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THE CONSTRUCTION OF PERSISTENCY TABLES

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PERSISTENCY tables are frequently needed by the actuary in valuing future expenses and in various types of forecasting, including the building of model companies or model agencies. He may construct his own tables, or more often he may base his calculations upon a published table, provided of course tests show that the table chosen reflects satisfactorily the withdrawal and death rates that are appropriate for the particular circumstances. Some discussion of questions that arise in construction of persistency tables and some illustrative solutions, including a family of derived tables, may therefore be useful to members of the Society.

Existing Persistency Tables

The development of persistency tables on this continent was pioneered by two actuaries who used distinctly different approaches to the problem. Mr. P. C. H. Papps in 1919 (*RAIA* VIII, 13) used the method of establishing a desired value of the percentage surviving at the tenth policy year, and then deriving values for earlier durations by mathematical formulas. In 1924 Mr. M. A. Linton (*RAIA* XIII, 283) developed year-by-year withdrawal rates by direct investigation of published data, then combined them with select mortality rates at entry age 40 to compute his well-known "A" Table. To illustrate the importance of persistency he also constructed a "B" Table in which each year's voluntary withdrawal rate was double that of the "A" Table. Others have since computed extensions, blends and parallels of these original tables. The extensive use of Mr. Linton's tables throughout the 36 years since their publication testifies to their practical value.

Procedure for Construction of Tables

Apart from the question, to be considered later, of the actual withdrawal and mortality factors to be employed, there seem to be just two refinements that are clearly worth introducing into the procedure that Mr. Linton adopted. The first of these is to allow for voluntary withdrawals during each policy year arising from fractional premium business, thus creating a distinction between business that enters each policy year and business that pays a full premium for that policy year.

Introduction of an allowance for fractional premium business makes it important that tabular values be carefully defined. Among actuaries today there is lack of unanimity as to the meaning of some of the terms that must be used, particularly the meaning of the "*n*th year" lapse (or withdrawal) rate. Some define first year lapse, for example, as paid-for business that fails to pay the entire premium for the first policy year, others as business that fails to pay any part of the premium for the second policy year. In this paper the first of these two possible definitions is used.

The second refinement is to incorporate mortality rates that are blended from various issue ages rather than based upon a single average issue age. The difficulty with the latter is that the weighted average issue age which is appropriate at the low durations tends to produce mortality rates at the high durations that are too large because the average age of a group of policyholders does not increase by a full year for each year of duration. The blended-age method produces mortality rates that move up more slowly.

Some may feel that there is another desirable element that has not been taken into account either in Mr. Linton's tables or in the tables that appear in this paper, namely provision for (*a*) terminations by endowment maturity or term expiry, and (*b*) discontinuation of premium payments at the end of the premium period on limited payment plans of insurance. It is true that the actuary must make allowances for these occurrences, but nevertheless no such provision has been made here because to do so on the basis of one assumed combination of policy plans would greatly limit the practical usefulness of the results. Temporary annuities for valuation of such contracts can readily be used. Furthermore, the actuary frequently finds it convenient to make valuations and comparisons on the assumption that all business is on the ordinary life plan.

It may also be felt that allowance should be made for the obvious fact that the premium per thousand on a block of business tends to change at the longer durations as a result of differing withdrawal and mortality rates on business issued at different ages. This refinement can readily be introduced by the processes described in this paper.

Derivation of Formulas Used in This Paper

1. *Formulas for number of units of business remaining in force at end of each policy year.* To begin with, it is assumed that the withdrawal rates and mortality rates available for use in constructing the tables are annual rates of decrement $q_t'^{(w)}$ and $q_t'^{(d)}$ operating independently in policy year t, leading to a relationship between l_{t-1} and l_t, where l_t is defined as the number of units that remain in force at the end of policy

year t. Then, defining w_t and d_t as the numbers ceasing by withdrawal and death respectively in policy year t,[1]

$$l_t = l_{t-1} - w_t - d_t$$

$$w_t = q_t'^{(w)}\,(l_{t-1} - .5\,d_t)$$

$$d_t = q_t'^{(d)}\,(l_{t-1} - .72\,w_t) \text{ when } t = 1$$

$$d_t = q_t'^{(d)}\,(l_{t-1} - .84\,w_t) \text{ when } t > 1\,.$$

The factors .72 and .84 develop arithmetically from specific assumptions as to the relative proportions of annual, semiannual, quarterly and monthly premium business (using in these illustrations 35% A, 5% SA, 20% Q, 40% M) combined with the assumptions stated in the next paragraph as to the distributions of withdrawals in respect to premium due dates. The factor .5 comes of course from the assumption that deaths are distributed evenly throughout the year.

For withdrawals during the first policy year, the need for assumptions arises only on quarterly premium and monthly premium policies. On quarterlies the distribution of the withdrawals is assumed to be in the proportions 75:15:10 for the three premium due dates involved. On monthlies the corresponding eleven factors are 40:20:20:5:3:2:2:2:2:2:2. For the second and later policy years 50% of the withdrawals under each fractional mode of payment are assumed to occur on the policy anniversary, while the other 50% are evenly distributed over the remaining premium due dates.

