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INTRODUCTION TO NONPROPORTIONAL REINSURANCE

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CLASSIFICATION OF REINSURANCE METHODS

REINSURANCE methods can be divided into the following general classes determined by the procedures for sharing the claims by the original insurer and by the reinsurer:

A. Each claim for each insurance risk is shared in a proportion determined in advance.

B. For each claim for each insurance risk, the original insurer pays the amount of the claim up to an amount determined in advance and the reinsurer pays any amount of the claim in excess of that amount.

C. For each insurance risk, the original insurer pays the total amount of all claims in a specified period (such as one policy year) up to a total limit determined in advance for the period and the reinsurer pays the total amount in excess of the limit for the period.

D. For a collection of insurance risks which have a number of claims, as the result of one event or occurrence, the original insurer pays the total amount of all claims up to a total limit determined in advance for one event and the reinsurer pays the total amount in excess of the limit.

E. For a collection of insurance risks, the original insurer pays the total amount of all claims in a specified period (such as one calendar year) up to a total limit determined in advance for the period and the reinsurer pays the total amount in excess of the limit for the period.

These classes have more meaning for kinds of insurance (such as hospital insurance) for which the amount of each claim can be a variable and for which one risk can have more than one claim. For individual life insurance (which can have only one claim for a predetermined amount for each risk), there is no difference between Classes A and B, and Class C reinsurance does not apply. The kinds of reinsurance defined in Classes D and E can be used for life insurance for a collection of individual risks such as a group insurance policy or the Ordinary risks insured by one company.

Dr. S. Vajda (1)* refers to the method of Class A as "proportional re-

* Each number in parentheses is the number of the reference in the list at the end of this paper.

insurance" and uses the name "nonproportional reinsurance" for Classes B, D, and E, having in mind primarily insurance for which the amount of the claim is a variable. He further classifies methods of Classes B and D as limited nonproportional reinsurance and the method of Class E as complete or fully collective nonproportional reinsurance. The writer prefers the descriptive terms of Dr. Vajda. In this paper the term "nonproportional reinsurance" will mean complete nonproportional reinsurance unless a limited form is clearly indicated.

S. Bjerreskov (2) uses four titles in his discussion of reinsurance methods. Three of his titles can be applied to classes listed in this paper as follows:

Class A	Excess reinsurance
Class B	Loss excess reinsurance
Class E	Stop-loss reinsurance.

Mr. Bjerreskov uses the title "Quota Reinsurance" for reinsurance in which the reinsurer takes over an equal part of the insurance so that the original insurer and the reinsurer share equally in all claims. This seems to be his excess reinsurance with the proportion for reinsurance 50% of the amount of insurance for each risk. He does not consider reinsurance of the Class D type in his paper.

Names used in the field of property reinsurance in the United States can be assigned as follows:

Class A	(1) Pro-rata contributing and (2) Quota share
Class B	Specific excess of loss
Class D	Single occurrence excess of loss
Class E	(1) Excess of loss ratio and (2) Aggregate excess of loss.

In this field of property reinsurance, nonproportional reinsurance is referred to as excess of loss reinsurance. This classification of names is based on the writer's interpretation of terms used by Harold C. Crawford in his paper (3) published by the Conference of Actuaries and by Randolph C. Collins, an expert in the field of property insurance, in correspondence with the writer.

The name stop-loss reinsurance which is used by Mr. Bjerreskov for the kind of reinsurance for Class E is also given as a second title for this class by Dr. Vajda and Mr. Crawford.

Reinsurance of the kind included in Class D is sometimes referred to as catastrophe reinsurance.

The discussions that follow in this paper will deal primarily with the subject of nonproportional reinsurance for insurance benefits issued by life insurance companies in the United States and Canada.

REINSURANCE METHODS IN THE UNITED STATES

Practically all reinsurance among life insurance companies in the United States has been proportional reinsurance. By common usage, when reinsurance of benefits issued by a life insurance company is mentioned without any qualifying phrases, proportional reinsurance is indicated.

This regular proportional reinsurance is the subject of several papers that have been included in publications of the Society and of its precedent organizations. Among these papers are Irving Rosenthal's frequently referred to paper in *The Record* (4) and Edward A. Dougherty's more recent paper in the *Transactions* (5).

Life insurance companies have used proportional reinsurance for group insurance as well as for individual insurance. This reinsurance for group insurance may be for the excess amount for each person with insurance above a fixed retention limit or may be a proportion of the total insurance for the group, in which case the retention limit varies with the variations in the amounts of insurance for the individuals insured.

Stuart J. Kingston (6) has suggested proportional reinsurance for larger amounts of individual pensions granted by trusteed pension plans. It appears that there has not been any application of this reinsurance procedure to pension plans or to any other trustee-insured benefits.

There has been some catastrophe reinsurance (Class D method) for life insurance companies in the United States. This is the type of reinsurance discussed in Edward A. Green's recent paper in the *Transactions* (7). Some life insurance companies have sought this type of limited nonproportional reinsurance for group insurance policies to protect the company against excessive claims from one accident such as the Texas City disaster. It does not appear that any life insurance company in this country has ever granted or secured fully collective or stop-loss nonproportional reinsurance.

Both limited and complete nonproportional reinsurance have been used much more extensively for kinds of insurance other than life. Mr. Crawford's paper (3) contains information on this use and indicates some of the problems involved in the determination of premiums. Some casualty reinsurance companies are providing catastrophe accident insurance of the Class C type for group insurance and for workmen's compensation insurance.

Workmen's compensation cases have secured complete nonproportional reinsurance as well as the limited form of nonproportional reinsurance. The experience of some of the reinsurance companies providing the complete form has not been satisfactory. The causes for this probably should be studied and analyzed by any insurer that seriously considers providing nonproportional reinsurance. One possible source of difficulty may have been deficient premiums based on the results of a few favorable years for a case without sufficient consideration of probability distributions for the averages and deviations involved.

PURPOSE OF REINSURANCE

The principal purpose, or at least one of the principal purposes, of reinsurance is to eliminate excessive fluctuations in insurance costs. An excessive fluctuation is one which reduces surplus earnings to an extent that is objectionable to management.

Proportional reinsurance does this only to the extent that excess claim payments in any one year are the result of claims for the larger amounts of benefits. Proportional reinsurance does not protect against excess claims due to accidental or fortuitous variations in the claim rates and claim costs.

Catastrophe reinsurance and other forms of limited nonproportional reinsurance provide more protection against such variations, but this reinsurance protection is limited and thus deserves the name used by Dr. Vajda.

