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VALUING LIABILITIES AND LUMP SUMS UNDER PPA

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The Pension Protection Act (PPA) introduced a significant modification in the way pension liabilities and annuity payments, in general, are to be valued.

Very succinctly, PPA follows the contemporary idea that the present value of an annuity is the discounted value of a cash flow stream. As such, a present value can, in theory, be equated with a series of zero coupon bonds, each of which has a maturity date and face amount that corresponds to each expected cash flow payment. For example, the present value of $10,000 payable at age 65 to an active person currently age 61 is completely determined by the yield-to-maturity of a 4-year zero coupon bond and the probability that a 61-year-old will survive to age 65. Similarly, $10,000 payable at age 66 to the same 61-year-old would be based on the yield-to-maturity of a 5-year zero coupon bond and the probability of surviving to age 66. While it's permissible for valuation purposes to discount each future cash payment by the interest rate corresponding to a yield-to-maturity bond curve, PPA simplified the number of interest rates to three segment rates. The first segment rate applies to cash flows payable within the first five years of the determination date. The second segment rate applies to cash flows payable during the 15-year period that extends from five to twenty years from the determination date. The third segment rate applies to cash flows payable from twenty years and beyond with respect to the determination date.

Returning to our 61-year old active participant who is expected to retire at age 65, there is statistical data supporting the fact that actively working people are generally healthier and have lower mortality than those who are retired. PPA captured this notion by stipulating two mortality tables: an annuitant mortality table which applies to an individual when benefits go into pay status and a non-annuitant mortality table which applies for the period preceding payment status. In the previous paragraph, the expected second payment of $10,000 at age 66 would be computed as the probability of surviving four years to age 65 using non-annuitant mortality and one year to age 66 using annuitant mortality. (In accordance with IRS Revenue Ruling 2007-67, the mortality rates for payment forms subject to §417(e) (*i.e.*, lump sums) are based on the combined annuitant and non-annuitant tables rather than on separate tables which generally will be required for minimum funding.)

In general, as we all learned at one time, the annuity with annual payments of $10,000 can be formulated as:

where equals the first segment rate when t = 4; the second segment rate for ; and, the third segment rate for , and

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is derived using non-annuitant mortality prior to age 65, and annuitant mortality, thereafter.

It may be a matter of opinion, but this actuary concluded that with three interest rates and two mortality tables, it's easier to value the summation directly than to develop three sets of commutation functions in order to first compute and then sum the present value of payments for each of the three segment periods.

The summation formula denoted above can be expanded to reflect monthly payments. Before we completely abandon commutation functions, note that the present value of a 1-year temporary annuity can be approximated by:

If you consider an annuity as the discounted value of a series of 1-year temporary annuities the present value summation can be rewritten as

But wait. There's more…

Not only is it straightforward to evaluate the above summation formula by tabulating each component term in a column of a spreadsheet, but this first principles approach can be extended very nicely. For starters, there's no reason to be restricted to an annual payment of $1 payable monthly. Have you ever had to evaluate a temporary annuity where the number of payments is not a multiple of 12? Sure, you can do a linear interpolation between two temporary annuities. A better solution is to develop a general formula for a temporary annuity where the payment period is less than one year:

The reasonableness of is easily verified. Setting n = 1 results in

. Setting n = 12, results in the familiar

. Now we have complete flexibility in choosing any number of months in the payment period. Tabulating a column of means you can specify the age where payments begin and the point where payments end. In other words,

where: is computed using the appropriate segment rate;

is computed using non-annuitant mortality prior to the payment period and annuitant mortality, thereafter; and

is derived using annuitant mortality.

But wait. There's more…

In the original example, suppose that instead of being exact age 61, the participant were age 61 and 3 months, and we had to compute the present value of a $10,000 annual accrued benefit where the normal retirement age is 65. While it's possible to interpolate between annuity values, the PPA segmented interest rates and mortality table structure pose a knotty problem. The second and third segment interest rates begin at ages and , respectively. The breakpoint age between non-annuitant mortality and annuitant mortality is exact age 65. Given what we have learned, what's the best way to solve the problem? Answer: Interpolate the mortality rates. In other words, is really and, in general, 2 . Once the mortality rates are adjusted, the segmented interest rates coincide with the ages. But for one other adjustment discussed below, the annuity factor is derived the same way as it would for an exact age calculation. No further interpolation is required.

Having defined the beginning of 1-year time intervals to correspond with ages etc., age 65 is no longer an exact age with reference to the revised age intervals. Age 65 is nine months into the 1-year interval that began at age Moreover; from age 65 to age there are only three monthly payments. There may be other alternatives, but one prudent solution is to define in terms of the annuitant mortality at age 65, then discount it back to the beginning of the year with nine months of interest and non-annuitant mortality.

Accordingly, if we define the term as

where is based on non-annuitant mortality, we can further generalize the basic PV summation formula:

where equals 1 for all years, except in the year where the first payment is in not at the beginning of a year.

Conclusion

Years ago, when I started in the field, my manager was an actuary who was an engineer by training. I was humored by the fact that he did not have an electric calculator on his desk (yes, electric not electronic). He worked with a slide rule. He was comfortable with it, and it served him well right up until his retirement.

Commutation functions have been in the actuarial literature for about 200 years. They have served us well by simplifying to a division of , what would have been a completely unworkable calculation without the aid of a computer. Life was simple when we had one interest rate and one mortality table. We now have multiple interest rates combined with non-annuitant and annuitant generational mortality tables. Just as my first manager was comfortable with his slide rule, actuaries are comfortable with commutation functions. It may take a while, but eventually we all will have to part with our old ways. I actually framed my high school slide rule several years ago. I might have to consider doing the same with my copy of Wallace Jordan's *Life Contingencies.*

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Notes

[1] The present value of a n-payment temporary annuity of $1 per annum payable monthly equals

.

Let and approximate with a linear function over the range . Note, and . Hence, the slope of the line over the range is .

Consequently,

. So, the summation can be approximated as:

[2] (Interpolating Mortality Rates)
Except for early ages, the mortality curve is concave. Consequently, a linear interpolation between any two points will result in a line segment that is above the mortality curve.

A geometric interpolation, *i.e.*, results in an interpolated rate that is equal to or less than a linearly interpolated rate:

. The conclusion, therefore, is that a geometrically interpolated rate "fits" closer to the underlying mortality curve.

[3] (Proposed formula to evaluate)

Let .

Note, and .

Since ,

As a further note, upon expanding the expression , we have

Within the brackets, the sum of the third and subsequent terms is negative.

Consequently, for . Therefore, consistent with the observation in NOTE 2, the proposed formula results in a larger than would be derived using a UDD assumption.

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