Solving these simultaneous equations produces the following expressions for w_t and d_t.

$$w_t = \frac{l_{t-1}\,q_t'^{(w)}\,(1 - .5\,q_t'^{(d)})}{1 - .36\,q_t'^{(w)}\,q_t'^{(d)}} \text{ when } t = 1$$

$$w_t = \frac{l_{t-1}\,q_t'^{(w)}\,(1 - .5\,q_t'^{(d)})}{1 - .42\,q_t'^{(w)}\,q_t'^{(d)}} \text{ when } t > 1$$

[1] To make the tables (particularly Exhibit 2) most readily comprehensible it has been found desirable to use the subscript t for decrements in the tth year, a departure from the notation that results from omission of issue age from the familiar subscript of standard notation. This variation is for the special circumstances of this paper and is, of course, not intended to set a standard.

$$d_t = \frac{l_{t-1} q_t'^{(d)} (1 - .72 q_t'^{(w)})}{1 - .36 q_t'^{(w)} q_t'^{(d)}} \text{ when } t = 1$$

$$d_t = \frac{l_{t-1} q_t'^{(d)} (1 - .84 q_t'^{(w)})}{1 - .42 q_t'^{(w)} q_t'^{(d)}} \text{ when } t > 1 \ .$$

2. *Formulas for number of units of premium paid in each policy year.* The function l_t', the number of units of premium paid in policy year t, comes directly from the value of l_{t-1} on the assumptions stated above together with the additional assumption in these illustrations that premiums on the deaths are paid only to the date of death. The relationship is

$$l_t' = l_{t-1} - .5 d_t - .72 w_t \text{ when } t = 1$$

$$l_t' = l_{t-1} - .5 d_t - .84 w_t \text{ when } t > 1 \ .$$

The factors .72 and .84 appear unchanged in these expressions because for both withdrawals and deaths the premium-paying period and the exposure period are identical under the assumptions here employed.

3. *Formulas for withdrawal and death rates to be used when combining tables.* The problem discussed in this section arises when separate tables are developed for different ages at issue, and it is desired to combine these into a single table weighted in specific proportions. In this paper the problem is primarily that of obtaining weighted death rates since it was only at the high durations that any variation in withdrawal rates by issue age was assumed.

The procedure followed is first to obtain values of the combined l_t weighted in the desired proportions, then to work back to the values of the combined $q_t'^{(w)}$ and $q_t'^{(d)}$.

$q_t'^{(w)}$, in the few cases for which this was needed, was taken as the withdrawal rate that emerges by weighting the corresponding withdrawal rates in the individual tables in proportion to the values of l_{t-1}. The following expressions for $q_t'^{(d)}$ are then obtained by substituting the formulas already stated for w_t and d_t in the relationship $l_t = l_{t-1} - w_t - d_t$, and solving for $q_t'^{(d)}$.

$$q_t'^{(d)} = \frac{l_{t-1} (1 - q_t'^{(w)}) - l_t}{l_{t-1} [1 - .86 q_t'^{(w)}] - .36 q_t'^{(w)} l_t} \text{ when } t = 1$$

$$q_t'^{(d)} = \frac{l_{t-1} (1 - q_t'^{(w)}) - l_t}{l_{t-1} [1 - .92 q_t'^{(w)}] - .42 q_t'^{(w)} l_t} \text{ when } t > 1 \ .$$

4. *Formulas for present values of units of premium paid in each policy year.* In deriving values of the function E'_t, the present value at date of issue of a premium paid in year t, interest adjustment was made for fractional premiums paid on their due dates on policies remaining in force during the year. Tests showed that the corresponding interest adjustment for withdrawals and deaths among fractional premium policies was small enough to be ignored. On the distribution by modes of payment assumed in this study and the 10,000 radix employed, the relationship is

$$E'_t = \frac{v^{t-1} l'_t \times r}{10{,}000},$$

where $r = .99336$ at $2\frac{1}{2}\%$ interest, $r = .98951$ at 4% interest.

Factors for Construction of Illustrative Tables

In assembling the factors to be used in putting the principles already stated into practice the aim has been to produce tables that stand a good chance of being useful in practical conditions that actuaries may encounter. No pretense whatever is made that these are standard tables that fit any single known experience, and certainly no inference that they represent industry averages or yardsticks of any kind is justified. The most that can be suggested is that sufficient variety has been created to provide a spectrum of choices. The withdrawal factors used are not even derived precisely from any persistency investigation. However, to repeat a phrase used admirably by Mr. Linton in 1924, their selection was "largely influenced" by a study of two sets of figures.

Early policy years. At the suggestion of the author of this paper, the Compensation Committee of the Life Insurance Agency Management Association requested that a persistency study be made from an already existing sample of 12,000 policies sold in May 1949 by the ordinary agents of 54 companies. The results covering the 9-year period from issue to May 1958 have been published by the Association in its *Research Report, 1960–3* entitled "*Persistency 1949–1958.*"

In that study the material from the 54 companies was combined into three roughly equal groups, based upon the ranking of each company in terms of two-year persistency by number of policies. From the following table it will be observed that the proportions of business on which premiums are assumed to be paid for years 1–9 inclusive in this present author's Table R, Table S and Table T bear a similarity to those reported for whole life, continuous payments, on page 7 of the *Research Report.*

Later policy years. It has often been pointed out– for example, by Mr. C. F. B. Richardson on page 364 of his paper "Lapse Rates" (*TSA* III, 338)—that lapse rates at later policy durations are extremely volatile. In Mr. Richardson's illustration the aggregate termination rate for the third and later policy years was shown to be easily capable of doubling or halving within a period of two or three calendar years. This being the case, it appears unfruitful to measure with apparent precision the rates that happen to exist at any particular moment. What is needed is some measurement of the size of the practical variation that may be experienced under present-day conditions.

Number of Full Years' Premiums Paid	Percentages of New Business Remaining on Premium-Paying Status					
	LIAMA "X" Cos.*	Table R of This Paper	LIAMA "Y" Cos.*	Table S of This Paper	LIAMA "Z" Cos.*	Table T of This Paper
1	92.9%	92.86%	87.5%	87.37%	80.2%	79.88%
2	86.1	88.03	78.2	78.47	64.9	63.78
3	83.4	84.74	75.5	74.75	60.5	59.17
4	81.0	81.94	73.1	71.91	57.8	56.33
5	79.0	79.41	71.0	69.51	56.6	53.89
6	75.9	77.03	68.1	67.25	53.6	51.74
7	73.9	74.87	67.2	65.14	51.8	49.90
8	71.8	72.92	65.9	63.12	50.0	48.11
9	70.1	71.12	64.5	61.16	48.8	46.35

* As defined in LIAMA *Research Report 1960-3*, *"Persistency 1949-1958."*

A letter was therefore sent to the actuaries of 65 companies inviting them to furnish from their own experience the values of ${}_{1959}Z_{t+1}/{}_{1958}Z_t$ and also ${}_{1956}Z_{t+1}/{}_{1955}Z_t$ for just four values of t, $t = 12$, $t = 17$, $t = 22$, $t = 27$, where ${}_yZ_n$ is the amount of business on the ordinary life plan that was in force on December 31 of calendar year y and which completed its nth policy year in calendar year y. The reason for specifying the ordinary life plan was simply to avoid the disturbing effect of maturing endowments and of limited payment policies transferred in valuation records from premium paying to paid-up.