The kind of reinsurance that protects against all fortuitous variations in claim rates and claim costs is complete nonproportional reinsurance. The maximum amount of claim costs that an insurer can conveniently pay in one year for a specified collection of risks can be established in advance as the retention limit for the nonproportional reinsurance. This amount can be expressed as a percentage of the expected claim costs for the insurance benefits in force during the year. The amount paid by the reinsurance company is based on the excess claims over this limit. This amount can be all of the excess, all of the excess subject to a maximum limit, or a percentage of the excess. Any reduction below 100% of all excess claims does limit the effectiveness of nonproportional reinsurance. This reinsurance can be said to be no longer complete, but for purposes of this paper fully collective reinsurance, except for reasonable percentage and maximum claim limits, will be classed as complete nonproportional reinsurance.

A reinsurance contract which requires the payment of a small portion of the excess losses by the original insurer may be helpful as a deterrent for any tendency to liberalize claim procedures after the claim cost retention limit is passed. The original insurer can offset the financial effect to some extent by reducing the maximum claim limit to be paid before the nonproportional reinsurance becomes effective.

The limits for nonproportional reinsurance included in the previous discussion will be defined for the purposes of this paper as follows:

"Maximum retention limit" is the factor to be applied to the expected losses to determine the maximum losses that the original insurer will pay before the nonproportional reinsurance applies.

"Excess losses" are the amount of actual losses in excess of the maximum retention limit.

"Percentage reinsurance limit" is the percentage of the excess losses that will be paid by the reinsurer.

"Maximum reinsurance limit" is the factor to be applied to the expected losses to determine the total losses (maximum retention limit plus excess losses) that will be considered in the nonproportional reinsurance payments to be made by the reinsurer. Losses in excess of the maximum reinsurance limit must be paid entirely by the original insurer.

GENERAL CONSIDERATIONS

Nonproportional reinsurance can be used for all types of life, accident and sickness insurance and can be used by trustees of self-insured trusteed groups as well as by life insurance companies. In theory, nonproportional reinsurance can be used for any number of individual risks in a group or company, but if the number is very small the uncertainties involved in determining premium rates do not make such reinsurance a practical solution.

For life insurance, the claim costs for a given claim rate can vary only by having the amount of benefits vary for the individuals included in the group. For sickness insurance, claim costs can vary not only because of variation in the number of units of insurance (such as the daily allowance for hospital care) but also because the cost for each unit can be a variable (for example, the number of days in the hospital). In order to keep the nonproportional reinsurance premium rate at a lower level and in order to keep the probability distribution formulas reasonably simple, other reinsurance methods should be used so as to limit the maximum amount of an individual claim that is included in the nonproportional reinsurance. Another reason for doing this is that it is not desirable to have the nonproportional reinsurance premium unduly influenced by very large amounts of insurance on a comparatively small number of lives. The traditional proportional reinsurance methods (Class A) should therefore be continued for a very large amount of life insurance or for a very large number of units of sickness insurance on one life. Limited nonproportional reinsurance of Class B also could be used for health insurance, but a market for such reinsurance would need to be developed in the United States. A company that secures complete nonproportional reinsurance can, of course, adopt a considerably higher retention limit for proportional reinsurance of standard risks than would be considered advisable if the nonproportional reinsurance were not to be used.

In order to simplify the probability distribution formulas for the mean claim rates, it is necessary to have an accurate determination of the expected mean claim rates for a collection of risks. Uncertainties in the claim rates for some classes of risks can add to the problems of nonproportional reinsurance. An example is a class of extremely substandard lives for which little experience is available for determining claim rates. For such risks, a larger amount of regular reinsurance may be advisable so as to reduce the amount of insurance that will be subject to nonproportional reinsurance. Irving Rosenthal was considering such risks in his discussion of Edward Dougherty's paper (5).

Under nonproportional reinsurance, the group of individual risks is considered as a collective risk subject to a variable amount of total claim payments. The original insurer of the group does not need insurance against average losses because he can provide for such losses in the original cost calculations. It is excess losses in a short period of time for which the original insurer needs protection.

There is no extensive discussion of nonproportional reinsurance in the publications of actuarial organizations in the United States. Two papers dealing with the subject have been submitted to the Conference of Actuaries in Public Practice, but neither paper includes any information on the mathematics of premium calculations. Portions of American papers on experience rating for group insurance and for workmen's compensation apply to this subject, and Mr. Rosenthal's paper on reinsurance (4) gives useful information on the problems of calculating the standard deviations for mean claim rates for life insurance.

Papers directly on the subject of nonproportional reinsurance have been published in Europe, and the subject has been a frequent topic of an International Congress of Actuaries. Much of the discussion in these papers is for kinds of insurance other than those issued by United States life insurance companies. The kinds of insurance usually considered in these papers can have a large variation in the amount of each claim and are subject to a considerable catastrophe hazard. An example is fire insurance. The writer found that the mathematical statistical discussions in these papers were not easy to follow.

The bibliography lists the papers, both European and American, that have been consulted in the writing of this paper.

The two most important statistical problems for nonproportional reinsurance are the determination of the expected mean claim rate for the collection of risks and the measure of the variation in that rate. These problems are essentially sampling problems, with each period of insurance (usually a year) for each group being a sample of the experience for the very large generalized collection of risks. Claim rates will vary during one year among the groups and will vary for the same group from year to year because of chance variations. The causes of these chance variations are large in number, are substantially independent, and are unpredictable; and the variations resulting from each cause are a relatively small part of the total variation.

There are also causes of variation in insurance claim rates and costs from year to year for a group of risks that do not meet the requirements for chance variations. Among these causes are the following:

1. Age and sex.
2. Occupation.
3. Geographical location.
4. Change in group exposure.
5. Variations in individual claim costs.
6. Long term improvement in mortality or morbidity rates.
7. Countrywide variations in claim rates and claim costs.
8. Monetary inflation.
9. Concentration of risks in a limited area.

Actuarial procedures should be used that will either eliminate or greatly reduce the effect of such causes of variation so that the effect of each cause on claim rates is comparable to that of each of the many unpredictable causes of chance variations in claim rates. The first requirement is, of course, satisfactory actuarial investigations to determine the exact effect of the larger measurable and predictable causes of variation in claim rates. The results combined with an accurate census of a group can be used to determine an accurate estimate of average claim cost for each unit of exposure. This average is the true net premium (or pure net premium) per unit of exposure for the insurance and is also the mean claim cost rate per unit of exposure. With the mean claim cost rate calculated accurately for a group, the formula for measuring the possible variations in that mean rate can be simplified and the premium for nonproportional reinsurance can be determined more accurately.

