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GROSS PREMIUM RATES FOR RENEWABLE TERM INSURANCE

HENRY S. HUNTINGTON

THIS paper is written as a sequel to that on Derivation of Premium Rates for Renewable Term Insurance.[1] In that paper a method is presented for determining renewable term net premiums which are independent of the proportion of the eligible term policyholders who renew on the occasion of each increase in premium. In Mr. Leggett's discussion of this paper and in the author's review of the discussions, methods for determining renewable term gross premiums are given; these methods, however, are not fully consistent with the rationale of the paper itself in that they necessitate the use of assumed renewal rates and in that the resulting premium rates vary materially in relation to the specific renewal rates assumed.

It is the purpose of this paper to present a method for deriving gross premiums for renewable term insurance of contemporary design, which are virtually independent of the rates of renewal.

DERIVATION OF GROSS PREMIUMS

In the development of this gross premium basis, essentially the same assumptions are made with respect to (i) the policy conditions of the renewable, convertible term policy, (ii) the exercise of the renewal and conversion options, and (iii) the presence of antiselection not involving the exercise of these options, as were made in the paper dealing with net premiums. These assumptions may be summarized as follows (those not involving expenses are given in greater detail in the earlier paper):

x = original age at issue.

y = attained age on the policy anniversary marking the end of the period during which the term insurance may be converted to permanent insurance at standard premium rates without evidence of insurability.

m = term period measured from the original date of issue or from any renewal date, during which the gross annual premium rate remains constant.

s = curtate duration measured from beginning of current term period; hence $0 \leq s < m$.

n chosen to satisfy the inequality, $x + nm < y \leq x + (n + 1)m$.

[1] *TSA* X, 329.

$\tau = rm + s.$

${}_{y}P'^{\,1}_{[x]+rm:\overline{m}|}$ = gross annual premium rate payable over the $(r + 1)$th term period of m years for a renewable term policy of unit sum insured issued at age x, convertible to age y and providing for $t - 1$ successive renewals for term periods of m years each, where $y \leq x + tm$. The policy will finally expire at the end of the last of the $t - 1$ renewal periods, except as to any then existing conversion right. (It should be noted here that provision is made for the gross premium rate, corresponding to a given combination of $x + rm$, m and y, to vary in relation to the original age at issue, x. In the net premium development, on the other hand, no such variation in relation to the original issue age is provided.)

$P'_{[y]}$ = standard gross annual premium rate for a whole life policy of unit sum insured issued at age y to a freshly selected risk; this is the rate applying in case of conversion at this age under the option included in the renewable term policy.

$l_{[x]}$ = the number of lives originally insured at age x on the specified renewable term insurance basis for unit amount each.

$q_{[x]+rm+s}$ = the rate of mortality for the $(rm + s + 1)$th policy year among the $l_{[x]+rm+s}$ survivors of the original $l_{[x]}$ lives, including those who are not insured during this policy year as well as those who are then insured. (As a practical matter it may be reasonable to assume, with respect to the period from the issue date to the expiry date, that such mortality rates are equal to, or slightly greater than, the corresponding rates experienced under corresponding level premium, nonrenewable, convertible term policies.)

${}_{rm}^{\;y}g_{x:\overline{m}|}$ = the proportion of the lives insured on the renewable term basis at the end of the rth term period (where $x + rm < y$) who fail to renew their term insurance at that time. It will be assumed that these nonrenewing lives may be treated for mortality experience purposes as then being freshly select lives and that none of them convert their insurance.

${}_{y-x}^{\;\;y}g_{x:\overline{m}|}$ = the proportion of the lives insured on the renewable term basis at the end of the $(y - x)$th year (the end of the conversion period) who fail to convert at that time. It will be similarly assumed that these nonconverting lives may be treated for mortality experience purposes as then being freshly select lives.

(In connection with the definitions of ${}_{rm}^{\ y}g_{x:\overline{m}|}$ and ${}_{y-x}^{\ y}g_{x:\overline{m}|}$, specific reference should be made to the background material given in *TSA* X, pages 331 and 334.)

${}_y l^{(\varrho)}_{[x]+rm+s:\overline{m}|}$ = the number of lives insured on the renewable term basis at the end of the sth year of the $(r+1)$th m-year period out of the $l_{[x]}$ original entrants at age x.

Accordingly,

$$ {}_y l^{(\varrho)}_{[x]+rm+s:\overline{m}|} = l_{[x]+rm+s} - \sum_{\rho=1}^{r} {}_{\rho m}^{\ y}g_{x:\overline{m}|} \cdot {}_y l^{(\varrho)}_{[x]+\rho m:\overline{m}|} \cdot {}_{(r-\rho)m+s}p_{[x+\rho m]} \, . $$

Now, let

${}_y e_{[x]+rm+s:\overline{m}|}$ = the total expense, excluding claim expense, incurred per unit sum insured during the $(s+1)$th year of the $(r+1)$th term period,

${}_y e_{y+\sigma}$ = the corresponding expense incurred per unit sum insured during the $(\sigma+1)$th year in which the whole life conversion policy issued at age y is in force, and

k_4 = death claim expense per unit sum insured of claim proceeds paid.