Of the 65 companies, 49 furnished values of the requested ratio for the year 1959, and 41 companies furnished values for the year 1956. When the results are ranked in order of size of the ratio (for each of the 4 policy durations independently of each other) and are divided into three blocks by number of companies, the comparisons with Tables R, S and T of this paper are as shown in the following table.

The tendency of the ratios furnished by each company to differ in superiority at the various durations is marked. For example, the coefficients of rank correlation by company between the ratio for $t = 12$ and for $t = 17$ were only 0.34 for the 1959 result and 0.42 for the 1956 result. Also there was no positive relationship apparent between the quality of the results at these longer durations and the early persistency shown by

	Median Values of Z_{t+1}/Z_t			
	$t=12$	$t=17$	$t=22$	$t=27$
High One-Third of Companies				
1959 Result	98.0%	97.6%	97.3%	96.1%
1956 Result	98.0	98.1	98.2	97.1
Table R of This Paper*	(97.5)	(97.3)	(96.9)	(96.1)
Middle One-Third of Companies				
1959 Result	97.1	96.8	95.9	95.1
1956 Result	97.3	97.5	96.4	95.3
Table S of This Paper*	(96.8)	(96.6)	(96.4)	(95.2)
Low One-Third of Companies				
1959 Result	96.2	95.4	95.0	92.7
1956 Result	96.7	96.2	95.3	94.3
Table T of This Paper*	(96.0)	(95.6)	(95.0)	(93.8)

* Values quoted here are $\frac{1}{2}(a + b)$, where a is the proportion of business persisting from the end of the tth to the end of the $(t + 1)$th policy year, and b is the proportion of business persisting from the end of policy year $t + 1$ to the end of policy year $t + 2$.

the LIAMA results. The unweighted averages of the 1959 results for companies that contributed to both studies were as follows:

	Unweighted Average Values of Z_{t+1}/Z_t			
	$t=12$	$t=17$	$t=22$	$t=27$
14 LIAMA "X" Companies*	96.6%	96.1%	95.3%	94.4%
12 LIAMA "Y" Companies*	96.8	97.2	96.4	96.0
7 LIAMA "Z" Companies*	97.3	96.6	97.6	92.9

* Same definition as earlier in this paper.

Persistency Tables Constructed by Methods Described in This Paper

Tables, designated R, S and T, as already indicated, have been constructed embodying the voluntary withdrawal rates shown in Exhibit 1 following.

Mortality rates used in conjunction with these withdrawal rates were 125% of the select rates for issue ages 5, 30 and 50 respectively from Mr. Norman F. Buck's Ordinary Select 1950–54 Mortality Table (*TSA* IX, 38–39). The added 25% was to allow for a proportion of substandard business.

The steps in the construction were as follows:

(1) Three "R" tables for ages 5, 30 and 50, respectively, were constructed, and these were combined in the proportions: 5% at age 5, 65% at age 30, 30% at age 50. Likewise three "S" and three "T" tables, and combinations thereof in the same proportions as for the "R" tables, were constructed. The mortality rates actually used to construct the combined-age tables were the averages of the three sets of rates that emerged, since these differed slightly from each other because of the limited size of the radix that was adopted.

(2) In view of the apparent absence of correlation between persistency experience of early and late policy years, four blended tables were constructed, designated R/S, S/R, S/T and T/S respectively. In each case the first letter indicates the withdrawal rates used in the first four policy years, and the second letter indicates the withdrawal rates used in durations eight and later. Values for the three intervening durations are blends of the two in the respective proportions 75:25, 50:50, and 25:75.

The balance of this paper gives the withdrawal and death rates employed, and the resulting family of persistency tables, as follows:

Exhibit 1. Voluntary Withdrawal Rates and Death Rates.
Exhibit 2. Numbers ceasing by withdrawal (w_t) and death (d_t) and numbers completing each policy year (l_t).
Exhibit 3. Numbers of premium units paid in each policy year (l_t').
Exhibits 4*a*, 4*b*, 4*c*. Present values of premium units.

Exhibit 4*a*	No interest.
Exhibit 4*b*	$2\frac{1}{2}\%$ interest.
Exhibit 4*c*	4% interest.

The essential contributions to this paper by Charles A. Yardley, F.S.A., who developed the formulas and supervised the arithmetical work, are gratefully acknowledged.

EXHIBIT 1

VOLUNTARY WITHDRAWAL RATES AND DEATH RATES

WITHDRAWAL RATES DURING POLICY YEAR t ($100\ q_t^{\prime(w)}$)

