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DERIVATION OF PREMIUM RATES FOR RENEWABLE TERM INSURANCE

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It is the purpose of this paper to include in our *Transactions* a description of a method for deriving premiums for renewable term insurance of contemporary design, under conditions where experience under such insurance is not available.

The rationale underlying this derivation may be viewed as the counterpart, with respect to renewability, of that underlying the formula for attained age conversion costs, presented by Mr. E. E. Cammack[1] and stemming from the work of Dr. T. B. Sprague. Conceivably, renewable term premiums could also be derived by an approach representing the counterpart of Mr. Frank L. Griffin's treatment of conversion costs.[2] This approach will be left to a more courageous spirit.

The paper will first show that net premium rates for renewable term insurance, including a conversion option, may be derived through use of a select and ultimate mortality basis, on the assumption that all eligible lives renew their term insurance on each renewal date before the end of the conversion period, then convert to whole life insurance at such latter time. It will then be shown that, under given assumptions, which are reasonable and conservative, as to the conditions under which such renewal and conversion take place, these net premium rates will also apply for any such renewal and conversion rates lying above certain minimum levels, which levels depend on the select and ultimate mortality basis used. These minimum levels with respect to current mortality experience are, probably, well below the normally experienced rates of renewal and of conversion.

Also included is a relatively brief treatment on the subject of the funds which must be held in connection with a solvent renewable term insurance operation.

[1] *TASA* XX, 406. The formula is

$$[P_{[x]+n} - P_{[x+n]}] \cdot \frac{{}_{n|}\ddot{a}_{[x]}}{\ddot{a}_{[x]:\overline{n|}}},$$

where x is the age at issue of the term policy and n is the period from the date of issue of the term policy to the end of the conversion period.

[2] *RAIA* XXXI, "A New Approach to the Problem of Term Insurance Conversion Costs."

The paper does not treat the impact of expense considerations on the costs of renewable term insurance. Nevertheless, it is clear that, to the degree the excess initial expense for new applicants for renewable term insurance at a given age exceeds the corresponding extra expense attendant upon the renewal at such age of eligible renewable term insurance, there are margins available to offset the excess mortality costs associated with the renewals.

DERIVATION OF NET PREMIUMS

The following policy conditions are assumed with respect to the renewable, convertible term policy:

- x = age nearest birthday at issue.
- y = age nearest birthday on the policy anniversary which marks the end of the period during which the policy (or any renewal thereof) may be converted without evidence of insurability. The option provided permits such conversion at the attained age to a policy of equal sum insured on a permanent plan at the then applicable standard premium rate. The cost of this provision may be assumed to reach its maximum level when conversion is effected at the end of the conversion period to the whole life plan.
- m = term period, measured from date of issue, during which initial premium rate applies.
- ${}^{\infty}_{y}\bar{\pi}^{1}_{x:\overline{m}|}$ = net level continuous yearly premium rate payable over the initial term period of m years for a renewable term policy issued at age x, convertible to age y as specified above, and providing for $t-1$ successive renewals for term periods of m years each, where $y \leq x + tm$. The premium rate applicable over each such renewal period shall be the same as that for a corresponding renewable term policy newly issued at the attained age at the beginning of such renewal period. The policy will finally expire at the end of the last of the $t-1$ renewal periods, except as to any then existing conversion right.

Inevitably, considerable uncertainty exists with respect to the rates of renewal and of conversion, and the corresponding rates of mortality, which will be experienced under renewals and conversions of this policy. It is, nevertheless, possible to determine a set of net premium rates for such a policy, which will cover the claim costs to be experienced under all such renewals and conversions regardless of the actual rates of renewal and of conversion, if the following assumptions are made with respect to the exercise of the renewal and conversion options and to the presence of antiselection not involving the exercise of these options:

a) On any renewal date falling before the end of the conversion period, the renewal option is exercised by each individual who is then eligible for renewal and unacceptable for new standard insurance. (It seems evident that it is in the interest of each such person to renew, rather than to withdraw or to convert, since by renewing he maximizes the risk which the insurer is obligated to bear on his life at "standard" costs.) This assumption may be considered practically equivalent to the assumption that each individual then eligible for renewal who effects either conversion or withdrawal, the only alternatives to renewal, may be treated for mortality experience purposes as then being a freshly select life.

This assumption will apply at all such renewal dates, whether or not the renewable term plan is actually made available to new entrants at the particular attained age involved.

b) At the end of the conversion period, either the right to convert or the right to continue on the renewable term basis, whichever involves the larger deficiency in present value of future premiums, is exercised by each individual then eligible for conversion and unacceptable for new standard insurance. This assumption may be considered practically equivalent to the assumption that each individual then eligible for conversion, who does not continue his insurance on the basis involving such larger deficiency, may be treated for mortality purposes as then being a freshly select life. Since the conversion option will normally involve the larger such deficiency, it will be assumed in what follows that all such "nonselect" lives convert. To the degree that highly impaired lives eligible for conversion, with respect to whom the larger such deficiency is associated with a continuing of the term insurance, do continue their term insurance, the total deficiency in present value of future premiums may exceed that on the assumption that all the "nonselect" lives convert. The consequent understatement of costs, on the assumption that all the "nonselect" lives convert, is undoubtedly of little importance under the term and conversion periods involved in policies of contemporary design, and will hereinafter be ignored.

c) Antiselection, otherwise than as assumed in (*a*) and (*b*) above with respect to renewal dates and to the end of the conversion period, (*i.e.*, antiselection exercised on premium due dates when the premium rate does not increase) does not differ materially from that correspondingly experienced under level premium, nonrenewable, convertible term policies with conversion periods and final expiry dates corresponding to those under the renewable term policies. If the selection standards applied to renewable term policies are consistent with (this may mean slightly stricter than) those applying to the above-mentioned non-renewable term policies, the mortality experience under such non-renewable policies may be used with suitable provision for the anti-selection assumed in (*a*) and (*b*) above, as the basis for determining the set of net premium rates for renewable term policies.

In order to derive the desired set of net continuous yearly premiums of the form ${}^{\infty}_{y}\bar{\pi}^{1}_{x:\overline{m}|}$ we shall first derive a corresponding set of net continuous yearly premiums, ${}^{\infty}_{y}\bar{\pi}'^{1}_{x:\overline{m}|}$, on the assumption that all eligible lives renew at the end of each term period where the attained age is less than y and that all eligible lives convert to the whole life plan at attained age y. We shall then show that if renewal, conversion and other antiselection take place as assumed in (*a*), (*b*) and (*c*) above, the desired set of premiums, ${}^{\infty}_{y}\bar{\pi}^{1}_{x:\overline{m}|}$, may be taken as identical with the set, ${}^{\infty}_{y}\bar{\pi}'^{1}_{x:\overline{m}|}$, determined as specified in this paragraph.

