the present value of liability at various percentiles:



An interesting observation from the graph is that the change in present values is not uniform as confidence level increases. The increase in assets required tends to be higher at both ends. The high increase at a low confidence level indicates there is a certain minimum cost for providing the benefits. The high increase at a high confidence level indicates that the cost to provide a complete guarantee is extremely expensive.

Note that mortality risk may be the most significant risk in this mathematical example because the plan created for illustration purposes involves one retiree only. The most significant types of risk driving the cost in other cases will be different. For example, termination risk may be the most significant type of risk for a Supplemental Executive Retirement Plan (SERP), and investment return risk may be the most significant type of risk for a pension plan heavily invested in equity. The choice of the risk factors that will be used to determine the VaR is crucial. Actuaries must apply their pension knowledge and professional judgment in choosing the risk factors and to determine the VaR amount. Future research in this area is likely to be required.

In the end, a plan sponsor should select a single or range of confidence level(s) that it feels comfortable with and fund the pension plan accordingly. The plan sponsor (or group) that is taking the risk should make the decision about risk. The actuaries' role is to assist their clients in making well-informed funding decisions.

3.4 Funding Based on Hypothetical Wind-up

Recently, it has been suggested that pension plan funding should be *solely* based on the hypothetical wind-up situation. It is true that the wind-up position represents the minimum actuarial liability required to secure the pension benefits accrued up to the date of the valuation if the employer elects to settle the obligations at the valuation date. However, hypothetical wind-up valuation normally ignores items such as future salary increases, early retirement enhancements for those members not yet eligible due to age or service requirements, equity premiums, short-term volatility of the market value of assets between valuation and settlement date, etc. If pension plans are to be funded based on a hypothetical wind-up basis, the liability calculated ignores some risk factors that the plan will face in the future. Since pension plans are more likely to remain ongoing for many years, ignoring certain known future contingencies in the valuation may lead to significant experience losses in the future. These future experience losses will surprise the plan sponsor, and the plan actuary may be blamed for failing to communicate the known risks in funding pension plans in advance. Funding *solely* based on hypothetical wind-up completely ignores future benefit accrual, which is a very important component of the member's retirement benefits. Members rely on both past and future accruals to cover their postretirement living expenses. *If we, as human beings, don't think we should live a life as if there is no tomorrow, why should we fund a pension plan as if the plan has no tomorrow?*

4. What Is a Typical Life Cycle of a Pension Plan?

Sometimes I view pension plans as human beings. The plan sponsor gives birth to a baby. The baby grows up and enters various stages of life. First is youth. A pension plan is said to be young when a high proportion of its members are accruing benefits and a small proportion are receiving pension benefits. The pension plan further matures when its active members start to retire. A pension plan grows up and enters middle age when it converges to a stable population of membership. At this stable stage, the proportions of active members and retirees in the pension plan remain constant, even though the number of plan members may continue to increase. Unlike human beings, a pension plan can stay in its middle age for an extended period of time. This may occur when the plan sponsor's business is growing steadily.

When the pension plan passes the stable stage, it enters the retirement stage. A pension plan enters this stage when the proportion of members accruing benefits decreases and the proportion of members receiving pension benefits increases. This can be caused by the plan sponsor's closing the plan to new entrants or the sponsor's significantly reducing its workforce. Alternatively, the plan sponsor may decide to convert all future benefit accruals to defined contribution. Another example is that all active members join the new employer's pension plan after a purchase and sale with the existing retirees remaining in the old plan. If the plan has no more new entrants and all members have been paid out, the pension plan comes to a natural death. During the life cycle, pension plan experience fluctuates much like the ups and downs in a typical human life. We refer to these fluctuations as gains and losses. Some pension plans may also experience life-threatening situations. These life-threatening situations can cause temporary hardship to the pension plan, such as becoming insolvent or, in the worst case, leading to the sudden death of a pension plan through bankruptcy of the plan sponsor.

After reviewing the life cycle of a typical pension plan, we observed that the life of a pension plan can be very complicated and unpredictable. Therefore, we need a funding approach that can balance the various concerns and has flexibility so that some of its key components can be adjusted automatically along with the maturation of a pension plan to reflect the changing concerns in funding at various stages. This funding approach needs to stand the test of time. The approach developed should be subject to refinement over time with advancement in computing technology and risk management theories.

