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Gambler's Fallacy: Probability of Reversion

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An odd event does not necessarily imply an immediate reversion.

Gambler's Fallacy

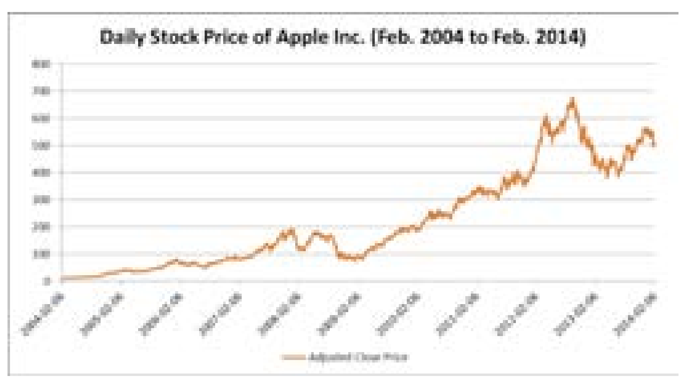
After a gambler loses a game many times, he/she may mistakenly believe that the chance of losing it again is smaller than normal. However, if the outcomes of the game are independent, it is a false belief. This is called the gambler's fallacy. For example, when tossing a fair coin, the probability of getting five heads in five tosses in a row is small. When making the sixth toss after getting five heads, the player may think that the probability of getting six heads in six tosses is very small. Therefore it is more likely that the sixth toss will get a tail than a head. Here, the player considers the conditional probability as the unconditional probability. The probability of getting six heads in the first six tosses is small. But the probability of getting six heads in the first six tosses given five heads in the first five tosses is not small.

Event	**Probability**
Five heads in the first five tosses	(½).5 = 1/32
Six heads in the first six tosses	(½).6 = 1/64
At least one tail in the first six tosses	**1-(½).6=63/64**
A head in the sixth toss given five heads in the first five tosses	½
A tail in the sixth toss given five heads in the first five tosses	½

Probability of Reversion

The gambler's fallacy can be seen in investment activities as well. Some investors speculate that a reversion of a trend will occur simply because the chance of the long-term trend happening is small. For example, investors may expect a price decrease in the next day after several days' stock price increases. They may be optimistic about a rising interest rate environment after a long period of low interest rates. There might be some in-depth analysis to support their conclusions. But in many cases, it is the gambler's fallacy that leads to them. Sometimes the probability of reversion is not affected by an unlikely trend in the past.

Figure 1 shows the daily close price of Apple Inc.'s stock in the past ten years. It increased a lot during that period but with many fluctuations. Table 2 lists the historical experience of continuous increases or decreases of daily closed stock price of Apple Inc. If the stock price had increased for five days, there are 55 cases that the stock price increased in the next day. And there are 59 cases that the stock price decreased in the next day. Comparing the number of cases that price increased in the next day to that the price decreased, in many situations, it is not obvious that the probability of reversion is higher than the probability of continuum of the trend. Similar to the gambler's fallacy, it does not imply that the chance of having a price decrease in the sixth day is high after having the stock price increasing for five days.



Source: Adjusted close share prices of Apple Inc. from Yahoo! Finance. Close prices are adjusted for dividends and splits.

Psychological Explanation

The gambler's fallacy is not rational, but it is appealing to human cognition. Tversky and Kahneman (1974)[1] explained

Table 2: Daily Price Movement Summary of Apple Inc. (Sept. 7, 1984 ~ Feb. 6, 2014)Source: Adjusted close share prices of Apple Inc. from Yahoo! Finance. Close prices are adjusted for dividends and splits.

# of days with continuous price **increase**	Next Day Price Movement		# of days with continuous price **decrease**	Next Day Price Movement	
	Down	Up		Down	Up
1	942	903	1	856	990
2	451	452	2	494	496
3	237	215	3	274	222
4	101	114	4	106	116
5	59	55	5	56	60
6	30	25	6	38	22
7	8	17	7	13	9
8	7	10	8	4	5
9	5	5	9	3	2
10	4	1	10	1	1
11	0	1	11	1	0
12	1	0	12	0	0

Source: Adjusted close share prices of Apple Inc. from Yahoo! Finance. Close prices are adjusted for dividends and splits.

that the gambler's fallacy is a cognitive bias caused by representativeness heuristic. When people are asked to assess the probability of an event, they compare it to their experiences and knowledge to find out the similarity. And they may expect that short run outcomes should be representative of long run outcomes. But in reality it is not. Using the coin-tossing example, let's compare the mean and volatility of 10 tosses' outcome and those of 100 tosses' outcome. The *Volatility/Mean* of the 10 tosses' outcome is relatively high compared to that of the outcome of 100 tosses. Furthermore, the probability of having heads more than 70% of the tosses for 10 tosses is much higher than that for 100 tosses. A long run outcome is not likely to happen in the short run. Assessing the probability of the short run outcome based on the understanding of the long run outcome will certainly lead to a biased conclusion.

Event	10 Tosses	100 Tosses
# of heads: Mean *(np)*	5	50
# of heads: Volatility ($\sqrt{np(1-p)}$)	1.58	5
Volatility/Mean	.32	.1
Probability that the # of heads is greater than 70% of the number of tosses	5.469%	0.002%

Solutions

It is unlikely to completely remove the gambler's fallacy as it is part of human nature. However, several approaches can be taken to mitigate its effects.

1. Educating the investors about the existence, the causes, and the potential impact of the gambler's fallacy. With an increasing awareness of this cognitive bias, people may adjust their forecast to offset the impact of the

[1] Tversky, Amos and Daniel Kahneman, "Judgment under Uncertainty: Heuristics and Biases." (1974) Science, V185, No. 4157: 1124-1131.

bias intentionally.

2. Conducting analysis to understand the underlying drivers of market change. Fundamental analysis can be used to predict stock price movements. Macroeconomic analysis can be used to forecast the movement of interest rates. Although exploring cause-and-effect relationships is very difficult, it is very useful for prediction. On the other hand, considering the market movement as a random process will make the prediction subject to the gambler's fallacy. In addition, when comparing the predictions using several approaches, it is easier for people to realize and correct their biases.

3. Due to the lack of knowledge, random processes may be used to analyze some issues. In those situations, historical experience other than current status does not need to be provided when asking for people's prediction. This can reduce the impact of the gambler's fallacy.

Changing a cognitive bias can be very difficult and take a long time. But with sufficient training and appropriate tools in place, its impact can be immaterial.

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2 $\sum_{i=x}^{n} \binom{n}{i} p^{i}(1-p)^{n-i}$

n: # of tosses. ***p:*** the chance of getting a head. It is assumed 0.5 in the example. ***x***=70%×n