TRANSACTIONS OF SOCIETY OF ACTUARIES
1958 VOL. 10 NO. 28

THE PREPARATION OF INTERIM ACCOUNTING STATEMENTS USING ELECTRONIC DATA PROCESSING EQUIPMENT

J. C. DAVIDSON AND J. T. BIRKENSHAW

LIFE insurance companies have expressed varying opinions on the necessity of preparing accurate statements during the calendar year as a means of determining the emergence of surplus. Some companies go through elaborate procedures in order to prepare proper income and balance sheet statements at the end of every month. Other companies make independent estimates of the emergence of surplus during the year from a review of mortality, interest earnings and expenses, while others depend solely on calculations which are made at the year end. It would appear that the principal reason why most companies produce accurate statements only once a year is that extensive work is required to obtain certain figures. If it were possible to prepare interim statements during the year with little or no extra work, it would seem that any company would be interested in determining how its surplus is emerging.

Recent actuarial literature contains little if any reference to methods by which interim accounting statements can be produced during the year without doing considerable extra work. The estimation of surplus earnings was one of the subjects discussed at the XIIIth International Congress of Actuaries, and there were important contributions made to this by American and Canadian actuaries. The approaches outlined require a fair amount of special work during the year, and the results give varying degrees of accuracy in the estimates of certain items. However, if the earnings as they develop during the year are expected to be a true indication of surplus, interim statements must be prepared with substantially the same accuracy as the final year end statement.

Early in the development of electronic systems it became apparent that the bulk of the items in interim statements which would normally require estimates or extensive special calculations could be determined by the electronic system as accurately at interim periods as at the year end. However, to do this without considerable additional clerical and machine time requires careful consideration in setting up the various electronic systems which would be necessary to carry on the regular operations of a

life insurance company. Each of the items in the balance sheet for which estimates must be made during the year, but which are determined on an accurate and different basis at the year end, must be reviewed carefully. By installing suitable procedures in the electronic system for such items, accurate figures can be obtained automatically at the end of each month. It is the purpose of this paper to present actuarial bases for such procedures.

Generally speaking, most of the procedures outlined in this paper would be applicable to any company whose system includes a review each month of all transactions which have occurred during the month, together with special calculations for policies whose anniversaries fall in such month. In other cases for the procedures to be applicable a special processing of all policies once a year may be required. As each procedure is outlined the related assumptions will be indicated. Many of these interim calculations are a direct build-up of year end figures which will subsequently be required in any event. Also, with the tremendous calculating ability of large-scale electronic systems a few additional calculations cost little, if any, computer time.

All references to statement items will be to the United States Convention Blank. The following is a classification of items in the Convention Blank for the statement of Assets and Liabilities which normally require special work at each year end and for which interim figures may be determined on a correspondingly accurate basis with electronic systems. Reference is made only to Ordinary insurance operations. Similar procedures may be developed for other lines of business.

a) *Actuarial Items*
   1. Reserves for life insurance.
   2. Reserves for nondeduction of deferred fractional premiums.
   3. Reserves for excess of valuation net premiums over corresponding gross premiums.
   4. Life insurance premiums deferred.
   5. Provision for policyholders' dividends payable in following calendar year.

b) *Accrued Items*
   1. Interest and other investment income accrued.
   2. Policyholders' accumulations accrued.

c) *Premiums and Commissions*
   1. Life insurance premiums uncollected.
   2. Cost of collection of premiums deferred and uncollected in excess of total loading thereon.

d) Miscellaneous

1. Net adjustment in assets and liabilities due to foreign exchange rates.
2. Taxes, licenses and fees due and accrued.

The remainder of the items in the Convention Blank are primarily derived from the accounts of the Company and are readily available at any interim duration as well as at the year end. There are a few, however, which require special calculations at the year end, and it is assumed that parallel procedures are followed at interim statement dates. Typical examples of such items are the value of real estate owned by the Company and policy and contract claims incurred but unreported.

For the actuarial items enumerated in (*a*) the first three reserve items are specifically referred to in the Liabilities of the Convention Blank and the fourth, deferred premiums, arises in the Assets as a result of the procedures used in order to calculate the first two. These four items will be considered as a group.

RESERVE ITEMS INCLUDING DEFERRED PREMIUMS

The generally accepted practice in most countries and jurisdictions has been to hold a reserve at the end of the year, called a medial reserve, which assumes that the anniversary of the policy falls halfway through the year and that linear interpolation between the initial and terminal reserves is satisfactory. The effect of these assumptions is that simple interest is assumed on the accumulation of reserves and premiums for any interim duration and that the cost of insurance for the first and second halves of the calendar year is given by the expressions $\frac{1}{2}q_{x+t}(1 - {}_{t+1}V)$ and $\frac{1}{2}q_{x+t+1}(1 - {}_{t+2}V)$. This practice has no doubt arisen as a result of the complexities of determining a more theoretically accurate reserve at the year end and generally the medial reserve is a sufficiently close approximation for all practical purposes.

With the computing ability of present day electronic systems the problem of calculating a more theoretically accurate reserve has been substantially removed. Also, it is quite feasible to introduce many refinements into the reserve calculations if it is so desired. The determination of reserves at interim durations will be considered from two basic points of view:

I. Holding a reserve which recognizes the actual policy anniversary and does not assume that it falls halfway through the year.

II. Holding a reserve which is consistent with the holding of the usual medial reserve at each year end.

Although formulas may be developed for all plans of insurance, the various procedures will be illustrated by considering specifically an n year endowment with premiums payable for s years, the frequency of premium payment being m times a year. It will be assumed that true premiums are payable.

GENERAL FORMULAS APPLICABLE TO VARIOUS INTERIM RESERVE PROCEDURES

Let ${}_s\pi^{(m)}_{x:\overline{n}|}$ be the gross annual premium payable m times a year and ${}_sP^{(m)}_{x:\overline{n}|}$ be the corresponding net valuation premium. It is assumed that $k/m + r$ represents the fraction from the policy anniversary to date of valuation, which will usually be the end of a month, where $k < m$ and $r < 1/m$ so that $0 < (k/m + r) < 1$, and $t \leq s$. Then

$$ {}_{t+k/m+r}{}^{s}V^{(m)}_{x:\overline{n}|} = \left(1 - \frac{k}{m} - r\right) {}_{t}{}^{s}V^{(m)}_{x:\overline{n}|} + \left(\frac{k}{m} + r\right) {}_{t+1}{}^{s}V^{(m)}_{x:\overline{n}|} + \left(\frac{1}{m} - r\right) {}_{s}P^{(m)}_{x:\overline{n}|} \tag{1} $$

in accordance with formula 5.33 of *Life Contingencies* by Jordan. This formula assumes linear interpolation between the terminal reserves involved, which is considered satisfactory for the purposes of this paper.

$$ {}_{s}P^{(m)}_{x:\overline{n}|} = {}_{s}P_{x:\overline{n}|} + \frac{m-1}{2m}\left(P^{1}_{x:\overline{s}|} + d\right) {}_{s}P^{(m)}_{x:\overline{n}|} $$

and

$$ {}_{t}{}^{s}V^{(m)}_{x:\overline{n}|} = {}_{t}{}^{s}V_{x:\overline{n}|} + {}_{s}P^{(m)}_{x:\overline{n}|} \frac{m-1}{2m} {}_{t}V^{1}_{x:\overline{s}|}. $$

Making the above substitutions and rearranging, the reserve becomes

$$ {}_{t+k/m+r}{}^{s}V^{(m)}_{x:\overline{n}|} = $$

$$ + \overline{{}_{t}{}^{s}V_{x:\overline{n}|} + {}_{s}P_{x:\overline{n}|}} + \left(\frac{k}{m} + r\right)\left({}_{t+1}{}^{s}V_{x:\overline{n}|} - \overline{{}_{t}{}^{s}V_{x:\overline{n}|} + {}_{s}P_{x:\overline{n}|}}\right) \tag{2} $$

$$ + {}_{s}P^{(m)}_{x:\overline{n}|} \frac{m-1}{2m}\left[\overline{{}_{t}V^{1}_{x:\overline{s}|} + P^{1}_{x:\overline{s}|}} + \left(\frac{k}{m} + r\right)\left({}_{t+1}V^{1}_{x:\overline{s}|} - \overline{{}_{t}V^{1}_{x:\overline{s}|} + P^{1}_{x:\overline{s}|}}\right)\right] \tag{3} $$

$$ - \left(1 - \frac{k+1}{m}\right) {}_{s}P_{x:\overline{n}|} \tag{4} $$

$$ - \frac{m-1}{2m} {}_{s}P^{(m)}_{x:\overline{n}|}\left[\left(1 - \frac{k+1}{m}\right) P^{1}_{x:\overline{s}|} - \left(\frac{1}{m} - r\right) d\right], \tag{5} $$

where (2) represents the reserve assuming annual premiums are payable;
(3) represents the reserve for nondeduction of deferred fractional premiums;

(4) represents, taken positively, the deferred premium asset ignoring the reserve for nondeduction of deferred fractional premiums;

(5) represents, taken positively, the deferred premium asset for the premium for the nondeduction of deferred fractional premiums less the loss of interest every year.

