SECTION II. FEMALE EXTENSION OF THE 1958 CSO AND CET MORTALITY TABLES

INTRODUCTION

Your committee concluded that it could not justifiably publish monetary tables based upon the new mortality tables without first introducing appropriate refinements to eliminate two anomalies in assumed juvenile female mortality rates which would result from a strict application of the "3-year age setback": (1) female mortality rates would be higher than male mortality rates at ages 10 and under, and (2) there would not be any clearly defined female mortality rates at ages 0, 1 and 2.

CONSTRUCTION OF FEMALE EXTENSION TO 1958 CSO TABLE

After considering various possible solutions, it was agreed that the best approach was the straightforward one of constructing an extension of the 1958 CSO Table for females at ages 14 and under that would:

(1) grade smoothly into the 1958 CSO Table set back 3 years at ages 15 and over (*i.e.*, male ages 12 and over),

(2) be based on the same general method as that used in the construction of the 1958 CSO Table, and

(3) start with our best estimate of female experience mortality rates that correspond to the experience mortality rates shown in the 1958 CSO Basic Table.

Starting with (3) above, we used the United States Life Tables 1949–51 (Vital Statistics—Special Reports—Vol. 41, No. 1) to obtain ratios of white female mortality rates to total white mortality rates by individual age. These mortality rates and ratios are shown in Table 1. In general, these ratios would appear to be slightly conservative because the proportion of females in the population is undoubtedly higher than the proportion of females in an Ordinary life insurance experience. We believe, however, that any distortion because of this factor should be relatively small at the juvenile ages and could be ignored.

We also investigated the possibility that the age 0 ratio derived from population data is not applicable to insured lives data because of differences in the distribution of exposures by exact age during the first year of life. This problem was analyzed by following the same approach for white females as that described in C. M. Sternhell's paper "The New Standard Ordinary Mortality Table" (*TSA* IX, 7–9). This analysis indicated that the age 0 ratio derived from population data would not be significantly

affected by an adjustment for differences between insured lives and population distributions of exposures by exact age and it was agreed that no such adjustment should be made.

The next step was to obtain a female counterpart to the 1958 CSO Basic Table by applying the ratios from Table 1 to the 1958 CSO Basic Table mortality rates. This calculation is shown in Table 2. The Table 1 ratios did not appear to require any graduation, as the mortality rates

TABLE 1

UNITED STATES LIFE TABLES 1949–51
MORTALITY RATES AT AGES 0 TO 14 FOR
WHITE FEMALES AND TOTAL WHITES

Age	White Females (1)	Total Whites (2)	Ratio Col. (1) ÷ Col. (2) (3)
0	.02355	.02722	86.5%
1	.00189	.00201	94.0
2	.00112	.00125	89.6
3	.00087	.00097	89.7
4	.00069	.00080	86.3
5.............	.00060	.00071	84.5
6.............	.00053	.00064	82.8
7.............	.00048	.00058	82.8
8.	.00044	.00054	81.5
9.	.00041	.00051	80.4
10.............	.00040	.00050	80.0
11.	.00039	.00051	76.5
12.............	.00041	.00054	75.9
13.............	.00043	.00060	71.7
14.	.00048	.00069	69.6

they produced exhibited satisfactory smoothness after only slight adjustment (1 in last place) at a few ages. The third differences also indicate a very smooth junction between the female mortality rates produced by this method at ages 0 to 14 and the female mortality rates at ages 15 and over obtained by using the 1958 CSO Basic Table with a 3-year age setback.

The last step was to obtain a female counterpart to the 1958 CSO Table by loading the mortality rates of the female counterpart to the 1958 CSO Basic Table from Table 2 in a manner consistent with the procedure followed in the construction of the 1958 CSO Table. This calculation is shown in Table 3. Since the loading in the 1958 CSO Table increased by .01 per year of age up to age 38, the use of a 3-year age setback for females from age 15 to age 38 results in a loading in this range of

ages .03 less than the loading at the same male ages. This loading pattern has been extended for ages under 14 in order to produce a smooth series of loadings.