Much of the complication in formulas developed for nonproportional reinsurance and the resulting comparatively high premium rates for this reinsurance appear to be due to use of a mean claim cost rate (true net premium rate) that is not calculated so as to accurately reflect these measurable causes in total claim costs for a group. Premium rates that have been quoted for nonproportional reinsurance of group insurance cases in the United States (primarily by European reinsurers) are substantially equal to the group insurance charges quoted for the cases by domestic life insurance companies. The group insurance charges are frequently referred to as "group retentions" which gives a different meaning to the word "retention" than is true in the reinsurance business. A proper group insurance retention limit must not only provide for a nonproportional reinsurance charge but also for costs of administrative services and forms and for commissions and taxes. Apparently the formulas used for the quoted nonproportional premiums are based on assumed variations in claim costs that are greater than are justified by selectively determined average claim costs per unit of exposure.

Let us assume that all group life insurance in the United States is combined as one large population and the average claim cost per year is determined for each individual in the group regardless of age, occupation, amount of insurance on each individual and similar factors. Let us further assume that random samples of individuals are taken from this large population and the claim cost for each group determined by multiplying the number of individuals in each group by the average claim for each individual in the large population. Nonproportional reinsurance premiums based on such estimated claim costs will involve complications that will lead to complex formulas unless the samples are very large.

Because of underwriting limitations for group life insurance, the actual group cases that would be combined into the proposed total population would not be random samples from the generalized population. For example, the maximum amount of insurance for one life and the variation in amounts of insurance for individuals has a substantial correlation with the number in each group and this would not be true of random samples. The average ages of the individuals in random samples theoretically will also vary more than the average ages for group insurance cases issued in accordance with the underwriting rules used for most of the group insurance in the United States.

The suggested treatment of major causes of possible variation from the estimated true claim cost rates can be compared with the procedures used by scientific public poll takers. The professionals in this field do not determine changes in public sentiment by questioning the first 1,000 people they meet on two different days separated by a period of time. By use of stratified or representative random sampling procedures, these professionals in sampling make sure that the two samples properly represent the same generalized population.

DETERMINATION OF PREMIUMS

In this discussion, the nonproportional reinsurance premium will be expressed as a percentage of the true mean claim cost rate (the true net premium) per unit of exposure. Consideration will first be given to a group of 6,319 persons insured for life insurance with each life in the group insured for the same amount. The age distribution for this group by 5 year age groups is given in Table 1. Having the same amount of insurance on each life simplifies the calculation and the explanation of the nonproportional reinsurance premium.

The determination of a nonproportional reinsurance premium for this group is not difficult to explain. Ignoring interest, the insurance cost for a risk is the product of the probability of a loss times the amount of the loss. For nonproportional reinsurance, the amount of the loss is measured by the amount of the actual mean claim cost rate in excess of the true mean claim cost rate. In terms of statistics, the claim rate is the mean rate and the excess of the actual claim rate over the true mean rate is expressed in terms of the standard deviation of the true mean rate.

TABLE 1

6,319 PERSONS IN LIFE INSURANCE GROUP
AGE DISTRIBUTION

Attained Age	Percent	Attained Age	Percent
18	0.1%	48	17.8%
23	2.2	53	15.3
28	5.3	58	12.0
33	8.1	63	7.8
38	13.6	68	2.7
43	14.6	73	0.5
		Total	100.0%

If the specified amounts of deviations are taken at sufficiently small intervals, an approximate differential nonproportional reinsurance premium can be calculated by use of the basic insurance cost formula for each interval of deviation. The total of the differential premiums so determined for the deviations above a specified amount of deviation will give the reinsurance cost (or premium) for the desired nonproportional reinsurance.

For the group of 6,319 persons being considered, it has been assumed that the true mortality rates are 120% of the 1946–1950 group life insurance experience mortality rates. This gives a total of 58.87 deaths in one year for the group. The annual mean death rate per person in the group is .00932. Since the amount of insurance on each life is the same, the variation in insurance costs is the same as the variation in the number of deaths. Using the standard formula

$$\sqrt{Npq} \div N$$

as the measure of deviations, the standard deviation in the mean claim cost rate is determined to be .00121. This standard deviation equals 13.0% of the mean claim cost rate. Table 2 gives examples of excess claim cost rates and of probabilities that actual claim costs will exceed the expected costs in accordance with the expected rate.

TABLE 2

LIFE INSURANCE GROUP
PROBABILITIES THAT ACTUAL CLAIM COST RATES WILL EXCEED EXPECTED CLAIM COST RATES BY AMOUNTS OF EXCESS SPECIFIED

(Expected Claim Cost Rate = .00932)

Percentage of Excess	Amount of Excess Rate	Probability
0%	.00000	.5000
13	.00121	.1587
20	.00186	.0618
25	.00233	.0268
35	.00326	.0036

The number of deaths for this group for one year is sufficiently large to justify the use of the normal distribution for measuring the possible variations in the annual mean claim cost rate. The nonproportional reinsurance premium that is to be determined is to be for a maximum retention limit equal to 113% of the expected death claim costs. This is equivalent to an excess of one standard deviation above the expected annual claim cost rate. The first interval that will be used for the calculation will be for excess losses between one standard deviation and 1.3 standard deviations. The probability that the actual mean claim cost rate for one year will exceed the expected rate by one standard deviation is .1587 and by 1.3 standard deviations is .0968. The probability that the excess will be between one and 1.3 standard deviations is the difference in these two probabilities, or .0619. The actual claim cost rate will be .00121 above the expected claim cost rate if the excess is one standard deviation and will be .00157 above the expected death rate if the excess is 1.3 standard deviations. Assuming an even distribution of claims over the interval, the average excess for the actual claim rate will be the mean of these excess figures, or .00139. This is .00018 in excess of the proposed limit for the commencement of nonproportional reinsurance. Multiplying this excess by .0619 gives a differential premium cost factor of .0000111 for this interval.

Following similar assumptions for other intervals so as to cover all possible claim rates and adding the resulting differential cost factors for the several intervals gives a total cost factor of .0001026. This total is 1.10% of the expected claim cost rate of .00932. The nonproportional reinsurance premium rate for this case for a retention limit of 113% for one year on the basis of the assumptions used in this discussion is therefore 1.10% of the net one year term insurance premiums for this group of risks.

TABLE 3

NONPROPORTIONAL REINSURANCE PREMIUMS FOR LIFE INSURANCE GROUP

Maximum Retention Limit	Premium as Percentage of Expected Claim Costs	
	Finite Method	Continuous Method
100%	5.21%	5.18%
113	1.10	1.08
120	0.36	0.35
125	0.14	0 13
135	0.01	0.01

The nonproportional reinsurance premium rates for this case for five different minimum retention limits are given in Table 3.

The same procedures can be used to secure premiums for nonproportional reinsurance with a percentage reinsurance limit and with a maximum reinsurance limit.

An approximate procedure using finite differences has been used because (for the writer at least) the method is easier to follow and thus helps to make clear the basic principles of nonproportional reinsurance net premium calculations.

A more exact procedure using continuous functions is given in the illustration for life insurance that follows:

N = Total number of lives.