Letting $x + nm < y \leq x + (n + 1)m$, where n is integral, it is evident that:

$$\sum_{r=0}^{n-1} {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+rm}:\overline{m}|} \cdot {}_{rm|m}\bar{a}_{[x]} + {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m}|} \cdot {}_{nm|y-x-nm}\bar{a}_{[x]} + {}^{\infty}\bar{\mathrm{P}}_{[y]} \cdot {}_{y-x|}\bar{a}_{[x]} - \bar{\mathrm{A}}_{[x]} = 0 \,. \tag{1}$$

In order to determine the individual net premiums involved in Equation (1), let us first set $x + (n + 1)m = y$. The premium with respect to new entrants at age $x + nm$ may now be determined by means of the following equation, setting $r_1 = 0$:

$$\sum_{r=0}^{r_1} {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+(n-r)m}:\overline{m}|} \cdot {}_{(r_1-r)m|m}\bar{a}_{[x+(n-r_1)m]} + {}^{\infty}\bar{\mathrm{P}}_{[y]} \cdot {}_{(r_1+1)m|}\bar{a}_{[x+(n-r_1)m]} - \bar{\mathrm{A}}_{[x+(n-r_1)m]} = 0 \,. \tag{1a}$$

Next, setting $r_1 = 1$ and making use of the value of ${}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m}|}$ just determined, Equation (1*a*) may now be used to solve for ${}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+(n-1)m}:\overline{m}|}$, the net premium applicable to new entrants at age $x + (n - 1)m$ on the assumption that, upon renewal at age $x + nm$, net premium ${}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m}|}$ will apply.

By successive applications of Equation (1*a*), increasing the value of r_1

by one (*i.e.*, stepping the original age at entry back by m years) with each such application, the whole chain of values of ${}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+rm}:\overline{m}|}$ may be determined for all applicable values of r, where $x + (n+1)m = y$.

We are now ready to determine each of the other $m - 1$ corresponding chains of values of the form ${}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+rm}:\overline{m}|}$ where $x + (n+1)m = y + s$ and $1 \leq s \leq m - 1$. The premium with respect to new entrants at age $x + nm$ may now be determined by means of the following equation:

$$ {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m}|} \cdot \bar{a}_{[x+nm]:\overline{m-s}|} + {}^{\infty}\bar{\mathrm{P}}_{[y]} \cdot {}_{m-s|}\bar{a}_{[x+nm]} - \bar{\mathrm{A}}_{[x+nm]} = 0 \,. \quad (1b) $$

Finally, by a process similar to that described in the preceding paragraph [involving Equation (1*a*)], the calculation of these $m - 1$ chains may be completed by successive applications of the following equation:

$$ \sum_{r=1}^{r_1} {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+(n-r)m}:\overline{m}|} \cdot {}_{(r_1-r)m|m}\bar{a}_{[x+(n-r_1)m]} + {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m}|} \cdot {}_{r_1 m|m-s}\bar{a}_{[x+(n-r_1)m]} \quad (1c) $$
$$ + {}^{\infty}\bar{\mathrm{P}}_{[y]} \cdot {}_{(r_1+1)m-s|}\bar{a}_{[x+(n-r_1)m]} - \bar{\mathrm{A}}_{[x+(n-r_1)m]} = 0 \,. $$

Having determined the set of premiums, ${}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+rm}:\overline{m}|}$, on the assumption that all eligible lives renew on each renewal date before attained age y and convert to whole life at attained age y, let us finally determine the corresponding values of ${}^{\infty}_{y}\bar{\pi}_{\overset{1}{x+rm}:\overline{m}|}$, assuming that renewal, conversion and other antiselection take place as specified in (a), (b) and (c) above, respectively. For this purpose let us assume that of those eligible to renew at the end of the rth term period, a proportion, ${}^{y}_{rm}g_{x:\overline{m}|}$, do not so renew and that this group who do not renew their term insurance may be treated for mortality experience purposes as then being freshly select lives. Similarly, it will be assumed that of those eligible to convert at attained age y, a proportion, ${}^{y}_{y-x}g_{x:\overline{m}|}$, all then freshly select lives, do not so convert. And let us define ${}_{y}l^{(g)}_{[x]+rm+s:\overline{m}|}$ as being the number of lives insured under the renewable term basis at the end of the sth year of the $(r+1)$th m-year period out of a group of $l_{[x]}$ original entrants at age x, on these assumptions.

Then,

$$ {}_{y}l^{(g)}_{[x]+rm+s:\overline{m}|} = l_{[x]+rm+s} - \sum_{\rho=1}^{r} {}^{y}_{\rho m}g_{x:\overline{m}|} \cdot {}_{y}l^{(g)}_{[x]+\rho m:\overline{m}|} \cdot {}_{(r-\rho)m+s}p_{[x+\rho m]} \,, \quad (2) $$

where $0 < s \leq m$ and s is integral.

It should be pointed out here that the function ${}_{rm}^{y}g_{x:\overline{m}|}$ has been defined as a rate of *non*renewal (or *non*conversion) rather than of *renewal* (or *conversion*), because this course simplifies the definition of the function and the algebraic expressions in which it is used. However, in discussing the implications of these expressions and the renewable term insurance operation in general, it seems easier and more natural to write in terms of rates of *renewal* (or *conversion*) rather than of *non*renewal (or *non*conversion), and this usage has accordingly been adopted in the paper. The rate of renewal (or conversion) is, of course, always the complement of the rate of nonrenewal (or nonconversion); with this background it is felt there will be no confusion on this score.

At this point a condition must be specified with respect to the values of ${}_{rm}^{y}g_{x:\overline{m}|}$. The statement was made earlier that the net premium rates to be derived will apply for any rates of renewal (on renewal dates before the end of the conversion period) and of conversion (at the end of the conversion period) lying above certain minimum levels. These minimum levels may be defined as being the minimum rates of such renewal and conversion, varying with respect to the attained age and to the corresponding rates at earlier renewal dates, which are compatible with the assumptions given in (*a*) and (*b*) above. In other words, the rate of such renewal or conversion must always be great enough to provide for renewal or conversion, as the case may be, of all those who are both (i) eligible for such renewal or conversion and (ii) unacceptable for new standard insurance.

It seems inevitable that any attempt to determine quantitative values of these minimum renewal and conversion rates must involve somewhat arbitrary assumptions as to the proportion of eligible lives who are unacceptable for new standard insurance and their distribution by degree of extra mortality. Accordingly, the matter of determining specific criteria to ensure that the applicable renewal and conversion rates fall above the corresponding minimum rates will not be treated here. As a practical matter, however, an examination of the rates of mortality implicit in any series of values of ${}_{y}l^{(g)}_{[x]+rm+s:\overline{m}|}$ may usually serve to indicate whether the underlying renewal or conversion rates fall below the "minimum" levels to any important degree.[3]

Let us now investigate the results obtained under a renewable term in-

[3] An examination along these lines is indicated by the mortality ratios shown in the last column of Table 2. Since none of the ratios seemed inordinately high, it was felt that the renewal proportions assumed in Basis B were not below the "minimum" levels to any important degree. Clearly, individual judgment is involved in deciding whether any mortality rates or ratios so obtained are inordinately high.

surance operation on the assumptions stated above, if premiums are payable on the basis of the rates determined by using Equation (1). These assumptions may be recapitulated briefly as follows:

(i) $l_{[x]}$ lives are originally insured at age x on the specified renewable term insurance basis for unit amount each;
(ii) at the end of each m-year period which ends before attained age y, a proportion, ${}^{y}_{rm}g_{x:\overline{m|}}$, who may be treated as freshly select lives, do not renew;
(iii) similarly, at attained age y, a proportion, ${}^{y}_{y-x}g_{x:\overline{m|}}$, who may be treated as freshly select lives, do not convert (all conversions being to the whole life plan);
(iv) the proportions specified in (ii) and (iii) above are not greater than those corresponding to the "minimum" levels described above.