The reserve for excess of valuation net premiums over corresponding gross premiums, if such is applicable, is given by

$$({}_{s}\mathrm{P}^{(m)}_{x:\overline{n}|} - {}_{s}\pi^{(m)}_{x:\overline{n}|})\,{}_{1/m-r|}\ddot{a}^{(m)}_{x+t+k/m+r:\overline{s-t-(k+1)/m}|}. \tag{6}$$

Formulas (2) to (6) for these various items are accurate on the assumptions stated. In practice, many obvious simplifications would no doubt be made by the actuary and be considered sufficiently accurate for all practical purposes. However, because of the great calculating ability of electronic systems the above formulas can be applied directly provided all the basic information is available. This will not normally be the case and thus the formulas will require some modifications. In addition, the amount of information which is retained on individual policy records for the sole purpose of obtaining figures at interim durations requires careful consideration, since this will have the effect of increasing the processing time of records by the electronic system. By careful planning, however, and the retention of accumulated total figures this may be reduced to a minimum.

In any consolidated functions approach which includes valuation it would be reasonable to suppose that at least the following basic data will be contained in each individual policy record:

a) Date of issue
b) Plan of insurance
c) Age at issue
d) Gross premium payable
e) Mode of premium payment
f) Valuation interest rate and mortality table.

It will be assumed that the electronic system is programmed to carry out the following calculations:

a) Valuation premiums
b) Terminal reserves assuming annual premiums
c) Annual gross premiums, corresponding to the gross premiums payable.

The actuarial functions stored will probably be tables of N_x, q_x or l_x depending on the formulas used to calculate various functions required.

In certain instances it may be considered advisable to include in each individual policy record certain of the functions which actually could be calculated when required—for example, the annual valuation premiums. Also, if it is not considered worth while to calculate or include in the basic policy record the valuation premium payable m times a year, the annual valuation premium could be substituted where required in the formulas with little loss of accuracy, at the discretion of the actuary.

APPROACH I

ASSUMES INTERPOLATION BETWEEN INITIAL AND TERMINAL RESERVES

With basic information available and programs written to carry out the calculations indicated previously, the various reserve calculations can be made in accordance with formulas (2) to (6) or as modified by individual preference. Thus $k/m + r$ may be deduced from the policy anniversary and the date of valuation. Subsequently, k/m may be determined and hence $1 - (k + 1)/m$ from the frequency of premium payment, which will also give $(m - 1)/2m$ and so forth.

The terminal reserve on the last policy anniversary may be retained on the basic policy record or calculated each time it is required. By calculating the next terminal reserve and comparing the initial and terminal reserve the daily increment may be determined. This might be recorded on policy detail. If so, it would be necessary to process all policies once a month if interim statements were required at monthly intervals and multiply the daily increments by the appropriate time intervals since the last policy anniversary to determine the reserve liability at the required interim statement date. On the other hand, if the interim statement dates are fixed such as at the end of each month, then when the increment is calculated at the policy anniversary, the appropriate reserves could be calculated for each of the future statement dates by making use of the daily increment, multiplying by the appropriate time intervals and then distributing such reserves to a special series of accounts, one for each interim statement date. This would require that for all nonscheduled transactions[1] which occur during the year the appropriate valuation accounts be adjusted accordingly.

Thus, assuming actual policy anniversaries are being considered, there are several ways by which it is possible to determine interim reserves

[1] For the purposes of this paper a nonscheduled transaction is defined as a noncontractual change affecting the status of the policy, including the issuance and termination thereof.

on all policies at any point of time consistent with year end figures. Certain methods require the retention of special data on policy detail which will have some effect on the computer processing time while others require the accumulation of valuation figures in a special series of accounts. The particular method adopted will depend on the requirements and preference of the individual actuary.

The determination of the deferred premium asset is a special problem in itself. Formulas (4) and (5) show that the only variable during a policy year for a particular plan and mode of premium payment is the factor $1 - (k + 1)/m$ which, as explained previously, can be determined at any point of time. Three possible procedures are outlined:

1. If all policies are processed at the statement date the variable factors may be determined and substituted in the appropriate formulas for the determination of the deferred premium asset.
2. The deferred premium asset may be individually calculated for each of the anticipated interim statement dates and a separate account set up for each such date. If such a procedure is followed these accounts must be adjusted for all nonscheduled transactions. As each interim statement date is reached, figures are automatically available. Such a procedure removes the necessity of processing the whole file at each valuation date in order to determine this asset.
3. If premium billing information is available, then at each policy anniversary, when the reserves are calculated, a deferred premium equal to the total annual valuation premium on the policy could be set up in a special account. At each premium due date, including the policy anniversary, a proportionate part of the valuation premium corresponding to the mode of premium payment is deducted from the total deferred premium. Again it will be necessary to adjust this account for all nonscheduled transactions as they occur.

APPROACH II

ASSUMES MEDIAL INTERPOLATION

The following are the corresponding formulas and procedures if it is decided to continue the holding of the usual medial reserve at each year end.

Even when medial reserves are held at the year end it is general practice to hold an accurate deferred premium asset. In other words, no assumption is made that all policies are issued at the middle of the year. Thus, formulas (4) and (5) insofar as they refer to deferred premiums will be applicable. Hence, the only ones to which the usual assumption of medial reserves is applicable are formulas (2), (3) and (6).

If it is assumed $r = 0$ and $k/m = \frac{1}{2}$ and these values are substituted in formulas (2), (3) and (6), the reserve becomes ${}_{t+1/2}V^{(m)}_{x:\overline{n}|} =$

$$+ \tfrac{1}{2}\,({}_tV^{s}_{x:\overline{n}|} + {}_sP_{x:\overline{n}|} + {}_{t+1}V^{s}_{x:\overline{n}|}) \tag{7}$$

$$+ {}_sP^{(m)}_{x:\overline{n}|} \cdot \frac{m-1}{2m} \cdot \tfrac{1}{2}\,({}_tV^{1}_{x:\overline{s}|} + P^{1}_{x:\overline{s}|} + {}_{t+1}V^{1}_{x:\overline{s}|}) \tag{8}$$

$$+ ({}_sP^{(m)}_{x:\overline{n}|} - {}_s\pi^{(m)}_{x:\overline{n}|})\,{}_{1/m}|\,\ddot{a}^{(m)}_{x+t+1/2:\overline{s-t-1/2-1/m}|}. \tag{9}$$

For any individual policy the error resulting from the medial reserve assumption at the year end may be determined by comparing formulas (2), (3) and (6) with (7), (8) and (9). Thus, subtracting the latter from the former gives the following expressions:

$$\left(\frac{1}{2} - \frac{k}{m} - r\right)({}_tV^{s}_{x:\overline{n}|} + {}_sP_{x:\overline{n}|} - {}_{t+1}V^{s}_{x:\overline{n}|}) \tag{10}$$

$${}_sP^{(m)}_{x:\overline{n}|} \cdot \frac{m-1}{2m}\left(\frac{1}{2} - \frac{k}{m} - r\right)({}_tV^{1}_{x:\overline{s}|} + P^{1}_{x:\overline{s}|} - {}_{t+1}V^{1}_{x:\overline{s}|}) \tag{11}$$

$$({}_sP^{(m)}_{x:\overline{n}|} - {}_s\pi^{(m)}_{x:\overline{n}|})\left({}_{1/m-r}|\,\ddot{a}^{(m)}_{x+t+k/m+r:\overline{s-t-(k+1)/m}|} - {}_{1/m}|\,\ddot{a}^{(m)}_{x+t+1/2:\overline{s-t-1/2-1/m}|}\right). \tag{12}$$

If the medial reserve assumption is accepted, it is necessary to move during the calendar year in some manner from one medial reserve to the next in order to establish sufficiently accurate reserve figures for the preparation of interim accounting statements. Several possible procedures will be outlined for the case of a policy with annual premiums. The corresponding formulas for the reserve for the nondeduction of deferred fractional premiums and reserve for excess of valuation net premiums over corresponding gross premiums may be deduced in a similar manner. Possible system routines to handle the various procedures will be outlined and examples will be furnished of the application of the formulas developed to a sample of business.