Policy Year t	Table R	Table S	Table T	Table R/S	Table S/R	Table S/T	Table T/S
1	7.00	12.50	20.00	7.00	12.50	12.50	20.00
2	5.00	10.00	20.00	5.00	10.00	10.00	20.00
3	3.50	4.50	7.00	3.50	4.50	4.50	7.00
4	3.00	3.50	4.50	3.00	3.50	3.50	4.50
5	2.75	3.00	4.00	2.81	2.94	3.25	3.75
6	2.50	2.75	3.50	2.62	2.63	3.12	3.13
7	2.25	2.60	3.00	2.51	2.34	2.90	2.70
8	2.00	2.50	3.00	2.50	2.00	3.00	2.50
9	1.80	2.45	3.00				
10	1.70	2.40	3.00				
11	1.60	2.35	3.00				
12	1.55	2.30	3.00				
13	1.50	2.25	3.00				
14	1.45	2.20	3.00				
15	1.40	2.15	3.00	Same as Table S	Same as Table R	Same as Table T	Same as Table S
16	1.35	2.10	3.00				
17	1.30	2.05	3.00				
18	1.25	2.00	3.00				
19	1.20	1.90	3.00				
20	1.15	1.80	3.00				
21	1.10	1.70	3.00				
22	1.05	1.60	3.00				
23	1.00	1.50	3.00				
24	1.00	1.50	3.00				
25	1.00	1.50	3.00				
26	1.00	1.54	3.04				
27	1.06	1.74	3.24				
28	1.11	1.93	3.43				
29	1.16	2.12	3.62				
30	1.21	2.30	3.84				

DEATH RATE DURING POLICY YEAR t ($100\ q_t^{\prime(d)}$)

Policy Year t	Issue Age 5	Issue Age 30	Issue Age 50	Weighted Average	Policy Year t	Issue Age 5	Issue Age 30	Issue Age 50	Weighted Average
1	.065	.081	.335	.157	16	.106	.502	3.451	1.235
2	.059	.098	.506	.218	17	.109	.556	3.776	1.342
3	.054	.108	.605	.247	18	.111	.615	4.135	1.449
4	.050	.121	.760	.313	19	.112	.682	4.530	1.568
5	.048	.132	.875	.350	20	.115	.758	4.958	1.695
6	.046	.176	1.364	.520	21	.116	.840	5.412	1.821
7	.049	.191	1.501	.568	22	.119	.931	5.886	1.960
8	.054	.210	1.652	.625	23	.122	1.026	6.375	2.075
9	.059	.234	1.819	.683	24	.125	1.128	6.876	2.203
10	.064	.262	1.999	.746	25	.130	1.240	7.404	2.336
11	.069	.295	2.196	.807	26	.135	1.364	7.975	2.460
12	.076	.330	2.410	.891	27	.141	1.501	8.606	2.586
13	.084	.369	2.640	.972	28	.148	1.652	9.315	2.749
14	.094	.410	2.888	1.055	29	.155	1.819	10.115	2.913
15	.101	.454	3.156	1.142	30	.165	1.999	10.999	3.094

EXHIBIT 2

NUMBERS CEASING BY WITHDRAWAL (w_t) AND DEATH (d_t) AND NUMBERS COMPLETING EACH POLICY YEAR (l_t) PER 10,000 THAT START THE FIRST POLICY YEAR

Policy Year t	Table R w_t	Table R d_t	Table R l_t	Table S w_t	Table S d_t	Table S l_t	Table T w_t	Table T d_t	Table T l_t
1......	699	15	9,286	1,249	14	8,737	1,999	13	7,988
2......	464	19	8,803	873	17	7,847	1,596	14	6,378
3......	308	21	8,474	353	19	7,475	446	15	5,917
4......	254	26	8,194	261	23	7,191	266	18	5,633
5......	225	28	7,941	215	25	6,951	225	19	5,389
6......	198	40	7,703	191	35	6,725	188	27	5,174
7......	173	43	7,487	174	37	6,514	155	29	4,990
8......	149	46	7,292	162	40	6,312	149	30	4,811
9......	131	49	7,112	154	42	6,116	144	32	4,635
10......	120	52	6,940	146	45	5,925	139	34	4,462
11......	111	55	6,774	139	47	5,739	133	35	4,294
12......	105	60	6,609	131	50	5,558	128	37	4,129
13......	99	63	6,447	124	53	5,381	123	39	3,967
14......	93	67	6,287	118	56	5,207	118	41	3,808
15......	88	71	6,128	111	58	5,038	114	42	3,652
16......	82	75	5,971	105	61	4,872	109	44	3,499
17......	77	79	5,815	99	64	4,709	104	46	3,349
18......	72	83	5,660	94	67	4,548	100	47	3,202
19......	67	88	5,505	86	70	4,392	95	49	3,058
20......	63	92	5,350	78	73	4,241	91	51	2,916
21......	58	97	5,195	71	76	4,094	87	52	2,777
22......	54	101	5,040	65	79	3,950	83	53	2,641
23......	50	104	4,886	59	81	3,810	78	53	2,510
24......	48	107	4,731	57	83	3,670	74	54	2,382
25......	47	110	4,574	54	85	3,531	71	54	2,257
26......	45	112	4,417	54	86	3,391	68	54	2,135
27......	46	113	4,258	58	86	3,247	68	54	2,013
28......	47	116	4,095	62	88	3,097	68	54	1,891
29......	47	118	3,930	65	89	2,943	67	53	1,771
30......	47	120	3,763	67	89	2,787	67	53	1,651
Total	4,067	2,170		5,475	1,738		7,153	1,196	
	Cumulative Numbers Completing Policy Years Indicated $\left(\sum_{r=1}^{t} l_r\right)$								
1- 5....			42,698			38,201			31,305
1-10....			79,232			69,793			55,377
1-15....			111,477			96,716			75,227
1-20....			139,778			119,478			91,251
1-25....			164,204			138,533			103,818
1-30....			184,667			153,998			113,279

EXHIBIT 2—*Continued*

NUMBERS CEASING BY WITHDRAWAL (w_t) AND DEATH (d_t) AND NUMBERS COMPLETING EACH POLICY YEAR (l_t) PER 10,000 THAT START THE FIRST POLICY YEAR