Table 4 shows values of l_x, d_x, and commutation columns at $2\frac{1}{2}\%$ interest for the female extension of the 1958 CSO Table (set back 3 years). All of the values for age 15 are exactly the same as the corresponding values for age 12 on the 1958 CSO Table. The values of l_x and d_x at ages

TABLE 2

CONSTRUCTION OF FEMALE COUNTERPART TO
1958 CSO BASIC TABLE

Age	1958 CSO Basic Table $1,000q_x$ (1)	Ratio Col. (3) Table 1 (2)	1958 CSO Female Basic Table $1,000q_x$ Col. (1)×Col. (2) (3)	Δ Col. (3) (4)	Δ^2 Col. (3) (5)	Δ^3 Col. (3) (6)
0...	6.33	86.5%	5.48	−4.54	4.27	−4.07
1...	1.00	94.0	.94	− .27	.20	− .20
2...	.75	89.6	.67	− .07	.00	.01
3...	.68	89.7	(.61) .60*	− .07	.01	− .01
4...	.61	86.3	.53	− .06	.00	.01
5...	.55	84.5	(.46) .47*	− .06	.01	.00
6...	.49	82.8	.41	− .05	.01	.01
7...	.44	82.8	.36	− .04	.02	− .01
8...	.40	81.5	(.33) .32*	− .02	.01	.00
9...	.37	80.4	.30	− .01	.01	.00
10...	.36	80.0	.29	.00	.01	.00
11...	.37	76.5	(.28) .29*	.01	.01	.00
12...	.39	75.9	.30	.02	.01	.00
13...	.44	71.7	.32	.03	.01	.00
14...	.50	69.6	.35	.04	.01	.00
15...			.39†	.05	.01	− .01
16...			.44†	.06	.00	.01
17...			.50†	.06	.01	
18...			.56†	.07		
19...			.63†			

* Adjusted to improve grading.
† Equals 1958 CSO Basic Table $1,000q_x$ with 3-year age setback.

under 15 were obtained by applying the mortality rates in column (3) of Table 3 backward from l_{15}. As the values of D_x and N_x at female ages 15 and over involve the factor v^{-3} because of the 3-year age setback, the values of D_x at ages under 15 were obtained by multiplying l_x by v^{x-3}. Similarly, the values of C_x at ages under 15 were obtained by multiplying d_x by v^{x-2}. The values of N_x and M_x were obtained in the traditional manner by summing the values of D_x and C_x, respectively. This method

TABLE 3

CONSTRUCTION OF FEMALE COUNTERPART
TO 1958 CSO TABLE

Age	1958 CSO Female Basic Table 1,000q_x Col. (3) Table 2 (1)	Loading* (2)	1958 CSO Female Table 1,000q_x Col. (1) +Col. (2) (3)	Δ Col. (3) (4)	Δ² Col. (3) (5)	Δ³ Col. (3) (6)
0	5.48	.72	6.20	−4.53	4.27	−4.07
1	.94	.73	1.67	− .26	.20	− .20
2	.67	.74	1.41	− .06	.00	.01
3	.60	.75	1.35	− .06	.01	− .01
4	.53	.76	1.29	− .05	.00	.01
5	.47	.77	1.24	− .05	.01	.00
6	.41	.78	1.19	− .04	.01	.01
7	.36	.79	1.15	− .03	.02	− .01
8	.32	.80	1.12	− .01	.01	.00
9	.30	.81	1.11	.00	.01	.00
10	.29	.82	1.11	.01	.01	.00
11	.29	.83	1.12	.02	.01	.00
12	.30	.84	1.14	.03	.01	.00
13	.32	.85	1.17	.04	.01	.00
14	.35	.86	1.21	.05	.01	.00
15	.39	.87	1.26†	.06	.01	− .01
16	.44	.88	1.32†	.07	.00	.01
17	.50	.89	1.39†	.07	.01	
18	.56	.90	1.46†	.08		
19	.63	.91	1.54†			

* Loading used in constructing 1958 CSO Table less .03.
† Equals 1958 CSO Table 1,000q_x with 3-year age setback.

