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REFUND ANNUITIES WITHOUT "TRIAL AND ERROR"—ACTUARIAL NOTE

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The purpose of this note is to outline a rather simple technique that can be used in determining premium rates for cash refund and instalment refund annuities, and in converting other forms of annuity to either of the refund forms. The method can be applied to deferred annuities, irrespective of the preretirement death benefit, and to immediate annuities.

Before proceeding, a brief explanation of refund annuities might be helpful. Under refund annuities a death benefit is payable if the annuitant dies before the sum of the annuity payments received exceeds a predetermined refund amount. This death benefit is equal to the excess of the refund amount over the sum of the annuity payments received. If it is paid in a lump sum, the term "cash refund annuity" applies, whereas if it is paid in instalments at the same frequency and in the same amount as annuity payments during the annuitant's lifetime, the term "instalment refund annuity" applies. The refund amount is generally the preretirement death benefit calculated immediately prior to the commencement of annuity payments. This preretirement death benefit would in most situations be a return of the premium or premiums paid, either with or without interest.

Let us now look separately at the problems of (i) determining refund annuity premium rates and (ii) converting other forms of annuity to the refund forms.

Premium Rates for Refund Annuities

The determination of premium rates for refund annuities is complex because the death benefits after retirement are related to the premium rates themselves. Generally, in the determination of such rates, a "trial and error" method is used to calculate the premiums required for the two successive integral refund periods which are such that the amount of annuity payments made during the refund period is less than the premium required for the shorter period and more than the premium required for the longer period. The premium for the refund period which will return the required premium exactly is then found by interpolation.

To avoid the difficulties involved in communicating a "trial and error" process to a clerical staff, the method described below was developed.

The first step is to calculate the premium rates for the required amount of monthly annuity on the life annuity basis, including the required pre-retirement death benefit and loading for expenses. Next, the life annuity premium is transposed into the appropriate refund annuity premium.

In order to explain the method of effecting the transposition certain symbols will be used. These are defined as follows:

R = Refund Amount. This is the life annuity premium accumulated for the deferred period at the death benefit rate of interest, if any. For immediate annuities, R is the single premium for the immediate monthly life annuity.

A = 12 times the amount of monthly life annuity provided by the premium involved in the determination of R.

n = Number of years and completed months.

G_n = Factor to apply to a monthly life annuity to convert it to a monthly refund annuity involving an n-year refund period.

For instalment refund annuities using retirement age x, G_n would be of the form

$$\frac{\ddot{a}_x^{(12)}}{\ddot{a}_{\overline{n}|}^{(12)}+\frac{D_{x+n}}{D_x}\left(\ddot{a}_{x+n}^{(12)}\right)}.$$

For cash refund annuities using retirement age x, G_n would be of the form

$$\frac{\ddot{a}_x^{(12)}}{\ddot{a}_x^{(12)}+n\cdot\frac{M_x}{D_x}-\frac{M_{x+n}}{D_x}-\frac{13}{24}\left(\frac{M_x-M_{x+n}}{D_x}\right)-\left(\frac{R_{x+1}-R_{x+n}}{D_x}\right)}.$$

The denominator for the cash refund factor is the net single premium for an immediate cash refund annuity, involving an n-year refund period. This is a modification of formula (7.3) on page 145 of the Society of Actuaries' *Textbook on Life Contingencies*.

In each case, the values of G_n would be prepared on the basis of the same mortality and interest as the life annuity premium.

The transposition is effected by first finding the value of n such that

$$n\cdot A\cdot G_n=R\ .$$

In other words, we must find the period which is such that the equivalent refund annuity ($A\cdot G_n$) when multiplied by the period (n) will equal the refund amount (R). Since we know both A and R, it follows that we are searching for the value of n which is such that $n\cdot G_n = R/A$; and entering a predetermined table of the following form we select the value of

n which satisfies the above condition. Separate tables must be made up for each combination of sex, retirement age and refund form.

In practice, the value of n selected was the lowest value of n such that

$$n \cdot G_n > \frac{R}{A}.$$

Having found the value of n that meets our requirements, we then divide the life annuity premium by G_n in order to complete the transposition to the refund annuity premium.