First Suggested Procedure

If ${}_{t+1/2}V^{s}_{x:\overline{n}|}$ and ${}_{t+1\ 1/2}V^{s}_{x:\overline{n}|}$ represent two successive medial reserves, the assumption could be made that the reserve fund moves in one step from ${}_{t+1/2}V^{s}_{x:\overline{n}|}$ to ${}_{t+1\ 1/2}V^{s}_{x:\overline{n}|}$ at the policy anniversary.

The formula connecting two medial reserves is as follows:

$${}_{t+1/2}V^{s}_{x:\overline{n}|}(1+i) + {}_sP_{x:\overline{n}|}\left(1 + \frac{i}{2}\right) - \tfrac{1}{2}q_{x+t}(1 - {}_{t+1}V^{s}_{x:\overline{n}|}) - \tfrac{1}{2}q_{x+t+1}(1 - {}_{t+2}V^{s}_{x:\overline{n}|}) = {}_{t+1\ 1/2}V^{s}_{x:\overline{n}|}.$$

Rearranging terms,

$$ {}_{t+1\ 1/2}{}^{s}V_{x:\overline{n}|} - {}_{t+1/2}{}^{s}V_{x:\overline{n}|} - {}_{s}P_{x:\overline{n}|} = i\,({}_{t+1/2}{}^{s}V_{x:\overline{n}|} + \tfrac{1}{2}\,{}_{s}P_{x:\overline{n}|}) - \tfrac{1}{2}\,q_{x+t}\,(1 - {}_{t+1}{}^{s}V_{x:\overline{n}|}) - \tfrac{1}{2}\,q_{x+t+1}\,(1 - {}_{t+2}{}^{s}V_{x:\overline{n}|})\,. $$

Thus, this assumption would be true when

a) the interest required to maintain reserves is equal to the cost of insurance during the calendar year and

b) the only increment to the reserve received during the year is the valuation premium.

When this assumption is true,

$$ {}_{t+1/2}{}^{s}V_{x:\overline{n}|} + {}_{s}P_{x:\overline{n}|} = {}_{t+1\ 1/2}{}^{s}V_{x:\overline{n}|}\,. $$

For the whole range of a company's business this may be considered as a sufficiently accurate assumption. However, for any individual policy this is incorrect except at possibly a few isolated points.

If the interest required to maintain reserves is not equal to the cost of insurance, the assumptions made imply that the reserve funds at the policy anniversary receive an increment of

$$ {}_{s}P_{x:\overline{n}|} + i\,({}_{t+1/2}{}^{s}V_{x:\overline{n}|} + \tfrac{1}{2}\,{}_{s}P_{x:\overline{n}|}) - \tfrac{1}{2}\,q_{x+t}\,(1 - {}_{t+1}{}^{s}V_{x:\overline{n}|}) - \tfrac{1}{2}\,q_{x+t+1}\,(1 - {}_{t+2}{}^{s}V_{x:\overline{n}|})\,, $$

or the valuation premium plus the interest required to maintain reserves less the cost of insurance, and that no increment occurs at any other point.

Second Suggested Procedure

If, on the average, at any particular point of time at which the reserve liability may be required it cannot be assumed that the interest required to maintain reserves balances the cost of insurance for the corresponding period as outlined in the first suggested procedure, other assumptions and approximations must be made.

One possible approach is that if the reserve on an individual policy is required at duration h past the end of the calendar year, $k/m + r$ represents the period from the policy anniversary to the end of the calendar year and $h < 1 - k/m - r$ (*i.e.*, the policy anniversary has not been passed), then formula (2) can be written as

$$ {}_{t+k/m+r+h}{}^{s}V_{x:\overline{n}|} = {}_{t+1/2}{}^{s}V_{x:\overline{n}|} + \left(\frac{1}{2} - \frac{k}{m} - r\right)({}_{t}{}^{s}V_{x:\overline{n}|} + {}_{s}P_{x:\overline{n}|} - {}_{t+1}{}^{s}V_{x:\overline{n}|}) - h\,({}_{t}{}^{s}V_{x:\overline{n}|} + {}_{s}P_{x:\overline{n}|} - {}_{t+1}{}^{s}V_{x:\overline{n}|})\,. \qquad (13) $$

When the middle term

$$\left(\frac{1}{2}-\frac{k}{m}-r\right)({}_{t}{}^{s}V_{x:\overline{n}|}+{}_{s}P_{x:\overline{n}|}-{}_{t+1}{}^{s}V_{x:\overline{n}|})$$

is compared with formula (10) it will be seen that this corresponds with the error caused by the medial reserve assumption. If this is not taken into account the formula for the reserve becomes

$${}_{t+k/m+r+h}{}^{s}V_{x:\overline{n}|}={}_{t+1/2}{}^{s}V_{x:\overline{n}|}-h\,({}_{t}{}^{s}V_{x:\overline{n}|}+{}_{s}P_{x:\overline{n}|}-{}_{t+1}{}^{s}V_{x:\overline{n}|})\,. \quad (14)$$

If $h > 1 - k/m - r$ (*i.e.*, the policy anniversary has been passed), the formula corresponding to (13) is

$$\begin{aligned}{}_{t+k/m+r+h}{}^{s}V_{x:\overline{n}|}&={}_{t+1\ 1/2}{}^{s}V_{x:\overline{n}|}\\&\quad+\left(\frac{1}{2}-\frac{k}{m}-r\right)({}_{t+1}{}^{s}V_{x:\overline{n}|}+{}_{s}P_{x:\overline{n}|}-{}_{t+2}{}^{s}V_{x:\overline{n}|}) \quad (15)\\&\quad+(1-h)\,({}_{t+1}{}^{s}V_{x:\overline{n}|}+{}_{s}P_{x:\overline{n}|}-{}_{t+2}{}^{s}V_{x:\overline{n}|})\,.\end{aligned}$$

Similarly, if the middle term in formula (15) is not taken into account the formula for the reserve becomes

$${}_{t+k/m+r+h}{}^{s}V_{x:\overline{n}|}={}_{t+1\ 1/2}{}^{s}V_{x:\overline{n}|}+(1-h)\,({}_{t+1}{}^{s}V_{x:\overline{n}|}+{}_{s}P_{x:\overline{n}|}-{}_{t+2}{}^{s}V_{x:\overline{n}|})\,. \quad (16)$$

The difference in reserves produced by formulas (14) and (16) at the anniversary may be determined by substituting the value $h = 1 - k/m - r$.

Formula (14) produces the expression

$${}_{t+1/2}{}^{s}V_{x:\overline{n}|}-\left(1-\frac{k}{m}-r\right)({}_{t}{}^{s}V_{x:\overline{n}|}+{}_{s}P_{x:\overline{n}|}-{}_{t+1}{}^{s}V_{x:\overline{n}|}) \quad (17)$$

and formula (16) produces the expression

$${}_{t+1\ 1/2}{}^{s}V_{x:\overline{n}|}+\left(\frac{k}{m}+r\right)({}_{t+1}{}^{s}V_{x:\overline{n}|}+{}_{s}P_{x:\overline{n}|}-{}_{t+2}{}^{s}V_{x:\overline{n}|})\,. \quad (18)$$

Subtracting expression (17) from expression (18) and simplifying gives the following:

$${}_{s}P_{x:\overline{n}|}-\left(\frac{1}{2}-\frac{k}{m}-r\right)(2_{t+1}{}^{s}V_{x:\overline{n}|}-{}_{t}{}^{s}V_{x:\overline{n}|}-{}_{t+2}{}^{s}V_{x:\overline{n}|})\,. \quad (19)$$

If the latter term in this expression is compared with formula (10) it will be seen that it represents the difference in the error in the medial reserve assumption at durations $t + \frac{1}{2}$ and $t + 1\frac{1}{2}$.