5. Pension Plan Life-Cycle Funding Approach

5.1 Min-Max Funding

After reviewing the fluctuations of the pension plan liabilities at different scenarios, I developed a funding approach that I call the Pension Plan Life-Cycle Funding Approach. This approach is built around two key risks in the typical life cycle of a pension plan:

1. A “sudden death” situation (i.e., the plan sponsor becomes insolvent, and the plan is forced to wind up), and
2. A “horrible lengthy life” situation (i.e., the plan is kept ongoing but faces consecutive bad luck: it has experienced more losses than gains, and that makes the cost of funding the pension plan higher than anticipated).

To protect against a sudden death situation, a minimum funding valuation is required. The minimum funding valuation should be related to a hypothetical wind-up situation. *This provides protections to the members so that they will not lose both their jobs and pension benefits at the same time.*

To protect against continual bad luck, a maximum funding valuation is suggested. The maximum funding valuation should utilize the VaR and economic capital concepts. *Capital that is in addition to the minimum funding assets (or liabilities) should be set aside within the pension plan to provide protection against unforeseeable losses, and maybe even provide a certain level of protection against losses due to catastrophic deviations.* Therefore, economic capital in funding pension plans is redefined to be the amount of assets required under the VaR approach used in maximum funding valuation less the assets required under the hypothetical wind-up approach used in minimum funding valuation.

One possible approach to performing a maximum funding valuation is to use stress testing to measure the amount of economic capital (or assets) required in the plan to secure the benefits. The idea here is that we want to ensure, say, 80 percent of the time that there will be enough assets accumulated in the plan to cover expected liabilities and additional liabilities caused by experience losses. We also want to advise the plan sponsor about the level of potential higher cost for funding the pension plan if unanticipated losses occur, which substantially increases the cost.

The minimum funding valuation results provide a short-term view of the life of the pension plan that answers the question “What if I die tomorrow?” The maximum funding valuation results provide a long-term view of the life of the pension plan that answers the question “If things go wrong in my life and I need to take some losses, how bad might it be?” By combining the minimum and maximum funding valuation results, we develop a range of possible funding positions that is capped on both ends by short- and long-term considerations. However, this range may be too wide to be useful for the plan sponsor to make appropriate funding decisions. Therefore, an actuary may consider providing an estimate of the median cost for funding the pension plan to the sponsor. This median cost, of course, should be within the min-max range.

In summary, results related to the following scenarios should be communicated to the plan sponsor:

1. A (mandatory) valuation based on a hypothetical plan wind-up
2. A (optional) valuation to estimate the median cost for funding the pension plan and
3. A (mandatory) valuation to reflect the extreme cost for funding the pension plan.

Depending on whether the plan sponsor is risk-averse or a risk taker, the sponsor should decide on a capital level based on its funding philosophy and risk tolerance after reviewing the results generated from these two or three scenarios. The funding decision belongs to the plan sponsor because the sponsor has the ownership of the pension plan and is responsible to make additional contributions to fund any deficits. The actuary who has been hired as the consultant or advisor of the pension plan has the responsibility to show the plan sponsor the complete picture, to assist the sponsor in understanding the risks for funding a pension plan, and to assist the sponsor in making appropriate funding decisions. A benchmark about the appropriate capital level or range of capital levels should then be determined. This funding approach is designed to provide both benefit security to the members and, to the extent the amount of capital is greater than zero (or assets are greater than minimum funding liability), flexibility in changing the contribution level (or stability of contributions) to the plan sponsor.

5.2 Minimum Funding Valuation and Normal Cost

A minimum funding valuation protects employees from plan failure. A minimum funding valuation should be determined based on hypothetical wind-up liability plus a provision for adverse deviations. Rather than adjusting the value of assets to make the presentation of the results more complicated (you will see the reason behind this point from the graph in the Generational Funding Impacts subsection), the provision for adverse deviations should be reflected in the liability. This provision should reflect the liquidation cost (the difference between market value of assets and liquidation value of assets), a cushion for potential decrease in asset value between the valuation date and expected settlement date, and estimated wind-up expense.

To enhance benefit security through encouraging the plan sponsor to build up capital (or assets) in advance, the minimum funding requirement should include some penalty provisions. For example, if the amount of capital falls to zero (or plan assets fall below the minimum funding liability), we can consider requiring the sponsor to fund the shortfall immediately or within a very short period. Given that, under the proposed funding approach, the sponsor can view the full range of potential costs for providing the pension benefits and has the flexibility to change the contribution levels, the sponsor can avoid the penalty provisions by choosing a higher contribution level to build up capital (or assets), which can reduce the probability that the amount of capital falls to zero (or that the value of assets falls below the minimum funding liability).