POLICY YEAR t	TABLE R/S			TABLE S/R		
	w_t	d_t	l_t	w_t	d_t	l_t
1	699	15	9,286	1,249	14	8,737
2	464	19	8,803	873	17	7,847
3	308	21	8,474	353	19	7,475
4	254	26	8,194	261	23	7,191
5	230	28	7,936	211	25	6,955
6	207	40	7,689	182	35	6,738
7	192	43	7,454	157	38	6,543
8	186	46	7,222	130	40	6,373
9	176	48	6,998	114	43	6,216
10	167	51	6,780	105	46	6,065
11	159	54	6,567	97	48	5,920
12	150	57	6,360	91	52	5,777
13	142	61	6,157	86	55	5,636
14	135	64	5,958	81	59	5,496
15	127	67	5,764	77	62	5,357
16	120	70	5,574	72	65	5,220
17	114	74	5,386	67	69	5,084
18	107	77	5,202	63	73	4,948
19	98	80	5,024	59	77	4,812
20	90	84	4,850	55	81	4,676
21	82	87	4,681	51	84	4,541
22	74	91	4,516	47	88	4,406
23	67	93	4,356	44	91	4,271
24	65	95	4,196	42	93	4,136
25	62	97	4,037	41	96	3,999
26	61	98	3,878	40	98	3,861
27	67	99	3,712	40	99	3,722
28	71	100	3,541	41	101	3,580
29	74	101	3,366	41	103	3,436
30	76	102	3,188	41	105	3,290
Total	4,824	1,988		4,811	1,899	
	Cumulative Numbers Completing Policy Years Indicated $\left(\sum_{r=1}^{t} l_r\right)$					
1- 5			42,693			38,205
1-10			78,836			70,140
1-15			109,642			98,326
1-20			135,678			123,066
1-25			157,464			144,419
1-30			175,149			162,308

EXHIBIT 2—*Continued*

NUMBERS CEASING BY WITHDRAWAL (w_t) AND DEATH (d_t) AND NUMBERS COMPLETING EACH POLICY YEAR (l_t) PER 10,000 THAT START THE FIRST POLICY YEAR

Policy Year t	Table S/T w_t	Table S/T d_t	Table S/T l_t	Table T/S w_t	Table T/S d_t	Table T/S l_t
1	1,249	14	8,737	1,999	13	7,988
2	873	17	7,847	1,596	14	6,378
3	353	19	7,475	446	15	5,917
4	261	23	7,191	266	18	5,633
5	233	24	6,934	211	19	5,403
6	216	35	6,683	169	27	5,207
7	193	37	6,453	140	29	5,038
8	193	39	6,221	126	31	4,881
9	186	41	5,994	119	33	4,729
10	179	44	5,771	113	35	4,581
11	172	45	5,554	107	36	4,438
12	166	48	5,340	102	39	4,297
13	159	51	5,130	96	41	4,160
14	153	53	4,924	91	43	4,026
15	147	55	4,722	86	45	3,895
16	141	57	4,524	81	47	3,767
17	135	59	4,330	77	50	3,640
18	129	61	4,140	72	52	3,516
19	123	63	3,954	66	54	3,396
20	118	65	3,771	61	57	3,278
21	112	67	3,592	55	59	3,164
22	107	69	3,416	50	61	3,053
23	101	69	3,246	45	63	2,945
24	96	70	3,080	44	64	2,837
25	91	70	2,919	42	65	2,730
26	88	70	2,761	42	66	2,622
27	88	69	2,604	45	67	2,510
28	88	70	2,446	48	68	2,394
29	87	69	2,290	50	69	2,275
30	87	69	2,134	52	69	2,154
Total	6,324	1,542		6,497	1,349	
	Cumulative Numbers Completing Policy Years Indicated $\left(\sum_{r=1}^{t} l_r\right)$					
1–5			38,184			31,319
1–10			69,306			55,755
1–15			94,976			76,571
1–20			115,695			94,168
1–25			131,948			108,897
1–30			144,183			120,852

EXHIBIT 3

NUMBERS OF PREMIUM UNITS PAID IN EACH POLICY YEAR (l_t') PER 10,000 PREMIUM UNITS THAT START THE FIRST POLICY YEAR

Policy Year t	Table R	Table S	Table T	Table R/S	Table S/R	Table S/T	Table T/S
1	9,489	9,094	8,554	9,489	9,094	9,094	8,554
2	8,887	7,995	6,640	8,887	7,995	7,995	6,640
3	8,534	7,541	5,996	8,534	7,541	7,541	5,996
4	8,248	7,244	5,685	8,248	7,244	7,244	5,685
5	7,991	6,998	5,435	7,987	7,001	6,983	5,446
6	7,755	6,773	5,218	7,742	6,785	6,735	5,248
7	7,536	6,560	5,029	7,506	6,587	6,502	5,075
8	7,339	6,358	4,850	7,275	6,414	6,271	4,917
9	7,157	6,162	4,674	7,050	6,256	6,044	4,765
10	6,985	5,971	4,501	6,832	6,105	5,822	4,617
11	6,819	5,785	4,333	6,619	5,960	5,604	4,473
12	6,656	5,604	4,168	6,413	5,818	5,391	4,333
13	6,494	5,427	4,006	6,210	5,677	5,181	4,196
14	6,335	5,254	3,847	6,012	5,538	4,975	4,062
15	6,178	5,085	3,691	5,818	5,400	4,773	3,931
16	6,022	4,919	3,538	5,628	5,264	4,575	3,803
17	5,867	4,757	3,389	5,441	5,129	4,381	3,677
18	5,713	4,597	3,242	5,258	4,995	4,191	3,554
19	5,560	4,441	3,098	5,080	4,860	4,005	3,434
20	5,406	4,290	2,956	4,906	4,725	3,822	3,316
21	5,253	4,143	2,817	4,738	4,591	3,643	3,202
22	5,099	4,000	2,681	4,573	4,458	3,468	3,092
23	4,946	3,860	2,549	4,413	4,324	3,297	2,984
24	4,792	3,721	2,421	4,254	4,189	3,130	2,876
25	4,637	3,582	2,295	4,095	4,054	2,969	2,769
26	4,480	3,443	2,173	3,937	3,916	2,810	2,662
27	4,322	3,299	2,051	3,772	3,778	2,653	2,551
28	4,161	3,151	1,929	3,602	3,637	2,495	2,436
29	3,997	2,998	1,808	3,428	3,494	2,338	2,318
30	3,831	2,842	1,688	3,251	3,349	2,182	2,197