Equation (1) will now be set down to reflect the condition that the present value at issue of all premiums to be received less that of all benefits and expenses to be paid will be zero. This objective is achieved by providing firstly for premiums, benefits and expenses each year with respect to the entire group then surviving from the original $l_{[x]}$ entrants. From this expression is deducted provision for those premiums, benefits and expenses included above, which are associated with the nonrenewers (and nonconverters at attained age y) and are consequently not actually incurred. The assumption that the nonrenewers (and nonconverters) may be treated as freshly select lives on their respective dates of nonrenewal (or nonconversion) is incorporated in this equation.

(Because of the condition determining n, it will be evident that the method of this paper will not serve to determine renewal premium rates for any renewal period beginning at attained age y or later. It should, furthermore, be noted that when the final m-year term period considered ends after attained age y—*i.e.*, when $y < x + (n+1)m$—it is assumed that ${}_yP'^{\ 1}_{[x]+nm:\overline{m}|}$, the renewable term premium for this period, is paid only to attained age y.)

(To avoid additional complication in presenting the method, no provision is made for withdrawals off the renewal dates. Equation (1) could, of course, be modified to include such provision.)

$$
\begin{aligned}
l_{[x]} \Bigg[& \sum_{r=0}^{n-1} {}_{y}\mathrm{P}'_{\overset{1}{[x]+rm:\overline{m}|}} \cdot {}_{rm|m}\ddot{a}_{[x]} + {}_{y}\mathrm{P}'_{\overset{1}{[x]+nm:\overline{m}|}} \cdot {}_{nm|y-x-nm}\ddot{a}_{[x]} \\
& + \mathrm{P}'_{[y]} \cdot {}_{y-x|}\ddot{a}_{[x]} - (1+k_4)\,\bar{\mathrm{A}}_{[x]} - \sum_{\tau=0}^{y-x-1} {}_{y}e_{[x]+\tau:\overline{m}|} \cdot v^{\tau} \cdot {}_{\tau}p_{[x]} \\
& - \sum_{\sigma=0}^{\infty} {}_{y}e_{y+\sigma} \cdot v^{y-x+\sigma} \cdot {}_{y-x+\sigma}p_{[x]} \Bigg] - \sum_{r=1}^{n} v^{rm} \cdot {}^{y}_{rm}g_{x:\overline{m}|} \cdot {}_{y}l^{(g)}_{[x]+rm:\overline{m}|} \\
& \qquad \times \Bigg[\sum_{\rho=r}^{n-1} {}_{y}\mathrm{P}'_{\overset{1}{[x]+\rho m:\overline{m}|}} \cdot {}_{(\rho-r)m|m}\ddot{a}_{[x+rm]} \\
& + {}_{y}\mathrm{P}'_{\overset{1}{[x]+nm:\overline{m}|}} \cdot {}_{(n-r)m|y-x-nm}\ddot{a}_{[x+rm]} + \mathrm{P}'_{[y]} \cdot {}_{y-x-rm|}\ddot{a}_{[x+rm]} \qquad (1) \\
& - (1+k_4)\,\bar{\mathrm{A}}_{[x+rm]} - \sum_{\tau=rm}^{y-x-1} {}_{y}e_{[x]+\tau:\overline{m}|} \cdot v^{\tau-rm} \cdot {}_{\tau-rm}p_{[x+rm]} \\
& - \sum_{\sigma=0}^{\infty} {}_{y}e_{y+\sigma} \cdot v^{y-x-rm+\sigma} \cdot {}_{y-x-rm+\sigma}p_{[x+rm]} \Bigg] \\
& - v^{y-x} \cdot {}^{y}_{y-x}g_{x:\overline{m}|} \cdot {}_{y}l^{(g)}_{[x]+y-x:\overline{m}|} \Bigg[\mathrm{P}'_{[y]} \cdot \ddot{a}_{[y]} - (1+k_4)\,\bar{\mathrm{A}}_{[y]} \\
& - \sum_{\sigma=0}^{\infty} {}_{y}e_{y+\sigma} \cdot v^{\sigma} \cdot {}_{\sigma}p_{[y]} \Bigg] = 0\,.
\end{aligned}
$$

If it is now assumed that the value of the last term on the left side of Equation (1) is zero (the error involved in making this assumption will be considered in the next paragraph), it will become possible to achieve our purpose of determining for original issue age x a full series of renewable term gross premiums which are independent of the rates of renewal, ${}^{y}_{rm}g_{x:\overline{m}|}$. This result may be accomplished as follows:

a) Observe that Equation (1) involves $n+1$ different renewable term premium rates of the form, ${}_{y}\mathrm{P}'_{\overset{1}{[x]+rm:\overline{m}|}}$, there being one such rate for each value of r from 0 through n inclusive. It will, accordingly, be necessary to obtain $n+1$ simultaneous equations, involving these premium rates, in order to solve for the $n+1$ individual rates desired.

b) Of the $n+1$ needed equations, n may be obtained from Equation (1) by equating to zero the coefficient of $v^{rm} \cdot {}^{y}_{rm}g_{x:\overline{m}|} \cdot {}_{y}l^{(g)}_{[x]+rm:\overline{m}|}$ for each of the n values of r involved [these values run from 1 through n—see the second summation in the third line of Equation (1)]. The $(n+1)$th equation is obtained by equating to zero the coefficient of $l_{[x]}$, which occupies the first two and one-half lines of Equation (1).

(The first n equations described above may be viewed as having been obtained by the partial differentiation of Equation (1) with respect to each of the n functions, ${}_{rm}^{y}g_{x:\overline{m}|}$ (one for each value of r from 1 through n), and the equating of each of the resulting n functions to zero. Accordingly, we have introduced into this system of $n+1$ equations the condition that the solution will be independent of the value of each of the n ${}_{rm}^{y}g_{x:\overline{m}|}$ functions.)