S = Total amount of insurance for N lives.

n_{sr} = Number of lives included in N who are in rating class r, each of whom is insured for s dollars of life insurance.

q_N = Average annual expected claim rate for each dollar of insurance for the N lives.

M = Expected amount of claims for N lives in one year.

q_{sr} = Average annual expected claim rate for each dollar of insurance for the lives in rating classification n_{sr}.

σ_M = Standard deviation for M, the expected amount of claims.

l = Maximum retention limit.

h = Maximum reinsurance limit.

L = Absolute amount of retention limit = $l \times M$.

H = Absolute amount of maximum reinsurance limit = $h \times M$.

t = Actual amount of claims.

R = Percentage reinsurance limit.

${}_R\mathrm{P}_L^H$ = Nonproportional annual reinsurance premium rate expressed as a proportion of the true annual expected claim cost rate for the group for nonproportional reinsurance with a maximum retention limit of l, a percentage reinsurance limit of R and a maximum retention limit of h for a group of N lives with M expected amount of claims.

$f(t)$ = The frequency function for distribution of t, the actual dollars of death claims for S dollars of insurance for the N lives. The mean and the standard deviation for this distribution are respectively M and σ_M.

These symbols are established so as to permit variations by amounts of insurance and by rating classes as normally occurs for Ordinary insurance issued by life insurance companies in the United States. For group insurance that is issued under underwriting rules that prevent individual selection, the division of the individuals by rating class other than occupation is not available and is not needed.

Irving Rosenthal's paper (4) includes an extensive and valuable discussion of the determination of standard deviations for each n_{sr} classification by rating and amount of insurance and for the total grouping of N lives insured for S dollars of insurance. This discussion applies directly to the calculations for nonproportional reinsurance.

The mean expected claim cost rate for each n_{sr} classification is determined from a distribution by age of the amounts of insurance for the lives in the class. For Ordinary insurance, the distribution should also be by duration during the period that selection is effective. If there are significant differences in mortality by amount of insurance, this can also be reflected in the calculations. The amount of insurance for each life in each n_{sr} classification is not exactly the same, but the groupings of amounts of insurance for each classification must be such as not to significantly affect the calculations. The accurate determination of the mean expected claim cost rate is necessary so that the variations between the actual claim costs and the expected claim costs will be the result of a large number of small chance causes. In the opinion of the writer, this point is not adequately developed in most of the discussions of nonproportional reinsurance.

The formula for calculating nonproportional reinsurance premiums with continuous functions is as follows:

$$_{R}\mathrm{P}_{L}^{H} = \int_{t=L}^{t=H} (t-L)\, f(t)\, dt \times \frac{R}{M}. \tag{1}$$

The integral expression can be changed to two integral expressions as follows:

$$_{R}\mathrm{P}_{L}^{H} = \frac{R}{M}\left[\int_{t=L}^{t=\infty} (t-L)\, f(t)\, dt - \int_{t=H}^{t=\infty} (t-L)\, f(t)\, dt\right].$$

The first term gives the nonproportional reinsurance premium for a retention limit equal to l and the second term gives this premium for a retention limit equal to h. The further development will be for the first term but the results will apply to both expressions.

The function $f(t)$ is assumed to be the normal curve and can be converted to the standard form by use of the relationship $z = (t - M) \div \sigma_M$. With this change the first integral becomes the following:

$$\sigma_M \int_{(L-M)/\sigma_M}^{\infty} [\sigma_M(z) + M - L] \frac{1}{\sigma_M \sqrt{2\pi}}\, e^{-z^2/2} dz.$$

This expression can be changed to the following:

$$\sigma_M \left(\frac{e^{-1/2\{(L-M)/\sigma_M\}^2}}{\sqrt{2\pi}}\right) - \frac{L-M}{\sqrt{2\pi}} \int_{(L-M)/\sigma_M}^{\infty} e^{-z^2/2} dz.$$

This equals the following:

$$\sigma_M A_{(L-M)/\sigma_M} - (L-M) B_{(L-M)/\sigma_M}. \tag{2}$$

In this formula A is the ordinate and B is the area from a table of values for the normal curve with the number of standard deviations for the standard form taken as $(L - M) \div \sigma_M$.

Mr. Jackson's paper (8) gives a more complete discussion of the mathematical development of this formula for nonproportional reinsurance. His final form with the normal curve used for $f(t)$ is somewhat different from the one given above, but the two forms are equivalent.

Mr. Jackson gives an approximate method for determining the standard deviation for the average probability of death for a group of lives. The writer prefers the more extensive and more exact procedure provided in Mr. Rosenthal's paper (4), especially for collections of risks involving considerable variation in the amounts of insurance and in the rating classes for the individual risks.

Mr. Larson determines a nonproportional reinsurance premium for his paper (9) based on the Poisson distribution, but he does not give his formula for this premium nor for the statistical factors that he used in his calculations. Herbert J. Stark and Arthur G. Weaver in their discussions of Mr. Larson's paper pointed out that proper consideration should be given to differences in amounts of insurance and in rating classes for individuals included in a group. Mr. Rosenthal's methods should meet the requirements of these gentlemen.

Mr. Larson's maximum mortality cost is 150% of the basic premium. His charge is therefore for nonproportional reinsurance with a maximum retention limit equal to 150% of the basic premium.

As Irving Rosenthal demonstrates in his paper (4) and as Herbert Stark and Arthur Weaver indicated in their discussions of Robert Larson's paper (9), the standard deviation of a specified number of lives increases with increase in the distribution by size of the amounts of insurance for the lives in the group. Mr. Rosenthal's method does not permit the amount of insurance on each life to be a sampling variable.

Formula (2) has been used to calculate nonproportional reinsurance premiums for the case used in our approximate procedure based on finite differences. The results are given in Table 3.

The comparison in Table 3 of the nonproportional reinsurance premiums calculated by the two procedures indicates that the finite difference procedure is substantially accurate. This accuracy would be increased if the number of divisions were increased. For the approximate calculations reviewed in this paper, 17 divisions were used for all excess over the expected claims and only 6 divisions for the excess over 135% of the expected claims.

This suggests that the function selected to represent the distribution of mean claim cost rates can be broken into units and a commutation column system developed for the calculation of premium rates. Such a system will be particularly helpful if the mathematical function is a complicated one. Some of the functions that have been listed for nonproportional reinsurance premium calculations are very complicated. As a practical matter, it is as difficult to calculate premiums with some of them as it would be to calculate regular life insurance premiums by use of an integral expression for the mortality and interest rates involved.

Irving Rosenthal's paper (4) contains illustrative calculations for the life insurance in force in a life insurance company. The standard deviation and mean claim rate figures of this paper have been used to calculate nonproportional reinsurance premiums. These premiums are given in Table 4.

