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EXTENDED TERM INSURANCE CONSISTENT WITH FULLY PAID INSURANCE CALCULATED ON A DIFFERENT BASIS

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THE Standard Nonforfeiture Law requires that the mortality table and interest rate provided in the policy for the calculation of cash surrender values and of nonforfeiture paid-up insurance be the same, say Basis A. It also contemplates that, if desired, a more stringent Basis B may be provided for calculating the present value of extended term insurance, using mortality rates up to 130% those of Basis A.

The Problem

With the nonforfeiture paid-up insurance calculated on the same Basis A as the cash surrender values on premium default, there is no problem of consistency of the nonforfeiture paid-up insurance with fully paid insurance.

If, however, extended term insurance is calculated throughout on Basis B, there is inconsistency near the end of the premium period of limited payment life and limited payment endowment plans. This is because such extended insurance does not merge with fully paid insurance as the premium default date approaches the full paid date.

Thus, in (*c*) of Table 1 the limit of the extended insurance (20 Payment Life, Age 35, 130% CSO 2½%) is 28 years 172 days, however little of the premium remains to be paid on default. This compares with 45 years insurance if fully paid. In other words, if, for example, only the last quarterly premium is defaulted, the insurance expires before age 84 at latest, whereas if the premium payments are completed the coverage runs to age 100 (and in practice is paid then as a matured endowment).

Of course, it can be claimed that the nonmerging is to be expected (except perhaps in not so severe a degree)—that the situations are "poles apart," that one is a default and the other a performance of contract. Yet it is difficult to deny the appearance of harshness in these extreme cases. These cases would be expected to show but little of the antiselection which gives rise to higher mortality under extended term insurance in general.

It would seem desirable for the extended insurance calculation to include a process merging into Basis A as the default date approaches the full paid date, thus avoiding the inconsistency.

History

Attention was attracted to the desirability of a distinctive Basis B for extended insurance by the appearance of the American Men Table. See Dr. Hunter's paper in 1919, *TASA* XX, 36, also the discussion of the paper. This need was recognized January 1, 1930 by Section 88 of the New York Insurance Law, authorizing the assumption of higher mortality under extended term insurance up to 130%.

The inconsistency resulting from such a Basis B distinction received considerable discussion when the CSO Table was introduced, but no satisfactory conclusion was reached. There were various proposals, all of which were felt to be impracticable. Reference should be made to the discussion in the American Institute of Actuaries in 1943 as reported in *RAIA* XXXII, 240, also to Mr. November's paper in 1946 in the Actuarial Society, *TASA* XLVII, 33, which includes a summary of the Institute discussion.

Now that a new mortality table is again under active consideration, being X_{17} or some further version, it would be appropriate if the inconsistency problem could be disposed of.

Theory

The difficulty can be overcome if, in determining extended insurance of s years for limited payment plans, the charge with respect to no more than p years is according to Basis B and the remainder of the charge, if any, is according to Basis A, where p is some function so related to the limited payment period r that p vanishes as n, the policy duration on default, approaches r.

A simple function of this character is:

$$p = \text{a multiple of } (r - n) .$$

The method suggested in this paper is to use:

$$p = 2 (r - n) .$$

Suggested Solution

The suggested solution is to define a varying period of years, p, under limited payment life and limited payment endowment plans; and to depart from Basis B in the cases under these plans where the extension is more than p years, making these calculations by a varying blend of Bases B and A.

p is defined as double the period from the date of default to the full paid date. Thus, the full paid date is the midpoint of the period p which commences at the date of default.

The proposed blend is accomplished by calculating the single premium for s years' term insurance upon default at attained age y as follows:

$$A'_{y:\overline{s}|} \qquad \text{for } s \lessgtr p \quad (1)$$

$$A'_{y:\overline{p}|} + \frac{D'_{y+p}}{D'_y} \cdot A_{\substack{1 \\ y+p:\overline{s-p}|}} \qquad \text{for } s > p, \quad (2)$$

where primed symbols are by Basis B, and plain symbols are by Basis A.

The corresponding pure endowment single premiums for maturity at age $y + s$, according as s is within or greater than the value of p, are

$$\frac{D'_{y+s}}{D'_y}$$

and

$$\frac{D'_{y+p}}{D'_y} \cdot \frac{D_{y+s}}{D_{y+p}}.$$

Calculation

The process will be understood by examining Tables 1 and 2, of which section (*b*) shows the proposed calculations. Sections (*a*) and (*c*) show the corresponding calculations on Basis A and Basis B alone, respectively. The factors used in the Tables are the regular nonforfeiture functions from *Actuarial Tables, Volume III, CSC* 2½%, *Basic Values*, pages 84 to 143, and 148 to 177.

