# Actuarial Considerations in Establishing Gradual Retirement Pension Plans

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**ABSTRACT**

Traditional pension plans assume that at a certain age, the participant will cease active employment for the plan sponsor and will start receiving a retirement pension either immediately or at the normal retirement age. In this paper, we propose a new type of pension scheme where the retirement of the employee is a gradual event, increasing from 0 to 100% and active employment correspondingly decreases from 100% to 0. We look at the actuarial implications of such a retirement scheme from the employer's point of view in terms of the normal cost and also from the employee's side, the total income (revenue and retirement) he would receive under various gradual retirement scenarios. We discuss the advantages and disadvantages of this new type of plan and new issues the actuary will face.

# 1 Introduction

In Canada, the baby-boomers generation (those born between 1952 and 1966, aged between 40 and 54 in 2006, see Brown (1991)) are now almost all in the workforce, making it difficult for younger people to become employed or to obtain permanent positions. Statistics Canada figures show that in 1986, the people in the age group 15-24 accounted for a third of the unemployed. Beaujot (1991) reports that in 1986, 20% of male workers under 45 had more more than one job versus 9% of workers over 45.

One way of helping the younger people in their job search would be to reduce gradually the employement of older people; these might find more attractive the smooth transition from active employement to retirement, as declining health is often a reason for early retirement. We look at some issues the employee, the employer and the actuary would encounter with this new gradual retirement pension plan.

Pension actuaries are well aware of the fact that the normal cost under the accrued benefit cost method (ABCM) increases steeply with age and that a projected benefit cost method (PBCM) which keeps the normal cost constant (either in amount or as a percentage of salary) is therefore preferable for pension plan sponsors worried about increasing pension costs. Winklevoss (1977) and McGill (1984) have observed this by calculating the normal cost for these funding methods for a typical employee of a pension plan; they also compare various variants of these two methods, but the general conclusions remain the same. PBCM produce normal costs that are more stable over the long run. This explains why in this paper, we will only consider these methods.

Throughout the paper, we will use the notation of Winklevoss (1977) for pension mathematics, the same typical employee and illustrative plan. The plan provisions used for the numerical comparisons of section 5 will be the following. The retirement benefit payable from age 65 as a life annuity will be equal to 1.5% of the of the final five-year average salary times the number of years of service. Early retirement is possible at age 55 after 20 years of service without actuarial reduction, and there is no late retirement. The mortality table we will use is the 1979-81 US Life Table, and the service table for ages 20-65 is found in Winklevoss (1977, p.32). The interest rate is set equal to $i = 0.04$ and salaries are assumed to increase at 5% per year. The participant is a male employee who joined the plan at age 30.

The paper is organized as follows.

In section 2, we review some basic concepts of pension mathematics for the standard entry age normal method. In section 3, we introduce a new concept, gradual retirement of an employee with the cumulative retirement function, and its complement, the active employment function from which we define the total income function from pension and employment for the participant. We also look at some examples which could serve as typical functions for an employee who would take a gradual retirement. In section 4, we use these newly defined functions to define the normal cost for the employee member of a gradual retirement pension scheme under a modified PBCM. Section 5 computes the normal cost and the total income for the employee under various cumulative retirement functions, compared to the method presently used. We look at some issues not considered in our model and present some remarks and conclusions in section 6.

## 2 Review of Traditional Entry Age Normal Cost Method

For the standard entry age normal cost method, also called projected benefit cost method, we will consider the two versions of the method, one producing a constant amount (CA) and the other producing a constant percentage of salary (CS) for the normal cost. In both cases, an actuarial equivalence is established at the age of entry of the participant in the pension plan, equating the actuarial present value of future normal costs to the actuarial present value of future retirement benefits and soving for the resulting normal cost, either as a constant amount or a constant percentage of the salary.

For the constant amount version (${}^{CA}PBCM$), this actuarial equivalence is

$$^{CA}NC_x \times \ddot{a}_{y:\overline{r-y|}} = PVFB_y,$$

where ${}^{CA}NC_x$ is the constant normal cost, for all $x$ between entry age $y$ and retirement age $r$,
$PVFB_y = B_r \times v^{r-y}\ {}_{r-y}p_y^{(\tau)} \times \ddot{a}_r$ is the actuarial present value at age $y$ of the annual retirement pension $B_r$ payable for life from age $r$ (standard actuarial symbols are defined in Bowers et al. (1997)).

The annual pension could be a flat amount, an amount based on career earnings or a percentage of the average salary of the best 3 or 5 years of service times the number of years of service (in our example, 1.5% of the final 5-year average salary per year of service).

For the constant percentage of salary version, the normal cost will the fraction $k$ of the employee's salary calculated from the actuarial equivalence equation

$$k \times s_y \times^s \ddot{a}_{y:\overline{r-y|}} = PVFB_y,$$

where ${}^s\ddot{a}$ is the general symbol for the actuarial present value of an annuity increasing according to the salary scale,
and $s_y$ is the salary at entry age $y$.