Examination of the above-mentioned last term of Equation (1) will show that (i) the product, $v^{y-x}\cdot{}_{y-x}^{y}g_{x:\overline{m}|}\cdot{}_{y}l^{(g)}_{[x]+y-x:\overline{m}|}$, generally diminishes rapidly with increase in n, the number of renewal periods involved,[2] and (ii) the coefficient of this product is positive and relatively very small.[3] Accordingly, the gross premiums determined as described above generally involve only negligible error and may safely be considered adequate regardless of the rates of renewal experienced. (*Cf.* footnote to Table 2 for an indication of the size of the error introduced into the renewable term premium rates by assuming that this last term vanishes.)

At this point let us examine briefly what happens if the mortality among the nonrenewers (and nonconverters at attained age y) differs from that assumed for them, without change in the originally assumed values of $q_{[x]+rm+s}$ applicable to the entire group of survivors. If it is considered unlikely that the mortality among the nonrenewers (and nonconverters) would under these conditions go to levels below the corresponding select rates originally assumed for them, we may confine our attention to the situation where the level of mortality among the nonrenewers (and nonconverters) exceeds that originally assumed.

If, now, a full set of premium rates, ${}_{y}\mathrm{P}'_{\overset{1}{[x]+rm}:\overline{m}|}$, has been determined as described above on the original assumptions, what is the financial effect on the renewable term insurance operation of an increase in mortality among the nonrenewers (and nonconverters)? Since (i) the values of ${}_{y}\mathrm{P}'_{\overset{1}{[x]+rm}:\overline{m}|}$ originally satisfied the condition that the coefficient of

[2] If the rate of nonrenewal, ${}_{rm}^{y}g_{x:\overline{m}|}$, is relatively large, as is normally the case, the number of continuing lives, ${}_{y}l^{(g)}_{[x]+y-x:\overline{m}|}$, is reduced every m years by this relatively large percentage, in addition to the normal mortality decrement. On the other hand, if ${}_{rm}^{y}g_{x:\overline{m}|}$ is relatively small, it may normally be assumed that ${}_{y-x}^{y}g_{x:\overline{m}|}$ will similarly be relatively small; this latter condition alone is sufficient to make the product small.

[3] In practice, this coefficient is generally equal to the saving in selection costs under conversion policies.

$v^{rm} \cdot {}_{rm}^{\;y}g_{x:\overline{m}|} \cdot {}_{y}l_{[x]+rm:\overline{m}|}^{(g)}$ for each value of r is zero and (ii) we are now assuming an increase in the level of mortality among the nonrenewers (and nonconverters), it is clear that each of these coefficients will become negative (the changes in the two expense terms are in the positive direction but are small in relation to the negative changes in the premium and benefit terms).

Accordingly, an expression corresponding to the left side of Equation (1) would become positive under the assumed conditions. This means that the present value of the premiums would exceed that of the benefits and expenses—in other words, if the mortality among the nonrenewers (and nonconverters) is at higher levels than originally assumed, the premiums determined by this method would be redundant.

It is clear that the method set forth above would apply under any expense assumptions made. We shall now make the following assumptions as to the general character of the expenses, develop a specific formula, and proceed to some illustrative results in dollars and cents. Examination of these results will indicate how the method may be used in practice.

For $r = 0$ and $s = 0$,

$$ {}_{y}e_{[x]+rm+s:\overline{m}|} = c_1 \cdot {}_{y}P'_{\overset{1}{[x]}:\overline{m}|} + k_1 + k_2 \, ; $$

for $r \geq 1$ and $s = 0$,

$$ {}_{y}e_{[x]+rm+s:\overline{m}|} = c_1 \left({}_{y}P'_{\overset{1}{[x]+rm}:\overline{m}|} - {}_{y}P'_{\overset{1}{[x]+(r-1)m}:\overline{m}|} \right) + c_2 \cdot {}_{y}P'_{[x]+(r-1)m:\overline{m}|} + k_2 \, ; $$

for $r \geq 0$ and $s \geq 1$,

$$ {}_{y}e_{[x]+rm+s:\overline{m}|} = c_2 \cdot {}_{y}P'_{\overset{1}{[x]+rm}:\overline{m}|} + k_2 \, ; $$

for $\sigma = 0$,

$$ {}_{y}e_{y+\sigma} = c_3 P'_{[y]} + k_3 + k_2 \, ; $$

for $1 \leq \sigma \leq 10$,

$$ {}_{y}e_{y+\sigma} = c_2 P'_{[y]} + k_2 \, ; $$

and for $\sigma \geq 11$,

$$ {}_{y}e_{y+\sigma} = c_4 P'_{[y]} + k_2 \, . $$

(It will be noted that the commission at the first year rates is paid at renewal on the increase in premium only; the adjustment needed to provide for payment of such commission on the full premium due at renewal will be evident to the interested reader.)