In other words, formulas (14) and (16) assume a decrement to the reserve at the anniversary, other than the increment of the premium, of the

difference between the error of the medial reserve assumption at the end and beginning of the calendar year. If such an error after investigation can be ignored without great loss of accuracy these formulas provide a simple method of estimating reserves at any interim duration.

The latter expression in formula (19) may be considered in two parts:

$$a) \quad \left(\frac{1}{2} - \frac{k}{m} - r\right)$$

$$b) \quad (2\,{}_{t+1}{}^{s}V_{x:\overline{n}|} - {}_{t}{}^{s}V_{x:\overline{n}|} - {}_{t+2}{}^{s}V_{x:\overline{n}|}) .$$

If the business is uniformly distributed over the year, factor (a) will be zero. If the second expression is to be zero it must be assumed that a linear relationship exists between the three terminal reserves involved. This is not likely to be the case for a normal distribution of business since the bulk of policies produce a reserve curve which is concave upwards for premium paying business.

If this error in formula (19) is significant it could be assumed that it is uniformly distributed over the calendar year. If such an assumption is made, formula (14) becomes, for $h < 1 - k/m - r$,

$$\begin{aligned} {}_{t+k/m+r+h}{}^{s}V_{x:\overline{n}|} = {}& {}_{t+1/2}{}^{s}V_{x:\overline{n}|} - h\,({}_{t}{}^{s}V_{x:\overline{n}|} + {}_{s}P_{x:\overline{n}|} - {}_{t+1}{}^{s}V_{x:\overline{n}|}) \\ & - h\left(\frac{1}{2} - \frac{k}{m} - r\right)(2\,{}_{t+1}{}^{s}V_{x:\overline{n}|} - {}_{t}{}^{s}V_{x:\overline{n}|} - {}_{t+2}{}^{s}V_{x:\overline{n}|}) . \end{aligned} \tag{20}$$

Correspondingly if $h > 1 - k/m - r$ formula (16) becomes

$$\begin{aligned} {}_{t+k/m+r+h}{}^{s}V_{x:\overline{n}|} = {}& {}_{t+1\ 1/2}{}^{s}V_{x:\overline{n}|} + (1-h)\,({}_{t+1}{}^{s}V_{x:\overline{n}|} + {}_{s}P_{x:\overline{n}|} - {}_{t+2}{}^{s}V_{x:\overline{n}|}) \\ & - (1-h)\left(\frac{1}{2} - \frac{k}{m} - r\right)(2\,{}_{t+1}{}^{s}V_{x:\overline{n}|} - {}_{t}{}^{s}V_{x:\overline{n}|} - {}_{t+2}{}^{s}V_{x:\overline{n}|}) . \end{aligned} \tag{21}$$

It should be noted that both formulas (20) and (21) involve three terminal reserves.

Third Suggested Procedure

It could be assumed that the increase in the reserve over the calendar year, with the exception of the receipt of the valuation premium, is received uniformly over the year, that is

$${}_{t+1\ 1/2}{}^{s}V_{x:\overline{n}|} - {}_{t+1/2}{}^{s}V_{x:\overline{n}|} - {}_{s}P_{x:\overline{n}|}$$

is received uniformly over the year.

In other words, for the case of a policy prior to the policy anniversary

$${}_{t+k/m+r+h}{}^{s}V_{x:\overline{n}|} = {}_{t+1/2}{}^{s}V_{x:\overline{n}|} + h\,[\tfrac{1}{2}\,({}_{t+2}{}^{s}V_{x:\overline{n}|} - {}_{t}{}^{s}V_{x:\overline{n}|}) - {}_{s}P_{x:\overline{n}|}] . \tag{22}$$

Comparing this with formula (14) it will be noted that the two procedures give equal results if

$$\tfrac{1}{2}\,({}_{t+2}{}^{s}V_{x:\overline{n}|} - {}_{t}{}^{s}V_{x:\overline{n}|}) = {}_{t+1}{}^{s}V_{x:\overline{n}|} - {}_{t}{}^{s}V_{x:\overline{n}|}$$

or

$${}_{t+2}{}^{s}V_{x:\overline{n}|} - {}_{t+1}{}^{s}V_{x:\overline{n}|} = {}_{t+1}{}^{s}V_{x:\overline{n}|} - {}_{t}{}^{s}V_{x:\overline{n}|}\,.$$

This assumes that reserves increase in arithmetical progression. For the bulk of policies this is not so, and formula (22) will generally give a higher interim reserve than formula (14).

If the policy anniversary has been passed, formula (22) must be increased by the valuation premium and becomes

$${}_{t+k/m+r+h}{}^{s}V_{x:\overline{n}|} = {}_{t+1/2}{}^{s}V_{x:\overline{n}|} + \tfrac{1}{2}h\,({}_{t+2}{}^{s}V_{x:\overline{n}|} - {}_{t}{}^{s}V_{x:\overline{n}|}) + (1-h)\,{}_{s}P_{x:\overline{n}|}\,. \quad (23)$$

Formula (16) can be put in the form

$${}_{t+k/m+r+h}{}^{s}V_{x:\overline{n}|} = {}_{t+1/2}{}^{s}V_{x:\overline{n}|} + \tfrac{1}{2}h\,({}_{t+2}{}^{s}V_{x:\overline{n}|} - {}_{t}{}^{s}V_{x:\overline{n}|}) + (1-h)\,{}_{s}P_{x:\overline{n}|} + \tfrac{1}{2}\,(1-h)\,(2\,{}_{t+1}{}^{s}V_{x:\overline{n}|} - {}_{t}{}^{s}V_{x:\overline{n}|} - {}_{t+2}{}^{s}V_{x:\overline{n}|})\,.$$

Thus, after the policy anniversary this procedure generally gives a higher interim reserve than formula (16) for the bulk of policies.

Since for premium paying life and endowment plans the reserve curve is normally concave upwards it would appear that formulas (14) and (16) would follow the development of the reserve more accurately and formulas (22) and (23) would tend to overestimate reserves at interim durations.

System Routines for First Procedure

Records could be set up with two accounts, one for ${}_{t+1/2}{}^{s}V_{x:\overline{n}|}$ and the other ${}_{t+1\,1/2}{}^{s}V_{x:\overline{n}|}$. As each policy anniversary is reached and reserve calculations made, the first account is correspondingly decreased and the second account is correspondingly increased. The accounts would be adjusted for nonscheduled transactions at interim statement dates. Hence, the sum of the two accounts at any point of time would give the interim reserve figure. If such a procedure produces satisfactory interim reserve figures it would be the simplest to carry out in practice.

System Routines for Second Procedure

If interim reserves are required at each month end, in addition to the accounts outlined in the first procedure above it will be necessary to keep 12 accounts by monthly anniversary of the values of ${}_{t}{}^{s}V_{x:\overline{n}|} + {}_{s}P_{x:\overline{n}|} - {}_{t+1}{}^{s}V_{x:\overline{n}|}$ for policies that have not reached the policy anniversary and the values of ${}_{t+1}{}^{s}V_{x:\overline{n}|} + {}_{s}P_{x:\overline{n}|} - {}_{t+2}{}^{s}V_{x:\overline{n}|}$ for policies that have passed the anniversary, in accordance with formulas (14) and (16).

During each month it will be necessary to develop the value of the latter expression for all policies with anniversaries in that month. With this information available the reserve figures are readily determined by application of formulas (14) and (16) provided the various accounts are suitably adjusted for nonscheduled transactions.