The normal cost at age $x$ is equal to

$$NC_x = k \times s_x$$

Table 1: Normal Cost as a percentage of salary

| Age $x$ | ${}^{CA}PBCM$ | ${}^{CS}PBCM$ |
|---|---|---|
| 30 | 13.77% | 8.21% |
| 40 | 8.45% | 8.21% |
| 50 | 5.19% | 8.21% |
| 60 | 3.19% | 8.21% |

and will normally increase as the salary of the employee increases through merit, productivity and inflation.

For the illustrative plan of section 1, Table 1 gives the normal cost as a percentage of the salary for attained ages 30, 40, 50, 60 for the 2 versions of the PBCM.

For an age $x$ between $y$ and $r$, the actuarial liability at age $x$ is defined as the actuarial present value of future benefits minus the actuarial present value of future normal costs, where those two values are calculated at age $x$.

## 3 Gradual Retirement

Under any retirement pension plan, an employee, on his retirement day, would transfer from the active group to the retired group. Full retirement would happen on that day. In the last few years, many employers have offered early retirement to selected groups of employees, as a workforce management tool. In this section, we want to look at a new type of retirement, gradual retirement where the transition from active employment to retired status would be gradual, not abrupt as it is above. In the next section, we will look at the actuarial implications of such a gradual retirement.

## 3.1 Cumulative retirement and active employment functions

Let $x$ be the current age of the participant in a pension plan, and $r$ be the normal retirement age for this plan. Under traditional pension plans, we define the cumulative retirement function $R(x)$, as the indicator function representing the retired status of an individual,

$$R(x) = \begin{cases} 0 & \text{if } x < r \\ 1 & \text{if } x \geq r. \end{cases}$$

The active employment function $AE(x)$ is defined as $1 - R(x)$. It is also an indicator function

$$AE(x) = \begin{cases} 1 & \text{if } x < r \\ 0 & \text{if } x \geq r. \end{cases}$$

It shows whether the same individual is an active employee or not. An active employee is 100% active and 0% retired, while a retired employee is 0% active and 100% retired. The basic scenario of comparison, denoted Scenario 0, will be this previous $R(x)$ function with $r$=65.

However, under gradual retirement, other increasing functions $R(x)$ which go from 0 to 1 for $x$ between $e$ and $l$, where $e$ is the earliest retirement age permitted and $l$ the latest one permitted under the plan, would be allowed. An employee could partly belong to both groups at the same time.

For example (Scenario 1), for a participant who takes a half-retirement at age 62 and full retirement at 65, the function $R(x)$ would be defined as

$$R(x) = \begin{cases} 0 & \text{if } x < 62 \\ 0.5 & \text{if } 62 \leq x < 65 \\ 1 & \text{if } 65 \leq x. \end{cases}$$

His active employment function $AE(x)$ would be equal to

$$AE(x) = \begin{cases} 1 & \text{if } x < 62 \\ 0.5 & \text{if } 62 \leq x < 65 \\ 0 & \text{if } 65 \leq x. \end{cases}$$

The graph of $R(x)$ is a step function. It is seen that the cumulative retirement function has the same properties as the cumulative distribution function $F_X(x)$ of a random variable $X$: it is a non-decreasing function of $x$ going from 0 to 1. On the other hand, the active employment function $AE(x)$ is analogous to the survival function $S_X(x)$ in statistics: this non-increasing function of $x$ goes from 1 to 0.

For every individual, at any age $x$, we must have

$$AE(x) + R(x) = 1,$$

where $0 \leq AE(x), R(x) \leq 1$.

Let us define $r_1$ as the age at which the function $R(x)$ first becomes positive (age 65 in Scenario 0 and 62 in Scenario 1). We will assume that an employee stops accruing years of service for his retirement at age $r_1$ and that the contributions to a pension plan would be made from age $y$ to $r_1$. Under gradual retirement, an employee would receive a fraction $R(x)$ of his retirement pension at age $x$, in addition to a fraction $AE(x)$ of his salary as an active employee.

As another example (Scenario 2), an employee who takes a one third retirement at age 55, an additional one third retirement at age 60 and fully retires at age 65 would have a cumulative retirement function $R(x)$ equal to

$$R(x) = \begin{cases} 0 & \text{if } x < 55 \\ 0.33 & \text{if } 55 \leq x < 60 \\ 0.67 & \text{if } 60 \leq x < 65 \\ 1 & \text{if } 65 \leq x, \end{cases}$$

and an active employment function $AE(x)$ equal to

$$AE(x) = \begin{cases} 1 & \text{if } x < 55 \\ 0.67 & \text{if } 55 \leq x < 60 \\ 0.33 & \text{if } 60 \leq x < 65 \\ 0 & \text{if } 65 \leq x. \end{cases}$$

Let us now define the non-cumulative retirement function $p(x)$ at age $x$ describing where the jumps in $R(x)$ occur (corresponding to ages where an additional retirement percentage is taken) and the height of those jumps

$$p(x) = R(x) - R(x^-).$$

The function $p(x)$ shares the same properties as the probability mass function of a discrete random variable $X$. For all elements $r_i$ of the domain $D = \{r_1, \ldots, r_m\}$ of the function $p(x)$, $0 < p(r_i) \leq 1$, and $\sum_{r_i \in D} p(r_i) = 1$. We assume that at age $r_m$, the employee has fully retired, i.e. $R(r_m) = 1$.

