# Financial Economic Theory and Engineering Formula Sheet

# 2010

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee felt that by providing many key formulas, candidates would be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula sheet was developed sequentially by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not on the formula sheet.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

## General Formulas

**Lognormal distribution**:

$$f(x) \;=\; \frac{1}{x\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{(\ln(x)-\mu)^2}{\sigma^2}\right\}$$

$$E[X] \;=\; e^{\mu+\sigma^2/2} \qquad V[X] = e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$$

$$\Pr[X \le x] = \Phi\left(\frac{\ln(x)-\mu}{\sigma}\right) \quad x > 0$$

**Normal distribution:**

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$$

**Capital Asset Pricing Model:**

$$E[R_i] = R_f + \beta_i\left(E[R_m]-R_f\right) \qquad \text{where} \quad \beta_i = \frac{\mathrm{Cov}(R_i,R_m)}{\mathrm{Var}[R_m]}$$

**Weighted Average Cost of Capital:**

$$\mathrm{WACC} = (1-\tau_c)k_b\frac{B}{B+S} + k_s\frac{S}{B+S}$$

where $k_b$=return on debt, $k_s$ =return in equity.

**GARCH(1,1) Model**

$$Y_t = \mu + \sigma_t\epsilon_t \qquad \sigma_t^2 = \alpha_0 + \alpha_1(Y_{t-1}-\mu)^2 + \beta\sigma_{t-1}^2$$

**RSLN-2 Model**

$$Y_t|\rho_t \sim N\left(\mu_{\rho_t},\sigma_{\rho_t}^2\right) \text{ where } \{\rho_t\}, \, t=1,2,..., \text{ is a Markov process with 2-states.}$$

$$p_{ij} = \Pr[\rho_{t+1}=j|\rho_t=i]$$

**Conditional Tail Expectation**
For loss $L$, continuous at $V_\alpha = F_L^{-1}(\alpha)$:

$$\mathrm{CTE}_\alpha(L) = E\left[L|L > V_\alpha(L)\right]$$

If there is a probability mass at $V_\alpha$, define $\beta' = \max\{\beta : V_\alpha = V_\beta\}$, then

$$\mathrm{CTE}_\alpha(L) = \frac{(1-\beta')E\left[X|X > V_\alpha\right] + (\beta'-\alpha)V_\alpha}{1-\alpha}$$

## Vasicek Model

$$dr \;=\; a(b-r)dt + \sigma dz \qquad P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

$$B(t,T) \;=\; \frac{1 - e^{-a(T-t)}}{a} \qquad A(t,T) = \exp\left[\frac{\left(B(t,T) - T + t\right)\left(a^2 b - \sigma^2/2\right)}{a^2} - \frac{\sigma^2 B(t,T)^2}{4a}\right]$$

## Cox Ingersoll Ross Model

$$dr \;=\; a(b-r)dt + \sigma\sqrt{r}dz \qquad P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

$$B(t,T) \;=\; \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma} \qquad A(t,T) = \left[\frac{2\gamma e^{(a+\gamma)(T-t)/2}}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma}\right]^{2ab/\sigma^2}$$

## Ho-Lee Model

$$dr \;=\; \theta(t)dt + \sigma dz \qquad \theta(t) = F_t(0,t) + \sigma^2 t$$

$$P(t,T) \;=\; A(t,T)e^{-r(t)(T-t)}$$

$$\ln A(t,T) \;=\; \ln\frac{P(0,T)}{P(0,t)} + (T-t)F(0,t) - \frac{1}{2}\sigma^2 t(T-t)^2$$

## Hull-White (Extended Vasicek) Model

$$dr \;=\; (\theta(t) - ar)dt + \sigma dz \qquad \theta(t) = F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}\left(1 - e^{-2at}\right)$$

$$P(t,T) \;=\; A(t,T)e^{-B(t,T)r(t)}$$

$$B(t,T) \;=\; \frac{1 - e^{-a(T-t)}}{a}$$

$$\ln A(t,T) \;=\; \ln\frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{1}{4a^3}\sigma^2\left(e^{-aT} - e^{-at}\right)^2\left(e^{2at} - 1\right)$$

## Itô's Lemma
Let $X$ be an Itô process such that $dX_t = u(t, X_t)\,dt + v(t, X_t)\,dW_t$,

and let $g(t,x)$ denote a twice differentiable function. Then, for $Y_t = g(t, X_t)$,

$$dY_t = \frac{\partial g}{\partial t}(t, X_t)\,dt + \frac{\partial g}{\partial x}(t, X_t)\,dX_t + \frac{1}{2}\frac{\partial^2 g}{\partial x^2}(t, X_t)\,(dX_t)^2.$$

## Geometric Brownian Motion

$$\frac{dS_t}{S_t} = \mu\,dt + \sigma\,dZ_t$$

## Black-Scholes Pricing Formulas

$$c_t = S_t N(d_1) - Ke^{-r(T-t)}N(d_2)$$

$$p_t = Ke^{-r(T-t)}N(-d_2) - S_tN(-d_1)$$

where  $d_1 = \dfrac{\ln(S_t/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}$  and  $d_2 = d_1 - \sigma\sqrt{T - t}$

# Copeland, Weston, Shastri: Financial Theory and Corporate Policy

## Chapter 2

(2.2) $\quad Rev_t + m_t S_t = Div_t + (W\&S)_t + I_t$

(2.5) $\quad NI_t = Rev_t - (W\&S)_t - dep_t$

(2.7) $\quad S_0 = \sum_{t=1}^{\infty} \dfrac{NI_t - \Delta A_t}{(1 + k_s)^t}$

(2.13) $\quad FCF = EBIT(1 - \tau_c) + \Delta dep - \Delta I$

## Chapter 6

(6.34) $\quad R_{jt} = E(R_{jt}) + \beta_j \delta_{mt} + \varepsilon_{jt}$

(6.36) $\quad R_{jt} - R_{ft} = (R_{mt} - R_{ft})\beta_j + \varepsilon_{jt}$

(6.36) $\quad R'_{pt} = \gamma_0 + \gamma_1 \beta_p + \varepsilon_{pt}$

(6.37) $\quad R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$

(6.38) $\quad E(R_i) = E(R_Z) + [E(R_m) - E(R_Z)]\beta_i$

(6.40) $\quad E(R_i) - R_f = b_i [E(R_m) - R_f] + s_i E(\text{SMB}) + h_i E(\text{HML})$

(6.41) $\quad \lambda_i = E(R_m) - E(R_Z)$

(6.46) $\quad R_{it} = \widehat{\gamma}_{0t} + \widehat{\gamma}_{1t}\beta_{it} + \varepsilon_{it}$

(6.49) $\quad E(R_i) = E(R_{z,I}) + [E(R_I) - E(R_{z,I})]\beta_{i,I}$

(6.50) $\quad \widetilde{R}_i = E(\widetilde{R}_i) + b_{i1}\widetilde{F}_1 + \ldots + b_{ik}\widetilde{F}_k + \widetilde{\varepsilon}_i$

(6.57) $\quad E(\widetilde{R}_i) = \lambda_0 + \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik}$

(6.59) $\quad E(R_i) - R_f = \left[\bar{\delta}_1 - R_f\right] b_{i1} + \ldots + \left[\bar{\delta}_k - R_f\right] b_{ik}$

(6.60) $\quad b_{ik} = Cov(R_i, \delta_k)/Var(\delta_k)$

## Chapter 9

(9.1) $\quad C(S_A, S_B, T) = S_A N(d_1) - S_B N(d_2)$

$\quad$ where $d_1 = \left[\ln (S_A/S_B) + V^2 T\right]/V\sqrt{T}; \qquad d_2 = d_1 - V\sqrt{T}$

$\quad$ and $V^2 = V_A^2 - 2\rho_{AB}V_A V_B + V_B^2$

## Chapter 10

(10.1) $\quad V(\eta) \equiv \sum_m q(m) \max_a \sum_e p(e|m)U(a, e) - V(\eta_0)$