Then, letting $x + nm < y \le x + (n + 1)m$, where n is integral, the excess of the present value at issue of all premiums paid over that of all benefits paid is given by the following expression:

$$
\begin{aligned}
&\Big\{ l_{[x]}\Big[\sum_{r=0}^{n-1} {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+rm}:\overline{m|}} \cdot {}_{rm|m}\bar{a}_{[x]} + {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m|}} \cdot {}_{nm|y-x-nm}\bar{a}_{[x]} \\
&\qquad + {}^{\infty}\bar{\mathrm{P}}_{[y]} \cdot {}_{y-x|}\bar{a}_{[x]}\Big] - \sum_{r=1}^{n-1} v^{rm} \cdot {}^{y}_{rm}g_{x:\overline{m|}} \cdot {}_{y}l^{(g)}_{[x]+rm:\overline{m|}} \\
&\qquad\qquad \times \Big[\sum_{\rho=r}^{n-1} {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+\rho m}:\overline{m|}} \cdot {}_{(\rho-r)m|m}\bar{a}_{[x+rm]} \\
&\quad + {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m|}} \cdot {}_{(n-r)m|y-x-nm}\bar{a}_{[x+rm]} + {}^{\infty}\bar{\mathrm{P}}_{[y]} \cdot {}_{y-x-rm|}\bar{a}_{[x+rm]}\Big] \\
&\qquad - v^{nm} \cdot {}^{y}_{nm}g_{x:\overline{m|}} \cdot {}_{y}l^{(g)}_{[x]+nm:\overline{m|}} \cdot \big[{}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m|}} \cdot \bar{a}_{[x+nm]:\overline{y-x-nm|}} \\
&+ {}^{\infty}\bar{\mathrm{P}}_{[y]} \cdot {}_{y-x-nm|}\bar{a}_{[x+nm]}\big] - v^{y-x} \cdot {}^{y}_{y-x}g_{x:\overline{m|}} \cdot {}_{y}l^{(g)}_{[x]+y-x:\overline{m|}} \cdot {}^{\infty}\bar{\mathrm{P}}_{[y]} \cdot \bar{a}_{[y]}\Big\} \\
&\qquad - \Big\{ l_{[x]}\bar{\mathrm{A}}_{[x]} - \sum_{r=1}^{n-1} v^{rm} \cdot {}^{y}_{rm}g_{x:\overline{m|}} \cdot {}_{y}l^{(g)}_{[x]+rm:\overline{m|}} \cdot \bar{\mathrm{A}}_{[x+rm]} \\
&\qquad\qquad - v^{nm} \cdot {}^{y}_{nm}g_{x:\overline{m|}} \cdot {}_{y}l^{(g)}_{[x]+nm:\overline{m|}} \cdot \bar{\mathrm{A}}_{[x+nm]} \\
&\qquad\qquad - v^{y-x} \cdot {}^{y}_{y-x}g_{x:\overline{m|}} \cdot {}_{y}l^{(g)}_{[x]+y-x:\overline{m|}} \cdot \bar{\mathrm{A}}_{[y]}\Big\} .
\end{aligned}
\tag{3}
$$

Expression (3) may be rewritten as follows:

$$
\begin{aligned}
&l_{[x]}\Big[\sum_{r=0}^{n-1} {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+rm}:\overline{m}|}\cdot {}_{rm\,|\,m}\bar{a}_{[x]} + {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m}|}\cdot {}_{nm\,|\,y-x-nm}\bar{a}_{[x]} \\
&\quad + {}^{\infty}\bar{P}_{[y]}\cdot {}_{y-x\,|}\bar{a}_{[x]} - \bar{A}_{[x]}\Big] - \sum_{r=1}^{n-1} v^{rm}\cdot {}^{y}_{rm}g_{x:\overline{m}|}\cdot {}_{y}l^{(g)}_{[x]+rm:\overline{m}|} \\
&\qquad \times\Big[\sum_{\rho=r}^{n-1} {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+\rho m}:\overline{m}|}\cdot {}_{(\rho-r)m\,|\,m}\bar{a}_{[x+rm]} \\
&\quad + {}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m}|}\cdot {}_{(n-r)m\,|\,y-x-nm}\bar{a}_{[x+rm]} + {}^{\infty}\bar{P}_{[y]}\cdot {}_{y-x-rm\,|}\bar{a}_{[x+rm]} \\
&-\bar{A}_{[x+rm]}\Big] - v^{nm}\cdot {}^{y}_{nm}g_{x:\overline{m}|}\cdot {}_{y}l^{(g)}_{[x]+nm:\overline{m}|}\cdot\big[{}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+nm}:\overline{m}|}\cdot \bar{a}_{[x+nm]:\overline{y-x-nm}|} \\
&\qquad + {}^{\infty}\bar{P}_{[y]}\cdot {}_{y-x-nm\,|}\bar{a}_{[x+nm]} - \bar{A}_{[x+nm]}\big] \\
&\qquad - v^{y-x}\cdot {}^{y}_{y-x}g_{x:\overline{m}|}\cdot {}_{y}l^{(g)}_{[x]+y-x:\overline{m}|}\big[{}^{\infty}\bar{P}_{[y]}\cdot \bar{a}_{[y]} - \bar{A}_{[y]}\big]\,.
\end{aligned}
\tag{4}
$$

Now, the coefficient of each value of $v^t\cdot{}^{y}_{t}g_{x:\overline{m}|}\cdot{}_{y}l^{(g)}_{[x]+t:\overline{m}|}$ appearing in Expression (4) is 0. This may be seen by noting that each such coefficient is, in fact, Equation (1) as applied to the particular age involved [this is an age at entry in Equation (1), an age at exit as used in Expression (4)], except for the very last term, which is obviously also equal to 0.

It follows that the value of Expression (4) is equal to that of its first term which, after dividing through by $l_{[x]}$, is identical with the left-hand side of Equation (1), which is in turn equal to 0. Accordingly, under the assumptions made with respect to nonrenewals and nonconversions, the desired chain of net premiums, ${}^{\infty}_{y}\bar{\pi}_{\overset{1}{x+rm}:\overline{m}|}$, is equal, age for age, to the corresponding chain, ${}^{\infty}_{y}\bar{\pi}'_{\overset{1}{x+rm}:\overline{m}|}$, based on the assumption of 100% renewal before attained age y and 100% conversion to the whole life plan at attained age y.