Applying the procedure described in the paragraph following Equation

(1) and, for simplicity, confining our attention to the case where $y = x + (n+1)m$, we may now write Equation (2) and n equations of the form of Equation (3), one of the latter for each value of r from 1 through n, inclusive:

$$
\begin{aligned}
(1-c_2)\sum_{r=0}^{n} {}_yP'_{\overset{1}{[x]+rm}:\overline{m|}} \cdot {}_{rm|m}\ddot{a}_{[x]} &- (c_1-c_2) \\
\times\Big[{}_yP'_{\overset{1}{[x]}:\overline{m|}} + \sum_{r=1}^{n}\Big({}_yP'_{\overset{1}{[x]+rm}:\overline{m|}} - {}_yP'_{\overset{1}{[x]+(r-1)m}:\overline{m|}}\Big) v^{rm}\cdot {}_{rm}p_{[x]}\Big] \\
+ P'_{[y]}\,[\,(1-c_2)\cdot {}_{y-x|}\ddot{a}_{[x]} - (c_3-c_2)\,v^{y-x}\cdot {}_{y-x}p_{[x]} \\
+ (c_2-c_4)\cdot {}_{y+10-x|}\ddot{a}_{[x]}] - (1+k_4)\,\bar{A}_{[x]} \\
- k_2\ddot{a}_{[x]} - k_1 - k_3\,v^{y-x}\cdot {}_{y-x}p_{[x]} = 0\ .
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
(1-c_2)\sum_{\rho=r}^{n} {}_yP'_{\overset{1}{[x]+\rho m}:\overline{m|}} \cdot {}_{(\rho-r)m|m}\ddot{a}_{[x+rm]} &- (c_1-c_2) \\
\times\sum_{\rho=r}^{n}\Big({}_yP'_{\overset{1}{[x]+\rho m}:\overline{m|}} - {}_yP'_{\overset{1}{[x]+(\rho-1)m}:\overline{m|}}\Big) v^{(\rho-r)m}\cdot {}_{(\rho-r)m}p_{[x+rm]} \\
+ P'_{[y]}\,[\,(1-c_2)\cdot {}_{y-x-rm|}\ddot{a}_{[x+rm]} - (c_3-c_2) \\
\times\, v^{y-x-rm}\cdot {}_{y-x-rm}p_{[x+rm]} + (c_2-c_4)\cdot {}_{y+10-x-rm|}\ddot{a}_{[x+rm]}] \\
- (1+k_4)\,\bar{A}_{[x+rm]} - k_2\ddot{a}_{[x+rm]} - k_3\,v^{y-x-rm}\cdot {}_{y-x-rm}p_{[x+rm]} = 0\ .
\end{aligned}
\tag{3}
$$

From these $n+1$ simultaneous equations the value of ${}_yP'_{\overset{1}{[x]+rm}:\overline{m|}}$ may be determined for each of the $n+1$ values of r from 0 through n, inclusive. (It should be noted here that Formulas (2) and (3) involve the assumption that, for all values of r, ${}_yP'_{\overset{1}{[x]+rm}:\overline{m|}} < {}_yP'_{\overset{1}{[x]+(r+1)m}:\overline{m|}}$. Actually, this condition will obtain generally, except for the younger original issue ages where ${}_yP'_{\overset{1}{[x]}:\overline{m|}}$ may exceed ${}_yP'_{\overset{1}{[x]+m}:\overline{m|}}$. In this case appropriate adjustments should be made in the formulas to reflect the fact that no commission would be paid at the first year rate, c_1, on premium increments until the renewal premium first becomes greater than the initial premium.)

ILLUSTRATIVE RESULTS

In order to indicate the relative magnitudes of renewable term premiums determined on the basis outlined in this paper, and to investigate how this basis might be used practically, premium rates computed on the assumptions set forth below are shown on a 5 year term basis in Table 1:

a) The right to renew or to convert under the renewable term policy continues to the policy anniversary nearest the 60th birthday; there is no right to renew or to convert thereafter.

b) Interest is assumed to be earned at the rate of 3% per annum.

c) Mortality for select lives is assumed to follow a select and ultimate basis. The mortality rates are, for policy years after the 15th, those of the 1946–1949 Ultimate Basic Table, and, for the first 15 policy years, rates determined by applying to this table ratios appropriately interpolated (policy year by policy year, by applying the straight line method to the ratios for the neighboring "central" issue ages) from

TABLE 1

5 YEAR RENEWABLE AND CONVERTIBLE TERM INSURANCE
ILLUSTRATIVE GROSS ANNUAL PREMIUM PER 1,000 OF SUM INSURED*

AGE AT BEGINNING OF 5 YEAR PERIOD	ORIGINAL ISSUE AGE						
	25	30	35	40	45	50	55
25	3.02						
30	2.19	3.60					
35	3.46	3.33	4.93				
40	5.94	5.95	5.81	7.51			
45	10.53	10.53	10.54	10.40	12.31		
50	14.53	14.53	14.53	14.54	14.38	16.69	
55	18.59	18.59	18.59	18.59	18.60	18.41	21.09

* While these premiums are shown to two decimal places only, it should be noted that in performing the computations a greater number of places—six in the case of original issue age 25—were absolutely necessary in order to obtain reasonably good results.

those shown in *TSA* II, p. 512, for the 1946–1949 Select Basic Table.

d) $P'_{[y]}$ is computed by the formula,

$$\frac{(1+k_4)\,\bar{A}_{[y]}+k_2\ddot{a}_{[y]}+k_1}{(1-c_2)\,\ddot{a}_{[y]}+(c_2-c_4)\cdot{}_{10|}\ddot{a}_{[y]}-(c_3-c_2)}.$$

e) The following expense assumptions are used (these yield the same expense charges as resulted from use of the assumptions given on page 349 of *TSA* X):