(10.4) $\quad p(r - c_2) + (1 - p)(dr - c_2) = p\,(r/d - c_1) + (1 - p)(r - c_1)$

(10.7) $\quad$ Fair Game: $\varepsilon_{j,t+1} = \dfrac{P_{j,t+1} - P_{jt}}{P_{jt}} - \dfrac{E(P_{j,t+1}|\eta_t) - P_{jt}}{P_{jt}} = 0$

(10.9a) $\quad$ Submartingale: $\dfrac{E(P_{j,t+1}|\eta_t) - P_{jt}}{P_{jt}} = E(r_{j,t+1}|\eta_t) > 0$

(10.9b) $\quad$ Martingale: $\dfrac{E(P_{j,t+1}|\eta_t) - P_{jt}}{P_{jt}} = E(r_{j,t+1}|\eta_t) = 0$

(10.14) $\quad E(R_{jt}|\widehat{\beta}_{jt}) = R_{jt} + \left[ E(R_{mt}|\widehat{\beta}_{mt}) - R_{ft} \right] \widehat{\beta}_{jt}$

## Chapter 11

(11.2) $\quad R_{jt} = \alpha_j + \beta_{1j}(R_{mt} - R_{ft}) + \beta_{2j}(RLE_t - RSE_t) + \beta_{3j}(HBTM_t - LBTM_t) + \varepsilon_{jt}$

(11.3) $\quad \Delta NI_{jt} = \hat{a} + \hat{b}_j \Delta m_t + \varepsilon_{jt}$

(11.4) $\quad \widehat{\Delta NI}_{j,t+1} = \hat{a} + \hat{b}_j \Delta m_{t+1}$

## Chapter 12

(12.1) $\quad V(\alpha) = \dfrac{1}{(1+r)}[\mu(\alpha) - \lambda]$

(12.3) $\quad W_0 = X + \beta V_M + Y - (1 - \alpha)V(\alpha)$

(12.4) $\quad \widetilde{W}_1 = \alpha[\mu + \widetilde{\varepsilon} - \mu(\alpha) + \lambda] + \beta[\widetilde{M} - (1+r)V_M] + (1+r)(W_0 - X) + \mu(\alpha) - \lambda$

(12.5) $\quad \dfrac{\partial E\left( U\left( \widetilde{W}_1 \right) \right)}{\partial \alpha} = E\left[ U'\left( \widetilde{W}_1 \right) (\mu + \widetilde{\varepsilon} - \mu(\alpha) + \lambda + (1-\alpha)\mu_\alpha) \right] = 0$

(12.6) $\quad \dfrac{\partial E\left( U\left( \widetilde{W}_1 \right) \right)}{\partial \beta} = E\left[ U'\left( \widetilde{W}_1 \right) \left( \widetilde{M} - (1+r)V_M \right) \right] = 0 \qquad \text{where} \quad \mu_\alpha = \dfrac{\partial \mu}{\partial \alpha}$

(12.7) $\quad (1-\alpha)\mu_\alpha = -\dfrac{E\left[ U'\left( \widetilde{W}_1 \right) (\widetilde{\varepsilon} + \lambda) \right]}{E\left[ U'\left( \widetilde{W}_1 \right) \right]}$

(12.15) $\quad E(D) = \dfrac{1}{1+r} \left[ V(D) + \mu - \tau_p D - \beta \int_{\underline{X}}^{D} (X - D)f(X)dX \right]$

(12.16) $\quad E(D) = \dfrac{1}{1+r} \left[ V(D) + \dfrac{t}{2} - \tau_p D - \beta \dfrac{D^2}{2t} \right]$

(12.17) $\quad V'(D^*) = \tau_p + \beta \dfrac{D^*}{t}$

(12.18) $\quad V[D^*(t)] = \dfrac{1}{r} \left[ \dfrac{t}{2} - \tau_p D^*(t) - \beta \dfrac{[D^*(t)]^2}{2t} \right]$

(12.20)  $V[D^*(t)] = (\tau_p + \beta A)D^*(t)$

(12.21)  $A = -\left[\dfrac{\tau_p}{\beta}\right]\left[\dfrac{1+r}{1+2r}\right] + \left[\dfrac{\tau_p}{\beta}\right]\left[\dfrac{1+r}{1+2r}\right]\sqrt{1 + \dfrac{\beta(1+2r)}{\tau_p^2(1+r)^2}}$

(12.22)  $I + D = C + Np_e = C + P_e$

(12.24)  $\max_D \left(L - \tau_p D + \left[\dfrac{P + \tau_p D - L}{P + \tau_p D + I - C}\right]X\right)$

(12.25)  $\tau_p = \left(\tau_p + \dfrac{\partial P}{\partial D}\right)\dfrac{L + I - C}{P + \tau_p D + I - C}X$

(12.26)  $D(X) = \dfrac{1}{\tau_p}\max(I - C + L, 0)\ln X$

(12.30)  $V_0^{old} = \dfrac{P'}{P' + E}(E + S + a + b)$

(12.36)  $W^0(z) = \alpha\overline{P}(z) + \beta P - T(m, P)$

(12.37)  $W^s(n, z) = \alpha\widehat{P}(n, z) + \beta P - T(n, p)$

(12.38)  $W_n = \alpha\widehat{P}_n - \dfrac{t_2}{p^{\gamma-1}} = 0$

(12.40)  $\widehat{P}(n, z) = k[n + c(z)]^{1/\gamma}$  where  $k = (t_2\gamma/\alpha)^{1/\gamma}$

(12.41)  $M(n, z) = \widehat{P}(n, z) - T(n, P) = k(1 - t_1)[n + c(z)]^{1/\gamma} - t_2 nk^{1-\gamma}[n + c(z)]^{(1-\gamma)/\gamma}$

(12.45)  $V_0 = (1 - t)X_0$

(12.46)  $E(X|n) = \widehat{X}(n) = \dfrac{X_0 s_0 + \widehat{Y_m}(n)s_m}{s_0 + s_m}$

(12.47)  $V_1(n) = \widehat{X}(n) - T(n) - C$

(12.50)  $V_2(T, \overline{Y}) = \dfrac{X_0 s_0 + \widehat{Y}_m s_m + \overline{Y}FTs}{s_0 + s_m + FTs} - E\left[Xt(X/n)|T, \widehat{Y}_m, \overline{Y}\right] - C$

(12.51)  $E[V_2(T)|Y_m] = \dfrac{X_0 s_0 + \widehat{Y}_m s_m + \left(\frac{X_0 s_0 + Y_m s_m}{s_0 + s_m}\right)FTs}{s_0 + s_m + FTs} - T - C$

(12.54)  $\left(\dfrac{d\alpha}{dD}\right)[V(n) - B(n, D)] - \alpha(D)\left(\dfrac{\partial B}{\partial D}\right) = 0$

(12.55)  $\left(\dfrac{d^2\alpha}{dD^2}\right)[V(n) - B(n, D)] - 2\left(\dfrac{d\alpha}{dD}\right)\left(\dfrac{\partial B}{\partial D}\right) - \alpha(D)\left(\dfrac{\partial^2 B}{\partial D^2}\right) < 0$

(12.56)  $\alpha(D^*(n))[V(n) - B(n, D^*(n))] - [V(n) - I] = 0$

(12.57)  $\left[\dfrac{d^2\alpha}{dD^2}(V - B) - 2\dfrac{d\alpha}{dD}\dfrac{\partial B}{\partial D} - \alpha\dfrac{\partial^2 B}{\partial D^2}\right]\dfrac{dD}{dn} = \dfrac{d\alpha}{dD}\left(\dfrac{\partial B}{\partial n} - \dfrac{dV}{dn}\right) + \alpha\left(\dfrac{\partial^2 B}{\partial n\partial D}\right)$