That these two chains of net premiums should be equal under the stated assumptions may be seen by general reasoning. The crux of the matter is the assumption that those who do not renew may be treated as freshly select lives at the point of nonrenewal. Because for new (select) entrants at any age the value of the future premiums (on a 100% renewal, 100% conversion at age y, basis) is just sufficient to cover the claim costs, the solvency of the fund held on a renewal date, with respect to an existing group of lives so insured, is not affected by the withdrawal at that time of any lives who would then qualify as new entrants.

It may be noted that, where any value of ${}^{y}_{t}g_{x:\overline{m}|}$ is above that corresponding to the "minimum" renewal (or conversion) level, it follows that the corresponding group of nonrenewing (or nonconverting) lives must include some lives not qualifying as freshly select lives. Since this group of nonrenewing (or nonconverting) lives would, accordingly, be subject to mortality rates above the select level for the particular age at renewal (or conversion) involved, the coefficient of the corresponding value of $v^t \cdot {}^{y}_{t}g_{x:\overline{m}|} \cdot {}_{y}l^{(g)}_{[x]+t:\overline{m}}$ would become negative—that is, the present value of the premiums involved in this coefficient would become less than that of the corresponding benefits, instead of being equal to the benefits as was the case where the group of nonrenewing (or nonconverting) lives was subject to select mortality.

The result of this situation is seen to be an increase in the value of Expression (4). In other words, when any values of ${}^{y}_{rm}g_{x:\overline{m}|}$ lie above those corresponding to the "minimum" renewal (or conversion) level, the set of net premium rates, ${}^{\infty}_{y}\bar{\pi}^{\ 1}_{\overline{x+rm}:\overline{m}|}$, becomes redundant. Thus these net premium rates may be assumed to involve no deficiencies so long as the mortality experience among the whole group of original entrants, nonrenewers and renewers (and nonconverters and converters), follows the underlying select and ultimate mortality assumption, without regard to the incidence or the rates of nonrenewal (or nonconversion).

It is recognized that there may be a theoretical question as to whether the mortality experience among each group of nonrenewing lives encountered above will correspond to that among a group of newly insured lives of the same age, sex, class, etc.; however, as a practical matter this assumption appears reasonable—probably conservative (that is, it probably assumes a lighter mortality among those not renewing than will actually be experienced)—and it certainly has considerable value, if usefully and reasonably related to actual experience conditions, because it makes possible the derivation of renewable term insurance premium rates which are independent of the rates of renewal.

CONTINGENCY FUND IMPLICATIONS

The legal reserves to be held by the insurer on account of such m-year renewable term business would presumably be the same as those held on corresponding nonrenewable term business. Since the feature of renewability, *i.e.*, the "insurance of insurability," clearly involves costs in excess of those for claims during the current term period, it is evident that a solvent renewable term fund must exceed the corresponding legal reserves. Provision for the excess of the required fund over the legal reserve should be made in special or general contingency reserves.

On the assumption of a 100% rate of renewal on each renewal date before the end of the conversion period and of a 100% rate of conversion at attained age y, the end of the conversion period, the required fund at the end of the tth m-year term period for entry age x will be denoted by ${}_{tm}^{\overset{\infty}{y}}\bar{\phi}^{1}_{x:\overline{m}|}$.

Viewed retrospectively,

$$ {}_{tm}^{\overset{\infty}{y}}\bar{\phi}^{1}_{x:\overline{m}|} = \frac{1}{\mathrm{A}_{[x]:\overline{tm}|}^{\;1}} \cdot \left[\sum_{r=0}^{t-1} {}^{\infty}_{y}\bar{\pi}_{\overline{x+rm}:\overline{m}|}^{\;1} \cdot {}_{rm|}{}_{m}\bar{a}_{[x]} - \bar{\mathrm{A}}_{[x]:\overline{tm}|}^{\;1} \right], \qquad (5)$$

where

$$x + nm < y \leq x + (n+1)\, m$$

and

$$0 < t \leq n \,,$$

where n and t are both integral.

In the general situation, where allowance is made for the failure of some lives to renew (or to convert at the end of the conversion period) on the assumption that those who do not so renew (or convert) may be treated as then being freshly select lives, the corresponding fund at the end of the tth m-year period for entry age x will be denoted by ${}_{tm}^{\overset{\infty}{y}}\bar{\phi}^{1\,(g)}_{x:\overline{m}|}$.

Viewed retrospectively,

$$ {}_{y}l^{(g)}_{[x]+tm:\overline{m}|} \cdot {}_{tm}^{\overset{\infty}{y}}\bar{\phi}^{1\,(g)}_{x:\overline{m}|} = l_{[x]+tm} \cdot {}_{tm}^{\overset{\infty}{y}}\bar{\phi}^{1}_{x:\overline{m}|} - \sum_{r=1}^{t-1} {}_{rm}^{\;y}g_{x:\overline{m}|} \cdot {}_{y}l^{(g)}_{[x]+rm:\overline{m}|} \cdot {}_{(t-r)m}p_{[x+rm]} \cdot {}_{(t-r)m}^{\overset{\infty}{y}}\bar{\phi}_{\overline{x+rm}:\overline{m}|}^{\;1}, \qquad (6)$$

since the total fund on hand is clearly the excess of the corresponding fund on the 100% renewal basis over the sum of the funds associated with those who withdrew on the successive renewal dates, assuming those so withdrawing to have been select lives on their respective withdrawal dates.

It may be noted that Equation (6) clearly, and for obvious reasons, parallels Equation (2).

ILLUSTRATIVE RESULTS

For the purpose of giving some indication of the relative magnitudes of the various functions developed in this paper, certain net premiums and associated "funds" are shown on a 5-year term basis in Tables 1 and 2, respectively, together with certain mortality rates which are representative of those underlying the premiums and "funds" shown. The following assumptions are incorporated in the derivation of these illustrative results:

a) The right to renew or to convert under the renewable term policy continues to the policy anniversary nearest the 60th birthday; there is no right to renew or to convert thereafter. The nonrenewable, convertible policy may be converted at any time before expiry.

b) Interest is assumed to be earned at the rate of 3% per annum.

c) Mortality for select lives is assumed to follow a select and ultimate basis. The mortality rates are, for policy years after the 15th, those of the 1946–1949 Ultimate Basic Table, and for the first 15 policy years, rates determined by applying to this table ratios appropriately interpolated (policy year by policy year, by applying the straight line method to the ratios for the neighboring "central" issue ages) from those shown in *TSA* II, p. 512, for the 1946–1949 Select Basic Table.