TABLE 4

NONPROPORTIONAL REINSURANCE PREMIUMS FOR A LIFE INSURANCE COMPANY FOR AMOUNTS OF INSURANCE AT RISK PREMIUMS AS PERCENTAGES OF TRUE ONE YEAR TERM COSTS FOR AMOUNTS AT RISK

Maximum Amounts of Insurance at Risk on One Life and Number of Lives as Specified

MAXIMUM RETENTION LIMIT	MAXIMUM INSURANCE ON ONE LIFE		
	$25,000	$50,000	$100,000
A. 10,000 Lives			
100%	7.4818%	8.9807%	10.4148%
113	2.7105	3.9380	5.1802
120	1.3747	2.3090	3.3297
125	0.7957	1.5127	2.3560
135	0.2261	0.5816	1.0897
B. 50,000 Lives			
100%	3.3459%	4.0161%	4.6575%
113	0.2191	0.4667	0.7800
120	.0237	0.0884	0.2068
125	.0034	0.0211	0.0673
135	....	0.0006	0.0044
C. 100,000 Lives			
100%	2.3659%	2.8398%	3.2934%
113	.0296	0.1145	0.2035
120	.0005	0.0052	0.0209
125	.0001	0.0005	0.0029
135	....	....	....

HEALTH INSURANCE

The previous section of this paper has been primarily in regard to determination of nonproportional reinsurance premiums for life insurance issued by companies in the United States and Canada.

Nonproportional reinsurance is also applicable to health insurance benefits issued by these companies. Health insurance differs from life insurance because for health insurance the amount of a claim is a variable and also because for health insurance an individual insured can have more than one claim.

Because of the resulting variability in claim costs, health insurance reinsurance can be by the limited nonproportional methods of Classes B and C as well as by the complete nonproportional method of Class E. Because of comparatively low maximum limits for benefits, there has been little reinsurance of health insurance in the United States. This is true of the limited nonproportional methods of Classes B and C as well as the traditional proportional method of Class A. The catastrophe type (Class D) limited nonproportional reinsurance has some appeal for health insurance, especially for a group of risks concentrated in a limited area.

A deductible health insurance policy can be compared to the Class B reinsurance method. The deductible limit, the maximum claim payment, and the coinsurance ratio are similar respectively to the amount of retention limit, the amount of maximum reinsurance limit, and the percentage reinsurance limit. The procedures for determining the reinsurance premium for a Class B kind of reinsurance agreement are the same as those for determining premiums for the deductible health insurance policies such as the major medical policies.

The procedures for securing premiums for complete nonproportional reinsurance are similar to those discussed in the previous topic for life insurance. Calculations have been made for a group of employees and dependents. The employees have benefits numbered 1 to 4, inclusive, and the dependents have benefits numbered 1 to 3 and 5 in the following list:

1. Hospitalization with a daily benefit of $13 for a maximum of 31 days (no maternity).
2. Hospital extras to a maximum of $220 (no maternity).
3. Surgical benefits based on a standard $250 maximum schedule.
4. Weekly disability income of $35 per week for a maximum of 13 weeks after the first day for an accident and after the eighth day for an illness.
5. Maternity benefits of $100 for hospital costs and a $75 schedule for surgical costs.

Information for the above benefits has been secured from a group of 1,000 employees and 745 dependent units. Figures are not available for the number of children and wives in the dependent units. The age distribution of the employees is given in Table 5. This distribution is not the same as the age distribution for the life group of 6,319 lives considered previously, as can be seen by a comparison of Tables 1 and 5.

The calculations for this paper were based on the original group of

1,000 employees and 745 dependent units and also on an assumed group of 6,300 employees and 4,700 dependent units. This larger group has approximately the same number of employees as the life insurance group, so that a more direct comparison can be made for the nonproportional reinsurance premiums for the two kinds of insurance.

The rates and averages selected for this group are satisfactory for the illustrative purposes of this paper. For an actual nonproportional reinsurance contract, a much more extensive investigation of experience is needed in order to have reliable figures for the true expected claim costs.

TABLE 5

1,000 EMPLOYEES IN HEALTH INSURANCE GROUP
AGE DISTRIBUTION

Attained Age	Percent	Attained Age	Percent
18	0.1%	48	17.2%
23	3.0	53	9.0
28	8.0	58	4.8
33	14.5	63	1.7
38	19.7	68	1.0
43	21.0		
		Total	100.0%

In determining claim costs and rates, all benefits were grouped together for the employees and for the dependents. The four types of benefits (for each of the two classifications for employees and for dependents) were grouped together because the claim costs for the benefits are not statistically independent.

The following statistical averages were assumed to apply to this group:

	Employees	Dependents
Annual Claim Cost Rate	.1583	.4349
Average Claim Cost	$237.17	$158.94
Annual Claim Cost per Unit	$ 37.54	$ 69.12

The method of calculation for these health insurance claim cost rates differs from that for life insurance claim cost rates used in this paper. The q_x rate for life insurance claim costs is the ratio of the amount of claims to the initial amount of exposure for the year, assuming no additions to the original risk. The r_x annual claim cost rate for health insurance is the ratio of the amount of claims for the year to the average exposure for the year. The number of employees and the number of dependent units given previously are actually the average number included in the exposure for one year and not the initial number at the beginning of the year.

A ratio of the annual claim costs to initial exposure can be calculated for health insurance, but that ratio is not satisfactory for calculating nonproportional reinsurance premiums. For life insurance, if there are l_x people alive at age x subject to an annual mortality rate of q_x^d, the number of people alive at the end of the year is $l_x(1 - q_x^d)$. For health insurance, if there are l_x people alive and well at age x subject to an annual morbidity rate of q_x^i, the number of people alive and well at the end of the year will not be $l_x(1 - q_x^i)$.

The reason for this (apart from the fact that there is another decrement, death) is that some of the individuals who become sick and qualify for health insurance benefits can recover and return to the exposure. It is because of this return to the exposure that one individual can have more than one claim under a health insurance policy.

The annual claim cost rate for health insurance is the sum of a very large number of small interval claim rates. For example, if the interval is assumed to be one day, the annual claim cost rate is 365 times the daily claim cost rate for average daily exposure for the year.

Arthur Bailey develops this point in his paper (11). He explains that the binomial distribution, $(q + p)^n$, cannot properly be used for health insurance and demonstrates that the Poisson distribution is satisfactory for measuring the variability in the health insurance claim cost rates when all claims are for the same amount. Each small interval claim cost rate under these assumptions has a second moment of nr_s with r_s equal to the claim cost rate for the small interval and n equal to the exposure. Since the total annual rate r is equal to the sum of the r_s rates, the second moment for the annual rate is taken as the sum of the second moments for the small interval rates, or as nr.