If it is decided to make allowance for the refinements contained in formulas (20) and (21) the values of ${}_{t+\frac{1}{2}}^{s}V_{x:\overline{n}|}$ and ${}_{t}^{s}V_{x:\overline{n}|}$ are required in addition to the terminal reserves included in the two expressions above. These would normally not be available. A sufficiently accurate approximation could no doubt be made by expressing the missing terminal reserve as a percentage of the terminal reserves available, which percentage would be determined by special investigation. These results should be sufficiently accurate since the whole correction itself is small in value.

To apply these refinements requires the initial determination of the correction for each policy at the beginning of the calendar year. If interim reserves are required at each month end, accounts could be kept for each month and the corresponding corrections initially calculated for the subsequent 12 months. These accounts would have to be adjusted for all nonscheduled transactions.

However, it would probably not be considered worth while to go to such extensive work for this small correction. By an analysis of previous years' results, suitable percentages could be developed for each month end which when applied to the reserves given by formulas (14) and (16) would no doubt produce satisfactory results.

System Routines for Third Procedure

Carrying out of this procedure would require that early in the year the next medial reserve on all policies be calculated. From this information the interim reserves for the required durations could be calculated initially and placed in separate accounts. If these accounts are adjusted for all nonscheduled transactions, interim reserve figures would always be available at the appropriate times.

Suggested electronic methods have been developed in detail for the basic reserve benefit only. Corresponding formulas and procedures can be developed along parallel lines for the reserve for nondeduction of deferred fractional premiums and for reserve for excess of valuation net premiums over corresponding gross premiums.

The above are certain suggestions which could be used in order to estimate the reserve liabilities at various interim durations when it is assumed that a medial reserve will be held at the year end. Many others can be

developed. With some years' testing of actual results the actuary should be able to arrive at simple procedures by which the reserve liability may be estimated to almost any degree of accuracy at selected interim durations.

NUMERICAL EXAMPLES OF APPLICATION OF RESERVE FORMULAS

In order to test some of the formulas developed, a representative sample of business was obtained and analyzed. It was divided into three main sections subsequently referred to as A, B and C.

A is a mature block of business which has been closed for 25 years.

B is a fairly mature block of business which is still open.

C is a fairly recent block of business which is still open.

Three terminal reserves designated as T_0, T_1 and T_2 and the two intermediate medial reserves designated as $T_{1/2}$ and $T_{1\,1/2}$ were calculated with the results shown in Table 1.

Since the date of issue shown on the detail cards in the samples included only month and year, it will be assumed in all subsequent calculations that all business for any one month was issued in the middle of that month.

It is interesting to note the variations in the distribution of reserves by month for each block of business. This is shown in Table 2.

Formula (2) for reserves, which makes a correct interpolation between the initial and terminal reserves, was used to calculate the corresponding values of the reserves to be held at two successive year ends. These are compared with the medial reserves at the corresponding points of time in Table 3.

By similar procedures the reserves may easily be determined at any other duration.

This table illustrates the error and change in error resulting from assuming medial reserves at the end of the year.

In order to test the various formulas based on the assumption that medial reserves are held at the year end, the reserves at the middle of the central calendar year have been determined. The formulas outlined in the first, second and third suggested procedures have been used and the results compared with the reserves determined by the application of formula (2) which, it should be remembered, does not produce medial reserves at the year end. The comparison is shown in Table 4.

It should be noted that the various procedures give results which are extremely close even though the actual distribution of the business over the year is by no means uniform. The numerical values derived from the sample confirm those cases where theory would indicate that certain procedures would produce higher values than others.

TABLE 1

Block of Business	Amount of Insurance	Gross Premium	Valuation Premium	T_0	$T_{1/2}$	T_1	$T_{1\ 1/2}$	T_2
A.....	4,063,052	42,558	34,047	2,609,876	2,655,743	2,666,806	2,713,151	2,724,688
B.....	95,317,251	2,827,011	2,401,573	16,218,401	18,592,222	18,557,591	20,960,493	20,960,992
C.....	19,184,420	267,351	230,465	2,300,082	2,511,860	2,493,035	2,705,083	2,686,390
Totals....	118,564,723	3,136,920	2,666,085	21,128,359	23,759,825	23,717,432	26,378,727	26,372,070

ANALYSIS OF INCREASE IN RESERVES

When the reserves are known accurately at any interim duration, it is a simple procedure to complete the analysis of increase in reserves.

It is necessary to analyze the various nonscheduled transactions occurring to determine the reserves released by death and other terminations net. In addition, it is necessary to develop figures for the annual valuation premiums for those policies which pass their anniversaries. This can be accomplished if reserves on policies are calculated on their monthly anniversary, at which time the valuation premiums would be credited to special accounts, probably by valuation basis. These accounts would have

TABLE 2

DISTRIBUTION OF RESERVES (T_1) BY MONTH

MONTH	BLOCK OF BUSINESS						TOTAL	
	A		B		C			
	Amount	%	Amount	%	Amount	%	Amount	%
1.....	184,695	6.9	1,597,131	8.6	199,004	8.0	1,980,830	8.4
2.....	231,833	8.7	1,509,179	8.1	227,678	9.1	1,968,690	8.3
3.....	261,361	9.8	1,537,200	8.3	155,274	6.2	1,953,835	8.2
4.....	235,577	8.8	1,426,931	7.7	212,810	8.6	1,875,318	7.9
5.....	214,191	8.0	1,823,739	9.8	169,594	6.8	2,207,524	9.3
6.....	284,876	10.7	1,623,741	8.8	220,608	8.9	2,129,225	9.0
7.....	210,678	7.9	1,182,079	6.4	216,180	8.7	1,608,937	6.8
8.....	224,109	8.4	1,058,281	5.7	153,891	6.2	1,436,281	6.1
9.....	215,732	8.1	1,664,148	9.0	237,689	9.5	2,117,569	8.9
10.....	178,031	6.7	1,469,432	7.9	182,420	7.3	1,829,883	7.7
11.....	209,484	7.9	1,631,763	8.8	337,443	13.5	2,178,690	9.2
12.....	216,239	8.1	2,033,967	10.9	180,444	7.2	2,430,650	10.2
Totals..	2,666,806	100.0	18,557,591	100.0	2,493,035	100.0	23,717,432	100.0

TABLE 3

COMPARISON OF INTERPOLATED RESERVES WITH MEDIAL RESERVES

BLOCK OF BUSINESS	FIRST YEAR-END RESERVE			SECOND YEAR-END RESERVE		
	Interpolated	Medial	Difference	Interpolated	Medial	Difference
A.....	2,656,235	2,655,743	492	2,716,895	2,713,151	3,744
B.....	18,586,781	18,592,222	−5,441	20,957,012	20,960,493	−3,481
C.....	2,515,450	2,511,860	3,590	2,708,686	2,705,083	3,603
Totals...	23,758,466	23,759,825	−1,359	26,382,593	26,378,727	3,866

to be appropriately adjusted for nonscheduled transactions during the year. With this basic information the cost of insurance and the interest required to maintain reserves could be calculated by the usual formulas and the analysis of increase in reserves completed.

Normally, there will be some type of reserve check carried out from year to year to ensure accuracy and consistency. If each successive reserve is calculated independently, it may be considered worth while to calculate the cost of insurance and the interest required to maintain reserves to carry out such a check. If these were accumulated in appropriate accounts and properly adjusted for nonscheduled transactions, it would be possible

TABLE 4

RESERVES AT CENTRAL POINT OF CALENDAR YEAR BY VARIOUS FORMULAS

BLOCK OF BUSINESS	RESERVE FORMULA OR PROCEDURE				
	Procedure 1	Procedure 2		Procedure 3	Formula (2)
	$T_{1/2}$ to $T_{1\,1/2}$	Formulas (14) and (16)	Formulas (20) and (21)	Formulas (22) and (23)	
A.....	2,687,173	2,684,289	2,684,452	2,684,911	2,685,488
B.....	19,773,901	19,765,860	19,775,181	19,785,372	19,771,701
C.....	2,596,781	2,592,594	2,592,653	2,592,764	2,596,355
Totals.....	25,057,855	25,042,743	25,052,286	25,063,047	25,053,544

to complete all items of the analysis of increase in reserves and use this analysis as a check on the over-all accuracy of the reserve routines.

PROVISION FOR POLICYHOLDERS' DIVIDENDS PAYABLE IN THE FOLLOWING CALENDAR YEAR

Several approaches could be used to determine this item with an electronic system.