(12.58)  $\widehat{\varepsilon}(\alpha, D) = \left[(a - D_0)^2 + D^2\dfrac{1 - \alpha}{\alpha^2}\right]$

$$(12.59) \qquad \overline{D} = X_1 \left[ \frac{\overline{\alpha}(\epsilon - \gamma)}{(1 - \overline{\alpha})(1 - \epsilon)\gamma} \right]$$

$$(12.60) \qquad \max_{c(s,p),a} \int \int U\left[s - c(s,p)\right] f(s,p|a) ds \, dp$$

$$(12.61) \qquad \int \int V\left[c(s,p)\right] f(s,p|a) ds \, dp - G(a) \geq \underline{V}$$

$$(12.63) \qquad \frac{U'[s - c(s,p)]}{V'[c(s,p)]} = \lambda$$

$$(12.65) \qquad \max_{a} U(k) + \lambda \left[ \int \int V[c(s,p)] f(s,p|a) ds \, dp - G(a) - \underline{V} \right]$$

$$(12.68) \qquad \max_{c(s),a} \int U[s - c(s)] f(s|a) \, ds + \lambda \left[ \int V[c(s)] f(s|a) \, ds - G(a) - \underline{V} \right]$$

$$+ \mu \left[ \int V[c(s)] f_a(s|a) ds - G'(a) \right]$$

$$(12.70) \qquad \frac{U'[s - c(s)]}{V'[c(s)]} = \lambda + \mu \frac{f_a(s|a)}{f(s|a)}$$

$$(12.73) \qquad \max_{c(s,p),a} \int \int U\left[s - c(s,p)\right] f(s,p|a) ds \, dp + \lambda \left[ \int \int V[c(s,p)] f(s,p|a) \, ds \, dp - G(a) - \underline{V} \right]$$

$$+ \mu \left[ \int \int V[c(s,p)] f_a(s,p|a) \, ds \, dp - G'(a) \right]$$

$$(12.74) \qquad \frac{U'\left[s - c(s,p)\right]}{V'[c(s,p)]} = \lambda + \mu \frac{f_a(s,p|a)}{f(s,p|a)}$$

$$(12.75) \qquad \frac{U'\left[s - c(\mathbf{p})\right]}{V'[c(\mathbf{p})]} = \lambda + \mu_1 \frac{f_{a_1}(\mathbf{p}|a)}{f(\mathbf{p}|a)} + \mu_2 \frac{f_{a_1}(\mathbf{p}|a)}{f(\mathbf{p}|a)} + \ldots + \mu_n \frac{f_{a_1}(\mathbf{p}|a)}{f(\mathbf{p}|a)}$$

$$(12.81) \qquad \max_{c(s,\mathbf{p},m),a(m),m(m)} E_{s,\mathbf{p},m}\left[ U\left[s - c(s,\mathbf{p},m)\right] | a(m) \right]$$

$$\text{Subject to (for all m)} \quad E_{s,\mathbf{p}|m}\left[ [V[c(s,\mathbf{p},m)] - G[a(m)]] | a(m) \right] \geq \underline{V}$$

$$(12.82) \qquad \frac{\partial V(X^*)}{\partial X} = \frac{\partial P(X^*)}{\partial X} - \frac{\partial C(X^*)}{\partial X} = 0$$

$$(12.83) \qquad S = \frac{(1 - \beta)E(s) - \alpha - \lambda(1 - \beta)\text{Cov}(s, R_M)}{1 + r_f}$$

$$(12.86) \qquad \max_{\alpha,\beta} \frac{(1 - \beta)E(s) - \alpha - \lambda(1 - \beta)\text{Cov}(s, R_M)}{1 + r_f}$$

$$+ \mu \left( a\left[E(W) + \alpha + \beta E(s)\right] - b\left[\text{Var}(W) + 2\beta\text{Cov}(W, s) + \beta^2 \text{Var}(s)\right] - \underline{V} \right)$$

$$(12.90) \qquad \beta = \frac{\lambda a \text{Cov}(s, R_M)}{2b\text{Var}(s)} - \frac{\text{Cov}(W, s)}{\text{Var}(s)}$$

$$(12.94) \qquad B_{new} - B = D \left\{ -\left[ P\left( \frac{V + dB}{D + dD}, 1, T, r_f, \sigma_V \right) \right] + P\left( \frac{V}{D}, 1, T, r_f, \sigma_V \right) \right\} = D(-P_X + P_Y)$$

## Chapter 15

(15.2)  $V_U = \dfrac{E(\widetilde{FCF})}{\rho}$  or  $V_U = \dfrac{E(\widetilde{EBIT})(1-\tau_c)}{\rho}$

(15.3)  $\widetilde{NI} + k_d D = \left(\widetilde{Rev} - \widetilde{VC} - FCC - dep\right)(1-\tau_c) + k_d D \tau_c$

(15.4)  $V^L = \dfrac{E(\widetilde{EBIT})(1-\tau_c)}{\rho} + \dfrac{k_d D \tau_c}{k_b}$

(15.6)  $V_L = V_U + \tau_c B$

(15.9)  $\dfrac{\Delta V_L}{\Delta I} = \dfrac{\Delta S^0}{\Delta I} + \dfrac{\Delta S^n + \Delta B^0}{\Delta I} = \dfrac{\Delta S^0}{\Delta I} + 1$

(15.11)  $\dfrac{(1-\tau_c)\Delta E(EBIT)}{\Delta I} > \rho\left(1 - \tau_c \dfrac{\Delta B}{\Delta I}\right)$

(15.18)  $k_s = \Delta NI/\Delta S = \rho + (1-\tau_c)(\rho - k_b)\dfrac{\Delta B}{\Delta S}$

(15.20)  $G = V_L - V_U = \tau_c B$

(15.21)  $V_U = \dfrac{E(EBIT)(1-\tau_c)(1-\tau_{ps})}{\rho}$

(15.23)  $V_L = V_U + \left[1 - \dfrac{(1-\tau_c)(1-\tau_{ps})}{(1-\tau_{pB})}\right] B$    where $B = k_d D(1-\tau_{pB})/k_b$

(15.28)  $\beta_L = \left[1 + (1-\tau_c)\dfrac{B}{S}\right]\beta_U$

(15.33)  $R_f S^L + \lambda^*(1-\tau_c)\text{Cov}(EBIT, R_m) - \lambda^*(1-\tau_c)B\left[\text{Cov}(R_{bj}, R_m)\right]$
$= \text{E}(EBIT)(1-\tau_c) - \text{E}(R_{bj})B(1-\tau_c)$

(15.38)  $dS = \dfrac{\partial S}{\partial V}dV + \dfrac{\partial S}{\partial t}dt + \dfrac{1}{2}\dfrac{\partial^2 S}{2\partial V^2}\sigma^2 V^2 dt$

(15.40)  $r_s = \dfrac{\partial S}{\partial V}\dfrac{V}{S}r_V$

(15.43)  $S = VN(d_1) - e^{-r_f T}DN(d_2)$

(15.46)  $\beta_S = \dfrac{1}{1 - (D/V)e^{-r_f T}\left[N(d_2)/N(d_1)\right]}\beta_V$

(15.47/8)  $k_s = R_f + (R_m - R_f)N(d_1)\dfrac{V}{S}\beta_V = R_f + N(d_1)(R_v - R_f)\dfrac{V}{S}$

(15.52)  $k_b = R_f + (\rho - R_f)N(-d_1)\dfrac{V}{B}$

Note: This is a correction to the formula in the reading.