TABLE 1

5-YEAR TERM INSURANCE

NET CONTINUOUS YEARLY PREMIUM PER $1,000 OF SUM INSURED

Age at Beginning of 5-Year Period	Nonrenewable Nonconvertible (1)	Nonrenewable Convertible (2)	Renewable Convertible (3)	(2) − (1) (4)	(3) − (2) (5)
20	$ .90	$ 1.24	$ 1.27	$ .34	$ .03
25	.86	1.23	1.26	.37	.03
30	1.00	1.59	1.66	.59	.07
35	1.38	2.62	2.79	1.24	.17
40	2.20	4.64	5.00	2.44	.36
45	3.46	8.14	8.84	4.68	.70
50	5.22	12.31	12.97	7.09	.66
55	7.99	16.78	16.78	8.79	0

d) Premiums shown in Table 1 for the renewable term insurance were computed by means of Equation (1); those shown on the nonrenewable, convertible basis were computed by the formula,

$$\frac{\bar{A}_{[x]} - {}^{\infty}\bar{P}_{[x+m]} \cdot {}_{m|}\bar{a}_{[x]}}{\bar{a}_{[x]:\overline{m}|}},$$

on the assumption that all "nonselect" lives convert to the whole life plan at the end of the conversion period. Funds shown in Table 2 were computed for Basis A by Equation (5) and for Basis B by Equation (6). Accordingly, the assumptions involved in deriving these equations, and in the demonstration that the net premiums are substantially independent of the rate of renewal, underlie the results given in Tables 1 and 2.

The death rates shown in Table 2 under Basis A are those of policy years 5, 10, etc., for entry age 25 on the assumed select and ultimate mortality basis. Those correspondingly shown under Basis B are the corresponding death rates derived from the appropriate values of ${}_{y}l^{(g)}_{[x]+rm+s:m|}$, determined by means of Equation (2) using the specified rates of nonrenewal. The ratio of the rate on Basis B to that on Basis A, shown beside the two corresponding mortality rates, accordingly indicates the degree to which the assumed antiselection affects the mortality among the lives who continue their renewable term insurance. The rates of renewal assumed for Basis B are, of course, arbitrary.

TABLE 2

5-YEAR RENEWABLE AND CONVERTIBLE TERM INSURANCE—ORIGINAL ENTRY AGE 25

FUND NEEDED AT END OF EACH 5-YEAR TERM PERIOD (BEFORE WITHDRAWALS) TO COVER DEFICIENCY IN FUTURE PREMIUMS PER $1,000 OF SUM INSURED THEN ELIGIBLE FOR RENEWAL (OR CONVERSION)

Attained Age	Basis A		Basis B		Death Rate per 1,000 for Year Ending at Specified Age		
	Proportion Renewing	Fund Needed	Proportion Renewing	Fund Needed	Basis A	Basis B	Ratio B to A
30.......	100%	$ 2.14	70%	$ 2.14	1.03	1.03	100%
35.......	100	5.09	70	5.73	1.33	1.34	101
40.......	100	11.23	65	13.34	2.32	2.69	116
45.......	100	22.89	60	28.27	4.02	4.81	120
50.......	100	45.05	50	59.43	6.84	9.20	135
55.......	100	74.68	40	120.74	11.44	19.20	168
60.......	*	99.39	*	246.48	19.15	52.60	275

* Fund needed is independent of proportion converting to whole life provided all those not converting can qualify at age 60 for new standard insurance.

A comparison of Columns (4) and (5) of Table 1 brings out the striking degree to which the costs of providing convertibility exceed those of adding renewability. It should be mentioned here that on the basis of the assumptions hereinbefore set forth the net yearly premium rates for the renewable term insurance will, for entrants at any of the ages shown, just cover the claim costs under all renewals and conversions thereof.

It should be explicitly stated that the funds needed to cover future premium deficiencies, set forth in Table 2, are automatically identical with the corresponding funds on hand, representing the excess of the accumulation of the premiums received over the accumulated cost of paying the claims incurred.

The result that these funds, for each $1,000 of insurance eligible for renewal, are greater under Basis B (assuming antiselective withdrawals on renewal dates) than under Basis A (assuming all eligible lives renew) seems wholly reasonable. However, for each $1,000 of insurance originally issued, the aggregate fund held at any time after the end of the fifth year would generally be less under Basis B than under Basis A.

DISCUSSION OF PRECEDING PAPER

RICHARD A. LEGGETT:

Mr. Huntington's paper describes a conservative approach to the computation of net renewable term insurance premiums. In order to make practical use of this method for the computation of nonparticipating gross premiums it seems necessary to introduce assumptions quite different from those used in computing net premiums. For renewable term premiums, the expense element constitutes a very large part of the cost, as may be seen from the comparison of net and gross premiums later in this discussion.

Mr. Huntington's method for computing net premiums involves the assumption of 100% renewal at the end of each five-year period until the expiry of the conversion privilege, at which time 100% of eligibles convert. However, in determining the expense element of the premium it is necessary to consider rates of termination of all types. The high lapse rates on term insurance are important to expense considerations because they result in spreading excess initial expenses over a shorter average period than would be necessary without lapse. Since on each renewal and conversion actual expenses are less than expenses allowed for in premiums for newly issued business, an expense saving may be credited. However, conservatively low rates of renewal and conversion should be assumed.

One practical way to utilize Mr. Huntington's method for computing nonparticipating gross premiums is by computing premiums in two steps: first, determining the net annual premiums as described in the paper, and then computing gross premiums using the net premiums as the annual cost of the death benefits. In the latter step conservative estimates of lapse rates and rates of nonrenewal or nonconversion must be used. This is described more completely later.

In the past, we have worked these gross premiums by making assumptions as to the proportions of eligibles renewing and converting at the end of each five-year period and the mortality to be expected on such renewals and conversions. A recent sample indicated that at the end of five years from issue about 70% of policyholders renew at the higher rate. The proportion renewing decreased slightly at the higher ages, being 63% in the 50–60 age group. We do not have adequate experience on renewal and conversion mortality on this plan, but have made estimates based on experience on other plans of insurance. Computation of these gross premiums has been laborious. First, a conversion charge is determined for conversions at age 60 which together with premiums on converted policies will meet expected mortality, lapses, and reduced expenses on the converted policies. A renewal charge is similarly determined at age 60, based on a term premium for the final 5 years. We then compute a 5 Year Renewable Term premium at issue age 55 which provides for mortality, lapses, and expenses on the term policies together with an amount to meet the conversion and renewal charges on the appropriate proportion of eligibles converting or renewing at the end of five years.

This is then repeated by computing a conversion charge at age 55, and a renewal charge at age 55 based on the 5 Year Renewable Term premium just computed for age 55, with expected renewal mortality, lapses, and expenses; and conversion and renewal charges at age 60. These in turn are used for computation of the 5 Year Renewable Term premium for issues at age 50. This procedure is repeated down to the lowest issue age.