It should be pointed out that the refund annuity premiums thus determined will have the same characteristics as the life annuity premiums used in the transposition with regard to

(i) number of premiums refunded,
(ii) interest rate used to determine preretirement death benefits,

TABLE OF FACTORS—INSTALMENT REFUND
MALES RETIRING AT AGE 65

n	G_n	$n \cdot G_n$	
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12	.86365	10.36380	
12 1/12	.86200	10.41583	
12 2/12	.86035	10.46759	
12 3/12	.85870	10.51908	*Basis*
12 4/12	.85706	10.57041	
12 5/12	.85541	10.62134	Mortality—*Ga*-1951 Table
12 6/12	.85376	10.67200	
12 7/12	.85211	10.72238	
12 8/12	.85046	10.77249	Interest—3%
12 9/12	.84882	10.82246	
12 10/12	.84717	10.87201	(Nonintegral values of G_n found by straight line interpolation)
12 11/12	.84552	10.92130	
13	.84387	10.97031	
13 1/12	.84219	11.01865	
13 2/12	.84050	11.06658	
13 3/12	.83882	11.11436	
13 4/12	.83714	11.16187	
13 5/12	.83546	11.20909	
13 6/12	.83378	11.25603	
13 7/12	.83209	11.16387	
13 8/12	.83041	11.34894	
13 9/12	.82873	11.39504	
13 10/12	.82705	11.44086	
13 11/12	.82537	11.48640	
14	.82368	11.53152	
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(iii) time at which interest accumulation commences for purposes of calculating preretirement death benefit, *i.e.*, beginning or end of year, and
(iv) loading for expenses.

Converting Other Forms of Annuity to Refund Forms

The steps are similar to those involved in the calculation of refund annuity premiums. First, we convert the alternate form to an equivalent amount of monthly life annuity. Next, on the basis of a method related to that outlined above, we transpose this life annuity into the desired form of refund annuity using the following definitions of R and A:

R = initial refund amount
A = 12 times equivalent amount of monthly life annuity.

Following selection of the value of n, which is such that $n \cdot G_n = R/A$, the equivalent amount of monthly life annuity is multiplied by G_n in order to determine the amount of monthly refund annuity.

Observations

The method is particularly useful in the preparation of supporting tables for contributory group annuity contracts, since a single table of $n \cdot G_n$ (cash or instalment refund) for a particular retirement age can be used as a basis for determining

(i) premium rates for immediate or deferred annuities,
(ii) deferred annuity premium rates which have a with-interest-from-beginning-of-year, with-interest-from-end-of-year, or without-interest preretirement death benefit, and
(iii) net or gross premium rates.

In the calculation of single premium rates for deferred annuities, it is simplest to work in terms of the amount of annuity per \$1,000 single premium, since R then is of the form $1{,}000\,(1+i)^{r-x}$, where r is retirement age and x is age from which interest is calculated in determination of R. The results per \$1,000 of single premium can easily be converted into any desired form.

In practice, the method produced gross single premium deferred annuity rates which were within 2/100 of 1% of the rates found using the "trial and error" process.

DISCUSSION OF PRECEDING PAPER

CHARLES GREELEY:

Mr. Reid is to be complimented for the ingenuity of his approach. He has, in effect, replaced a difficult trial and error method by a two step process which consists of:

(1) a calculation of something akin to a commutation column, and
(2) a straightforward finding process, which in effect is a "trial without error" process.

Another approach is available to a company that has electronic equipment at its disposal. Such equipment is ideally suitable for large-scale calculations. For example, the calculations of premiums for immediate or deferred instalment refund annuities by electronic equipment can be made in a matter of minutes. While the first writing of a program may take several days of a skilled programmer's time, the same program may be used for varying issue and retirement ages, interest rates, mortality tables, and so on, thus achieving a great economy.

It is interesting to speculate whether the progress of actuarial science as such may change in emphasis with the continued increase in the use of electronic equipment. In the past, practical necessity led to the development of innumerable ingenious working techniques, definitions of new symbolism, methods of approximation, etc. With the use of electronic equipment, the time saved by approximations is in general negligible, and elementary symbolism is as effective as more complicated symbolism. Therefore, with the inevitable increased reliance on electronic equipment, there will be decreased economic motivation for the type of development in actuarial techniques that is designed to facilitate a clerk's task. Developments must instead facilitate a programmer's task, and will therefore not only take a different direction, but will require a knowledge of both programming and actuarial principles.

These remarks are not intended to frighten any actuary into thinking that he is about to be replaced by automation.

(AUTHOR'S REVIEW OF DISCUSSION)

DONALD H. REID:

Mr. Greeley, is of course, quite right in stating that the use of electronic equipment will, in most cases, eliminate the practical necessity for employing actuarial short cuts and methods of approximation.

As a result, I would expect that the method I have outlined would seldom be used for premium calculations. However, I do think that predetermined tables of the necessary factors (perhaps calculated by electronic computer) would be very helpful to those who have the problem of converting other forms of annuity to either of the refund forms. This function will probably be handled on a manual basis for many years to come.