The cash surrender values used in column (2) after ten years are equal to the full reserve and before that duration they are representative of actual practice.

The bases of the factors in section (*b*) are:

Cols. (4), (6) —Basis B
(10) —Basis A
(12), if (5) to (10) is blank—Basis B
(12), if (5) to (10) is used—Basis A

Advantages of the Suggested Method

1. No new tables are needed for the calculations.
2. The additional calculations arise in only a fraction of the limited payment plan cases only, *viz.*, those where the extension is longer

than the period p; hence the additional calculations are only a small fraction of the total extended insurance work.

3. The additional calculations are in the form of a natural extension of the regular calculation.

Effect of Blend

The blend comes into operation only in those cases under limited payment life and limited payment endowment plans where the extension is longer than p years. In all other cases under those plans, and in all cases under all other plans, the extension is calculated purely on Basis B.

It is thought that limited payment endowments, being infrequent, are sufficiently illustrated by Table 2.

The effect of the blending for limited payment life plans may be judged from the pattern exhibited in Table 3. It is also shown graphically in the Chart.

Policy Provision

Let us assume that the existing policy provision, which uses Basis A throughout, is:

> The .....(A)..... Mortality Table with interest at .....% per annum shall be used to establish the reserves and net single premiums in this policy. (3)

Then, in applying Basis B to extended term insurance in policies other than on limited payment plans, (3) might become:

> The .....(A)..... Mortality Table with interest at .....% per annum shall be used to establish the reserves and net single premiums in this policy, except that in establishing the reserves and net single premiums for extended term insurance the .....(B)..... Mortality Table shall be used. (4)

In adopting the method proposed in this paper for maintaining consistency with fully paid insurance, which involves limited payment plans only, and assuming that the policy form defines "full paid date," (4) might become:

> The.....(A).....Mortality Table with interest at .....% per annum shall be used to establish the reserves and net single premiums in this policy, except that in establishing the reserves and net single premiums for extended term insurance, to the extent that the calculation involves some or all of the period from the date of default to the full paid date or of an immediately succeeding period of equal duration, the .....(B)..... Mortality Table shall be used. (5)

Comment: Referring back to formula (2), it will be observed that Basis B applies not only to the cost of the first p years' term insurance, but also to the discount factor to be used during that period for the

TABLE 1

$1,000 20 PAYMENT LIFE — AGE 35 AT ISSUE — BASIS A: CSO 2½% — BASIS B: 130% CSO 2½%

DEFAULT END OF YEAR (1)	CASH SURRENDER VALUE (2)	s, p IF LESS (3)	TERM INS. S.P. (4)	IF s IS GREATER THAN p: (2)-(4) (5)	P.E. Factor (6)	(5)×(6) (7)	Att. Age (8)	$s-p$ (9)	Term Ins. S.P. (10)	(7)-(10) IF ANY, OTHERWISE (2)-(4) (11)	DAY FACTOR (12)	EXTENDED TERM: s Yrs. (13)	EXTENDED TERM: Days (14)
(*a*) CSV and Term Extension on Basis A													(No problem of inconsistency)
5.....	$129.85	16	$127.70							$ 2.15	34.112	16	74
10.....	291.43	23	288.26							3.17	22.014	23	70
15.....	461.42	27	461.09							.33	21.817	27	8
18.....	573.51	30	570.33							3.18	32.074	30	102
19.....	612.94	32	609.31							3.63	47.900	32	174
20*.....	653.56	45	653.56							0	.....	45	0
(*b*) CSV on Basis A and Term Extension on Bases B and A as Proposed													(Inconsistency avoided)
5.....	$129.85	13	$125.05							$ 4.80	30.265	13	146
10.....	291.43	19	278.86							12.57	19.902	19	251
15.....	461.42	10	180.46	$280.96	1.61725	$454.38	60	15	$434.11	20.27	13.111	25	266
18.....	573.51	4	81.99	491.52	1.20943	594.46	57	25	584.89	9.57	24.156	29	232
19.....	612.94	2	42.88	570.06	1.09957	626.82	56	29	617.16	9.66	37.321	31	361
20*.....	653.56	0	0	653.56	1.00000	653.56	55	45	653.56	0	.....	45	0
(*c*) CSV on Basis A and Term Extension on Basis B Throughout													(Inconsistency disregarded)
5.....	$129.85	13	$125.05							$ 4.80	30.265	13	146
10.....	291.43	19	278.86							12.57	19.902	19	251
15.....	461.42	22	441.97							19.45	17.919	22	349
18.....	573.51	25	569.08							4.43	23.072	25	103
19.....	612.94	26	606.23							6.71	27.568	26	185
20*.....	653.56	28	649.44							4.12	41.572	28	172