The individual claim costs for health insurance do vary and for the illustrative case of this paper the claim costs are assumed to be distributed as shown in Tables 6 and 7. The claim cost rates and average claim costs in the list of statistical averages are based on some experience, but the distributions in Tables 6 and 7 are based entirely on the writer's ideas secured from discussions of fire claim distributions in Mr. Beard's paper (16) and of auto property damage claims in Mr. Bailey's paper (11). No attempt was made to secure mathematical formulas to represent the claim distributions. The distributions produce the average claim costs given above and the maximum claim amounts appear reasonable.

A distribution of claims by amounts similar to Tables 6 and 7 but based on actual experience would be very useful in determining premiums for deductible health insurance policies. The claims considered in the distribution, of course, must be for the benefits to be granted by the deductible contract.

The dollar units of exposure were determined as the average employee claim amount times the average number of employees for the year, and as the average dependent claim amount times the average number of dependent units for the year. Ignoring interest, the annual claim cost rate

TABLE 6

EMPLOYEE BENEFIT CLAIMS

Individual Claim Payments	Claim Distribution	Annual Claim Rate	Average Claim
Below $75..........	20,855	.033014	$ 42.41
$ 75–$ 125......	13,554	.021456	99.31
125– 175......	13,213	.020918	149.59
175– 225......	11,571	.018317	199.56
225– 275......	9,824	.015552	249.39
275– 325......	7,658	.012122	299.06
325– 375......	5,678	.008988	349.11
375– 425......	4,238	.006709	399.16
425– 475......	3,088	.004888	449.04
475– 525	2,381	.003770	499.32
525– 575......	1,870	.002961	549.13
575– 625......	1,182	.001871	596.91
625– 675......	650	.001028	648.86
675– 725	509	.000805	699.40
725– 775......	432	.000684	749.57
775– 825......	389	.000616	799.75
825– 875......	364	.000573	849.76
875– 925......	341	.000540	899.84
925– 975......	322	.000510	949.85
975– 1,025......	305	.000483	999.87
1,025– 1,075......	290	.000459	1,049.79
1,075– 1,125......	276	.000437	1,099.81
1,125– 1,175......	263	.000417	1,149.89
1,175– 1,225......	252	.000399	1,199.82
1,225– 1,275......	242	.000383	1,249.83
1,275 and over....	253	.000400	1,287.43
All Claims......	100,000	.158300	$ 237.17

is the net annual premium per dollar of expected claims, and the annual claim cost per unit is the net annual premium for one year of exposure for one employee or for one dependent unit.

The claim cost rates and the distribution of claim costs have been established as explained in previous paragraphs. The standard deviation must reflect the variations in claim costs as well as variations in the number of claims. The calculation is made by a modification of the third formula in Irving Rosenthal's paper (4). Instead of dividing the individuals in the group into subgroup amounts with each subgroup of individuals exposed for the entire claim cost rate, the claim cost rate is subdivided as shown in Tables 6 and 7 and the entire group is exposed for each claim amount subdivision of the total claim rate.

TABLE 7

DEPENDENT BENEFIT CLAIMS

Individual Claim Payments	Claim Distribution	Annual Claim Rate	Average Claim
Below $60	24,757	.107669	$ 33.74
$ 60–$ 100	15,426	.067087	79.33
100– 140	14,487	.063002	119.53
140– 180	11,972	.052067	159.50
180– 220	9,582	.041671	199.38
220– 260	7,032	.030580	239.09
260– 300	4,900	.021310	279.13
300– 340	3,433	.014932	319.16
340– 380	2,347	.010206	359.06
380– 420	1,695	.007373	399.29
420– 460	1,247	.005424	439.16
460– 500	740	.003217	477.37
500– 540	379	.001649	518.94
540– 580	277	.001206	559.39
580– 620	220	.000958	599.43
620– 660	185	.000806	639.64
660– 700	168	.000732	679.84
700– 740	156	.000680	719.90
740– 780	146	.000636	759.77
780– 820	137	.000597	799.86
820– 860	130	.000564	839.77
860– 900	123	.000535	879.79
900– 940	117	.000508	919.98
940– 980	112	.000485	959.91
980– 1,020	107	.000464	1,000.00
1,020 and over	125	.000542	1,041.69
All Claims	100,000	.434900	$ 158.94

Another modification of Mr. Rosenthal's formula is that the second moment for each subdivision is $a_s^2 n_s r_s$ and not $a_s^2 n_s q_s p_s$. This is correct because the Poisson distribution applies to each r_s subdivision of the total annual claim rate in the same way as to the total annual claim cost rate.

Since the number of lives (the n_s of Mr. Rosenthal's formula) is the same for each multiplication under the radical, this number can be factored out. The modifications change the formula to the following:

$$\sigma = \sqrt{N} \times \frac{\sqrt{a_1^2 r_1 + a_2^2 r_2 + \ldots + a_s^2 r_s}}{AC}.$$

In this formula the a's are the average claim figures and the r's are the claim cost rates of Tables 6 and 7, and AC is the symbol for the value of the average claim. The long expression under the second radical does not involve the number of individuals in the group. It is therefore necessary to calculate this function only once for different numbers of individuals if there is no change in the distribution of claim amounts.

In order to show the effect on the standard deviation of the distribution of claim amounts, values have been calculated with the above formula using the values in Tables 6 and 7 and using the formula $\sqrt{Nr}$ on the assumption that all claims were for the average amount. The standard deviations calculated by the two procedures are as follows:

Classification	Number	Standard Deviations	
		Claim Distribution	Average Claim
Employees	1,000	16.86	12.58
Employees	6,300	42.32	31.58
Dependent Units	745	23.86	18.00
Dependent Units	4,700	59.93	45.21
Combined Group	1,745	29.86	22.39
Combined Group	11,000	74.96	56.23

The standard deviations for the combined group (employees plus dependents) were calculated by the following formula:

$$(AC \times \sigma)^2_C = (AC \times \sigma)^2_E + (AC \times \sigma)^2_D \,.$$

This comparison brings out that with claim amounts distributed as assumed by the writer, it is necessary to calculate the standard deviation with the longer formula using the distribution of claim amounts.

Table 8 gives examples of excess claim cost rates and of probabilities that actual claim costs will exceed the expected costs in accordance with the expected rate. The variations for these values were determined by use of the standard deviations calculated with the claim distributions.

Using formula (2), nonproportional reinsurance premiums have been calculated for the health insurance benefits for the group of risks we have been considering. The premiums were calculated for both the standard deviations calculated with the long formula and those calculated with the short formula. The premiums secured are given in Table 9.

The premiums for nonproportional reinsurance are expressed as percentages of the expected annual claim costs which are the same as percentages of the annual one year term net premiums. It is obvious from these premiums that if the claim amounts are distributed as assumed for Tables 6 and 7, ignoring these distributions can result in serious understatement of the premiums for nonproportional reinsurance.