(15.55)  $\dfrac{dV}{V} = \mu(V,t)dt + \sigma dW$

(15.57) $\quad \frac{1}{2}\sigma^2 V^2 F_{VV}(V) + rV F_V(V) - rF(V) + C = 0$

(15.58) $\quad F(V) = A_0 + A_1 V + A_2 V^{-(2r/\sigma^2)}$

(15.59) $\quad V = V_B \Rightarrow B(V) = (1-\alpha)V_B \qquad V \to \infty \Rightarrow B(V) \to C/r$

(15.61) $\quad B(V) = (1-p_B)C/r + p_B[(1-\alpha)V_B] \quad$ where $\quad p_B = (V/V_B)^{-2r/\sigma^2}$

(15.62) $\quad V = V_B \Rightarrow DC(V) = \alpha V_B \qquad V \to \infty \Rightarrow DC(V) \to 0$

(15.63) $\quad DC(V) = \alpha V_B (V/V_B)^{-2r/\sigma^2}$

(15.67) $\quad V_L(V) = V_U(V) + T_c B(V) - DC(V) = V_U(V) + T_c B - p_B T_c B - \alpha V_B p_B$

(15.68) $\quad M = (1+r)\gamma_0 V_0 + \gamma_1 \begin{cases} V_1 & \text{if } V_1 \geq D \\ V_1 - C & \text{if } V_1 < D \end{cases}$

(15.69) $\quad M_a = \begin{cases} \gamma_0(1+r)\frac{V_{1a}}{1+r} + \gamma_1 V_{1a} & \text{if } D^* < D \leq V_{1a} \\ \gamma_0(1+r)\frac{V_{1b}}{1+r} + \gamma_1 V_{1a} & \text{if } D < D^* \end{cases}$

(15.70) $\quad M_b = \begin{cases} \gamma_0(1+r)\frac{V_{1a}}{1+r} + \gamma_1(V_{1b} - C) & \text{if } D^* \leq D \leq V_{1a} \\ \gamma_0(1+r)\frac{V_{1b}}{1+r} + \gamma_1 V_{1b} & \text{if } D < D^* \end{cases}$

## Chapter 16

(16.2) $\quad V_i(t) = \dfrac{Div_i(t+1) + n_i(t)P_i(t+1)}{1 + k_u(t+1)}$

(16.8) $\quad \widetilde{V}_i(t) = \dfrac{\widetilde{EBIT}_i(t+1) - \widetilde{I}_i(t+1) + \widetilde{V}_i(t+1)}{1 + k_u(t+1)}$

(16.9) $\quad \widetilde{Y}_{di} = \left[\left(\widetilde{EBIT} - rD_c\right)(1-\tau_c) - rD_{pi}\right](1-\tau_{pi})$

(16.10) $\quad \widetilde{Y}_{gi} = (\widetilde{EBIT} - rD_c)(1-\tau_c)(1-\tau_{gi}) - rD_{pi}(1-\tau_{pi})$

(16.26) $\quad S_1 - E(S_1) = \varepsilon_1\left[1 + \dfrac{\gamma}{1+k}\right] = [EBIT_1 - E_0(EBIT_1)]\left[1 + \dfrac{\gamma}{1+k}\right]$

(16.27) $\quad \Delta Div_{it} = a_i + c_i(Div_{it}^* - Div_{i,t-1}) + U_{it}$

(16.33) $\quad \Delta Div_t = \beta_1 Div_{t-1} + \beta_2 NI_t + \beta_3 NI_{t-1} + Z_t$

(16.35) $\quad P_{it} = a + bDiv_{it} + cRE_{it} + \varepsilon_{it}$

(16.38) $\quad \widetilde{R}_j = \gamma_0 + \left[\widetilde{R}_m - \gamma_0\right]\beta_j + \gamma_1\left[DY_j - DY_m\right]/DY_m + \varepsilon_j$

(16.43) $\quad \dfrac{\Delta W}{N_0 P_0} = (1 - F_P)\left(\dfrac{P_E - P_0}{P_0}\right) + F_P\dfrac{P_T - P_0}{P_0}$

# Hull: Options Futures and Other Derivatives

## Chapter 11: Binomial Trees

(11.1) $\quad \Delta = \dfrac{f_u - f_d}{S_0 u - S_0 d}$

(11.5/6) $\quad f = e^{-r\Delta t}\left[pf_u + (1-p)f_d\right] \quad$ where $\quad p = \dfrac{e^{r\Delta t} - d}{u - d}$

(11.13/14) $\quad u = e^{\sigma\sqrt{\Delta t}} \qquad d = e^{-\sigma\sqrt{\Delta t}}$

## Chapter 12: Wiener Processes and Itôs Lemma

(12.17) $\quad G = \ln S \qquad dG = \left(\mu - \sigma^2/2\right)dt + \sigma\,dz$

## Chapter 13: The Black-Scholes-Merton model

(13.9) $\quad df = \left(\dfrac{\partial f}{\partial S}\mu S + \dfrac{\partial f}{\partial t} + \dfrac{1}{2}\dfrac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \dfrac{\partial f}{\partial S}\sigma S dz$

(13.16) $\quad \dfrac{\partial f}{\partial t} + rS\dfrac{\partial f}{\partial S} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 f}{\partial S^2} = rf$

## Chapter 17: Greek Letters

(17. ) $\quad \Delta(\text{call}) = N(d_1) \qquad \Delta(\text{put}) = N(d_1) - 1$

(17.2) $\quad \Theta(\text{call}) = \dfrac{-S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$

(17.2) $\quad \Theta(\text{put}) = \dfrac{-S_0 N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$

(17. ) $\quad \Gamma(\text{call}) = \Gamma(\text{put}) = \dfrac{N'(d_1)}{S_0\,\sigma\sqrt{T}}$

(17.4) $\quad \Theta + rS\Delta + \dfrac{1}{2}\sigma^2 S^2\Gamma = r\Pi$

(17. ) $\quad \mathcal{V}(\text{call}) = \mathcal{V}(\text{put}) = S_0\sqrt{T}N'(d_1)$

(17. ) $\quad \text{rho}(\text{call}) = KTe^{-rT}N(d_2)$

(17. ) $\quad \text{rho}(\text{put}) = -KTe^{-rT}N(-d_2)$

## Chapter 19: Basic Numerical Procedures

(19.8) $\quad \Delta = \dfrac{f_{1,1} - f_{1,0}}{S_0 u - S_0 d}$

(19.9)  $\Gamma = \dfrac{(f_{2,2} - f_{2,1})/(S_0u^2 - S_0) - (f_{2,1} - f_{2,0})/(S_0 - S_0d^2)}{h}$

(19.10)  $\Theta = \dfrac{f_{2,1} - f_{0,0}}{2\Delta t}$

(19.12)  $p = \dfrac{e^{(f(t)-g(t))\Delta t} - d}{u - d}$

(19.27)  $a_j f_{i,j-1} + b_j f_{i,j} + c_j f_{i,j+1} = f_{i+1,j}$

where $a_j = \dfrac{1}{2}(r-q)j\Delta t - \dfrac{1}{2}\sigma^2 j^2 \Delta t, \ \ b_j = 1 + \sigma^2 j^2 \Delta t + r\Delta t$

and $c_j = -\dfrac{1}{2}(r-q)j\Delta t - \dfrac{1}{2}\sigma^2 j^2 \Delta t$

(19.34)  $f_{i,j} = a_j^* f_{i+1,j-1} + b_j^* f_{i+1,j} + c_j^* f_{i+1,j+1}$

where $a_j^* = \dfrac{1}{1+r\Delta t}\left(-\dfrac{1}{2}(r-q)j\Delta t + \dfrac{1}{2}\sigma^2 j^2 \Delta t\right), \quad b_j^* = \dfrac{1}{1+r\Delta t}\left(1 - \sigma^2 j^2 \Delta t\right)$

and $c_j^* = \dfrac{1}{1+r\Delta t}\left(\dfrac{1}{2}(r-q)j\Delta t + \dfrac{1}{2}\sigma^2 j^2 \Delta t\right)$