TABLE 4

FEMALE EXTENSION AT AGES 0 TO 14 OF 1958
CSO TABLE (SET BACK 3 YEARS)
COMMUTATION COLUMNS 2½%

Age	l_x	d_x	D_x*	N_x	C_x†	M_x
0	10 014 660	62 091	10 784 693.4666	356 121 626.4210	65 234.3569	2 098 800.1462
1	9 952 569	16 621	10 456 417.8060	345 336 932.9544	17 036.5250	2 033 565.7893
2	9 935 948	14 010	10 184 346.6998	334 880 515.1484	14 009.9999	2 016 529.2643
3	9 921 938	13 395	9 921 938.0000	324 696 168.4486	13 068.2927	2 002 519.2644
4	9 908 543	12 782	9 666 871.2196	314 774 230.4486	12 166.0916	1 989 450.9717
5	9 895 761	12 271	9 418 927.7811	305 107 359.2290	11 394.8433	1 977 284.8801
6	9 883 490	11 761	9 177 802.9921	295 688 431.4479	10 654.8855	1 965 890.0368
7	9 871 729	11 352	8 943 299.2530	286 510 628.4558	10 033.5139	1 955 235.1513
8	9 860 377	11 044	8 715 136.4892	277 567 329.2028	9 523.2066	1 945 201.6374
9	9 849 333	10 933	8 493 048.9781	268 852 192.7136	9 197.5528	1 935 678.4308
10	9 838 400	10 921	8 276 703.8883	260 359 143.7355	8 963.3733	1 926 480.8780
11	9 827 479	11 007	8 065 869.6893	252 082 439.8472	8 813.6170	1 917 517.5047
12	9 816 472	11 191	7 860 327.5429	244 016 570.1579	8 742.3913	1 908 703.8877
13	9 805 281	11 472	7 659 869.8463	236 156 242.6150	8 743.3250	1 899 961.4964
14	9 793 809	11 851	7 464 300.4276	228 496 372.7687	8 811.8808	1 891 218.1714
15	9 781 958	12 325	7 273 432.4866	221 032 072.3411	8 940.8062	1 882 406.2906

* $D_x = v^{x-2} l_x$ † $C_x = v^{x-2} d_x$

of constructing the female commutation columns satisfies the requirements that all monetary values for female issue ages 15 and over will be exactly equal to the corresponding values on the 1958 CSO Table with a 3-year age setback and that all monetary values for female issue ages under 15 will be consistent with those for female issue ages 15 and over.

Table 5 compares net annual premiums computed on the female extension of the 1958 CSO Table (set back 3 years) and 2½% interest with

TABLE 5

NET ANNUAL PREMIUMS
1958 CSO TABLE 2½%

Age	ORDINARY LIFE			20 PAYMENT LIFE			20 YEAR ENDOWMENT		
	Male	Male (−3)	Fe-male	Male	Male (−3)	Female	Male	Male (−3)	Female
0.........	$6.39	......	$5.89	$13.23	......	$12.37	$39.29	......	$39.19
1.........	6.38	......	5.89	13.13	......	12.31	38.96	......	38.89
2.........	6.52	......	6.02	13.37	......	12.53	38.94	......	38.87
3.........	6.69	$6.39	6.17	13.62	$13.23	12.77	38.93	$39.29	38.86
4.........	6.85	6.38	6.32	13.89	13.13	13.02	38.93	38.96	38.86
5.........	7.03	6.52	6.48	14.17	13.37	13.28	38.93	38.94	38.86
6.........	7.22	6.69	6.65	14.46	13.62	13.55	38.94	38.93	38.86
7.........	7.41	6.85	6.82	14.76	13.89	13.83	38.95	38.93	38.87
8.........	7.61	7.03	7.01	15.07	14.17	14.12	38.97	38.93	38.89
9.........	7.83	7.22	7.20	15.40	14.46	14.43	38.99	38.94	38.91
10.........	8.05	7.41	7.40	15.73	14.76	14.74	39.01	38.95	38.93
11.........	8.28	7.61	7.61	16.07	15.07	15.06	39.04	38.97	38.95
12.........	8.52	7.83	7.82	16.42	15.40	15.39	39.07	38.99	38.98
13.........	8.76	8.05	8.05	16.78	15.73	15.73	39.10	39.01	39.01
14.........	9.02	8.28	8.28	17.15	16.07	16.07	39.13	39.04	39.04
15.........	9.29	8.52	8.52	17.52	16.42	16.42	39.16	39.07	39.07

corresponding values on the 1958 CSO Table and on the 1958 CSO Table set back 3 years. This comparison covers net annual premiums for the ordinary life plan, the 20 payment life plan and the 20 year endowment plan. In every case, net annual premiums computed on the basis of the female extension of the 1958 CSO Table (set back 3 years) are equal to or lower than corresponding net annual premiums computed on the basis of either the 1958 CSO Table or the 1958 CSO Table set back 3 years.