Because of the design of this funding approach and the penalty provisions imposed, the plan is almost always fully funded on a hypothetical wind-up basis. Normally, there will not be any unfunded liability, and no amortization schedule is required to be established. By the same token, due to the impact of the penalty provisions of this approach, the plan sponsor should make the current service contributions equal to the normal cost determined based on a hypothetical wind-up basis. Certain assumptions must be made by the actuary so that the normal cost calculated can appropriately reflect the expected change in hypothetical wind-up liability between the current and next valuation dates. The assumptions should reflect the expected increase in benefit entitlements (i.e., increase in salary and credited service, eligibility to receive early retirement enhancements based on age/service requirements, etc.) during the intervaluation period. The expected change in demographics of the pension plan may be implicitly reflected by expressing the minimum funding normal cost calculated as a percentage of members' earnings.

5.3 Median Funding Valuation

Technically, it is very difficult to calculate the median value of the liabilities because such a calculation requires estimating the distribution of the anticipated outcomes. Even if we use approximations, the calculation is likely to be time-consuming and expensive. Since the median funding valuation results will not affect the minimum and maximum funding limits, and those are used only to assist plan sponsors in drawing a line in deciding the appropriate capital level within the contribution range, the median funding liability may be estimated using best-estimate going-concern actuarial assumptions (without margin).

There is no single right actuarial method to be adopted here. Actuaries should select the appropriate method based on the specific situation. My preliminary thought is that the Aggregate Method may not be adoptable under this approach, but the Entry Age Normal or Projected Unit Credit funding methods may be considered. New methods to be developed in the future to determine the liability may also be considered.

Stress testing can then be performed by changing the actuarial assumptions in the median funding valuation to derive the maximum funding valuation liability. This means median funding valuation and maximum funding valuation can be performed based on a very similar computer program.

There is a certain flexibility in determining the median funding liability. Rather than showing the median value, the actuary may modify this scenario to show the capital required based on the plan sponsor's specific preference. For example, the median funding valuation liability can be replaced by a benchmark capital amount that is equal to 20 percent of the minimum funding liability (or target the amount of assets to equal 120 percent of the minimum funding liability) if that is the target capital level selected by the plan sponsor. Then the plan sponsor can compare the value of plan assets with the benchmark and make a contribution decision accordingly. Please keep in mind that the key purpose for adding a median funding liability is to assist the sponsor in making decisions about contribution levels. Any benchmark amount that the sponsor may find helpful can be used.

5.4 Maximum Funding Valuation

Maximum funding valuation shows the plan sponsor the VaR and assists the sponsor in planning for rainy days. The objective of using stress testing and scenario analysis in a maximum funding valuation is to determine the size of potential future

experience losses on a proactive basis. It shows the stress level if the plan sponsor wants to maintain the plan on a going-forward basis.

A scenario analysis examines the effect of large deviations in key assumptions affecting the cost for funding the pension plan. A scenario may contain deviations in several key assumptions at the same time. The calculation that provides the amount of potential losses in a given scenario is the stress test. An actuary should prepare scenarios that capture the specific characteristics of the pension plan being valued as they are more familiar with the plan than other people and have a better understanding of the contingencies that may affect the funding cost. When constructing a scenario, it is important to make sure that the scenario itself makes sense. Historical experience can be used as a guide to the future. However, in selecting scenarios, the actuary should remember that history is unlikely to repeat itself exactly.⁴ It is important to combine historical data with an actuary's forecast of potential future changes in developing a sound scenario. In practice, only a relatively small number of scenarios can be analyzed. Relevant scenarios require careful planning. The usefulness of results derived using a stress test is highly subjective to the professional judgment used by the actuary on the scenario analysis. Bad or implausible scenarios will lead to wrong measures of VaR.⁵ As such, the calculation for maximum funding valuation actually requires more professional judgment than science.

Rather than using stress testing, another practical method that can be considered in determining the VaR for maximum funding valuation is to rely on the use of stochastic projection or simulation to examine the worst (extremely high actual) cost in funding a pension plan or to estimate the distribution of cost. Simulation is flexible and powerful, but one of its major weaknesses is that the results derived are sensitive to the parameters chosen, which may not properly reflect the potential deviations in reality. Another drawback is that, in reality, pension actuaries face limited budgets, which may make it a challenge to create customized simulations for each pension plan.