The nonproportional reinsurance premiums for health insurance in Table 9 for 6,300 employees are obviously less than the corresponding premiums for life insurance in Table 3 for a group of similar size.

TABLE 8

HEALTH INSURANCE GROUP
PROBABILITIES THAT ACTUAL COST RATES WILL EXCEED EXPECTED CLAIM COST RATES BY AMOUNTS OF EXCESS SPECIFIED

Percentage of Excess	Amount of Excess Rate	Probability	Amount of Excess Rate	Probability
	Employee Benefits (Employee Expected Claim Cost Rate=.1583)			
	1,000 Employees		6,300 Employees	
0	.000000	.50000	.000000	.50000
13	.020579	.11123	.020579	.00111
20	.031661	.03005	.031661	....
25	.039576	.00939	.039576	....
35	.055406	.00050	.055407	....
	Dependent Benefits (Dependent Expected Claim Cost Rate=.4349)			
	745 Dependents		4,700 Dependents	
0	.000000	.50000	.000000	.50000
13	.056538	.03836	.056537	....
20	.086981	.00326	.086979	....
25	.108726	.00034	.108724	....
35	.152216	....	.152215	....
	Combined Benefits (Combined Expected Claim Cost Rate=.2504)			
	Smaller Group		Larger Group	
0	.000000	.50000	.000000	.50000
13	.032554	.01786	.032564	....
20	.050083	.00062	.050099	....
25	.062603	.00003	.062624	....
35	.087644	....	.087674	....

The standard deviation for the health insurance for the 6,300 employees also is smaller than the standard deviation for the life insurance for the group of 6,319 lives. This agrees with the comparison of standard deviations for life insurance and for weekly indemnity policies as given in Arthur Weaver's discussion of Paul Jackson's paper (8).

The premiums calculated for nonproportional reinsurance do not allow for the catastrophe hazard resulting from concentration of risks in a limited area. A separate determination is needed for this cause of variation if this cause is sufficiently large as compared with other causes of varia-

TABLE 9

NONPROPORTIONAL REINSURANCE PREMIUMS FOR HEALTH INSURANCE GROUP

Maximum Retention Limit	Premiums as Percentages of Expected Claim Costs			
	Determined by Standard Deviations from Claim Distributions		Determined by Standard Deviations from Average Claims	
	Smaller Group	Larger Group	Smaller Group	Larger Group
	Employee Benefits			
100%	4.2490%	1.6928%	3.1707%	1.2632%
113	0.5728	0.0013	0.1698	....
120	.1247	....	.0151	....
125	.0339	....	.0017	....
135	.0002	....	....	....
	Dependent Benefits			
100%	2.9377%	1.1696%	2.2163%	.8824%
113	0.1146	0.0001	0.0181	....
120	.0075	....	.0002	....
125	.0006	....	.0001	....
135	.0001	....	....	....
	Combined Benefits			
100%	2.4691%	.9833%	1.8521%	.7376%
113	0.0400	....	0.0035	....
120	.0010	....	....	....
125	....	....	....	....
135	....	....	....	....

tion. Edward Green's paper (7) provides procedures for evaluating the catastrophe hazard. Catastrophe hazards are also considered in William T. Fee's paper (20) submitted to the Conference of Actuaries, but he does not explain the methods used by his company for determining the cost of catastrophe (or disaster) reinsurance.

The function, $f(t)$, of formula (1) was assumed to be the normal distribution for nonproportional reinsurance calculations for health insurance. This is the same assumption as used for the life insurance calculations. This assumption is justified for the collections of risks used for this paper because of the number of claims involved.

FREQUENCY DISTRIBUTIONS FOR NONPROPORTIONAL REINSURANCE PREMIUMS

A frequency distribution is necessary for measuring the variation in the claim costs. The normal curve has been used for the calculations of this paper on the assumption that all measurable causes of variation among different groups or collections of risks have been eliminated so that any variation in the actual claim costs from the expected claim costs is due to a large number of small causes. This requires an accurate determination of the expected claim cost rate and the standard deviation must be calculated to reflect the variation in actual claim payments.

The normal curve is sufficient and proper under these conditions if the number of claims involved is sufficiently large. The problem is essentially the same as the one for determining the probable error in mortality rates for the 1951 Impairment Study. The rule adopted for that Study appears to give a satisfactory standard, at least until further investigations can be made. Use the normal distribution if the number of claims in one year is more than 35 and the Poisson distribution if the number of expected claims is less than 35.

The writer would question the second part of this rule more than the first part. Pearson's Type III distribution probably is better for a small number of claims when claims are grouped in amount groups as is done in Irving Rosenthal's paper (4) and as is done in this paper for health insurance claims. If the number of expected claims is less than 10, a more complex formula is probably needed. One of the Gram-Charlier distributions may be satisfactory for such a small number of expected claims. The Gram-Charlier distributions discussed in most statistical books are Type A using the normal distribution and Type B using the Poisson distribution. A third type using the Pearson Type III distribution probably is better than either of these two usual types. The discussion and tables prepared by Dr. Louis R. Salvosa and published in a book (21) in about

1935 will be of great help to anyone desiring to use the Type III distribution or a Gram-Charlier series based on this distribution. The Type A series cannot satisfactorily represent a distribution of relatively large skewness as is true when the number of expected claims is less than 10. The Type III distribution is a skewed distribution and this distribution or a Gram-Charlier series using this distribution provides a better approximation of these skewed distributions.

Paul Jackson in his paper (8) indicates that for large exposure size cases the normal curve provides a satisfactory measure of the dispersion of actual mean claim rates. Robert Larson used the Poisson distribution in his paper (9) and Ralph Keffer a Pearson's Type III distribution in his paper (10). William R. Williamson, in his discussion of Mr. Keffer's paper, explains that the true expected mean mortality rate for a group depends on many factors, such as age, industry and geographical location. In his reply, Mr. Keffer states that if, by taking all of these factors into account, he determined the true expected mean claim rate for a group, the probability of the occurrence of d deaths when the true expected is c is given by the Poisson distribution. Hans Ammeter uses the compound Poisson (also called the negative binomial) distribution for the distribution of the number of claims in his theoretical development. In his paper in the book (1) edited by Dr. Vajda, his calculations are based on a Type A Gram-Charlier distribution formula with two terms. In his paper (15) presented to the XV International Congress of Actuaries he used the Poisson distribution. The tables in this second paper illustrate six cases with the number of expected claims from zero to ten, plus one case with an infinite number of claims. As can be demonstrated mathematically, the Poisson distribution, the Type III distribution and the compound Poisson distribution all approach the normal distribution as the exposure is increased, and become the normal distribution when the exposure is infinite.