$c_1 = .42$ $k_1 = \$5.033$ per $1,000

$c_2 = .095$ $k_2 = .30$ " "

$c_3 = .62$ $k_3 = 2.783$ " "

$c_4 = .05$ $k_4 = 1.00$ " "

A brief examination of the premium grid shown in Table 1 indicates that, for a given attained age at the beginning of a term period, the premium is substantially greater in the case of an initial term period than in that of a renewal term period. Furthermore, while the theory provides that the premium for such a given attained age always depends on the original issue age (the premium being of the form, ${}_{y}P'_{\overset{1}{[x]+rm}:\overline{m|}}$), nevertheless, in practice, for renewal periods after the first the premium variation in relation to the original issue age is negligible. Indeed, for such a given attained age the premium for the first renewal period may, in practice, be taken as equal to that for subsequent renewal periods, since Table 1 shows the former premiums to be slightly less than the latter.

Accordingly, a practical method of implementing this premium basis—at least, under expense assumptions of the general character of those used

TABLE 2

5 YEAR TERM INSURANCE

ILLUSTRATIVE GROSS ANNUAL PREMIUM PER 1,000 OF SUM INSURED*

AGE AT BEGINNING OF 5 YEAR PERIOD	NONRENEWABLE CONVERTIBLE	RENEWABLE AND CONVERTIBLE		(2)-(1)	(2)-(3)
		First Period	Renewal Period		
	(1)	(2)	(3)	(4)	(5)
25	2.59	3.02		.43	
30	3.03	3.60	2.30	.57	1.30
35	4.26	4.93	3.45	.67	1.48
40	6.66	7.51	5.94	.85	1.57
45	10.82	12.31	10.53	1.49	1.78
50	15.79	16.69	14.53	.90	2.16
55	21.09	21.09	18.59	0	2.50

* In developing both the nonrenewable and the renewable term premiums in this table it has been assumed that all eligible insurance is converted at the end of 5 years in the former case and at attained age 60 in the latter. The resulting premiums differ as described below from what they would be on realistic assumptions with regard to renewal and conversion rates (the assumptions as to conversion and renewal rates used for this purpose are those set forth on page 350 of *TSA* X in columns (3) and (5), respectively):

(i) The nonrenewable term premium rates shown in Table 2 would be increased by amounts ranging from .40 for issue age 25 down to .29 for issue age 55.

(ii) The renewable term premium rates for the first period, shown in column (2), would be unchanged for issue ages 35 and under, and increased by .01, .03, .09 and .29 for issue ages 40, 45, 50 and 55, respectively.

(iii) The renewable term premium rates for renewal periods, generally, would be unchanged.

here—is to use, for each attained age at the beginning of a term period, (i) the initial premium produced by the formula and (ii) a single renewal premium for all applicable original issue ages determined on the basis of the youngest such age considered. Renewable term insurance premium rates so determined are shown in Table 2, together with corresponding premium rates for nonrenewable, convertible term insurance computed on the basis of the same assumptions with respect to interest, mortality and expense, as were used in developing the renewable term insurance premiums. The nonrenewable term premiums involve the assumption that all such insurance is converted at the end of the 5 year term period, and were derived by means of the formula,

$$
{}_{x+m}P'^{\,1}_{[x]:\overline{m}|} = \{ (1+k_4)\bar{A}_{[x]} + k_2\ddot{a}_{[x]} + k_1 + k_3 v^5 \cdot {}_5p_{[x]}
$$

$$
- P'_{[x+5]} [(1-c_2) \cdot {}_{5|}\ddot{a}_{[x]} + (c_2 - c_4) \cdot {}_{15|}\ddot{a}_{[x]} - (c_3 - c_2) v^5 \cdot {}_5p_{[x]}] \}
$$

$$
\div [(1-c_2)\, \ddot{a}_{[x]:\overline{5}|} - (c_1 - c_2)] .
$$

It should be mentioned specifically that none of the premiums shown in Tables 1 or 2 include provision for loss on withdrawals occurring during the earliest policy years (nor for possible gain on later withdrawals). The effect of providing for such losses would probably be (i) to increase moderately the premium rates shown in column 1 of Table 2, (ii) to increase by smaller amounts those shown in column 2 and (iii) to leave those shown in column 3 virtually unchanged. Accordingly, we should expect the premium differences shown in column 4 to be slightly reduced and those shown in column 5 to be moderately increased, if these premiums provided for loss on early withdrawals.

CONCLUSIONS

Application of the method of the paper to the determination of renewable term gross premiums yields the results shown in Tables 1 and 2. The following conclusions may be inferred from these results:

a) Renewable term gross premium rates may be determined in such a way as to be virtually independent of the rates of renewal and conversion of the term insurance.

b) Such rates, varying by age at the beginning of a term period, may generally be expressed in terms of (i) a set of rates applicable during the first term period only and (ii) another set of rates applicable during any renewal period.[4]

c) For a given issue age the renewable term premium rate applicable during the first term period generally exceeds the corresponding premium rate for nonrenewable term insurance.

d) For a given attained age at the beginning of a term period the renewable term premium rate applicable during renewal periods is materially less than that applicable during the first term period; it is, further-

[4] It is possible that for some methods of paying renewal commissions under renewable term insurance a distinction may be needed between the rates applicable during the first renewal period and those applicable during subsequent renewal periods.

more, generally less than the corresponding premium rate for nonrenewable term insurance.