EXHIBIT 4a

PRESENT VALUES (SUMS) OF PREMIUM UNITS—NO INTEREST

Policy Year t	Table R E'_t	Table R $\sum_{r=1}^{t} E'_r$	Table S E'_t	Table S $\sum_{r=1}^{t} E'_r$	Table T E'_t	Table T $\sum_{r=1}^{t} E'_r$
1	.9489	.9489	.9094	.9094	.8554	.8554
2	.8887	1.8376	.7995	1.7089	.6640	1.5194
3	.8534	2.6910	.7541	2.4630	.5996	2.1190
4	.8248	3.5158	.7244	3.1874	.5685	2.6875
5	.7991	4.3149	.6998	3.8872	.5435	3.2310
6	.7755	5.0904	.6773	4.5645	.5218	3.7528
7	.7536	5.8440	.6560	5.2205	.5029	4.2557
8	.7339	6.5779	.6358	5.8563	.4850	4.7407
9	.7157	7.2936	.6162	6.4725	.4674	5.2081
10	.6985	7.9921	.5971	7.0696	.4501	5.6582
11	.6819	8.6740	.5785	7.6481	.4333	6.0915
12	.6656	9.3396	.5604	8.2085	.4168	6.5083
13	.6494	9.9890	.5427	8.7512	.4006	6.9089
14	.6335	10.6225	.5254	9.2766	.3847	7.2936
15	.6178	11.2403	.5085	9.7851	.3691	7.6627
16	.6022	11.8425	.4919	10.2770	.3538	8.0165
17	.5867	12.4292	.4757	10.7527	.3389	8.3554
18	.5713	13.0005	.4597	11.2124	.3242	8.6796
19	.5560	13.5565	.4441	11.6565	.3098	8.9894
20	.5406	14.0971	.4290	12.0855	.2956	9.2850
21	.5253	14.6224	.4143	12.4998	.2817	9.5667
22	.5099	15.1323	.4000	12.8998	.2681	9.8348
23	.4946	15.6269	.3860	13.2858	.2549	10.0897
24	.4792	16.1061	.3721	13.6579	.2421	10.3318
25	.4637	16.5698	.3582	14.0161	.2295	10.5613
26	.4480	17.0178	.3443	14.3604	.2173	10.7786
27	.4322	17.4500	.3299	14.6903	.2051	10.9837
28	.4161	17.8661	.3151	15.0054	.1929	11.1766
29	.3997	18.2658	.2998	15.3052	.1808	11.3574
30	.3831	18.6489	.2842	15.5894	.1688	11.5262

EXHIBIT 4a—*Continued*

PRESENT VALUES (SUMS) OF PREMIUM UNITS—NO INTEREST

Policy Year t	Table R/S		Table S/R		Table S/T		Table T/S	
	E'_t	$\sum_{r=1}^{t} E'_r$	E'_t	$\sum_{r=1}^{t} E'_r$	E'_t	$\sum_{r=1}^{t} E'_r$	E'_t	$\sum_{r=1}^{t} E'_r$
1	.9489	.9489	.9094	.9094	.9094	.9094	.8554	.8554
2	.8887	1.8376	.7995	1.7089	.7995	1.7089	.6640	1.5194
3	.8534	2.6910	.7541	2.4630	.7541	2.4630	.5996	2.1190
4	.8248	3.5158	.7244	3.1874	.7244	3.1874	.5685	2.6875
5	.7987	4.3145	.7001	3.8875	.6983	3.8857	.5446	3.2321
6	.7742	5.0887	.6785	4.5660	.6735	4.5592	.5248	3.7569
7	.7506	5.8393	.6587	5.2247	.6502	5.2094	.5075	4.2644
8	.7275	6.5668	.6414	5.8661	.6271	5.8365	.4917	4.7561
9	.7050	7.2718	.6256	6.4917	.6044	6.4409	.4765	5.2326
10	.6832	7.9550	.6105	7.1022	.5822	7.0231	.4617	5.6943
11	.6619	8.6169	.5960	7.6982	.5604	7.5835	.4473	6.1416
12	.6413	9.2582	.5818	8.2800	.5391	8.1226	.4333	6.5749
13	.6210	9.8792	.5677	8.8477	.5181	8.6407	.4196	6.9945
14	.6012	10.4804	.5538	9.4015	.4975	9.1382	.4062	7.4007
15	.5818	11.0622	.5400	9.9415	.4773	9.6155	.3931	7.7938
16	.5628	11.6250	.5264	10.4679	.4575	10.0730	.3803	8.1741
17	.5441	12.1691	.5129	10.9808	.4381	10.5111	.3677	8.5418
18	.5258	12.6949	.4995	11.4803	.4191	10.9302	.3554	8.8972
19	.5080	13.2029	.4860	11.9663	.4005	11.3307	.3434	9.2406
20	.4906	13.6935	.4725	12.4388	.3822	11.7129	.3316	9.5722
21	.4738	14.1673	.4591	12.8979	.3643	12.0772	.3202	9.8924
22	.4573	14.6246	.4458	13.3437	.3468	12.4240	.3092	10.2016
23	.4413	15.0659	.4324	13.7761	.3297	12.7537	.2984	10.5000
24	.4254	15.4913	.4189	14.1950	.3130	13.0667	.2876	10.7876
25	.4095	15.9008	.4054	14.6004	.2969	13.3636	.2769	11.0645
26	.3937	16.2945	.3916	14.9920	.2810	13.6446	.2662	11.3307
27	.3772	16.6717	.3778	15.3698	.2653	13.9099	.2551	11.5858
28	.3602	17.0319	.3637	15.7335	.2495	14.1594	.2436	11.8294
29	.3428	17.3747	.3494	16.0829	.2338	14.3932	.2318	12.0612
30	.3251	17.6998	.3349	16.4178	.2182	14.6114	.2197	12.2809