By using net premiums computed as in Mr. Huntington's paper the process is shortened considerably. The premium for any issue age may be computed by a formula of the type:

$$ {}_{y}P'_{[\overset{1}{x}]:\overline{5}|} = $$

$$ \{ (1+i/2+c)\, {}^{\infty}_{y}\bar{\pi}_{[\overset{1}{x}]:\overline{5}|} N^{[x]+5}_{[x]} + k_1 D_{[x]} + k_2 N^{[x]+5}_{[x]+1} + [\,{}_{r}h_{x+5}\cdot K_r $$

$$ + {}_{c}h_{x+5}\cdot K_c]\, D_{[x]+5} \} / [\, (1-p)\, N^{[x]+5}_{[x]} - c_1 D_{[x]} - c_2 N^{[x]+5}_{[x]+1}] \ . $$

where

c = claim expense factor

${}^{\infty}_{y}\bar{\pi}_{[\overset{1}{x}]:\overline{5}|}$ is the net premium as determined in the paper

$N^{[x]+5}_{[x]} = N_{[x]} - N_{[x]+5}$

k_1 and k_2 are expense constants

K_r = renewal saving per M

K_c = conversion saving per M

${}_{r}h_x$ = rate of renewal at age x

${}_{c}h_x$ = rate of conversion at age x

p, c_1, and c_2 are percentage expenses.

For conservatism the commutation functions should include provision for lapse as a decrement. The conversion saving per M may be computed at certain ages as the excess of first year expenses on a new issue over those on a conversion at same age. As a further refinement this difference may be adjusted for the different lapse and mortality expected on conversions versus new issues. The renewal saving per M may be computed as the excess of the present value of gross premiums over the present value of renewal expenses and net premiums, less renewal and conversion savings for the proportions renewing and converting 5 years later. Thus, to work these premiums we have to start at the top ages and work down as described in the earlier method, but the task is far shorter.

As a matter of interest we have worked scales of approximate premiums by this latter method, using 1946–1949 Basic Mortality, 3% interest, and lapse rates and expenses based on our experience. Mr. Huntington's net premiums are shown in column (1). In column (2) is shown a scale based on our adaptation of Mr. Huntington's method, using his net premiums as the annual provision for mortality costs, but with expenses included as described above. We have assumed that at the end of each five-year period 70% of eligibles renew and 20% convert, except at attained age 60 when 40% renew and 30% convert. For these rates in column (2) we used a

Issue Age	Net Premiums	Gross Premiums		
		Huntington's Net Premiums with Expense Loading		Conventional Method
	(1)	(2)	(3)	(4)
20	1.27	4.80	4.80	5.78
25	1.26	5.00	5.00	5.73
30	1.66	5.18	5.18	5.73
35	2.79	6.25	6.25	6.70
40	5.00	8.52	8.52	8.71
45	8.84	12.98	12.98	12.21
50	12.97	17.68	17.65	17.73
55	16.78	24.87	20.07	20.42
60	13.36	15.63	34.74	34.63

renewal premium at age 60 based on select mortality and renewal expenses. Although such a premium is consistent with the principles of the paper, we suspect that the actual experience will indicate substantially higher than select mortality. If we arbitrarily pick a rate at 60 which is close to those charged by most companies, there is a great expense saving on renewals at 60, and we develop the scale shown in column (3).

As an indication of the level of a scale of premiums computed by our usual method with approximately the same expense assumptions and making provision for certain proportions renewing and converting and our usual assumptions as to mortality thereon, we show such premiums in column (4). The principal reason for my discussion is to point out that an adaptation of Mr. Huntington's method will produce premiums at approximately the level of those we might otherwise compute by more laborious methods. It also supports an impression we have had that our current assumptions are somewhat conservative. Even the premiums in columns (2) and (3) are quite conservative, particularly at the higher ages. The assumptions in the paper imply that the policyholders select perfectly against the company, which is probably far from the actual experience. Furthermore, my method itself involves some error on the safe side. However, I am sure that we can make practical use of this method for computing nonparticipating gross premiums.

The principles described in Mr. Huntington's paper should also be useful in computing premiums for the guaranteed insurability option which is now gaining popularity. This has enough possibilities, however, to be the basis for another paper.

WARD VAN B. HART:

Although several of us have struggled with various ways of attacking the cost of convertibility in term insurance policies, this paper is the first effort to tackle the more formidable job of renewability superimposed on convertibility.

I suppose most of us would agree with Mr. Milligan's statement (*TASA* XXII, 475) that when general reasoning tells us that potential adverse selection exists it will later be found that it usually materializes. Following up this thought, it would logically follow that a term policy which is both renewable and convertible would automatically require a higher cost for the two options than for merely the convertibility option. The question which we all face is how much to charge for the addition of the second option. In the absence of empirical figures, Mr. Huntington naturally turns to his mathematics and on the basis of certain premises adopted by him, finds, as he shows in Table 1, that renewability adds relatively little to the cost once convertibility has already been provided. Or, by general reasoning, since no one policyholder can convert and renew a policy simultaneously, it would seem to follow that most of the renewable cost can be defrayed by the relief from the conversion cost.

Although his figures are derived from certain premises adopted by him, it is difficult to conceive of any other realistic premises which would not produce about the same relationship as in his Table 1, namely, a comparatively small excess in column 5; for instance, it would be hard to believe that a situation might exist where the mortality on renewed lives might be double or triple that on the corresponding converted lives on the converted policy. Therefore, I can accept quite comfortably the results which he obtains as to the relative value of the renewability and convertibility options without necessarily endorsing the absolute value of either.

Perhaps a more vital problem to be solved is the determination of whether there will be perceptibly more antiselection at the time of purchase of a renewable and convertible policy than at the time of purchase of a policy which is merely convertible. A good many of us have come to believe that if term insurance is sold rather than bought, it need not contain the speculative elements which were felt two generations ago. Today the trained agent sells short term insurance as an option on permanent insurance, term riders (whether level or decreasing) as part of a carefully thought out program, while long term insurance (expiry at age 65, for instance) has been sold for more than thirty years by several companies without too much evidence of severe antiselection at the time of purchase. In our rate making, therefore, we have little hesitation in assuming a mortality on term insurance differing little, if any, from that on life and endowment forms. It may well be, however, that renewable term insurance with its certainty of future periodical jumps in rates, each one more severe than the previous one, may attract the least thrifty cross-section of the public, and, therefore, in making rates the necessity for an adequate provision for this possibility may easily outweigh any refinements employed in assessing the cost of the renewability feature.

(AUTHOR'S REVIEW OF DISCUSSION)

HENRY S. HUNTINGTON:

First I wish to express my appreciation to Messrs. Hart and Leggett for their discussions of my paper. Both discussions add materially to its value. It is particularly gratifying to me that they have both accepted the results presented in the paper as giving reasonable cost levels for the renewability feature.

Mr. Hart brings out the vital point that renewable term insurance may well be the plan which is most vulnerable to antiselection at the time of purchase. Perhaps both stricter underwriting of such applicants as mentioned under assumption (*c*) in the "Derivation of Net Premiums" section of the paper and "additional" premium margins are desirable. At any rate the method of the paper may, perhaps, be viewed as indicating minimum levels of the extra mortality costs associated with the addition of the renewability feature.

I feel particularly indebted to Mr. Leggett for his treatment of the ultimate problem of determining nonparticipating gross premium rates for renewable term insurance. It would appear that his formulas and assumptions have the practical value of being close to the actual facts and his results are of particular interest on this account.