Let us now compare these conclusions with the suggested conclusions of the earlier paper that a renewable, convertible term policy would cost little, if any, more than a corresponding nonrenewable, convertible policy, and that the former, being more valuable to the buyer, would consequently tend to displace the latter from the market place (*cf. TSA* X, 351).

A priori, it would be expected that a renewable term premium structure developed by the method of this paper would be preferable to one developed by the earlier method, simply because the method of this paper involves no assumptions as to the rate of renewal, while the earlier method necessarily involves such assumptions and will be materially influenced by the particular choice made.

Moving now to the actual results, it has been seen that the method of this paper yields premiums for renewal periods which are at a lower level than those for corresponding initial periods, in contrast with the assumption of the earlier method that the premiums at a given attained age would be at the same level for renewal as for initial periods. Furthermore, the premiums for initial periods generally exceed the corresponding premiums for nonrenewable, convertible term insurance, in contrast with the earlier result that the former tended usually to be less than the latter.

From the buyer's standpoint it seems evident that a convertible term policy which includes a renewal right is worth more, and accordingly should cost more, than one which does not. Conversely, a term buyer who desires convertibility but not renewability would expect to pay less than if he were also buying renewability. (In this connection we are, of course, comparing the renewable term premium for the initial period with the corresponding nonrenewable term premium.) From the point of view that reasonable relationships should obtain between premiums and benefits, the new method thus seems distinctly preferable to the earlier one, because, firstly, it substantiates a higher initial cost for convertible term insurance with renewal rights than without them and, secondly, it places a price tag on these rights.

It seems evident that, as compared to earlier methods, the new method would have a favorable effect on the persistency of renewable term policies, because it introduces a premium-rate level for renewing policies which is below the corresponding rate level for newly issued convertible term policies, either renewable or nonrenewable.

Since insurers have generally charged higher initial premiums for renewable than for nonrenewable term policies of this kind, and since generally the differentials have been roughly comparable with those shown in column (4) of Table 2, the basic question posed by this paper is whether the renewal premium rates should be reduced to the degree called for by the method presented. It should be mentioned here that certain mutual companies, by paying higher dividends (tantamount to charging lower premiums) under renewal renewable term insurance than under corresponding new issues, actually do provide such reduced cost levels during renewal periods.

Finally, let us consider the reasonableness of the observed result that the renewal premiums are at a lower level than the corresponding initial premiums. The nature of the results shown in Table 1, *viz.*, a higher level for the initial premiums followed by a lower renewal premium level which is practically independent of the original issue age, points strongly to the influence of expenses (higher for the initial period and uniformly lower for all renewal periods), not of mortality (the level of which for the continuing policyholders of a given attained age would depend on the number of renewals which had occurred), as being responsible for the observed excess of initial over renewal premium levels. Computation of the excess of the expense for the initial period over that for the corresponding renewal period confirms that this expense differential is practically equal to the differential between the corresponding premiums determined by this method and set forth in Table 1.

The foregoing result suggests that premiums based on this method could be computed in practice by first determining the whole chain of gross premiums needed for the youngest issue age. Each of the resulting renewal premiums would then be increased to provide, in the case of policies issued at the corresponding attained age, for (i) the excess initial expense incurred and (ii) the loss on early withdrawals.

DISCUSSION OF PRECEDING PAPER

ALVIN B. NELSEN:

The determination of premium rates and dividend margins for renewable term insurance is one area where we need to rely on hypothesis as to the effects of selection. Only by observing experience over a very long period will it be possible to determine the effects that selection has on the mortality rates of the latest available attained age conversions. Even if such observations were available, the period covered would be so long and relate to such old issues that the experience would probably not be a reliable guide. In his two papers on this subject, Mr. Huntington has given us some ingenious techniques for determining the maximum cost of this benefit. The current paper is a valuable supplement to his previous one in providing formulas for determining maximum renewable term gross premiums, which are independent of the rate of renewal. In his previous paper on this subject, renewal premiums were assumed to be at the level of initial premiums, which had the effect of amortizing initial expenses over successive renewals, taking account of assumed rates of renewal. In this paper initial expenses are amortized over the initial period, which gives larger initial premiums, and renewal premiums are based on renewal expenses only, producing lower renewal premiums.

The effect of Mr. Huntington's method is to assess in each term period an annual charge for the cost of future renewals corresponding to the excess of the renewal costs, if all policyholders renewed, over the assumed renewal premiums. The assumed renewal premiums are calculated as the payments required for a freshly select life at the time of each renewal. Thus the results are independent of the rate of renewal, since it does not matter whether a policyholder renews or not, if the nonrenewing policyholder is assumed to be a freshly select life on the renewal date. On a net basis, these annual charges for the right to renew (and convert) at select rates are equal to the excess of (i) the net annual premiums derived in Mr. Huntington's previous paper for renewable and convertible term insurance over (ii) the net annual premiums for nonrenewable and nonconvertible term insurance. While on a net basis these annual charges for the right to renew are the same whether the term period is an initial term period or a renewal term period, for the gross premiums derived in the current paper these charges for the renewal privilege will be greater for an initial term period.