(19.35)  $\alpha_j f_{i,j-1} + \beta_j f_{i,j} + \gamma_j f_{i,j+1} = f_{i+1,j}$

where $\alpha_j = \dfrac{\Delta t}{2\Delta Z}\left(r - q - \dfrac{\sigma^2}{2}\right) - \dfrac{\Delta t}{2\Delta Z^2}\sigma^2 \qquad \beta_j = 1 + \dfrac{\Delta t}{\Delta Z^2}\sigma^2 + r\Delta t$

and $\gamma_j = \dfrac{-\Delta t}{2\Delta Z}\left(r - q - \dfrac{\sigma^2}{2}\right) - \dfrac{\Delta t}{2\Delta Z^2}\sigma^2$

(19.36)  $\alpha_j^* f_{i+1,j-1} + \beta_j^* f_{i+1,j} + \gamma_j^* f_{i+1,j+1} = f_{i,j}$

where $\alpha_j^* = \dfrac{1}{1+r\Delta t}\left[-\dfrac{\Delta t}{2\Delta Z}\left(r - q - \dfrac{\sigma^2}{2}\right) + \dfrac{\Delta t}{2\Delta Z^2}\sigma^2\right]$

and $\beta_j^* = \dfrac{1}{1+r\Delta t}\left(1 - \dfrac{\Delta t}{\Delta Z^2}\sigma^2\right)$

and $\gamma_j^* = \dfrac{1}{1+r\Delta t}\left[\dfrac{\Delta t}{2\Delta Z}\left(r - q - \dfrac{\sigma^2}{2}\right) + \dfrac{\Delta t}{2\Delta Z^2}\sigma^2\right]$

## Chapter 21: Estimating Volatilities and Correlations

(21.7)  $\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$

(21.8)  $\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$

(21.12)  $\displaystyle\sum_{i=1}^{m}\left[-\ln(v_i) - \dfrac{u_i^2}{v_i}\right]$

(21.13)  $\mathrm{E}\left[\sigma_{n+t}^2\right] = V_L + (\alpha + \beta)^t\left(\sigma_n^2 - V_L\right)$

## Chapter 24: Exotic Options
Call on a call

$$S_0 e^{-qT_2} M\left(a_1, b_1; \sqrt{T_1/T_2}\right) - K_2 e^{-rT_2} M\left(a_2, b_2; \sqrt{T_1/T_2}\right) - e^{-rT_1} K_1 N(a_2)$$

$$\text{where} \quad a_1 = \frac{\ln(S_0/S^*) + (r - q + \sigma^2/2)T_1}{\sigma\sqrt{T_1}} \quad a_2 = a_1 - \sigma\sqrt{T_1}$$

$$b_1 = \frac{\ln(S_0/K_2) + (r - q + \sigma^2/2)T_2}{\sigma\sqrt{T_2}} \quad b_2 = b_1 - \sigma\sqrt{T_2}$$

Put on a call

$$K_2 e^{-rT_2} M\left(-a_2, b_2; -\sqrt{T_1/T_2}\right) - S_0 e^{-qT_2} M\left(-a_1, b_1; -\sqrt{T_1/T_2}\right) + e^{-rT_1} K_1 N(-a_2)$$

Call on a put

$$K_2 e^{-rT_2} M\left(-a_2, -b_2; \sqrt{T_1/T_2}\right) - S_0 e^{-qT_2} M\left(-a_1, -b_1; \sqrt{T_1/T_2}\right) - e^{-rT_1} K_1 N(-a_2)$$

Put on a put

$$S_0 e^{-qT_2} M\left(a_1, -b_1; -\sqrt{T_1/T_2}\right) - K_2 e^{-rT_2} M\left(a_2, -b_2; -\sqrt{T_1/T_2}\right) + e^{-rT_1} K_1 N(a_2)$$

Chooser

$$\max(c, p) = c + e^{-q(T_2 - T_1)} \max\left(0, Ke^{-(r-q)(T_2 - T_1)} - S_1\right)$$

Barrier Options

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2} \qquad y = \frac{\ln\left[H^2/(S_0 K)\right]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$x_1 = \frac{\ln(S_0/H)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \quad y_1 = \frac{\ln(H/S_0)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

If $H \leq K$

$$c_{di} = S_0 e^{-qT}(H/S_0)^{2\lambda} N(y) - Ke^{-rT}(H/S_0)^{2\lambda-2} N\left(y - \sigma\sqrt{T}\right)$$

If $H \geq K$

$$c_{do} = S_0 N(x_1)e^{-qT} - Ke^{-rT} N\left(x_1 - \sigma\sqrt{T}\right)$$
$$- S_0 e^{-qT}(H/S_0)^{2\lambda} N(y_1) + Ke^{-rT}(H/S_0)^{2\lambda-2} N\left(y_1 - \sigma\sqrt{T}\right)$$

If $H > K$

$$c_{ui} = S_0 N(x_1)e^{-qT} - Ke^{-rT} N\left(x_1 - \sigma\sqrt{T}\right)$$
$$- S_0 e^{-qT}(H/S_0)^{2\lambda} \left[N(-y) - N(-y_1)\right]$$
$$+ Ke^{-rT}(H/S_0)^{2\lambda-2} \left[N\left(-y + \sigma\sqrt{T}\right) - N\left(-y_1 + \sigma\sqrt{T}\right)\right]$$

If $H \geq K$

$$p_{ui} = -S_0 e^{-qT}(H/S_0)^{2\lambda} N(-y) + K e^{-rT}(H/S_0)^{2\lambda-2} N\left(-y + \sigma\sqrt{T}\right)$$

If $H \leq K$

$$p_{uo} = -S_0 N(-x_1)e^{-qT} + K e^{-rT} N\left(-x_1 + \sigma\sqrt{T}\right)$$
$$+ S_0 e^{-qT}(H/S_0)^{2\lambda} N(-y_1) - K e^{-rT}(H/S_0)^{2\lambda-2} N\left(-y_1 + \sigma\sqrt{T}\right)$$

If $H \leq K$

$$p_{di} = -S_0 N(-x_1)e^{-qT} + K e^{-rT} N\left(-x_1 + \sigma\sqrt{T}\right) + S_0 e^{-qT}(H/S_0)^{2\lambda}\left[N(y) - N(y_1)\right]$$
$$- K e^{-rT}(H/S_0)^{2\lambda-2}\left[N\left(y - \sigma\sqrt{T}\right) - N\left(y_1 - \sigma\sqrt{T}\right)\right]$$

Lookback Options

$$c_{fl} = S_0 e^{-qT} N(a_1) - S_0 e^{-qT}\frac{\sigma^2}{2(r-q)}N(-a_1) - S_{\min}e^{-rT}\left(N(a_2) - \frac{\sigma^2}{2(r-q)}e^{Y_1}N(-a_3)\right)$$

where $\quad a_1 = \frac{\ln(S_0/S_{\min}) + (r-q+\sigma^2/2)T}{\sigma\sqrt{T}} \qquad a_2 = a_1 - \sigma\sqrt{T}$

$\quad\quad\quad\quad a_3 = \frac{\ln(S_0/S_{\min}) + (-r+q+\sigma^2/2)T}{\sigma\sqrt{T}}$

$\quad\quad\quad\quad Y_1 = \frac{2(r-q-\sigma^2/2)\ln(S_0/S_{\min})}{\sigma^2}$

$$p_{fl} = S_{\max}e^{-rT}\left(N(b_1) - \frac{\sigma^2}{2(r-q)}e^{Y_2}N(-b_3)\right) + S_0 e^{-qT}\frac{\sigma^2}{2(r-q)}N(-b_2) - S_0 e^{-qT}N(b_2)$$

where $\quad b_1 = \frac{\ln(S_{\max}/S_0) + (-r+q+\sigma^2/2)T}{\sigma\sqrt{T}}$

$\quad\quad\quad\quad b_2 = b_1 - \sigma\sqrt{T}$

$\quad\quad\quad\quad b_3 = \frac{\ln(S_{\max}/S_0) + (r-q-\sigma^2/2)T}{\sigma\sqrt{T}}$