EXHIBIT 4b

PRESENT VALUES OF PREMIUM UNITS—2½% INTEREST

Policy Year t	Table R E'_t	Table R $\sum_{r=1}^{t} E'_r$	Table S E'_t	Table S $\sum_{r=1}^{t} E'_r$	Table T E'_t	Table T $\sum_{r=1}^{t} E'_r$
1	.9426	.9426	.9034	.9034	.8497	.8497
2	.8613	1.8039	.7748	1.6782	.6435	1.4932
3	.8069	2.6108	.7130	2.3912	.5669	2.0601
4	.7608	3.3716	.6682	3.0594	.5244	2.5845
5	.7191	4.0907	.6298	3.6892	.4891	3.0736
6	.6809	4.7716	.5947	4.2839	.4581	3.5317
7	.6455	5.4171	.5619	4.8458	.4308	3.9625
8	.6133	6.0304	.5313	5.3771	.4053	4.3678
9	.5835	6.6139	.5024	5.8795	.3811	4.7489
10	.5556	7.1695	.4749	6.3544	.3580	5.1069
11	.5292	7.6987	.4489	6.8033	.3362	5.4431
12	.5039	8.2026	.4243	7.2276	.3156	5.7587
13	.4797	8.6823	.4009	7.6285	.2959	6.0546
14	.4565	9.1388	.3786	8.0071	.2772	6.3318
15	.4343	9.5731	.3575	8.3646	.2595	6.5913
16	.4130	9.9861	.3374	8.7020	.2427	6.8340
17	.3926	10.3787	.3183	9.0203	.2268	7.0608
18	.3730	10.7517	.3001	9.3204	.2116	7.2724
19	.3541	11.1058	.2829	9.6033	.1973	7.4697
20	.3359	11.4417	.2666	9.8699	.1837	7.6534
21	.3184	11.7601	.2512	10.1211	.1708	7.8242
22	.3016	12.0617	.2366	10.3577	.1586	7.9828
23	.2854	12.3471	.2227	10.5804	.1471	8.1299
24	.2698	12.6169	.2095	10.7899	.1363	8.2662
25	.2547	12.8716	.1967	10.9866	.1260	8.3922
26	.2400	13.1116	.1845	11.1711	.1164	8.5086
27	.2259	13.3375	.1725	11.3436	.1072	8.6158
28	.2122	13.5497	.1607	11.5043	.0984	8.7142
29	.1989	13.7486	.1492	11.6535	.0900	8.8042
30	.1860	13.9346	.1380	11.7915	.0819	8.8861

EXHIBIT 4b—*Continued*

PRESENT VALUES OF PREMIUM UNITS—$2\frac{1}{2}\%$ INTEREST

Policy Year t	Table R/S		Table S/R		Table S/T		Table T/S	
	E'_t	$\sum_{r=1}^{t} E'_r$	E'_t	$\sum_{r=1}^{t} E'_r$	E'_t	$\sum_{r=1}^{t} E'_r$	E'_t	$\sum_{r=1}^{t} E'_r$
1......	.9426	.9426	.9034	.9034	.9034	.9034	.8497	.8497
2......	.8613	1.8039	.7748	1.6782	.7748	1.6782	.6435	1.4932
3......	.8069	2.6108	.7130	2.3912	.7130	2.3912	.5669	2.0601
4......	.7608	3.3716	.6682	3.0594	.6682	3.0594	.5244	2.5845
5......	.7188	4.0904	.6300	3.6894	.6284	3.6878	.4901	3.0746
6......	.6797	4.7701	.5957	4.2851	.5913	4.2791	.4608	3.5354
7......	.6429	5.4130	.5642	4.8493	.5569	4.8360	.4347	3.9701
8......	.6080	6.0210	.5360	5.3853	.5241	5.3601	.4109	4.3810
9......	.5748	6.5958	.5101	5.8954	.4928	5.8529	.3885	4.7695
10......	.5434	7.1392	.4856	6.3810	.4631	6.3160	.3672	5.1367
11......	.5136	7.6528	.4625	6.8435	.4349	6.7509	.3471	5.4838
12......	.4855	8.1383	.4405	7.2840	.4081	7.1590	.3280	5.8118
13......	.4587	8.5970	.4193	7.7033	.3827	7.5417	.3099	6.1217
14......	.4332	9.0302	.3991	8.1024	.3585	7.9002	.2927	6.4144
15......	.4090	9.4392	.3796	8.4820	.3356	8.2358	.2764	6.6908
16......	.3860	9.8252	.3611	8.8431	.3138	8.5496	.2608	6.9516
17......	.3641	10.1893	.3432	9.1863	.2932	8.8428	.2460	7.1976
18......	.3433	10.5326	.3261	9.5124	.2736	9.1164	.2320	7.4296
19......	.3236	10.8562	.3095	9.8219	.2551	9.3715	.2187	7.6483
20......	.3048	11.1610	.2936	10.1155	.2375	9.6090	.2060	7.8543
21......	.2872	11.4482	.2783	10.3938	.2208	9.8298	.1941	8.0484
22......	.2705	11.7187	.2637	10.6575	.2051	10.0349	.1829	8.2313
23......	.2546	11.9733	.2495	10.9070	.1902	10.2251	.1722	8.4035
24......	.2395	12.2128	.2358	11.1428	.1762	10.4013	.1619	8.5654
25......	.2249	12.4377	.2226	11.3654	.1631	10.5644	.1521	8.7175
26......	.2109	12.6486	.2098	11.5752	.1506	10.7150	.1426	8.8601
27......	.1972	12.8458	.1975	11.7727	.1387	10.8537	.1333	8.9934
28......	.1837	13.0295	.1855	11.9582	.1272	10.9809	.1242	9.1176
29......	.1706	13.2001	.1738	12.1320	.1163	11.0972	.1153	9.2329
30......	.1578	13.3579	.1626	12.2946	.1059	11.2031	.1066	9.3395