RESERVES ON FEMALE EXTENSION OF 1958 CSO TABLE

The new model bills for the Standard Nonforfeiture and Standard Valuation laws include ". . . may be calculated according to an age not

more than three years younger than the actual age of the insured." The proposed female extension of the 1958 CSO Table (set back 3 years) was examined to see whether it would fulfill the requirements of the Standard Nonforfeiture and Standard Valuation laws. As to premium paying policies issued at female ages 3 and over, terminal reserves computed on the basis of the extension are always equal to or greater than the corresponding terminal reserves computed on the basis of the 1958 CSO Table set back 3 years. As to the question of the legal mortality rates for females under age 3, the law is rather ambiguous. The method used for constructing the female mortality rate extension through ages under 3 is consistent with the method used for constructing female mortality rates at ages 3 to 14, inclusive. There follows in the immediately succeeding paragraphs a more complete analysis of the suitability of the extension in these two juvenile female age brackets.

Reserves at attained ages 15 and over

Table 6 demonstrates that mortality rates on the proposed female extension of the 1958 CSO Table (set back 3 years) shown in column (3) are less than the corresponding mortality rates on the 1958 CSO Table set back 3 years, shown in column (2), at ages 3 to 14. Since the mortality rates at attained ages 15 and over on the 1958 CSO Female Table are

TABLE 6

COMPARISON OF MORTALITY RATES PER 1,000

Age	1958 CSO Table (1)	1958 CSO (−3) Table (2)	1958 CSO (F) Table (3)	Col. (2) − Col. (3) (4)
0........	7.08	....	6.20	....
1........	1.76	....	1.67	....
2........	1.52	....	1.41	....
3........	1.46	7.08	1.35	5.73
4........	1.40	1.76	1.29	.47
5........	1.35	1.52	1.24	.28
6........	1.30	1.46	1.19	.27
7........	1.26	1.40	1.15	.25
8........	1.23	1.35	1.12	.23
9........	1.21	1.30	1.11	.19
10.......	1.21	1.26	1.11	.15
11.......	1.23	1.23	1.12	.11
12.......	1.26	1.21	1.14	.07
13.......	1.32	1.21	1.17	.04
14.......	1.39	1.23	1.21	.02
15.......	1.46	1.26	1.26	0

exactly the same as the mortality rates on the 1958 CSO Table set back 3 years, the present value of future benefits at attained ages 15 and over will be the same on both tables. Therefore, a comparison of terminal reserves at attained ages 15 and over depends only on a comparison of net annual premiums. Since net annual premiums for issue ages 3 to 14 on the 1958 CSO Female Table are always equal to or less than net annual premiums on the 1958 CSO Table set back 3 years, it follows that terminal reserves computed on the basis of the 1958 CSO Female Table at attained ages 15 and over will always be equal to or greater than corresponding terminal reserves computed on the basis of the 1958 CSO Table set back 3 years.

Reserves at attained ages under 15

At attained ages under 15, Lidstone's Theorem (Society of Actuaries' *Textbook on Life Contingencies* by C. W. Jordan, Jr., page 114) was used as follows:

For an insurance of uniform amount with level premiums payable throughout the n year duration of the contract, reserves on the 1958 CSO Female Table will exceed reserves on the 1958 CSO (-3) Table at all durations t $(0 < t < n)$ provided that the critical function decreases as the duration t increases.

The critical function is defined as $(q_t - q'_t)(1 - _{t+1}V)$ for $0 \leq t < n$ where: q_t is the mortality rate at the attained age on the 1958 CSO (-3) Table, q'_t is the mortality rate at the same age on the 1958 CSO Female Table and $_{t+1}V$ is the terminal reserve on the 1958 CSO (-3) Table.

For coterminous insurance on which the reserves increase as the duration increases, it is apparent that if the differences in mortality rates decrease with increasing attained age, then the critical function must decrease with duration.