During the operating calendar year when dividends are normally calculated, an additional dividend account could be set up and the dividend payable in the subsequent calendar year calculated in addition to the current dividend. When this account is suitably adjusted for nonscheduled transactions and the actual payment of dividends, it would give a first approximation of the dividend liability for the next 12-month period.

Another method of handling the provision for dividends would be to process the entire file of policies at some point of time in the calendar year, calculating an average dividend per thousand on the business in force at

that time. This average rate per thousand could then be applied to the business in force at any particular interim duration.

In either of the above suggested procedures, if there is a revision in the dividend scales for the next dividend declaration period, it will be necessary to extract from the policy file the affected policies and adjust figures accordingly. In the first procedure described above, this could be done on both the old scale and the new scale, thus calculating the required liability for the next 12 months' dividends and at the same time checking that the proposed dividend scale will give effect to the changes in dividends estimated in setting the scale.

ACCRUED ITEMS

Accrued interest on assets other than those specifically related to policyholders is probably the most significant accrued item as far as amount is concerned. However, this item can be developed from a relatively small file of assets. Usually at each interim statement date the asset items will be processed to make various adjustments in their book values and at that time the accrued items could be calculated accurately for that particular interim statement.

Similarly, interest on policyholders' accrued items such as policy loans and dividends on deposit can be obtained accurately at any interim duration. At the beginning of the year it is assumed that the total capitalized amounts (which may or may not include the capitalization of interest at the policy anniversary) and the total accrued interest due at the following year end are available. During the year these amounts are adjusted for all transactions.

At each interim statement date the procedure would be to deduct from the accrued figure the amount derived by taking the interest rate applied to the capital for the period from the interim statement date to the year end.

PREMIUMS AND COMMISSIONS

Life Insurance Premiums Uncollected

In the same manner as for the deferred premium asset described earlier, it is possible to determine the asset item, Premiums Deferred and Uncollected, at the same time as the file is being processed for billing and payment of premiums, provided the information pertaining to billing premium, valuation premium and mode of premium payment is available on the one file.

A parallel operation between setting up the actual gross premiums outstanding on each policy and the corresponding valuation premiums outstanding can be developed. As noted earlier, it was assumed in the third method of establishing deferred premiums that the total deferreds are

decreased every time a premium is billed. In a like manner the total net premiums uncollected at the premium due date would be adjusted. Thus, from the total deferreds and net outstandings the required asset will be available at any interim statement date.

Cost of Collection of Premiums Deferred and Uncollected Loading Thereon in Excess of Total

Many companies will not be carrying any liability for this item because the excess loadings on the major portion of their policies will offset the deficiencies on others. However, some companies, particularly those writing nonparticipating business, will require data for this item. If such an item is required, it can be developed along the following lines. Assuming the gross premium, valuation premium, rates of commission and mode of premium payment are available, each time a policy anniversary is reached an amount would be determined which is the gross premium multiplied by the premium frequency, less the valuation premium, less percentage items such as commission and premium taxes, less an assumed administrative cost of collection. Such amounts would be accumulated in a special account. As each gross premium is collected, the account would be appropriately adjusted, involving for policies with premiums other than annual the appropriate fractional amount to be deducted as such premium is paid.

At any statement date, if there is a debit balance shown in the account, no amount will be entered in the statement. If it is a credit balance, the account figure can be taken directly into the statement.

MISCELLANEOUS

Net Adjustment in Assets and Liabilities Due to Foreign Exchange Rates

Since a consolidated file will be used for policyholder servicing work, amounts will of necessity be shown in currency of the policy. If the corresponding accounting procedures are set up in such currencies, it is a simple procedure to determine any statement item for different foreign currencies, using actual exchange rates as well as book rates. Thus, this item can be very readily determined at any interim statement date.

Taxes, Licenses and Fees Due and Accrued

The only item specifically considered is premium taxes. As premiums are paid, the appropriate tax rate is applied and the resultant tax is credited to a tax due and accrued account. As the taxes are actually paid, the account is debited and the net at any statement date will represent the liability for the premium taxes due.

CONCLUSION

This paper has outlined how the derivation of many balance sheet items can be improved both at the year end and at interim statement dates by a company which utilizes electronic data processing equipment. Items have been discussed individually with no attempt to correlate the individual procedures into an over-all system. However, many of the items must be combined with required operating procedures to obtain the desired interim statements with little extra work.

It would appear that, with the advent of electronic systems and careful planning of the over-all approach, it is now quite feasible to have as accurate accounting statements at interim periods as at the year end.

DISCUSSION OF PRECEDING PAPER

RALPH E. EDWARDS:

There is a striking disparity between the premise underlying this paper and what our industry says in public about its own operations. I refer to statements such as this one: "Measure of exact financial progress over such relatively short periods as a year is impossible."[1] If preparing financial statements at less than yearly intervals is prevalent among our companies, the public may find it hard to understand how we can say that the results are accurate enough to guide our operating policies but not accurate enough to be used as an income tax basis.

I have sought, without success, to find in actuarial literature an answer to this seeming contradiction. It is well known that exact valuation of policy reserves at five year intervals is considered adequate in England. Proceedings of actuarial meetings in this country and Canada contain descriptions of systems developed for individual companies to produce statements at less than yearly intervals, but these fail to analyze the significance of the figures so determined.

If interim statement data are, contrary to their appearance, neither valid nor meaningful, then we actuaries in developing these figures appear to be abdicating our responsibility. We are acting as mere technicians and not as competent professional advisors. On the other hand, if short-term statistics are meaningful, then we actuaries cannot ethically support our companies in taking the opposite view so as to justify a preferential federal income tax method.

Many industries sell their product and are done with it. Such is not the case with us. The first year of a new policy is a financial loss to the company, and the more new policies we sell the greater the strain on our surplus. It is in the renewal years that the expenses for the policy become low enough to make the business profitable. Under these circumstances it is obvious that our substantial fluctuations in the amount of business sold make earnings statements meaningless except over long periods of time.

[1] Federal Income Taxation of Life Insurance Companies: A memorandum of the Joint Committee of the American Life Convention, Life Insurance Association of America, and the Life Insurers Conference, 1957. See also Ways and Means Committee Report on H. R. 7201, July 11, 1958: "The annual accounting period, which is applied to business in general, is not a suitable measure of the income from issuing life insurance contracts, which by their nature may span many decades."

In a similar vein, the company will have allocated to a policy more in assets than it will pay out if the policy surrenders, yet the difference is less than the future margins in the premiums, so that it is against the company's long-range interest for the policy to be surrendered. Thus we live in a topsy-turvy world of large losses on new policies and low gains from surrender at a time when general business conditions are at their best, and low losses on new policies and high gains from surrender at a time when business is not so good. Our books of account do not show that in the first circumstance we have gained future profits and in the second circumstance lost them.

Equally as important, but not as striking, in this matter of long-term versus short-term financial figures, are the effects of variation in the rates of mortality, interest and expense. Particularly in the case of mortality, seasonal changes have a substantial effect on the earnings picture for periods of less than a year.

The practical situation just described convinces me that interim statements are not justified. However, I make an exception in the case of a brand new, rapidly expanding company lacking the margins on renewal business to support its losses on newly written policies. Such an insurer is concerned with remaining legally solvent and may require financial statements frequently so as to know how much new business it can afford to write.

Another situation involving financial statements at intervals of less than a year is in certain states which require such information from their domestic insurers. Such a requirement seems unjustified to me, but so long as it exists there is not much the companies in those states can do but comply.

I suspect that a few larger companies may feel that additions to surplus are very much in the public eye. They fear that any unusual increase may excite pressure from labor union sources, or criticism from persons having an uninformed viewpoint. For them it is possible to reconcile the use of interim statements to control the amount of year-end surplus addition, and at the same time to recognize that such surplus gain is not a true measure of the year's profits.

It can be argued that dividend equity justifies interim statements, so that the latest possible information is at hand when the adjustments in the former scale are made. I disagree. If interim statements are suitable as a base for dividends, then annual statements would surely seem suitable as a basis for tax.