* Actually fully paid but calculations made as if default.

TABLE 2

$1,000 20 PAYMENT 30 YEAR ENDOWMENT — AGE 35 AT ISSUE — BASIS A: CSO 2½% — BASIS B: 130% CSO 2½%

DEFAULT END OF YEAR (1)	CASH SURRENDER VALUE (2)	s, p IF LESS (3)	TERM INS. S.P. (4)	IF s IS GREATER THAN p: (2)-(4) (5)	P.E. Factor (6)	(5)×(6) (7)	Att. Age (8)	$s-p$ (9)	Term Ins. S.P. (10)	(7)-(10) IF ANY, OTHERWISE (2)-(4) (11)	DAY OR P.E. FACTOR (12)	EXTENDED TERM: s Yrs. (13)	Days or P.E. (14)
(*a*) CSV and Term Extension on Basis A											(No problem of inconsistency)		
5	$165.63	19	$160.91							$ 4.72	30.958	19	147
10	350.69	20	239.59							111.10	2.41746	20	$ 269
15	559.55	15	231.16							328.39	2.03229	15	667
18	699.61	12	217.16							482.45	1.81290	12	875
19	749.39	11	210.41							538.98	1.74139	11	939
20*	800.97	10	202.39							598.58	1.67063	10	1000
(*b*) CSV on Basis A and Term Extension on Bases B and A as Proposed											(Inconsistency avoided)		
5	$165.63	16	$162.36							$ 3.27	27.672	16	91
10	350.69	20	297.20	$ 53.49	2.72145	$145.57	65	0	$ 0.00	145.57	1.00000	20	$ 146
15	559.55	10	180.46	379.09	1.61725	613.08	60	5	136.61	476.47	1.32698	15	632
18	699.61	4	81.99	617.62	1.20943	746.97	57	8	181.87	565.10	1.53120	12	865
19	749.39	2	42.88	706.51	1.09957	776.86	56	9	192.94	583.92	1.60058	11	935
20*	800.97	0	0	800.97	1.00000	800.97	55	10	202.39	598.58	1.67063	10	1000
(*c*) CSV on Basis A and Term Extension on Basis B Throughout											(Inconsistency disregarded)		
5	$165.63	16	$162.36							$ 3.27	27.672	16	91
10	350.69	20	297.20							53.49	2.72145	20	$ 146
15	559.55	15	288.28							271.27	2.25349	15	611
18	699.61	12	272.11							427.50	1.98594	12	849
19	749.39	11	264.15							485.24	1.89864	11	921
20*	800.97	10	254.62							546.35	1.81224	10	990

* Actually fully paid, but calculation made as if default.

TABLE 3

(Corresponding to Cols. 13 and 14 of Table 1)