$\quad\quad\quad\quad Y_2 = \frac{2(r-q-\sigma^2/2)\ln(S_{\max}/S_0)}{\sigma^2}$

Exchange Options

$$(24.3) \qquad V_0 e^{-q_v T}N(d_1) - U_0 e^{-q_u T}N(d_2)$$

where $\quad d_1 = \dfrac{\ln(V_0/U_0) + (q_U - q_V + \hat{\sigma}^2/2)T}{\hat{\sigma}\sqrt{T}} \quad d_2 = d_1 - \hat{\sigma}\sqrt{T}$

and $\quad \hat{\sigma} = \sqrt{\sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V}$

## Chapter 26: More on Models and Numerical Procedures

$$(26.2, 3) \qquad \frac{dS}{S} = (r-q)dt + \sqrt{V}dz_S \quad dV = a(V_L - V)dt + \xi V^\alpha dz_V$$

## Chapter 27: Martingales and Measures

(27.4)     $\Pi = (\sigma_2 f_2)\, f_1 - (\sigma_1 f_1)\, f_2$

(27.5)     $\Delta\Pi = (\mu_1 \sigma_2 f_1 f_2 - \mu_2 \sigma_1 f_1 f_2)\, \Delta t$

(27.7)     $\dfrac{df}{f} = \mu dt + \sigma dz$

(27.8)     $\dfrac{\mu - r}{\sigma} = \lambda$

(27.13)    $\mu - r = \displaystyle\sum_{i=1}^{n} \lambda_i \sigma_i$

(27.14)    $d\left(\dfrac{f}{g}\right) = (\sigma_f - \sigma_g)\dfrac{f}{g}dz$

(27.20)    $f_0 = P(0,T)E_T(f_T)$

(27.23)    $s(t) = \dfrac{P(t,T_0) - P(t,T_N)}{A(t)}$

(27.25)    $f_0 = A(0)E_A\left[\dfrac{f_T}{A(T)}\right]$

(27.26)    $c = P(0,T)E_T\left[\max(S_T - K, 0)\right]$

(27.31)    $f_0 = U_0 E_U\left[\max\left(\dfrac{V_T}{U_T} - 1, 0\right)\right]$

(27.32)    $f_0 = V_0 N(d_1) - U_0 N(d_2)$

(27.35)    $\alpha_v = \rho \sigma_v\, \sigma_w$

## Chapter 28: Interest Rate Derivatives: The Standard Market Models

(28.1)     $c = P(0,T)\left[F_B N(d_1) - KN(d_2)\right]$

(28.2)     $p = P(0,T)\left[KN(-d_2) - F_B N(-d_1)\right]$

where   $d_1 = \dfrac{\ln(F_B/K) + \sigma_B^2 T/2}{\sigma_B \sqrt{T}}$   $d_2 = d_1 - \sigma_B \sqrt{T}$

(28.3)     $F_B = \dfrac{B_0 - I}{P(0,T)}$

(28.4)     $\sigma_B = D_{y_0} \sigma_y$

(28.7, Caplet)   $L\delta_k P(0, t_{k+1})\left[F_k N(d_1) - R_K N(d_2)\right]$

where   $d_1 = \dfrac{\ln(F_k/R_K) + \sigma_k^2 t_k/2}{\sigma_k \sqrt{t_k}}$   $d_2 = d_1 - \sigma_k \sqrt{t_k}$

(28.8, Floorlet)   $L\delta_k P(0, t_{k+1})\left[R_K N(-d_2) - F_k N(-d_1)\right]$

$$(28.10) \quad LA\left[s_0 N(d_1) - s_k N(d_2)\right]$$

## Chapter 29: Convexity, Timing, and Quanto Adjustments

$$(29.1) \quad E_T(y_T) = y_0 - \frac{1}{2}y_0^2\sigma_y^2 T \frac{G''(y_0)}{G'(y_0)}$$

$$(29.2) \quad E_T(R_T) = R_0 + \frac{R_0^2\sigma_R^2\tau T}{1 + R_0\tau}$$

$$(29.3) \quad \alpha_V = \rho_{VW}\sigma_V\sigma_W$$

$$(29.4) \quad E_{T^*}(V_T) = E_T(V_T)\exp\left[-\frac{\rho_{VR}\sigma_V\sigma_R R_0(T^* - T)}{1 + R_0/m}T\right]$$

## Chapter 30: Interest Rate Derivatives: Models of the Short Rate

$$(30.20, 21) \quad
\begin{aligned}
c &= LP(0,s)N(h) - KP(0,T)N(h - \sigma_p) \\
p &= KP(0,T)N(-h + \sigma_p) - LP(0,s)N(-h) \\
h &= \frac{1}{\sigma_p}\ln\frac{LP(0,s)}{P(0,T)K} + \frac{\sigma_p}{2} \\
\sigma_p &= \frac{\sigma}{a}\left[1 - e^{-a(s-T)}\right]\sqrt{\frac{1 - e^{-2aT}}{2a}}
\end{aligned}$$

$$(30.24) \quad P_{m+1} = \sum_{j=-n_m}^{n_m} Q_{m,j}\exp\left[-g\left(\alpha_m + j\Delta x\right)\Delta t\right]$$

## Chapter 32: Swaps Revisited

$$(32.1) \quad F_i + \frac{F_i^2\sigma_i^2\tau_i t_i}{1 + F_i\tau_i}$$

$$(31.2) \quad y_i - \frac{1}{2}y_i^2\sigma_{y,i}^2 t_i\frac{G_i''(y_i)}{G_i'(y_i)} - \frac{y_i\tau_i F_i\rho_i\sigma_{y,i}\sigma_{F,i}t_i}{1 + F_i\tau_i}$$

$$(32.3) \quad V_i + V_i\rho_i\sigma_{W,i}\sigma_{V,i}t_i$$

# Hardy: Investment Guarantees

## Chapter 6

(6.5)  $\quad F_{t^+} = F_{t^-}(1-m) = F_{(t-1)}(1-m)\dfrac{S_t}{S_{t-1}}$

(6.9)  $\quad M_t = (F_{t^-})\,m_c = m_c F_{0^-}\dfrac{s_t(1-m)^{t-1}}{S_0}$  *errata sheet*

(6.11)  $\quad C_n = {}_n p_x^\tau (G - F_n)^+$  *errata sheet*

(6.13)  $\quad C_t = -\,{}_t p_x^\tau M_t + {}_{t-1|1}q_x^d (G - F_t)^+$

(6.14)  $\quad C_t = -\,{}_t p_x^\tau F_{0^-}\,S_t(1-m)^t m_d + {}_{t-1|1}q_x^d\left(G - F_0 - S_t(1-m)^t\right)^+$  *errata sheet*

## Chapter 8

(8. )  $\quad F_T = F_0 \dfrac{S_T}{S_0}(1-m)^T$

(8.3)  $\quad P_0 = Ge^{-rT}\Phi(-d_2) - S_0(1-m)^T \Phi(-d_1)$

$\quad\quad\quad$ where  $d_1 = \dfrac{\log\left(S_0\,(1-m)^T/G\right) + (r+\sigma^2/2)T}{(\sigma\sqrt{T})}$  $\quad d_2 = d_1 - \sigma\sqrt{T}$

(8.8)  $\quad H(0) = \displaystyle\int_0^n \left(Ge^{-rt}\Phi(-d_2)\right) {}_t p_x^\tau\,\mu_{x,t}^{(d)}\,dt + \int_0^n \left(-S_0(1-m)^t\Phi(-d_1)\right) {}_t p_x^\tau\,\mu_{x,t}^{(d)}\,dt$

(8.9)  $\quad H(0,t_3) = \left(P_S(t_1) + S_0(1-m)^{t_1}\right)\times$

$\quad\quad\quad\quad\quad\quad \left\{1 + P(t_2 - t_1)\left(1 + P(t_3 - t_2)\right) + (1-m)^{t_2-t_1}\,P(t_3 - t_2)\right\}$