Perhaps the strongest demand for interim statements arises from officers and directors. They know of the more frequent reports in other

industries and in some of our own companies. From time to time I have discussed this problem with other actuaries, especially with those who furnished interim statements to company directors. In only a few instances did the actuary himself feel that the short-term changes were significant. At the other extreme, a few seemed to feel that the actuary had no responsibility in the matter, but merely furnished what was wanted without concern as to whether it was meaningless or misleading. The rest seemed to be genuinely troubled. They did not like their own part in the matter. They were concerned about directors not knowing the business well enough to realize that long-term, not short-term, trends are important. They worried about directors wasting time on this and neglecting really important matters. They feared decisions might be made which were justified by apparent results but not by true facts.

This paper is not designed for the brand new, rapidly growing company. This discussion will not change state laws requiring interim data. The present furor over income tax and the emphasis on long-term results will not convince officers and boards of directors that interim statements should be dispensed with. We shall continue to live with this problem no matter how we feel about it. Those who face the technical task of preparing interim statements will have the fine assistance of this most excellent paper.

I hope any negative tone in my remarks will not be considered as detracting from the merits of what Messrs. Davidson and Birkenshaw have prepared for us. I believe we should be grateful to them for their fine work and their able presentation.

J. ROSS GRAY:

The preparation of interim accounting statements has required approximations to the rate of growth of the reserves, or alternatively approximations to the tabular expected mortality and other items which contribute to the growth of the reserves. These have been none too satisfactory. The authors suggest methods by which electronic machines may be used to produce calculations of the amount of the reserves, which calculations are accurate within the limits described by them for their various methods. This is a big step forward.

I should like to introduce this discussion by making a suggestion that the United States form of annual statement, the Convention Blank, should not be allowed to restrict our thinking. The Convention Blank is based on the assumption that companies will calculate their reserve liabilities by the use of medial reserves, assuming annual premiums payable, and accordingly require credit as an asset for the net deferred

premiums to offset the overstatement of the reserves. The laws of the states which I have examined do not appear to require the use of medial reserves. They permit the superintendent, commissioner, etc., to use group methods and approximate averages for fractions of a year, but they do not appear to prohibit the calculation of accurate reserves, or reserves determined by other approximations than the usual medial approximation.

The Canadian form of annual statement recognizes that the net deferred premiums are a correction to the medial reserve liability and require that the net premiums be deducted from the reserves.

If we examine the authors' formulas from this point of view, we obtain some interesting results.

Combining formulas (2) and (4) we obtain

$$ {}_t^s\mathrm{V}_{x:\overline{n}|} + \left(\frac{k}{m} + r\right)\left({}_{t+1}^{s}\mathrm{V}_{x:\overline{n}|} - {}_t^s\mathrm{V}_{x:\overline{n}|}\right) + \left(\frac{1}{m} - r\right) \times {}_s\mathrm{P}_{x:\overline{n}|}, $$

which I shall call formula $2(a)$.

Combining formulas (3) and (5) we obtain

$$ {}_s\mathrm{P}_{x:\overline{n}|}^{(m)} \times \frac{m-1}{2m} \times \left[{}_t\mathrm{V}_{x:\overline{s}|}^{1} + \left(\frac{k}{m} + r\right)\left({}_{t+1}\mathrm{V}_{x:\overline{s}|}^{1} - {}_t\mathrm{V}_{x:\overline{s}|}^{1}\right) + \left(\frac{1}{m} - r\right)\left(\mathrm{P}_{x:\overline{s}|}^{1} + d\right) \right], $$

which I shall call formula $3(a)$. Note that the portion within the square brackets is in exactly the same form as formula $2(a)$.

If we take formula $2(a)$, and apply it to annual premiums, that is, $k = 0, m = 1$, and assume an even distribution of business so that $r = \frac{1}{2}$, the formula becomes

$$ {}_t^s\mathrm{V}_{x:\overline{n}|} + \tfrac{1}{2} \times \left({}_{t+1}^{s}\mathrm{V}_{x:\overline{n}|} - {}_t^s\mathrm{V}_{x:\overline{n}|}\right) + {}_s\mathrm{P}_{x:\overline{n}|} \times \tfrac{1}{2} = \frac{{}_t^s\mathrm{V}_{x:\overline{n}|} + {}_{t+1}^{s}\mathrm{V}_{x:\overline{n}|} + {}_s\mathrm{P}_{x:\overline{n}|}}{2}, $$

which is the usual medial reserve formula, but it may be more convenient to leave the result in the form

$$ \tfrac{1}{2} \times {}_t^s\mathrm{V}_{x:\overline{n}|} + \tfrac{1}{2} \times {}_{t+1}^{s}\mathrm{V}_{x:\overline{n}|} + \tfrac{1}{2} \times {}_s\mathrm{P}_{x:\overline{n}|}. $$

If we take formula $2(a)$, and apply it to half-yearly premiums, that is, $m = 2$, and assume an even distribution of business so that $k = 1$ on one-half the business, $k = 0$ on the other half of the business, and $r = \frac{1}{4}$, we obtain

$$ \tfrac{1}{2} \times {}_t^s\mathrm{V}_{x:\overline{n}|} + \tfrac{1}{2} \times {}_{t+1}^{s}\mathrm{V}_{x:\overline{n}|} + \tfrac{1}{4} \times {}_s\mathrm{P}_{x:\overline{n}|}. $$

Similarly, for quarterly premiums, assuming an even distribution of business, we obtain

$$\tfrac{1}{2} \times {}_t^s V_{x:\overline{n}|} + \tfrac{1}{2} \times {}_{t+1}^s V_{x:\overline{n}|} + \tfrac{1}{8} \times {}_s P_{x:\overline{n}|} .$$

Similarly, for monthly premiums, assuming an even distribution of business, we obtain

$$\tfrac{1}{2} \times {}_t^s V_{x:\overline{n}|} + \tfrac{1}{2} \times {}_{t+1}^s V_{x:\overline{n}|} + \tfrac{1}{24} \times {}_s P_{x:\overline{n}|} .$$

Similarly, for formula 3(*a*), we obtain for the four different methods of premium payment

Annual premiums

nil

Half-yearly

$${}_s P_{x:\overline{n}|}^{(2)} \times \tfrac{1}{4} \times \left[\tfrac{1}{2} \times {}_t V_{x:\overline{s}|}^{1} + \tfrac{1}{2} \times {}_{t+1} V_{x:\overline{s}|}^{1} + \tfrac{1}{4} \times (P_{x:\overline{s}|}^{1} + d) \right]$$

Quarterly

$${}_s P_{x:\overline{n}|}^{(4)} \times \tfrac{3}{8} \times \left[\tfrac{1}{2} \times {}_t V_{x:\overline{s}|}^{1} + \tfrac{1}{2} \times {}_{t+1} V_{x:\overline{s}|}^{1} + \tfrac{1}{8} \times (P_{x:\overline{s}|}^{1} + d) \right]$$

Monthly

$${}_s P_{x:\overline{n}|}^{(12)} \times \tfrac{11}{24} \times \left[\tfrac{1}{2} \times {}_t V_{x:\overline{s}|}^{1} + \tfrac{1}{2} \times {}_{t+1} V_{x:\overline{s}|}^{1} + \tfrac{1}{24} \times (P_{x:\overline{s}|}^{1} + d) \right]$$

Returning to formula 2(*a*), and assuming annual premiums, the result is exactly the same, assuming an even distribution of business, if we hold

$${}_t^s V_{x:\overline{n}|}$$

with respect to business with anniversaries in July to December,

$${}_{t+1}^s V_{x:\overline{n}|}$$

with respect to business with anniversaries in January to June, and

$$\tfrac{1}{2} \times {}_s P_{x:\overline{n}|}$$

with respect to all business.

If business is not evenly distributed the result is probably closer to the true reserve than is the result given by the medial reserve calculation, in that recognition is taken of the division of business between the two half-years. (The detailed formulas are omitted from this discussion.)

Calculation of the reserves for interim statements should not be too difficult on this basis. To move from the calculation at the end of Decem-

ber to the calculation at the end of January, we make adjustments for new business and cancellations, and then substitute the reserve ${}_{t+1}^{s}V_{x:\overline{n}|}$ for the reserve ${}_{t}^{s}V_{x:\overline{n}|}$ with respect to policies with anniversaries in July; and so on, throughout the year, for subsequent interim statements.