It should be appreciated that these annual charges for the right to renew at select rates will not actually be incurred in the period in which assessed in determining Mr. Huntington's gross premiums. For example, the renewal charges included in determining the annual premiums for the initial term periods will not actually be needed until the renewal years to cover the extra mortality contemplated. Thus, the differences between the initial period premiums and the renewal period premiums are inflated by the technique which Mr. Huntington has used to determine premiums which are independent of the rate of renewal. The initial premiums, and the differences from renewal premiums, would have been still further increased if account were taken of the effect of withdrawal and conversion rates in the initial five year period in arriving at the charge that must be made to cover first year expenses. This suggests that it may be desirable, in order to avoid having unreasonably large initial period premiums, to amortize some of the first year expenses over renewal periods—possibly to the extent of the renewal charge assessed in the initial term period. This would, of course, require assumptions as to the rates of renewal, which is a departure from the method of this paper.

Mr. Huntington compares nonrenewable and convertible term insurance premiums with renewable and convertible term insurance premiums, and concludes that a reasonable relationship from the buyer's point of view would be for the latter to cost more because of the additional renewal rights. While admittedly this has some logic, the actual situation is that on the nonrenewable form all costs for the conversion privilege must be assessed during the term period, whereas under the renewable form the conversion costs in the early years are likely to be much lower because of the tendency to defer conversions and the costs of the renewal privilege can be assessed as incurred after renewal.

I believe that Mr. Huntington will find that a close approximation to his results can be obtained by loading the net premiums given in his previous paper by the level annual equivalent of expenses in each period. The principal approximation involved is of the following form:

$$\frac{\sum_{\tau=rm}^{rm+4} {}_{y}e_{[x]+\tau:\overline{m}|}\, v^{\tau}\cdot {}_{\tau}p_{[x]}}{\sum_{\tau=rm}^{rm+4} v^{\tau}\cdot {}_{\tau}p_{[x]}} \approx \frac{\sum_{\tau=rm}^{m+4} {}_{y}e_{[x]+\tau:\overline{m}|}\, v^{\tau-rm}\cdot {}_{\tau-rm}p_{[x+rm]}}{\sum_{\tau=rm}^{rm+4} v^{\tau-rm}\cdot {}_{\tau-rm}p_{[x+rm]}}.$$

This would be particularly helpful where already determined participating premiums were being used and calculations were being made to measure the margins for dividends. Such an approximation facilitates the introduction of termination (withdrawal and conversion) rates during a term period.

The introduction of such termination rates during, say, the initial term period raises questions as to the effects of selection by those terminating prior to the end of this period. From our observed experience of 5 and 10 year nonrenewable and convertible term mortality, which should be similar to 5 year renewable term during the initial term period, I do not feel that there is much evidence of such selection. This experience for the period covering 1945 to 1950 and 1950 to 1955 policy anniversaries is as follows:

TERM MORTALITY EXPERIENCE COMPARED WITH CONTEMPORANEOUS EXPERIENCE BY AMOUNTS UNDER TOTAL STANDARD BUSINESS

POLICY YEARS	1945–1950 ANNIVERSARIES		1950–1955 ANNIVERSARIES	
	Term 5	Term 10	Term 5	Term 10
1 and 2	64%	111%	87%	122%
3 to 5	107	67	119	87
6 and 7	....	105	....	81
8 to 10	....	130	....	96
All	91%	97%	107%	94%

This experience is not conclusive but gives some support for assessing the renewal privilege charges for the initial five year period, only with respect to the policyholders persisting to the end of the initial five year period. This assumption has the effect of reducing the initial period premiums without affecting renewal premiums and would be an offsetting factor to the increase in loading that would result from the introduction of termination rates in the initial period. It assumes that the lives terminating during the initial term period are average risks (i.e., subject to average mortality pertaining to original issue age and duration) rather than select lives and that lives persisting to the end of the initial term period are therefore also average lives. This assumption also implies that provision should be made for the cost of conversion for the lives converting during the initial term period. As experience emerges under the 5 year renewable term, it may be possible to temper the results for the later renewal periods by introducing such actual experience as is available.

While Mr. Huntington's method is designed to produce maximum costs, it may be that his method is unduly conservative with respect to the cost of final attained age conversions. On page 340 of his previous paper he shows a net fund of $99.39 per $1,000 required per policyholder reaching age 60 to provide for conversion costs on Basis A assuming all policyholders persist, and $246.48 per $1,000 required per policyholder persisting on Basis B renewal rates. From our own observed experience and calculations on group conversion costs, the charge on Basis A would seem to require that more than 40% of the policyholders persist and convert at age 60 with experience comparable to group conversion experience and on Basis B that the sum accumulated would be more than enough to cover the cost if all policyholders persisting to age 60 converted with mortality thereafter comparable to group conversion experience. Since group conversions are generally all impaired lives, in view of the tendency to issue new insurance if qualified as a select risk, it would appear reasonable to assume that mortality for those converting as group conversions would in general exceed the mortality for those converting at the latest available attained age. This suggests that some reasonable pegging of the final attained age conversion costs might be made on the basis of observed group conversion experience using some conservative assumption as to the percentage of original entrants converting.

One last minor observation might be made. Since, as Mr. Huntington points out, the saving in selection costs under converted policies is relatively very small, he could have simplified his equation (1) by assuming net premiums collected on the final attained age conversions and omitting expenses after conversion. This would automatically have eliminated the last term in his equation (1). As contrasted with his result of providing for such savings in selection costs for policyholders who persist to age 60 but do not convert, this would err in the other direction by not providing for savings in selection costs for policyholders who persist to age 60 and do convert.

(AUTHOR'S REVIEW OF DISCUSSION)

HENRY S. HUNTINGTON:

We are indebted to Mr. Nelsen for proposing the possibility of deferring the collection of part of the renewable-term-insurance cost relative to the incidence inherent in the method of the paper. His primary purpose in raising this question is to justify a reduction in the relatively high level of initial premiums characteristic of this method.