5.5 Quantification Limitations

Is it possible to discover the actuarial liability at exactly the 80th percentile? I don't think we can precisely measure risks. There are many factors that can affect the actual cost of funding a pension plan. Some of the risks cannot be easily quantified. This also explains why it is often difficult to estimate the cost at the flat tail of the distribution of liabilities for funding pension plans. Even if we have sufficient relevant

⁴ Crouhy, Galai, and Mark 2001.

⁵ Jorion 2001.

historical data to build a perfect distribution, the future will likely be different from the past, which reduces the accuracy of our calculations. This is another reason that the use of professional judgment is necessary in funding due to the lack of perfect scientific solutions.

Why select an 80th percentile rather than a higher number such as 95 or 99 percent to provide better benefit security? As the actual cost most of the time is going to be lower than the VaR figure and the plan sponsor has the flexibility to increase contributions in the future to enhance benefit security, showing extreme VaR (VaR at an extremely high percentile) may present unrealistic results or show a liability level that will rarely be reached. In addition, one may argue that when the probabilities are too low, they do not affect the plan sponsor's decision-making process, no matter how significant the outcome is. To be realistic, I believe it is better to show numbers that the sponsor will surely use in their decision-making process than numbers that the sponsor will likely ignore.

Even though we may need to give up some accuracy when using VaR, VaR is still valuable for funding because it creates a *critical bridge between assisting the plan sponsor to understand the funding risks and to decide upon appropriate capital (or asset) level*. We can never perfectly predict the future. The best we can do is to estimate the potential deviations, understand the factors affecting the cost, accept that risk is part of life, and then proactively handle it.

5.6 Plan Sponsor Contribution Requirement and Surplus Ownership

To prepare for both the known and the unexpected future contingencies and to allow the plan sponsor to adjust the contribution level to survive from short-term cash-flow stress during a bad economic environment, the ideal funding approach must have a flexible contribution requirement. A significant increase in funding requirement combined with an economic slowdown creates a difficult environment for a plan sponsor to survive. In addition, the contributions should be more predictable so the contributions can be budgeted in advance.⁶ The funding approach should accommodate a business/economic cycle: allow higher contributions in good years so contributions could be reduced in difficult years. The funding approach should moderate contribution volatility: contributions should not change radically because of small or moderate changes in assets or interest rates. The decision to create and maintain a pension plan is an important decision for the plan sponsor. Retirement benefits are often a major part of the employee's compensation package. Funding a pension plan is a

⁶ American Academy of Actuaries 2005.

decision related to human resource investment that the sponsor needs to live with and be comfortable with.

According to this proposed funding approach, the plan sponsor should make contributions within the following minimum and maximum until the next valuation date:

Period After Valuation Date	First Year	Subsequent Years
Minimum plan sponsor contribution required	Normal cost	Normal cost
	(-)	(-)
	Expected employee contributions	Expected employee contributions
	(+)	(+)
	Minimum funding liability	Prior year plan sponsor minimum required contributions
	(-) Assets	(-) Prior year plan sponsor actual contributions
Maximum plan sponsor contribution allowed	Normal cost	Normal cost
	(-)	(-)
	Expected employee contributions	Expected employee contributions
	(+)	(+)
	Maximum funding liability	Prior year plan sponsor maximum allowed contributions
	(-) Assets	(-) Prior year plan sponsor actual contributions

Notes:

1. The total calculated should equal zero if it is negative.
2. "Prior year" refers to the period from the most recent valuation date to the end of the preceding year.
3. Assume that any shortfall under the minimum funding valuation is required to be funded immediately.
4. Actual capital equals assets less minimum funding liability.

In the minimum contribution calculation, the contribution level is increased by any shortfall on a minimum funding basis. This ensures the plan is fully funded on a hypothetical wind-up basis and avoids having a negative capital. On the other hand, the contribution level is reduced by any existing capital (or surplus available on a minimum funding basis). In the maximum contribution calculation, the plan sponsor can increase capital (or assets) to equal the economic capital (or maximum funding liability) through making additional contributions. If the value of assets is in excess of the maximum funding liability, the sponsor should cease contributing. Note that the median funding valuation results do not play a role in setting the contribution limits. Median funding valuation results are used only to assist plan sponsors in deciding appropriate capital and contribution levels within the limits. After considering the existing capital (or asset) level and target capital level, the plan sponsor can decide upon its contribution level accordingly.