Mr. Leggett assumes two different renewable term premium rates at age 60 to produce the sets of rates shown in columns (2) and (3), respectively, of his table. It is interesting to note that the difference between the two sets of rates becomes insignificant only two steps before the age 60 level. It should be mentioned in passing that the method of the paper is hardly applicable to the determination of such premium rates for age 60, since this age marks the end of the conversion period. The rationale of the method involves the condition, perhaps not clearly stated in the paper, that the renewable term insurance with respect to any term period for which a premium rate is so determined must be eligible for conversion after payment of the first premium at this rate. In the notation employed in the paper this condition may be stated as follows: values of ${}^{\infty}_{y}\bar{\pi}^{1}_{x:\overline{m}|}$ may be determined by this method only when $x < y$.

In this connection I should like to set forth a parallel treatment of the problem of determining gross nonparticipating annual premiums for renewable term insurance under the conditions and using the notation set forth in the paper. For convenience, traditional functions will be used instead of the continuous functions used in the paper.

Let ${}_{y}P'^{(g)}{}^{1}_{x:\overline{m}|}$ = gross annual premium rate corresponding to ${}^{\infty}_{y}\bar{\pi}^{1}_{x:\overline{m}|}$ as defined in the paper and assuming a specific set of rates of nonrenewal (or nonconversion), ${}^{\;y}_{rm}g_{x:\overline{m}|}$, as also defined therein;

c_1, c_2, c_3 and c_4 = appropriate percentage-of-premium expense factors;

α = average size of policy;

e_1 = extra first year costs per policy;

e_2 = every year costs per policy;

e_3 = extra first year costs per policy, excluding selection costs;

e_4 = claim costs per policy;

f_1 = extra first year costs incurred on a per $1,000 basis;

f_2 = every year costs incurred on a per $1,000 basis;

f_3 = extra first year costs incurred on a per $1,000 basis, excluding selection costs.

Then, if the assumptions set forth as (*a*), (*b*) and (*c*) in the "Derivation of Net Premiums" section of the paper are made here with respect to the exercise of the renewal and conversion options and to the presence of antiselection not involving the exercise of these options, Equation (7) may be written as the gross premium counterpart of Expression (4). This equation reflects the fact that, for those who do not renew on a renewal date falling before the end of the conversion period and are accordingly assumed then to be select lives, the excess of the premiums "lost" to the fund, because these select lives do not renew, over the corresponding benefits and expenses is equal to the excess of the expense for the year starting then under a newly issued policy over that under a renewing policy. This condition follows from the fact that for such a newly issued policy the present value of the premiums is equal to that of the benefits and expenses; hence with respect to the freshly select lives who renew, the present value on the renewal date of the premiums exceeds that of the benefits and expenses by the excess of the expenses under the newly issued policies over those under the renewing policies.

For consistency with the assumptions made and for the purpose of developing clear-cut comparisons between the costs of renewable term insurance and those of corresponding nonrenewable, convertible term insurance, it is assumed that no policies are converted before the end of the conversion period. Finally, it should be mentioned that the equation to be given assumes that commission at the first year rate is paid at renewal on the increase in premium only; the adjustment for payment of such commission on the full premium due on each renewal date may be easily made by the interested reader.

$$
\begin{aligned}
l_{[x]}\Big\{ &(1-c_2)\Big[\sum_{r=0}^{n-1} {}_yP'^{(g)}_{\overset{1}{x+rm}:\overline{m|}}\cdot {}_{rm|m}\ddot{a}_{[x]} + {}_yP'^{(g)}_{\overset{1}{x+nm}:\overline{m|}}\cdot {}_{nm|y-x-nm}\ddot{a}_{[x]}\Big] \\
&-(c_1-c_2)\Big[{}_yP'^{(g)}_{\overset{1}{x}:\overline{m|}} + \sum_{r=1}^{n}\Big({}_yP'^{(g)}_{\overset{1}{x+rm}:\overline{m|}} - {}_yP'^{(g)}_{\overset{1}{x+(r-1)m}:\overline{m|}}\Big)v^{rm}\cdot {}_{rm}p_{[x]}\Big] \\
&+P'_{[y]}\big[(1-c_2)\cdot {}_{y-x|}\ddot{a}_{[x]} + (c_2-c_4)\cdot {}_{y+10-x|}\ddot{a}_{[x]} \\
&\qquad\qquad - (c_3-c_2)\cdot v^{y-x}\,{}_{y-x}p_{[x]}\big] \\
&-\Big(1+\frac{e_4}{a}\Big)\bar{A}_{[x]} - \Big(\frac{e_2}{a}+f_2\Big)\ddot{a}_{[x]} - \Big(\frac{e_1}{a}+f_1\Big) \qquad\qquad (7) \\
&\qquad\qquad - \Big(\frac{e_3}{a}+f_3\Big)v^{y-x}\cdot {}_{y-x}p_{[x]}\Big\} \\
&-\sum_{r=1}^{n} v^{rm}\cdot {}_{rm}^{\;y}g_{x:\overline{m|}}\cdot {}_yl^{(g)}_{[x]+rm:\overline{m|}}\Big[(c_1-c_2)\cdot {}_yP'^{(g)}_{\overset{1}{x+(r-1)m}:\overline{m|}} + \frac{e_1}{a} + f_1\Big] \\
&- v^{y-x}\cdot {}_{y-x}^{\;y}g_{x:\overline{m|}}\cdot {}_yl^{(g)}_{[x]+y-x:\overline{m|}}\Big(\frac{e_1-e_3}{a}+f_1-f_3\Big) = 0\,.
\end{aligned}
$$

Each chain of values of the form ${}_yP'^{(g)}_{\overset{1}{x+rm}:\overline{m|}}$ may be determined for all applicable values of r in a manner corresponding to that described in the paper for determining the chains of the form ${}^{\infty}_{y}\bar{\pi}_{\overset{1}{x+rm}:\overline{m|}}$.

For the purpose of comparing gross premium rates for renewable term insurance with the corresponding rates for nonrenewable convertible term insurance, Table 3 presents certain gross premium rates on the basis of the policy conditions and net premium assumptions used in developing the illustrative results given in the paper, together with the expense and average size assumptions set forth below and the necessary rates of renewal and of conversion given in the table itself:

$c_1 = .42$	$e_1 =$ \$32.00 per policy	$f_1 =$ \$2.90 per \$1,000
$c_2 = .095$	$e_2 =$ 3.00 per policy	$f_2 =$ .10 per \$1,000
$c_3 = .62$	$e_3 =$ 24.50 per policy	$f_3 =$ 1.15 per \$1,000
$c_4 = .05$	$e_4 =$ 15.00 per policy	
	$a =$ \$15,000 per policy for all plans and ages encountered	