$\quad\quad\quad\quad\quad\quad - S_0(1-m)^{t_1}$

$\quad\quad\quad$ where  $P_S(t_1) = \mathrm{BSP}(S_0(1-m)^{t_1}, G, t_1)$

(8.15)  $\quad \alpha = \dfrac{B}{S_0\,\ddot{a}^\tau_{x:\overline{n}|i'}}$

(8.19)  $\quad \mathrm{HE}_t = H(t) + {}_{t-1}|q_x^d\left((G - F_t)^+\right) - H(t^-)$

(8.22)  $\quad \mathrm{TC}_t = \tau\, S_t\,|\Psi_t - \Psi_{t-1}|$

## Chapter 9

(9.2)  $\quad 100\beta\%$ Confidence Interval for $\alpha$-quantile:

$\quad\quad\quad (L_{0(N\alpha-A)},\, L_{0(N\alpha+A)});\ \ A = \Phi^{-1}\left(\dfrac{1+\beta}{2}\right)\sqrt{N\alpha(1-\alpha)}$

(9.4)  $\quad \xi = 1 - \Phi\left(\dfrac{\log G/S_0 - n(\mu + \log(1-m))}{\sqrt{n}\sigma}\right)$

$$(9.7) \qquad V_\alpha = \left(G - F_0 \exp\left(-z_\alpha \sqrt{n}\sigma + n(\mu + \log(1-m))\right)\right) e^{-rn}$$

$$(9.17) \qquad \mathrm{CTE}_\alpha(L) = e^{-rn}\left\{G - \frac{e^{n(\mu + \log(1-m) + \sigma^2/2)}}{1-\alpha}\Phi\left(-z_\alpha - \sqrt{n}\sigma\right)\right\}$$

## Chapter 12

$$(12.4) \qquad H_0 = B(0,n)E_Q\left[F_n\left(ga_{65}(n) - 1\right)^+\right]$$

$$(12.5) \qquad H_0 = F_0 E_Q\left[\left(\frac{ga_{65}^d(0,n)}{B(0,n)} - 1\right)^+\right]$$

$$(12.7) \qquad H_t = F_t\left\{ga_{65}(t)\Phi(d_1(t)) - \Phi(d_2(t))\right\}$$

$$\text{where} \quad d_1(t) = \frac{\log\left(ga_{65}(t)\right) + \sigma_y^2(n-t)/2}{\sigma_y\sqrt{n-t}}, \qquad d_2(t) = d_1(t) - \sigma_y\sqrt{n-t}$$

## Chapter 13

$$(13.3) \qquad \text{PTP:} \ H = \left(P\left(1 + \alpha\left(\frac{S_n}{S_0} - 1\right)\right) - G\right)^+$$

$$= \frac{\alpha P}{S_0}\left\{S_n - \frac{S_0}{\alpha}\left(\frac{G}{P} - (1-\alpha)\right)\right\}^+$$

$$(13.7) \qquad H_0 = \alpha P e^{-dn}\Phi(d_1) - (G - P(1-\alpha))\, e^{-rn}\Phi(d_2)$$

$$\text{where} \quad d_1 = \frac{\ln\frac{\alpha P}{G - P(1-\alpha)} + \left(r - d + \frac{\sigma^2}{2}\right)n}{\sigma\sqrt{n}}, \quad d_2 = d_1 - \sigma\sqrt{n}$$

$$(13.16) \qquad \text{CAR:} \ H = P\left\{e^{-r} + \alpha\left(e^{-d}\Phi(d_1) - e^{-r}\Phi(d_2)\right)\right\}^n$$

**FET-108-07 and FET-165-08: Doherty; Integrated Risk Management Chapter 13**

$$(p.\,465) \qquad \begin{aligned} V(E) &= V(F) - V(D) \\ &= V(F) - D_{DF} + P(V(F), D) \\ &= C(V(F), D) \end{aligned}$$

$$(p.\,481) \qquad V^*(F) = S + D\left(1 + \frac{m}{n}\right)$$

$$(p.\,494) \qquad \begin{aligned} V_R'(E) &= -C + V_R(F) - D + P\{V_r(F), D\} \\ &= -C + V_R - D + P_R \\ V_N'(E) &= V_N(F) - D + P\{V_N(F), D\} \\ &= V_N - D + P_N \end{aligned}$$

## Chapter 16

(16.1)  $T + (E + P)(1 + r_i) - L$

(16.2)  $E(T) = (E - P - R(I; S))(1 + E(R_i)) - E(L(a)) + hC(I; S) - a$

(16.3)  $\dfrac{\partial E(T)}{\partial a} = \dfrac{-\partial E(L(a))}{\partial a} - 1 + h\dfrac{\partial C}{\partial I}\dfrac{\partial I}{\partial L}\dfrac{\partial L}{\partial a} = 0$

## FET-159-08: Babbel and Fabozzi, Investment Management for Insurers
## Chapter 13

(8)  $\begin{aligned} C(t) = \ & P(t, TB)\chi^2\left(2r^*\left(\varphi + \psi + G(T, TB)\right), \tfrac{4c}{\sigma^2}, \tfrac{2\varphi^2 r e^{\gamma(T-t)}}{(\varphi + \psi + G(T, TB))}\right) \\ & - X\,P(t, T)\chi^2\left[2r^*(\phi + \psi), \tfrac{4c}{\sigma^2}, \tfrac{2\phi^2 r e^{\gamma(T-t)}}{(\phi + \psi)}\right] \end{aligned}$

## FET-106-07: Ho and Lee, The Oxford Guide to Financial Modeling
## Chapter 5: Interest Rate Derivatives: Interest Rate Models

(5.10)  $\begin{aligned} dr &= a_1 + b_1(l - r)dt + r\sigma_1 dZ \\ dl &= (a_2 + b_2 r + c_2 l)\,dt + l\sigma_2 dW \end{aligned}$

(5.12)  $\begin{aligned} dV &= M(t, r)dt + \Omega(t, r)dZ \\ M(t, r) &= V_t + \mu(t, r)V_r + \tfrac{1}{2}\sigma(t, r)^2 V_{rr} \\ \Omega(t, r) &= \sigma(t, r)V_r \end{aligned}$

## Chapter 6: Implied Volatility Surface: Calibrating the Models

(6.10)  $L(k, j + 1)$

$$= L(k, j)\exp\left[\left(\sum_{i=j+1}^{k} \tfrac{L(i,j)\Delta}{1 + L(i,j)\Delta}\Lambda_{i-j-1}\Lambda_{k-j-1} - \tfrac{\Lambda_{k-j-1}^2}{2}\right)\Delta + \Delta_{k-j-1}\sqrt{\Delta}Z\right]$$

(6.13)  $P(T^*, i\,; T) = \dfrac{P(T^* + T)}{P(T^*)}\,2\,\dfrac{\prod\limits_{t=T}^{T+T^*-1} h(t)}{\prod\limits_{t=1}^{T^*-1} h(t)}\,\delta^{Ti} \quad \text{where } h(t) = \dfrac{1}{1 + \delta t}$

## FET-145-08: The Cost of Capital for Financial Firms

p.27  $E_0 + F_0 = \sum_{t=1}^{\infty} \dfrac{D_t}{(1 + COE)^t}$

$$= \sum_{t=1}^{\infty} \frac{D_t + E_t - E_{t-1} - COE * E_{t-1}}{(1+COE)^t} - \sum_{t=1}^{\infty} \frac{E_t}{(1+COE)^t} + \sum_{t=1}^{\infty} \frac{E_{t-1}}{(1+COE)^{t-1}}$$

$$F_0 = \sum_{t=1}^{\infty} \frac{ROE_t - COE}{(1+COE)^t} E_{t-1}$$

p.36
$$(1 + R_f)\{A_0 - L_0 + F_0\} = A_0 - L_0 + (1 - k_T)\{A_0\,(R_f + m_A - k_A)$$
$$-L_0\,(R_f - m_L + k_L)\} + F_1$$
$$(1 + R_f)F_0 = (1 - k_T)\{A_0\,(m_A - k_A) + L_0\,(m_L - k_L)\} - k_T R_f(A_0 - L_0) + F_1$$

p.39
$$(1 + R_f)F_0 = A_0(1 - k_T)(m_A - k_A) + L_0(1 - k_T)(m_L - k_L) + (1 - s)F_1$$
$$-(R_f + s)k_T(A_0 - L_0)$$

p.40
$$(1 + R_f)(A_0 - L_0 + F_0) = A_0\,\{1 + (1 - k_T)(R_f - m_A - k_A)\}$$
$$-L_0\,\{1 + (1 - k_T)(R_f - m_L - k_L)\} + (1 - s)F_1 - sk_T(A_0 - L_0)$$

p.41
$$(R_f + s - g + sg)F_0 = A_0(1 - k_T)(m_A - k_A) + L_0(1 - k_T)(m_L - k_L)$$
$$-(R_f + s)k_T(A_0 - L_0)$$