It can be seen from Table 6 that the differences in mortality rates decrease with increasing attained age. Therefore, for all coterminous life and endowment plans issued at female ages 3 to 14, the reserves on the 1958 CSO Female Table will always be greater than the reserves on the 1958 CSO (-3) Table.

As the above theorem is not applicable to limited payment life plans issued at female ages 3 to 14, special comparisons were made of terminal reserves at attained ages under 15 for 10, 15 and 20 payment life plans issued at each female age from 3 to 14. These comparisons indicate that, during the premium paying period, terminal reserves on the 1958 CSO Female Table will always be equal to or greater than corresponding reserves on the 1958 CSO Table set back 3 years.

Our tests indicate that the only area where terminal reserves on the

TABLE 7

COMPARISON OF TERMINAL RESERVES ON 1958 CSO FEMALE TABLE AND 1958 CSO TABLE SET BACK 3 YEARS FOR POLICIES ISSUED AT FEMALE AGE 3—$1,000 BASIS

POLICY YEAR	10 PAYMENT LIFE			15 PAYMENT LIFE			20 PAYMENT LIFE		
	1958 CSO (F) (1)	1958 CSO (−3) (2)	Col. (1)− Col. (2) (3)	1958 CSO (F) (1)	1958 CSO (−3) (2)	Col. (1)− Col. (2) (3)	1958 CSO (F) (1)	1958 CSO (−3) (2)	Col. (1)− Col. (2) (3)
1.....	$ 21.86	$ 17.04	$4.82	$ 15.10	$ 10.01	$5.09	$ 11.75	$ 6.52	$5.23
2.....	44.36	39.78	4.58	30.66	25.56	5.10	23.87	18.52	5.35
3.....	67.49	63.35	4.14	46.67	41.77	4.90	36.36	31.06	5.30
4.....	91.28	87.60	3.68	63.15	58.45	4.70	49.22	44.00	5.22
5.....	115.73	112.55	3.18	80.10	75.64	4.46	62.46	57.34	5.12
6.....	140.84	138.21	2.63	97.53	93.32	4.21	76.07	71.07	5.00
7.....	166.62	164.58	2.04	115.41	111.52	3.89	90.05	85.22	4.83
8.....	193.07	191.67	1.40	133.76	130.23	3.53	104.39	99.77	4.62
9.....	220.21	219.51	.70	152.59	149.46	3.13	119.10	114.73	4.37
10.....	248.04	248.08	− .04	171.89	169.21	2.68	134.17	130.11	4.06
11.....	253.37	253.38	− .01	191.67	189.48	2.19	149.62	145.88	3.74
12.....	258.81	258.81	.00	211.94	210.27	1.67	165.44	162.06	3.38

POLICY YEAR	10 YEAR ENDOWMENT			15 YEAR ENDOWMENT			20 YEAR ENDOWMENT		
	1958 CSO (F) (1)	1958 CSO (−3) (2)	Col. (1)− Col. (2) (3)	1958 CSO (F) (1)	1958 CSO (−3) (2)	Col. (1)− Col. (2) (3)	1958 CSO (F) (1)	1958 CSO (−3) (2)	Col. (1)− Col. (2) (3)
1.....	$ 88.63	$ 84.08	$4.55	$ 55.13	$ 50.22	$4.91	$ 38.53	$ 33.43	$5.10
2.....	179.65	175.30	4.35	111.77	106.85	4.92	78.14	72.91	5.23
3.....	273.11	269.14	3.97	169.94	165.20	4.74	118.83	113.67	5.16
4.....	369.05	365.51	3.54	229.68	225.14	4.54	160.64	155.55	5.09
5.....	467.52	464.46	3.06	291.01	286.71	4.30	203.57	198.60	4.97
6.....	568.59	566.06	2.53	353.97	349.95	4.02	247.64	242.82	4.82
7.....	672.31	670.34	1.97	418.58	414.88	3.70	292.88	288.24	4.64
8.....	778.73	777.39	1.34	484.87	481.55	3.32	339.30	334.88	4.42
9.....	887.93	887.25	.68	552.90	549.98	2.92	386.93	382.77	4.16
10.....	1,000.00	1,000.00	.00	622.70	620.21	2.49	435.79	431.93	3.86
11.....				694.31	692.29	2.02	485.92	482.38	3.54
12.....				767.80	766.25	1.55	537.34	534.15	3.19