The higher the contribution level, the more benefit security is provided to the members. At the same time, the higher the contribution level, the more likely there would be excess funding that generates a significant amount of capital or surplus at plan wind-up. If future outcomes are more favorable than expected, assets reach levels that would be difficult for the plan sponsor to use up through taking contribution holidays or further enhancing benefit security. If the plan sponsor decides to increase the capital to the maximum funding level, the amount of capital in the pension plan (or surplus existing at wind-up) may possibly equal a significant percentage of the minimum funding liability (or actual wind-up liability), resulting in overprotection. The amount of capital accumulated to protect against the extremely high funding cost that occurs with low probability leads to a very high probability of having a significant amount of excessive capital (or surplus) at actual plan wind-up. To be fair to the plan sponsor, the sponsor should have ownership of the surplus at wind-up and the right to withdraw capital (or assets), if it can be justified that the amount of capital (or assets) is much more than sufficient to secure the pension benefits for an ongoing plan. *Failing to provide the plan sponsor access to capital (or assets in excess of the minimum funding liability) will impair plan sponsors' willingness to proactively fund their pension plans, and their rationale for setting effective investment policies.*⁷ It is to the members' disadvantage if plan sponsors decide to fund their pension plans at the minimum level required by legislation.

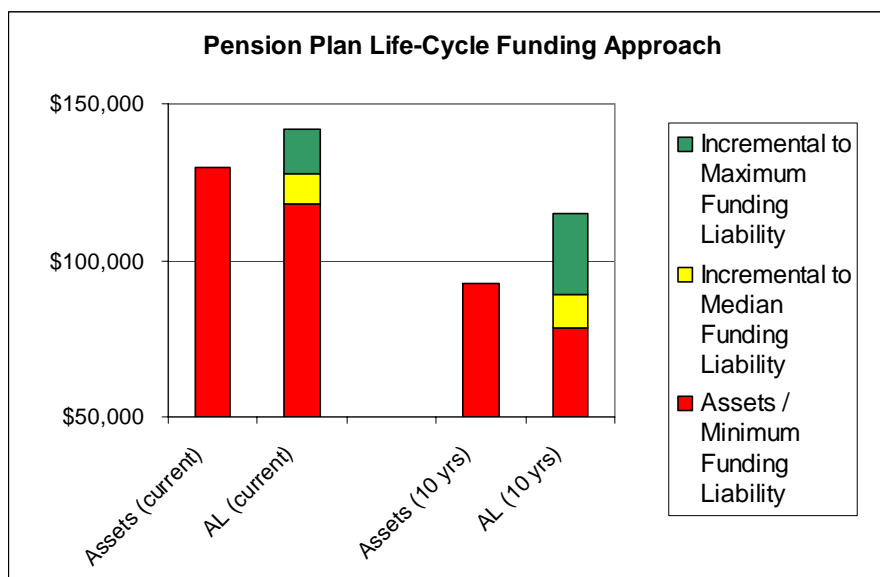
⁷ Towers Perrin 2005.

5.7 Generational Funding Impacts

A survey completed in mid-2004 with 100 chief financial officers (CFOs) from North American organizations with defined benefit or cash balance plans with assets of US\$20 million or more indicated that 96 percent of all the CFOs surveyed believed that it would be helpful to always have the ability to measure the long-term impact of their pension decisions on their company's corporate finances prior to making those decisions.⁸ As the value of assets in the pension plan is directly related to the amount of contributions made, the relationship between contributions in different cohorts should be properly explained to the plan sponsor. In communicating the valuation results, we should demonstrate the impact of making current contributions to the needs for future contributions. We should advise the sponsor that making a higher contribution today is more likely to result in a lower contribution requirement in future years and vice versa. To demonstrate the relationship between contributions, we should include the following subscenarios in each of the three scenarios described above upon presenting the valuation results:

1. Valuation results as at the valuation date and
2. Projected funding position at a future date based on one common set of assumed contribution levels.

No single number can be used to communicate the complexity of risks involved in funding pension plans easily. Pension plan funding risk is better explained through showing the complete picture of it using six sets of figures that are summarized in one graph. One possible format for the graph is as follows:



⁸ SEI Investments 2005.

*Assets are assumed to equal 110 percent of minimum funding liability at valuation date. Without active members, plan sponsor contributions and normal costs are assumed to equal zero in the projection.