In commenting on his approaches to this worthy objective I would like first to mention a possible basis for justifying slightly reduced premium levels and then to discuss the general proposition of deferring the incidence of renewable-term premium charges.

Within the framework set forth in the paper it might be possible, though it would certainly not be conservative, to assume that the nonrenewers (and nonconverters) experience mortality at some intermediate level between (i) the select rates which have been assumed for this purpose and (ii) the rates applicable to the entire group of survivors from the original entrants. If any such increased levels of mortality among the nonrenewers (and nonconverters) were justifiable, corresponding reductions in the levels of the renewable-term premiums would result. To my way of thinking, the key question here is whether any sound basis may be developed to support the assumption of any such increased mortality among nonrenewers. It seems evident that unsoundly high mortality assumptions here have the makings of a costly experience for the insurer.

Mr. Nelsen states that the "annual charges for the right to renew at select rates will not actually be incurred in the period in which assessed" under the method of the paper, and suggests that it may be desirable to reduce the initial-period premiums to the level of the nonrenewable-convertible-term premiums (assuming the renewal charge assessed in the initial term period under the method of the paper is the excess of the renewable-term premium for that period over the corresponding nonrenewable-convertible-term premium). Furthermore, he proposes, in the light of the term mortality experience he cites, that the initial-period premiums may be reduced without affecting renewal premiums. He also mentions that these suggestions "would, of course, require assumptions as to the rates of renewal which is a departure from the method of the paper."

It seems to me that Mr. Nelsen has not brought out the full import of introducing assumptions as to the rates of renewal. The very essence of the method is the development of a renewable-term cost structure which is proof against financial loss from antiselection at renewal (and conversion)—and the only practical way to achieve this result is to make the structure independent of the rates of renewal.

Once this condition (independence of rates of renewal) is imposed, the incidence of the charges for the right to renew is determined. The resulting incidence of premiums is the only such incidence which will meet the actual claims *regardless of what rates of renewal are actually experienced.* Any attempt to redistribute the premium charges from period to period must necessarily involve an assumption as to renewal rates; and to the extent that the assumed renewal rates affect the determination of premiums, the financial results of the renewable-term-insurance operation will be affected by any departures of the actual from the assumed renewal rates.

The tragic history of assessment insurance illustrates dramatically the consequences of deferring the charges for the renewal-right to the time when the extra claims occur. In the light of this extreme situation and of the above reasoning it would appear that any deferring of the premium incidence associated with the method of the paper involves a vulnerability to antiselection at renewal, and that the potential losses are greater, the more the premium incidence is deferred relative to this standard.

In fact, I would like to introduce the proposition—not yet proved—that any renewable-term premium series which (i) is determined on the basis of assumed renewal rates and (ii) involves a deferral of the premium incidence of the method of the paper is vulnerable to antiselection at renewal.

In the light of this background it seems to me that the degree, if any, to which the initial-period premiums may be reduced (below those produced by the method of the paper) involves a balancing of the advantages, particularly from a sales standpoint, of using a lower level against the disadvantages, namely, higher renewal premiums and vulnerability to antiselection at renewal.

I think that Mr. Nelsen's suggestion for obtaining premiums by loading net premiums obtained as described in the previous paper may be a useful one, particularly in its facilitation of the introduction of termination rates during a term period. The initial period should, of course, be treated separately from the renewal periods if the higher level of initial-period expense is to be recovered during that period.

In connection with his suggestion that the final attained age conversion costs are unrealistically high and that they be reasonably pegged through use of group conversion experience, I would like to make two comments.

Firstly, the nature of the method does not lend itself to the imposition of such restrictions—the only mortality rates appearing in Equations (2) and (3), by means of which the premiums are derived, are normal select and ultimate rates for individually selected lives.

And secondly, there are reasons to anticipate that the conversion experience under Basis B as described in *TSA* X, 340, would indeed be at levels corresponding to those of group conversion experience. Specifically, those eligible to convert at age 60 are the individuals who have stayed on the renewable-term basis after having had the opportunity to select against the insurer on each of *six* renewal dates. It may be noted that the successive product of the "Proportion Renewing" on Basis B (.70 × .70 × etc.) is less than .04. This result indicates that the cumulative effect of antiselection here may be considered as being roughly equivalent to that of a single and suitably intermediate opportunity for antiselection, following which only 4% of the eligibles continue the coverage; 4% is certainly a small enough proportion to warrant an expectation of mortality experience at group conversion levels. In this connection it may be noted on page 340 that the Basis B mortality rate for attained age 59 is 52.60 per 1,000. In relation to the corresponding freshly select rate of 5.90 per 1,000 (on the mortality basis used), 52.60 represents a mortality ratio of about 900%, a level typical of group conversion experience in the first policy year for this age range.

I have one last comment on his last minor observation. His suggestion to assume net premiums and no expenses on the final attained-age conversions introduces a departure from the true situation with respect to *all* those surviving to the final conversion age, whether or not they are then insured. Equation (1), on the other hand, reflects the true situation exactly—and the assumption that its last term is zero involves the same departure (as he suggests) from the true situation *only* with respect to those of the survivors who (i) are eligible for conversion and (ii) do not convert. Since the latter group will normally represent a very small proportion of the total number of survivors, it seems clear that the method of the paper is the more accurate.