POLICY YEAR	ORDINARY LIFE			SINGLE PREMIUM LIFE		
	1958 CSO (F) (1)	1958 CSO (−3) (2)	Col. (1)− Col. (2) (3)	1958 CSO (F) (1)	1958 CSO (−3) (2)	Col. (1)− Col. (2) (3)
1.....	$ 4.98	$ − .53	$5.51	$205.80	$207.26	$−1.46
2.....	10.15	4.26	5.89	209.93	211.06	−1.13
3.....	15.50	9.41	6.09	214.20	215.14	− .94
4.....	21.05	14.76	6.29	218.63	219.38	− .75
5.....	26.77	20.31	6.46	223.20	223.78	− .58
6.....	32.68	26.06	6.62	227.91	228.33	− .42
7.....	38.75	32.00	6.75	232.76	233.04	− .28
8.....	44.98	38.14	6.84	237.73	237.90	− .17
9.....	51.37	44.48	6.89	242.83	242.92	− .09
10.....	57.90	50.99	6.91	248.04	248.08	− .04
11.....	64.57	57.68	6.89	253.37	253.38	− .01
12.....	71.38	64.52	6.86	258.81	258.81	.00

1958 CSO Female Table are lower than terminal reserves on the 1958 CSO Table set back 3 years is on fully paid-up policies at attained ages under 15. This result reflects (1) that the terminal reserve on a fully paid-up policy is the single premium for future benefits and (2) that mortality rates on the 1958 CSO Female Table at ages under 15 are lower than corresponding mortality rates on the 1958 CSO Table set back 3 years.

Table 7 shows, for illustrative purposes, a comparison of terminal reserves on the 1958 CSO Female Table and on the 1958 CSO Table set back 3 years. This comparison covers the first 12 policy years for policies issued on various plans at female age 3.

NONFORFEITURE VALUES ON 1958 CSO FEMALE TABLE

Reduced Paid-up Insurance

Values of reduced paid-up insurance on the 1958 CSO Female Table will always be equal to or greater than corresponding values on the 1958 CSO Table set back 3 years because premium paying reserves on the Female Table are higher and single premiums on the Female Table are lower than corresponding values on the 1958 CSO Table set back 3 years.

TABLE 8

COMPARISON OF MORTALITY RATES PER 1,000

Age	1958 CSO Table (1)	1958 CSO Female Table (2)	1958 CET Table (3)	1958 CET Female Table (4)
0	7.08	6.20	7.83	6.95
1	1.76	1.67	2.51	2.42
2	1.52	1.41	2.27	2.16
3	1.46	1.35	2.21	2.10
4	1.40	1.29	2.15	2.04
5	1.35	1.24	2.10	1.99
6	1.30	1.19	2.05	1.94
7	1.26	1.15	2.01	1.90
8	1.23	1.12	1.98	1.87
9	1.21	1.11	1.96	1.86
10	1.21	1.11	1.96	1.86
11	1.23	1.12	1.98	1.87
12	1.26	1.14	2.01	1.89
13	1.32	1.17	2.07	1.92
14	1.39	1.21	2.14	1.96
15	1.46	1.26	2.21	2.01

Extended Term Insurance

An extension of the 1958 CET Table (set back 3 years) for females at ages 14 and under was constructed by following the same general method as that used in the construction of the 1958 CET Table. As the 1958 CET Table mortality rates at ages 14 and under were obtained by adding .75 deaths per 1,000 to the 1958 CSO Table mortality rates, this means that we merely have to add .75 deaths per 1,000 to the female extension of the 1958 CSO Table in order to obtain a consistent female extension of the 1958 CET Table.

Periods of extended term insurance on the 1958 CET Female Table will always be equal to or greater than corresponding periods on the 1958 CET Table set back 3 years because net single premiums on the 1958 CET Female Table are lower than corresponding values on the 1958 CET Table set back 3 years.

CONCLUSION

The above extension of the female mortality rates down to age 0 is being incorporated in the publication of the monetary tables based upon the 1958 CSO and CET Mortality Tables, and the Committee considers it appropriate if adopted as the legal standard for use in all calculations involving female lives.