## FET-146-08: Solvency Measurement for Property-Liability Risk-Based Capital Applications

(4a)
$$d_L = \frac{D_L}{L} = k\phi\left[\frac{-c}{k}\right] - c\Phi\left[\frac{-c}{k}\right]$$

$$d_A = \frac{D_A}{L} = \frac{1}{1 - c_A}\left[k_A\phi\left(\frac{-c}{k_A}\right) - c_A\Phi\left(\frac{-c_A}{k_A}\right)\right]$$

(5a)
$$d_L = \Phi(a) - (1 + c)\Phi(a - k)$$

(5b)
$$d_A = \Phi(b) - \frac{\Phi(b - k_A)}{1 - c_A}$$

where
$$a = (k/2) - (\ln(1 + c)/k) \qquad b = (k_A/2) + (\ln(1 + c_A)/k_A)$$

(6)
$$d_1 = \int_{-\infty}^{0} -zp(z)dz$$

(7)
$$\bar{c}_1 = c(1 + p) + [1 + c(1 + p)]\tilde{r} + pc\tilde{b} - \tilde{g}$$

(8)
$$C = \left[\sum_{i=1}^{n} c_i^2 + \sum_{i \neq j}^{n} \rho_{ij}c_ic_j\right]^{1/2}$$

(A1)
$$D = \sigma\phi\left(\frac{-\mu}{\sigma}\right) - \mu\Phi\left(\frac{-\mu}{\sigma}\right)$$

$$(A2) \quad d = \frac{D}{L} = k_T\phi\left(\frac{-c}{k_T}\right) - c\ \Phi\left(\frac{-c}{k_T}\right)$$

$$(A3) \quad d_L = k\phi\left(\frac{-c}{k}\right) - c\ \Phi\left(\frac{-c}{k}\right)$$

$$(A4) \quad d_A = \frac{D_A}{L} = \frac{1}{1-c_A}\left[k_A\phi\left(\frac{-c_A}{k_A}\right) - c_A\Phi\left(\frac{-c_A}{k_A}\right)\right]$$

$$(A5) \quad F = S\Phi(a) - Ee^{-it}\Phi\left(a - \sigma\sqrt{t}\right)$$

$$\text{where} \quad a = \frac{\ln(S/E) + (i + \sigma^2/2)t}{\sigma\sqrt{t}}$$

$$(A6) \quad D_L = L\Phi(a) - (1+c)L\Phi(a - \sigma_L)$$

$$(A7) \quad d_L = \Phi(a) - (1+c)\Phi(a - k)$$

$$(A8) \quad D'_L = A\Phi(a') - L\Phi(a' - \sigma_A)$$

$$(A9) \quad d_A = \Phi(b) - \frac{\Phi(b - k_A)}{1 - c_A} \quad \text{where } b = (k_A/2) + (\ln(1 - c_A)/k_A)$$

## FET-114-07: Capital Allocation in Financial Firms

p.116
$$\begin{aligned}
NPV &= (1-d)V\{S^+\} - (C - \mu) - (1+m)V\{S^-\} \\
&= \mu - (dV\{S^+\} + mV\{S^-\}) \\
V\{S^+\} &= \frac{\sigma(n(z) + zN(z))}{(1+r)} \\
V\{S^-\} &= \frac{\sigma(n(z) - zN(-z))}{(1+r)} \\
z &= C(1+r)/\sigma
\end{aligned}$$

## FET-151-08: Babbel and Merrill, Real and Illusory Value Creation by Insurance Companies

$$(2) \quad V(E) = F + V(A_T) - PV(L) + O$$

## FET-171-10: A Comparative Analysis of U.S. Canadian and Solvency II Capital Adequacy Requirements in Life Insurance, Sharara, Hardy and Saunders

p 8
$$CCM = i_{ccr}\sum_{t=1}^{T} SCR(t-1)e^{-r(t)t}$$

p 9
$$SCR = \sqrt{\sum_i \sum_j \rho(i,j)\,SCR_i\,SCR_j}$$

**Hardy, Freeland and Till, Valuation of Long-Term Equity Return Models for Equity-Linked Guarantees**

$$\text{MARCH} \qquad Y_t|\mathfrak{F}_{t-1} = \begin{cases} Q_1 & \text{w.p. } q \\ Q_2 & \text{w.p. } 1-q \end{cases}$$

$$\text{where } Q_1|\mathfrak{F}_{t-1} = \mu_1 + \sigma_t z_t$$

$$\sigma_t^2 = \alpha_{1,0} + \alpha_{1,1}(Y_{t-1} - \mu_1)^2 + \alpha_{1,2}(Y_{t-2} - \mu_1)^2$$

$$Q_2|\mathfrak{F}_{t-1} = \mu_2 + \alpha_{2,0} z_t \qquad z_t \sim N(0,1),\ i.i.d.$$

$$\text{RSDD} \qquad Y_t|\rho_t = \kappa_{\rho_t} + \varphi_{\rho_t} D_{t-1} + \sigma_{\rho_t} z_t$$

$$\text{where } D_{t-1} = \min(0, D_{t-2} + Y_{t-1})$$

$$\text{RSGARCH} \qquad Y_t|\rho_t = \mu_{\rho_t} + \sigma_{\rho_t,t} z_t$$

$$\text{where } \sigma_{\rho_t,t}^2 = \alpha_{\rho_t,0} + \alpha_{\rho_t,1}\varepsilon_{t-1}^2 + \beta_{\rho_t}\sigma_{t-1}^2$$

$$\varepsilon_t = Y_t - (p_1(t)\mu_1 + (1-p_1(t))\mu_2)$$

$$\sigma_t^2 = p_1(t)(\mu_1^2 + \sigma_{1,t}^2) + (1-p_1(t))(\mu_2^2 + \sigma_{2,t}^2) - (p_1(t)\mu_1 + (1-p_1(t))\mu_2)^2$$

$$\text{AAA Model} \quad Y_t = \mu_t/12 + (\sigma_t/\sqrt{12})z_{y,t}$$

$$\text{where } \mu_t = A + B\sigma_t + C\sigma_t^2$$

$$\log \sigma_t = \nu_t = (1-\varphi)\nu_{t-1} + \varphi\log\tau + \sigma_\nu z_{\nu,t}$$

$$(4.1) \qquad r_{t,1} = r_t|(\rho_t = 1) = \frac{y_t - \mu_1}{\sigma_1}$$

$$(4.2) \qquad r_{t,2} = r_t|(\rho_t = 2) = \frac{y_t - \mu_2}{\sigma_2}$$

$$(4.3) \qquad r_t = I_{\{\rho(t)>0.5\}}r_{t,1} + \left(1 - I_{\{\rho(t)>0.5\}}\right)r_{t,2}$$

**Smith: Investor & Management Expectations of the Return on Equity Measure vs. Some Basic Truths of Financial Accounting**

$$E_t - EV_t = \sum_{x=1}^{t}\left[(ROE_x - IRR)E_{x-1}(1+IRR)^{t-x}\right]$$