In determining the projected funding position, assets should be assumed to increase by the investment return related to the actual asset mix or asset mix per investment policy at the valuation date. In addition, the projection calculation should assume that the plan sponsor will make contributions that equal the normal cost (after being reduced by expected employee contributions, if any) calculated under the minimum funding valuation (or hypothetical wind-up scenario) to maintain the current benefit security level and to avoid hitting the penalty provisions in the projection period. Note that actual plan sponsor contributions, which are subject to a sponsor's funding decision and risk tolerance, will be higher or lower than the normal cost presented in the graph. As a result, other contribution levels may be assumed in the projection calculations when appropriate.

After reviewing the projected funding position, if the plan sponsor believes the existing capital is insufficient or would like to prefund the plan to lower future contribution requirements, the sponsor can decide to make contributions in excess of the minimum funding normal cost. On the other hand, if the plan sponsor believes the existing capital is more than sufficient, the sponsor can decide to use part of the existing capital to cover all or part of the future benefit accrual cost. If the plan sponsor wants to fund at the minimum level, this graph may be used to demonstrate the potential shortfall or diminishing margin in the future. In this case the sponsor should be reminded about the penalty provisions. In the end, the purpose of showing these six figures is to communicate a complete picture of the risks and the impact of the plan sponsor's funding decision today to the pension plan's funding position in the future. Similar to other projections, we should always keep in mind that the future will be different from the results of the deterministic projections computed today.

5.8 Funding Assumptions Mature Together with the Pension Plan

For all funding valuations, assumptions about future events must be chosen before calculations can be performed. I believe the selection of the assumptions should reflect the demographic composition between active members and retirees of a pension plan. The purpose of doing that is to take into account changes of risk tolerance during the maturation of the pension plan. *Over time, the pension fund should shift its asset mix to reduce risk— just as a prudent investor would shift toward lower risk investment vehicle with greater predictability as his or her retirement planning horizon shortens.*

To ensure there are sufficient assets to cover benefit payments due soon, the best way to invest the assets is to invest in money market or short-term deposits. For midterm payments, investing in a money market does not provide better returns compared to investing in a fixed-income market because of its lower risk and highly liquid natures. Thus, the assets that back the payments should be invested in a fixed-income market or bonds. In the long term, stocks almost always outperform bonds because of the equity risk premium. So the best way to fund long-term payments on a risk-return basis is to invest the assets in equity market or stocks. This asset allocation approach is consistent with the shift in an individual's investment mix for personal retirement savings. When one is young, one is encouraged to invest heavily in equity to take advantage of its long-term growth potential. Even though the equity value may fall in value in the short term, there are many years for the return to catch up. When one enters middle age, one generally has a relatively lower risk tolerance to accept investment losses compared to younger individuals. As such, the percentage of assets to be allocated to equity should decrease, and the percentage of assets to be allocated to fixed income should increase. After one retires and stops working, one normally has less physical ability to return to work. As he or she is not likely to earn further income or may have only the ability to earn a low income from working part-time, the retiree cannot afford to lose his or her retirement savings. The percentage of retirement assets that should be invested in equity should be significantly reduced, and a higher percentage of assets should be invested in the fixed-income market to minimize fluctuations with a portion of assets invested in a money market that can be used for monthly withdrawals.

Reflecting an individual's investment allocation in funding pension plans, it is clear that a pension plan with retirees only should invest mainly in fixed income. For pension plans with no actives or a small number, it is often challenging for the plan sponsor to find extra cash to fund any shortfalls arising resulting from investment losses. These challenges sometimes occur in funding mature union pension plans. In these plans, the retirees' liability can be a large proportion of the total actuarial liabilities. If there are any experience losses related to retirees' liabilities or retirees' portion of assets, plan sponsor contributions are required to increase. However, since contributions are often negotiated, or fixed, and further, there are not many active members in the plan, it is difficult to significantly increase the per-working-hour contribution rate to fund the shortfall. Often these pension plans fall into a black hole, and they may never be able to come back up to fully funded status. Immunization of assets is important for mature pension plans. On the other extreme, if all plan members are young actives who are not going to retire in the near future, investing all assets in fixed income fails to take advantage of the equity risk premium, which can increase the

long-term investment return of the pension plan. Therefore, for a pension plan with very young members, a high proportion of the assets should be invested in equity.