YEARS AND DAYS OF EXTENDED TERM INSURANCE

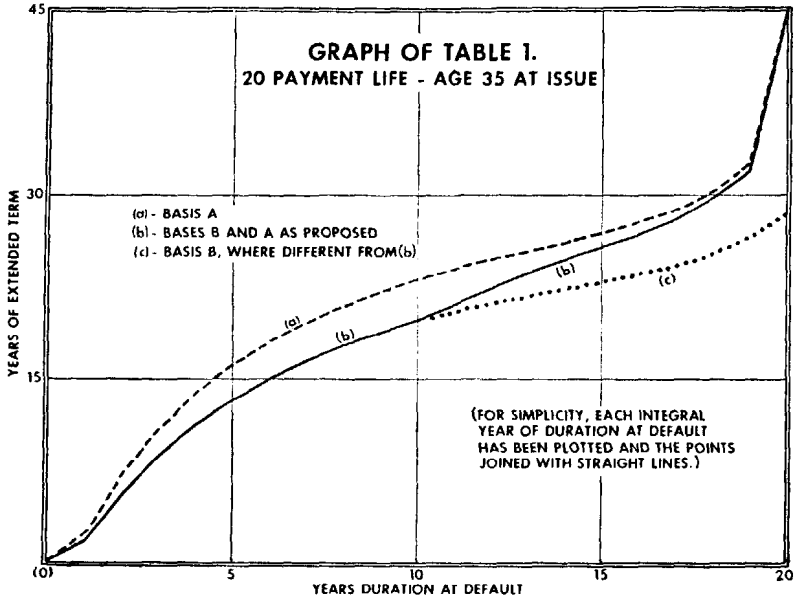
AGE AT ISSUE:	15			35			55		
Year	(a) Y D	(b) Y D	(c) Y D	(a) Y D	(b) Y D	(c) Y D	(a) Y D	(b) Y D	(c) Y D
	10 Payment Life								
1.....	8-265	6-274	6-274	6-239	5- 94	5- 94	2-218	2- 10	2- 10
5.....	42- 50	41-103	37- 01	25-335	24-266	22- 60	11-232	9-171	9-171
7.....	49-124	48-340	43-332	31-175	30-330	27- 65	15- 93	14- 61	12-190
8.....	52-286	52-184	46-350	34-137	33-357	29-237	17-134	16-244	14- 93
9.....	56-358	56-296	50- 79	38- 31	37-307	32-182	20-112	19-326	16-191
10*.....	75- 0	75- 0	54-203	55- 0	55- 0	36-284	35- 0	35- 0	20-316
	20 Payment Life								
1.....	1-215	1- 81	1- 81	2-173	1-336	1-336	1- 47	0-319	0-319
5.....	28- 88	23-235	23-235	16- 74	13-146	13-146	6-103	4-362	4-362
10.....	39-335	38- 10	35- 21	23- 70	19-251	19-251	9-156	7-212	7-212
15.....	45- 70	44-158	39-319	27- 8	25-266	22-349	11-141	9- 49	9- 49
17.....	47-184	46-358	41-224	28-331	28- 26	24-136	12-202	11- 59	10- 01
18.....	49- 30	48-247	42-226	30-102	29-232	25-103	13-183	12-196	10-256
19.....	51-195	51- 81	43-309	32-174	31-361	26-185	15- 49	14-196	11-289
20*.....	65- 0	65- 0	45-202	45- 0	45- 0	28-172	25- 0	25- 0	14-110
	30 Payment Life								
1.....	0- 0	0- 0	0- 0	1- 39	0-312	0-312	0-303	0-234	0-234
5.....	21-276	17-263	17-263	12-220	10- 96	10- 96	5- 71	4- 38	4- 38
10.....	33- 11	28-173	28-173	18-118	15-117	15-117	7-181	5-357	5-357
15.....	36-199	32-187	31-350	20- 69	16-355	16-355	8- 10	6-143	6-143
20.....	38- 79	36- 79	33-161	20-345	17-200	17-200	7-336	6- 92	6- 92
25.....	39-277	38-260	34-140	21-289	19-364	18- 0	7-305	6- 27	6- 27
27.....	40-345	40- 64	34-340	22-249	21-170	18-181	8- 52	6-126	6- 97
28.....	41-355	41-124	35-127	23-192	22-201	18-334	8-214	7-134	6-200
29.....	43-291	43-108	35-333	25- 28	24-132	19-226	9-197	8-289	7- 37
30*.....	55- 0	55- 0	36-284	35- 0	35- 0	20-316	15- 0	15- 0	8-347

* Actually fully paid, but calculation made as if default.

cost of the remaining term insurance. Also, after a part of the p years has elapsed, a similar situation applies to the residue of the p years in calculating the reserve referred to. This has of course been taken into account in the suggested language.

Basis B

From a public relations standpoint, it would seem better for Basis B not to be described in terms of Basis A as, for example, 130%. It



would appear preferable for it to bear a functional name as "Nonforfeiture Mortality Table," "Extended Term Insurance Mortality Table," or some such title.

Conclusion

It is realized that in order to be practical, the solution to the problem ought not to involve an excessive complication of administrative procedures and policy language. It is felt that the suggested solution meets this test. In this connection, it is to be remembered that the proposed solution is to be judged by comparing (5) not with (3), but with (4), and then only for limited payment plans. As for the change from (3) to (4), this is chargeable to the decision to use Basis B in the first place, which creates the problem.