The gross premium rates shown in columns (2) and (4) of the table were computed by the formula,

$$\Big\{\left(1+\frac{e_4}{a}\right)\bar{A}_x+\left(\frac{e_2}{a}+f_2\right)\ddot{a}_{[x]}+\left(\frac{e_1}{a}+f_1\right)+\left(\frac{e_3}{a}+f_3\right)v^5\cdot{}_5p_{[x]}$$

$$-P'_{[x+5]}\left[(1-c_4)\cdot{}_{5|}\ddot{a}_{[x]}+(c_2-c_4)\cdot{}_{15|}\ddot{a}_{[x]}-(c_3-c_2)\,v^5\cdot{}_5p_{[x]}\right]$$

$$+\,v^5\cdot{}^{x+5}_{\;5}g_{x:\overline{5|}}\cdot{}_5p_{[x]}\left(\frac{e_1-e_3}{a}+f_1-f_3\right)\Big\}$$

$$\div\left[(1-c_2)\,\ddot{a}_{[x]:\overline{5|}}-(c_1-c_2)\right],$$

on the assumption that all "nonselect" lives convert to the whole life plan at the end of the conversion period. The whole life gross premium rates were computed by the formula,

$$P'_{[x]}=\frac{(1+e_4/a)\,\bar{A}_{[x]}+(e_2/a+f_2)\,\ddot{a}_{[x]}+(e_1/a+f_1)}{(1-c_2)\,\ddot{a}_{[x]}+(c_2-c_4)\cdot{}_{10|}\ddot{a}_{[x]}-(c_3-c_2)}.$$

It may be helpful to set forth here the working formula used for ages under 55 in computing column (8) of Table 3, as follows:

$$\left[(1-c_2)\,N^{[x]+5}_{[x]}-(c_1-c_2)\left(D_{[x]}-D_{[x]+5}+{}^{g}_{60}D^{(g)}_{[x]+5:\overline{5|}}\right)\right]{}_{60}P'^{(g)}_{\overset{1}{x}:\overline{5|}}$$

$$=\left(1+\frac{e_4}{a}\right)\bar{M}_{[x]}+\left(\frac{e_2}{a}+f_2\right)N_{[x]}+\left(\frac{e_1}{a}+f_1\right)D_{[x]}+\left(\frac{e_3}{a}+f_3\right)D_{[x]+60-x}$$

$$-\sum_{r=1}^{10-.2x}{}_{60}P'^{(g)}_{\overset{1}{x+5r}:\overline{5|}}\left[(1-c_2)\,N^{[x]+5r+5}_{[x]+5r}-(c_1-c_2)\left(D_{[x]+5r}-D_{[x]+5r+5}\right)\right]$$

$$-{}_{60}P'^{(g)}_{\overset{1}{55}:\overline{5|}}\left[(1-c_2)\,N^{[x]+60-x}_{[x]+55-x}-(c_1-c_2)\,D_{[x]+55-x}\right]$$

$$-P'_{[60]}\left[(1-c_2)N_{[x]+60-x}+(c_2-c_4)N_{70}-(c_3-c_2)D_{[x]+60-x}\right]$$

$$+\left(\frac{e_1}{a}+f_1\right)\sum_{r=1}^{11-.2x}{}_{60}^{g}D_{[x]+5r:\overline{5}|}^{(g)}+\left(\frac{e_1-e_3}{a}+f_1-f_3\right){}_{60}^{g}D_{[x]+60-x:\overline{5}|}^{(g)}$$

$$+(c_1-c_2)\sum_{r=2}^{11-.2x}{}_{60}P'^{(g)}_{\overset{1}{x+5r-5:\overline{5}|}}\cdot{}_{60}^{g}D_{[x]+5r:\overline{5}|}^{(g)},$$

where

$${}_{y}^{g}D_{[x]+rm:\overline{m}|}^{(g)}={}_{rm}^{y}g_{x:\overline{m}|}\cdot v^{x+rm}\cdot{}_{y}l_{[x]+rm:\overline{m}|}^{(g)}.$$

It should be mentioned specifically that none of the premiums shown in Table 3 include provision for loss on withdrawals occurring before the end of the fifth policy year (nor for possible gain on such and later withdrawals occurring off the renewal dates). As far as the *relationships* among the four premiums shown for each age are concerned, however, it seems unlikely that the introduction of such provision on a realistic basis would introduce any material changes.

In comparing the illustrative premium rates in Table 3 for the nonrenewable, convertible term insurance with the corresponding rates for the

TABLE 3

5 YEAR TERM INSURANCE

ILLUSTRATIVE GROSS ANNUAL PREMIUM PER $1,000 OF SUM INSURED

AGE AT BEGINNING OF 5-YEAR PERIOD	NONRENEWABLE CONVERTIBLE				RENEWABLE (ASSUMING CONVERSIONS TAKE PLACE AT AGE 60 ONLY) WITH FIRST YEAR COMMISSION AT RENEWAL UPON			
	Basis A		Basis B		Full New Premium		Increase in Premium	
	Proportion Converting* (1)	Annual Premium (2)	Proportion Converting* (3)	Annual Premium (4)	Proportion Renewing* (5)	Annual Premium (6)	Proportion Renewing* (7)	Annual Premium (8)
20.....	100%	2.61	20%	3.00	70%	2.15	70%	2.00
25.....	100	2.59	20	2.99	70	2.14	70	1.97
30.....	100	3.03	20	3.42	70	2.61	70	2.40
35.....	100	4.26	25	4.63	65	3.95	65	3.64
40.....	100	6.66	30	7.00	60	6.58	60	6.11
45.....	100	10.82	35	11.14	50	11.22	50	10.56
50.....	100	15.79	40	16.08	40	16.43	40	15.98
55.....	100	21.09	40	21.38	40	21.38	40	21.38

* At the end of the 5-year period which begins at the age specified. For columns (5) and (7) the percentage shown for age 55 is the proportion converting; it is assumed that none renew at the end of this 5-year period.

renewable term insurance it should be borne in mind that the rates given in column (2), assuming 100% conversion, represent the lowest possible level for the nonrenewable insurance. The premium rates given in column (4) represent roughly the level for the nonrenewable term insurance which is appropriately comparable to the level for the renewable term insurance represented by the corresponding premium rates given in columns (6) and (8).

The most striking feature of this comparison is that, when commission at the first year rate is payable on each renewal date upon the increase in premium only [column (8)], the premium rate for the renewable term insurance is less than the corresponding rate for the nonrenewable term insurance at every age where the two plans actually differ. Even when such commission is payable on each renewal date upon the full new premium [column (6)], the renewable term insurance shows the lower premium rate for all ages under 45.

These results seem to point strongly to the general conclusion that, for current levels of mortality and expense and normal upper age limits for renewal and conversion, the cost levels for *renewable*, convertible term insurance should commonly be little, if any, higher than the corresponding levels for *nonrenewable*, convertible term insurance. If such a cost relationship actually does obtain between these two kinds of term policy, it would seem only natural that the renewable policies would tend to displace the corresponding (short term) nonrenewable policies.

In particular, it would seem reasonable to charge for renewable, convertible term insurance a premium based, age for age, on the greater of the two premiums, determined under the appropriate nonrenewable and renewable conditions, respectively. In order to provide in this connection for a possible excess mortality among all those originally accepted for renewable term insurance, it might be desirable, in computing the gross premiums under the renewable conditions, to assume a mortality basis appropriately increased over that assumed in the parallel computations made under the nonrenewable conditions.