After discussing both extreme ends, we will look at pension plans in the middle of the range. For a middle-aged pension plan, the allocation between equity and fixed income should be somewhere in the middle. This means that both investing assets in equity and fixed income are equally important. However, how are we going to allocate the assets?

One possible allocation method is to value the retiree liability using fixed income that matches the duration or expected retiree pension payment outflows. For actives and deferred vested members, the liability is to be valued using anticipated long-term equity returns. If this approach is taken, the liability of a pension plan automatically will be adjusted along with the maturation of the pension plan to reflect the change in risk tolerance and change in anticipated payment patterns over time. For example, let's look at a pension plan that contains a group of individuals with similar characteristics. The discount rate to be used in liability calculations when the group is young is based on a higher rate that reflects the equity risk premium. As the members get older, the liability increases to reflect anticipated investment returns over time (or reduction in the discounting period). Once this group starts to retire, the discount rate to be used in the liability calculations is now the bond rates, which reflect that the group now enters the payment stage, and a more conservative asset mix is recommended. Once the entire group becomes retirees, only bond rates are used in valuing the liabilities so that the ideal asset mix and liabilities can be matched. This method reflects the changing asset mix needed over the maturation of pension plan. The discount rate assumption in this funding approach is used to create a natural immunization along the life cycle of a pension plan.

After presenting the above concept, I would like to add one more component to the discount rate assumptions to complete it. You may notice that if the above is used in practice, there is a jump in actuarial liability in the year when a member retires because the default asset class backing the liability suddenly shifts from equity to fixed income, which leads to a sudden drop in discount rate assumptions. To proactively take into account these ideal asset mix shifts, what we should do is to apply the expected equity return in the preretirement period and apply the expected fixed-income return in the postretirement period to form the discount rate assumptions. In other words, for each individual member covered by the pension plan, we assume the assets backing the liability are invested in equity before his or her assumed retirement date and then shifted to fixed income after his or her assumed retirement date. The actuarial liability of the pension plan is then calculated based on the ideal asset mix, which reflected the

specific demographic composition of the pension plan. For each individual in the pension plan, the select and ultimate period (pre- and postretirement periods) for the discount rate assumption changes along with the member's increase in age at future valuation dates. Thus, the entire pension plan's discount rate assumptions are said to mature (change from mainly using equity returns to mainly using fixed-income returns) along with the aging (change in demographic composition) of the pension plan over time.

5.9 Impact of Asset Mix

Note that the actuarial liabilities are not linked to the actual asset mix of the pension plan at the valuation date. This may create concerns about asset and liability mismatch. This mismatch is allowed under this funding approach because it is the plan sponsor's decision to decide whether the assets are to be invested more aggressively or invested more conservatively than the ideal asset mix described above. The difference between the actual asset mix and the ideal asset mix is expected to be reflected in the maximum funding valuation. Actuaries should reflect the asset liability mismatch risk through stress testing in determining maximum funding liability. The amount of maximum funding liability should increase so that mismatch risks are factored in the calculations and the plan sponsor can be aware of the potential higher costs due to risk taking.

How does this funding approach work in practice? For example, how will the funding results change if the investment policy has been amended to increase equity exposure? First, the minimum funding liability should remain the same because it represents the cost of a hypothetical wind-up that is independent of asset mix (other than potential changes in provision for adverse deviations to reflect short-term volatility of asset value from valuation date to settlement date). The maximum funding liability will increase because the pension plan is now bearing higher risks. The projected asset value after 10 years in the graph will rise because of an increase in expected investment returns through an increasing proportion of equity investments. The plan sponsor can see the various impacts of the change through the six-figure graph.

5.10 Regulatory Constraints

A few actuaries suggest that the setting of minimum and maximum limits for the funding of pension plans is part of the mandate of legislators and regulators, not something to be prescribed by the actuarial profession.⁹ I hold a different point of view.

⁹ Canadian Institute of Actuaries 2005.

Legislators and regulators may define the minimum and maximum benefit levels and tax-deduction limits because these are often determined based on government policies. Nonetheless, the setting of funding limits should be the responsibility of the actuary. An actuary who provides funding recommendations to a plan sponsor should be encouraged to use a more sophisticated valuation approach and apply more professional judgment to measure and control the funding risks based on the specific needs of each individual pension plan. An oversophisticated regulatory environment will only restrict creativity and the development of innovative solutions. If people are forced to follow the book all the time, they will start to forget how to distinguish good and bad. Later, no one will know how to make the right decision and why it is right, when needed.