EXHIBIT 4c

PRESENT VALUES OF PREMIUM UNITS—4% INTEREST

Policy Year t	Table R E'_t	Table R $\sum_{r=1}^{t} E'_r$	Table S E'_t	Table S $\sum_{r=1}^{t} E'_r$	Table T E'_t	Table T $\sum_{r=1}^{t} E'_r$
1	.9389	.9389	.8999	.8999	.8464	.8464
2	.8456	1.7845	.7607	1.6606	.6318	1.4782
3	.7807	2.5652	.6899	2.3505	.5486	2.0268
4	.7256	3.2908	.6372	2.9877	.5001	2.5269
5	.6759	3.9667	.5919	3.5796	.4597	2.9866
6	.6307	4.5974	.5509	4.1305	.4244	3.4110
7	.5893	5.1867	.5130	4.6435	.3933	3.8043
8	.5519	5.7386	.4781	5.1216	.3647	4.1690
9	.5175	6.2561	.4455	5.5671	.3379	4.5069
10	.4856	6.7417	.4151	5.9822	.3129	4.8198
11	.4558	7.1975	.3867	6.3689	.2896	5.1094
12	.4278	7.6253	.3602	6.7291	.2679	5.3773
13	.4014	8.0267	.3354	7.0645	.2476	5.6249
14	.3765	8.4032	.3122	7.3767	.2286	5.8535
15	.3530	8.7562	.2906	7.6673	.2109	6.0644
16	.3309	9.0871	.2703	7.9376	.1944	6.2588
17	.3100	9.3971	.2513	8.1889	.1790	6.4378
18	.2902	9.6873	.2335	8.4224	.1647	6.6025
19	.2716	9.9589	.2169	8.6393	.1513	6.7538
20	.2539	10.2128	.2015	8.8408	.1388	6.8926
21	.2372	10.4500	.1871	9.0279	.1272	7.0198
22	.2214	10.6714	.1737	9.2016	.1164	7.1362
23	.2065	10.8779	.1612	9.3628	.1064	7.2426
24	.1924	11.0703	.1494	9.5122	.0972	7.3398
25	.1790	11.2493	.1383	9.6505	.0886	7.4284
26	.1663	11.4156	.1278	9.7783	.0807	7.5091
27	.1543	11.5699	.1177	9.8960	.0732	7.5823
28	.1428	11.7127	.1081	10.0041	.0662	7.6485
29	.1319	11.8446	.0989	10.1030	.0597	7.7082
30	.1216	11.9662	.0902	10.1932	.0536	7.7618

EXHIBIT 4c—*Continued*

PRESENT VALUES OF PREMIUM UNITS—4% INTEREST

Policy Year t	Table R/S		Table S/R		Table S/T		Table T/S	
	E'_t	$\sum_{r=1}^{t} E'_r$	E'_t	$\sum_{r=1}^{t} E'_r$	E'_t	$\sum_{r=1}^{t} E'_r$	E'_t	$\sum_{r=1}^{t} E'_r$
1.......	.9389	.9389	.8999	.8999	.8999	.8999	.8464	.8464
2.......	.8456	1.7845	.7607	1.6606	.7607	1.6606	.6318	1.4782
3.......	.7807	2.5652	.6899	2.3505	.6899	2.3505	.5486	2.0268
4.......	.7256	3.2908	.6372	2.9877	.6372	2.9877	.5001	2.5269
5.......	.6756	3.9664	.5922	3.5799	.5906	3.5783	.4606	2.9875
6.......	.6297	4.5961	.5518	4.1317	.5478	4.1261	.4268	3.4143
7.......	.5870	5.1831	.5151	4.6468	.5085	4.6346	.3969	3.8112
8.......	.5470	5.7301	.4823	5.1291	.4715	5.1061	.3697	4.1809
9.......	.5097	6.2398	.4523	5.5814	.4370	5.5431	.3445	4.5254
10.......	.4750	6.7148	.4244	6.0058	.4048	5.9479	.3210	4.8464
11.......	.4425	7.1573	.3984	6.4042	.3746	6.3225	.2990	5.1454
12.......	.4122	7.5695	.3740	6.7782	.3465	6.6690	.2785	5.4239
13.......	.3838	7.9533	.3509	7.1291	.3202	6.9892	.2593	5.6832
14.......	.3573	8.3106	.3291	7.4582	.2956	7.2848	.2414	5.9246
15.......	.3325	8.6431	.3086	7.7668	.2727	7.5575	.2246	6.1492
16.......	.3092	8.9523	.2892	8.0560	.2514	7.8089	.2090	6.3582
17.......	.2875	9.2398	.2710	8.3270	.2315	8.0404	.1943	6.5525
18.......	.2671	9.5069	.2537	8.5807	.2129	8.2533	.1805	6.7330
19.......	.2481	9.7550	.2374	8.8181	.1956	8.4489	.1677	6.9007
20.......	.2304	9.9854	.2219	9.0400	.1795	8.6284	.1557	7.0564
21.......	.2140	10.1994	.2073	9.2473	.1645	8.7929	.1446	7.2010
22.......	.1986	10.3980	.1936	9.4409	.1506	8.9435	.1343	7.3353
23.......	.1843	10.5823	.1805	9.6214	.1377	9.0812	.1246	7.4599
24.......	.1708	10.7531	.1682	9.7896	.1257	9.2069	.1155	7.5754
25.......	.1581	10.9112	.1565	9.9461	.1146	9.3215	.1069	7.6823
26.......	.1461	11.0573	.1454	10.0915	.1043	9.4258	.0988	7.7811
27.......	.1346	11.1919	.1348	10.2263	.0947	9.5205	.0910	7.8721
28.......	.1236	11.3155	.1248	10.3511	.0856	9.6061	.0836	7.9557
29.......	.1131	11.4286	.1153	10.4664	.0771	9.6832	.0765	8.0322
30.......	.1031	11.5317	.1063	10.5727	.0692	9.7524	.0697	8.1019