As pointed out by Dr. Vajda in his paper published in Finland (14), actual experience for a period of years (such as 10 years) does not provide a satisfactory basis for estimating the variations in claim cost rates and for calculating nonproportional reinsurance premiums. A proper pooling of the experience for all companies or groups in this country will give a much more reliable answer based on actual experience than is indicated in this paper. Until such an investigation is made, a theoretical curve based on probability theory must be used.

Mr. Whitney in his paper (12) states that his problem is to find the probability that x is the real hazard of the risk if P is the hazard indicated by experience for the hazard of the class, and p is the hazard indicated by the experience for the hazard of the risk within the class. On the assumption of a reasonably large exposure so as to reduce the skewness of the distribution, he then produces the normal distribution as the answer to his problem.

T. Pentikäinen, in his Fourteenth International Congress paper (18), investigates the use of the normal distribution for determining nonproportional reinsurance premiums. He concludes from his examples that the normal distribution is satisfactory for the usual cases that arise in practice although it is easy to find extreme examples where this distribution does not apply.

For insurance for which the amount of a claim is also a variable, a frequency distribution is needed for the number of claims by amounts of benefits. In his article in the publication (1) edited by Dr. Vajda, Hans Ammeter indicates that this frequency function can be based on observed distributions of sums paid out in recent years, but suggests graduation with a mathematical formula involving a geometrical factor. He uses a simplified form of this formula in paper (15) submitted to the XV International Congress. R. E. Beard in his paper (16) for this Congress uses a cumulant generating function which is a complex exponential function and tests some approximations for this function. Arthur Bailey in his 1942 paper (11) for the Casualty Actuarial Society uses a normal logarithmic distribution for his distribution of property damage liability claims by amount of claim. C. O. Segerdahl, in his paper on reinsurance retentions (17) submitted to the XV Congress, uses the sum of two functions, each involving a geometrical factor, for his distribution of Swedish nonindustry fire insurance claims by amount of claim.

As stated previously, no attempt was made to fit a mathematical formula to the distributions of claims by amounts as given in Tables 6 and 7. These distributions are satisfactory for the illustrative calculations of this paper, but cannot be taken as a correct representation of the actual distribution of claims based on experience for health insurance as issued by the life insurance companies in this country. As indicated by the calculations for this paper, it is not necessary that the distribution of claims by amounts must be expressed as a mathematical formula, although such a formula can provide an adequate means for graduating a set of distribution rates calculated from actual experience.

CONCLUDING STATEMENT

One of the fields in which nonproportional reinsurance can properly be used is that of employee benefit programs. This comment applies to insured plans as well as to self-insured or trusteed plans. The retention allowances for group cases should include an insurance charge equivalent to an adequate nonproportional reinsurance premium. Competition has reduced these retention allowances so low that some companies may find that there is more profit in providing nonproportional reinsurance together with administrative and claim services for self-insured groups.

The reinsurance business of life insurance companies is another field for nonproportional reinsurance. Many companies are finding that present proportional reinsurance methods are expensive both because of the premiums paid and because of administrative costs.

Considerable additional research is needed to establish a nonproportional reinsurance business on a satisfactory basis. The powerful calculating machines that have been developed today can be used to make investigations that previously were not possible. Such investigations can determine the usefulness of all the mathematical distributions that have been suggested for the claim rates and for the distribution of claims by size.

The emphasis in this paper has been on the fundamental principles of nonproportional reinsurance. Except for the comments in the preceding section on frequency distributions, the development has used general basic statistical functions, and advanced and complicated mathematical statistical formulas have been avoided. The writer hopes that the subject of nonproportional reinsurance has been introduced in this paper in such fashion as to provide a basis for a full and considered discussion of the subject.

BIBLIOGRAPHY

(1) *Non-Proportional Reinsurance* edited by Dr. S. Vajda (London), published by Arithbel, S.A., Brussels, Belgium.

(2) "On the Principles for the Choice of Reinsurance Method and for the Fixing of Net Retention for an Insurance Company" by S. Bjerreskov (Denmark), *Transactions XIVth International Congress of Actuaries*, Volume I, page 715.

(3) "Excess of Loss Reinsurance" by Harold C. Crawford, *Proceedings*, Conference of Actuaries in Public Practice, Volume III, page 78.

(4) "Limits of Retention for Ordinary Life Insurance" by Irving Rosenthal, *RAIA* XXXVI, 6.

(5) "Ordinary Life Insurance Limits" by Edward A. Dougherty, *TSA* V, 125.

(6) An "Application of the Principle of Reinsurance to Trusteed Pension Plans" by Stuart J. Kingston, *The Journal of the American Society of Chartered Life Underwriters*, Volume IX, page 270.

(7) "Concentration of Risk and the Catastrophe Accident Hazard" by Edward A. Green, *TSA* VI, 506.

(8) "Experience Rating" by Paul H. Jackson, *TSA* V, 239.

(9) "A Group Method of Calculating Group Term Dividends" by Robert E. Larson, *TSA* IV, 308.
(10) "An Experience Rating Formula" by Ralph Keffer, *TASA* XXX, 130.
(11) "Sampling Theory in Casualty Insurance" by Arthur L. Bailey, *Proceedings*, Casualty Actuarial Society, Volume XXIX, page 50.
(12) "The Theory of Experience Rating" by Albert W. Whitney, *Proceedings*, Casualty Actuarial Society, Volume IV, page 274.
(13) "The Practice of Experience Rating" by G. F. Michelbacher, *Proceedings*, Casualty Actuarial Society, Volume IV, page 293.
(14) "Analytical Studies in Stop-Loss Reinsurance" by S. Vajda, *1951 Skandinavist Aktuarietidskrift*, page 158.
(15) "A Rational Experience Rating Technique for Group Insurance on the Risk Premium Basis" by Hans Ammeter (Switzerland), *Transactions XVth International Congress of Actuaries*, Volume I, page 507.
(16) "Analytical Expressions of the Risks Involved in General Insurance" by R. E. Beard (England), *Transactions XVth International Congress of Actuaries*, Volume II, page 230.
(17) "Basis for Fixing Net Retention" by C. O. Segerdahl (Sweden), *Transactions XVth International Congress of Actuaries*, Volume II, page 280.
(18) "On the Reinsurance of an Insurance Company" by T. Pentikäinen (Helsinki), *Transactions XIVth International Congress of Actuaries*, Volume I, page 389.
(19) "A Method of Fixing the Premium of Excess of Loss Reinsurance in Fire Insurance" by G. Benktander (Stockholm), *Transactions XIVth International Congress of Actuaries*, Volume I, page 823.
(20) "Casualty, Fire and Disaster Reinsurance" by William T. Fee, *Proceedings*, Conference of Actuaries in Public Practice, Volume VI, page 159.
(21) *Tables of Pearson's Type III Functions* (a book) by Louis R. Salvosa of the University of Michigan, published about 1935 by Edwards Brothers, Inc., of Ann Arbor, Michigan.