It will, of course, be necessary to know at the end of each month the volume of premiums payable annually, half-yearly, quarterly and monthly in order to hold half of one net premium on each of these methods of payment, and in order to calculate the reserve for nondeduction of fractional premiums.

Business may not be distributed uniformly throughout the year, and the monthly variations in volume of business may be large. In such a case the increases in reserves caused by switching one month's business from the terminal reserve at the last anniversary to the terminal reserve at the next anniversary may vary too much from month to month to be suitable for use for monthly interim statements. In addition, there is a further problem, most easily demonstrated with respect to annual premiums. These premiums would be received in amounts which would vary from month to month, and would bear no relationship to the increase in reserves in the month, in that the reserves are changed six months in advance of the anniversary.

Let us return to formula 2(*a*) and write it with respect to a monthly premium policy, assuming that $r = \frac{1}{2}$ month:

$$ {}_{t+(k+1/2)/12}^{s}V_{x:\overline{n}|}^{(12)} = {}_{t}^{s}V_{x:\overline{n}|} + \left(\frac{k}{12} + \frac{1}{24}\right)\left({}_{t+1}^{s}V_{x:\overline{n}|} - {}_{t}^{s}V_{x:\overline{n}|}\right) + \tfrac{1}{24} \times {}_{s}P_{x:\overline{n}|}. \qquad (2b) $$

If we change our notation so that in all cases k represents the number of complete months since the last policy anniversary, and g represents the number of complete months since the last renewal date, the reserve for a policy with quarterly premiums is equal to that for one with monthly premiums, plus

$$\left(\frac{3}{12} - \frac{g+1}{12}\right) \times {}_{s}P_{x:\overline{n}|};$$

similarly, for a policy with half-yearly premiums, we add

$$\left(\frac{6}{12} - \frac{g+1}{12}\right) \times {}_{s}P_{x:\overline{n}|};$$

similarly, for a policy with annual premiums, we add

$$\left(\frac{12}{12} - \frac{g+1}{12}\right) \times {}_{s}P_{x:\overline{n}|} = \left(1 - \frac{k+1}{12}\right) \times {}_{s}P_{x:\overline{n}|}$$

We might note in passing that, given an even distribution of business over the year, any of these brings us back to the basic

$$\tfrac{1}{2} \times {}_t^s V_{x:\overline{n}|} + \tfrac{1}{2} \times {}_{t+1}^s V_{x:\overline{n}|} + \tfrac{1}{2} \times (\tfrac{1}{12} \times {}_s P_{x:\overline{n}|} \text{ or } \tfrac{1}{4} \times {}_s P_{x:\overline{n}|}$$

$$\text{or } \tfrac{1}{2} \times {}_s P_{x:\overline{n}|} \text{ or } {}_s P_{x:\overline{n}|}) .$$

The practical handling of formula (*2b*) will cause considerable subdivision of our material. For the issues of each month we shall set up totals of ${}_t^s V_{x:\overline{n}|}$ and ${}_{t+1}^s V_{x:\overline{n}|}$, and ${}_s P_{x:\overline{n}|}$. The figure for ${}_s P_{x:\overline{n}|}$ will require subdivision into policies whose premiums are payable monthly, quarterly, half-yearly and annually. All of these figures will require adjustment for nonscheduled transactions.

The reserve at the end of any month, including the end of the calendar year, will be found by taking the appropriate fractions of ${}_t^s V_{x:\overline{n}|}$ and ${}_{t+1}^s V_{x:\overline{n}|}$ according to the month involved, and adding $\frac{1}{24} \times {}_s P_{x:\overline{n}|}$ as for policies with monthly premiums. To this total will be added the appropriate further amounts of net premiums for policies with quarterly, half-yearly and yearly premiums.

R. F. RICHARDSON:

The emphasis in this paper on the use of electronic computers as the key to the production of interim statements calls for some comment. In our own operations as in those of many of the smaller companies, the use of such computers is definitely out of the question for the present and immediate future. I would suggest, however, that interim statements can be and are produced without these modern marvels and without the excessive work usually given as the reason for not preparing them.

In our own Company accurate interim statements are provided based on the use of a carefully integrated punch card system using no equipment more elaborate than the IBM 402 tabulator. No form of calculating punch is used.

Briefly, the main items are handled as follows:

The Actuarial Reserve.—Use of a Karup valuation method surmounts the equipment problem. It reduces the work of valuing unscheduled movements and eliminates the necessity of a calculating punch or computer, while taking full advantage of the adding features of the 402.

The office year of birth for valuation purposes is chosen so that policies issued in the first half of the year are assumed issued on January 1st and those in the second half of the year are assumed issued on December 31st. This takes account of policy distribution and is demonstrably more accu-

rate than the usual assumption of all policies being issued July 1st. It also makes possible the use of terminal rather than medial reserves throughout.

In-force at the beginning of the year is valued as at the end of the year. Unscheduled movement is summarized and valued monthly. At interim dates the reserve is adjusted by interest and death strain interpolations and by the actual net premium distribution. (Net premiums by month of issue are maintained.)

Deferred Premiums.—These are handled separately. The deferred premium file is a set of summary cards giving premium totals by frequency, month of issue, and premium due dates. Unscheduled movements are summarized monthly from our regular statistical file.

Provisions for Dividends Payable in Following Year.—Dividends are calculated a year in advance and appear in the dividend punch card file. This item is thus available from the cards and also from the dividend control totals which are adjusted monthly.

Accrued and Outstanding Items.—All or nearly all of our accounting functions are on punched cards. In some cases these are handled on a revenue basis daily (*e.g.*, claims), in others the necessary accrued or outstanding items are available from outstanding punch card files (premiums), and in still others such items are calculated from tabulation lists (investments).

We have been producing interim statements as outlined for several years, with quarterly revenue statements available for Board meetings by the 8th of the following month. We also produce most of the Canadian Insurance Blank exhibits for internal use. Our experience does not substantiate the long-standing claim that the excessive work and time required for interim statements justifies a Company in foregoing such timely information in analyzing emergence of surplus. No inordinate amount of work is required. It has resulted in streamlining and speeding up the production of the actual annual statement, which has now become part of a normal accounting routine rather than a special year-end job. It has improved general accounting efficiency throughout the year, and has greatly improved the internal cohesion between the record-keeping functions of various departments. (This internal integration may well prove invaluable in some future switch to magnetic tape and computer operations.)

In conclusion, one comment might be made on an interesting feature of interim statements. Once a normal emergence of surplus has been determined on basic interest, mortality and expense assumptions, the variation from normal on any interim statement can and should be traced. Since the variation from normal in interest should be very small or its cause

easily determinable, and since it is simple to adjust from the actual to a standardized death strain, any remaining deviations from normal emerging surplus generally point to unusual expense items or to accounting or valuation errors. This interim check on variations can be of particular value to any company going through a period of major expansion or other company upheaval.

(AUTHORS' REVIEW OF DISCUSSION)

J. C. DAVIDSON AND J. T. BIRKENSHAW

The authors appreciate very much the valuable comments of those who were kind enough to discuss this paper.

Mr. Edwards raises the fundamental question of the value or otherwise of interim accounting statements. The authors sidestepped this question, fully realizing that surplus as generated by regular annual statements may or may not be a reasonable reflection of the true surplus earned within one year's operations. Recognizing that the surplus earned over one year's operations may or may not be a true reflection of the earnings of that year, this paper provides suggestions as to how the year-end statement surplus figure may be built up at fractional periods during the year using electronic equipment. The fundamental question raised by Mr. Edwards is certainly worthy of an actuarial paper within itself and perhaps Mr. Edwards' provocative discussion will stimulate some actuary to make such a contribution.

Mr. Gray's discussion develops a different and quite unique approach to the determination of the growth of reserves. For certain companies his procedures may be simpler to apply and just as accurate as those developed by the authors.

Mr. Richardson outlines his company procedures for preparing interim statements without the use of electronic equipment. Such procedures could be applied to almost any relatively small company. The authors fully realize that interim statements can be and are developed without the aid of electronic equipment. However, the basic premise of the paper was ways and means of developing interim statements for those companies which are planning to install electronic procedures.