6. Remarks

The objective of this paper is to present the author's view on the fundamental challenges that pension actuaries are facing in providing recommendations on pension plan funding and propose a funding approach that attempts to resolve the existing concerns raised. In this paper I intended only to show this approach on a high-level basis. If this approach is accepted by the pension industry, I will prepare a subsequent paper to discuss further the details of how the calculations should be performed and my view on how special issues, such as the following, should be incorporated under this funding approach:

1. Transitional rules and funding past service liability of newly established pension plans
2. Impact of plan amendment
3. Terminal funding
4. Smoothing
5. Employee cost sharing
6. Employee surplus entitlement
7. Valuation for smaller pension plans (or individual's pension plans), and
8. Reporting of valuation results.

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Appendix

Mathematical Calculations

The assumptions used include an interest rate at 6.0 percent per annum and the 1994 Group Annuity Reserving Table (GAR94) unisex blended 50 percent males and 50 percent females. The pension amount is \$10,000, and it is payable annually in advance for the member's lifetime with no guarantee, indexing, or spousal protection. The member is at age 65 in 2005, the year of the valuation. The calculations are as follows:

Age	1 Year q_x	Accumulated Surviving Probability to Year End	Assets Required Based on Present Value of Future Payments				
			0%	25%	50%	75%	100%
			Quantile	Quantile	Quantile	Quantile	Quantile
65	0.010310	99.0%	\$10,000	\$102,950	\$127,641	\$142,105	\$169,905
66	0.011504	97.8		98,527	124,699	140,032	169,500
67	0.012649	96.6		93,838	121,581	137,834	169,070
68	0.013634	95.3		88,869	118,276	135,504	168,614
69	0.014688	93.9		83,601	114,773	133,034	168,131
70	0.015652	92.4		78,017	111,059	130,416	167,619
71	0.016785	90.9		72,098	107,122	127,641	167,076
72	0.018254	89.2		65,824	102,950	124,699	166,500
73	0.019733	87.4		59,173	98,527	121,581	165,890
74	0.021439	85.6		52,124	93,838	118,276	165,244
75	0.023420	83.6		44,651	88,869	114,773	164,558
76	0.025592	81.4		36,730	83,601	111,059	163,832
77	0.028783	79.1		28,334	78,017	107,122	163,062
78	0.032193	76.5		19,434	72,098	102,950	162,245
79	0.036016	73.8		10,000	65,824	98,527	161,380
80	0.040292	70.8			59,173	93,838	160,463
81	0.045056	67.6			52,124	88,869	159,491
82	0.050348	64.2			44,651	83,601	158,460
83	0.055056	60.7			36,730	78,017	157,368
84	0.060989	57.0			28,334	72,098	156,210
85	0.067211	53.1			19,434	65,824	154,982
86	0.074338	49.2			10,000	59,173	153,681
87	0.084225	45.0				52,124	152,302
88	0.094475	40.8				44,651	150,840

Age	1 Year q_x	Accumulated Surviving Probability to Year End	Assets Required				
			Based on Present Value of Future Payments				
			0% Quantile	25% Quantile	50% Quantile	75% Quantile	100% Quantile
89	0.105583	36.5				36,730	149,291
90	0.118364	32.2				28,334	147,648
91	0.129706	28.0				19,434	145,907
92	0.144634	23.9				10,000	144,062
93	0.160337	20.1					142,105
94	0.174408	16.6					140,032
95	0.193368	13.4					137,834
96	0.208715	10.6					135,504
97	0.228524	8.2					133,034
98	0.250233	6.1					130,416
99	0.266722	4.5					127,641
100	0.283481	3.2					124,699
101	0.315045	2.2					121,581
102	0.333711	1.5					118,276
103	0.353523	1.0					114,773
104	0.374435	0.6					111,059
105	0.395410	0.4					107,122
106	0.415408	0.2					102,950
107	0.433390	0.1					98,527
108	0.450960	0.1					93,838
109	0.468809	0.0					88,869
110	0.484535	0.0					83,601
111	0.495733	0.0					78,017
112	0.500000	0.0					72,098
113	0.500000	0.0					65,824
114	0.500000	0.0					59,173
115	0.500000	0.0					52,124
116	0.500000	0.0					44,651
117	0.500000	0.0					36,730
118	0.500000	0.0					28,334
119	0.500000	0.0					19,434
120	1.000000	0.0					10,000

* Assumed mortality table terminates at age 120.