For Scenario 0,

$$p(x) = \begin{cases} 1 & \text{if } x = 65 \\ 0 & \text{if not }, \end{cases}$$

while for Scenario 2,

$$p(x) = \begin{cases} 0.33 & \text{for } x = 55, 60, 65, \\ 0 & \text{for all other ages } x. \end{cases}$$

This function $p(x)$ will help us in the next section to define the normal cost under the modified entry age normal cost method. We will use Scenarios 1 and 2 and others defined analogously and compare the normal cost for the typical employee of section 1.

Note that we will consider Scenarios where the function $p(x)$ is discrete. We could construct a continuous function for $p(x)$, but this would only be of theoretical interest, since in practice, employees would be allowed to elect increased gradual retirement at certain ages only.

## 3.2 Total income

If $s_x$ is the annual rate of salary at age $x$, The total income the participant receives at age $x$, from his work and retirement pension, denoted $TI(x)$, is equal to

$$\begin{aligned} TI(x) &= R(x) \cdot B_{r_1} + AE(x) \cdot s_x \\ &= R(x) \cdot B_{r_1} + (1 - R(x)) \cdot s_x \\ &= s_x - R(x)[s_x - B_{r_1}]. \end{aligned}$$

For $x < r_1$, $R(x) = 0$ and $AE(x) = 1$, so that the participant only receives his employment income,

$$TI(x) = s_x, \text{ for } x < r_1.$$

For $r_1 \leq x < r_m$, $R(x) < 1$ and $s_x > B_{r_1}$, so that $TI(x)$ is always between $B_{r_1}$ and $s_x$.

For $x \geq r_m$, $R(x) = 1$ and $AE(x) = 0$: the former employee is fully retired and receives a full retirement pension

$$TI(x) = B_{r_1} \text{ for } x \geq r_m,$$

assuming that the pension is not indexed to inflation.

# 4 Modified Entry Age Normal Cost Method

## 4.1 Constant amount version

To determine the normal cost under the entry age method, we used in section 2 the equivalence, at the age of entry of the participant in the plan, between the actuarial present value of future retirement benefits and the actuarial present value of future normal costs. Remember that for gradual retirement, we assumed that service would accrue until age $r_1$, and the contributions would be made from age $y$ until age $r_1$.

Under the constant aount version of the modified $PBCM$ denoted $({}^{CA}MPBCM)$, the actuarial equivalence at entry age $y$ would now become

$$^{CA}MNC_x \times \ddot{a}_{y:\overline{r_1-y}|} = B_{r_1}\left[\sum_{r_i \in D} v^{r_i-y}\ {}_{r_i-y}p_y^{(\tau)} \times \ddot{a}_{r_i} \times p(r_i)\right].$$

The difference between the ${}^{CA}PBCM$ and the ${}^{CA}MPBCM$ to note are the following:

the modified normal cost $MNC_x$ is now paid from age $y$ until age $r_1$.

the future pension is determined at age $r_1$, since years of service and the final average salary are determined at that age.

At age $r_1$, the employee would start receiving a fraction $p(r_1)$ of that pension $B_{r_1}$ for life; at age $r_2 > r_1$, an additional fraction $p(r_2)$ of the amount $B_{r_1}$ would become payable, etc... It is seen that the total pension payable at any age $x \geq r_1$ can therefore be defined using the cumulative retirement function $R(x)$; it is equal to $R(x) \cdot B_{r_1}$ for all $x \geq r_1$.

As the percentage of the total pension becoming payable increases, the income the employee receives from his employment will correspondingly decrease as the active employment function $AE(x)$ decreases from 1 to 0. Note however that the employee could still benefit from partial salary increases through increases in the salary scale function.

In section 5, we will compare for certain Scenarios, the effect of various cumulative retirement functions $R(x)$ on the normal cost but also on the retirement pension and employment income the participant receives.

## 4.2 Constant percentage version

Under the constant percentage version of the modified $PBCM$ denoted $({}^{CS}MPBCM)$, we set up the same equation of value at the entry age, between the actuarial present value of future normal costs and the actuarial present value of future retirement benefits

$$ {}^{m}k \times s_y \times {}^{s}\ddot{a}_{y:\overline{r_1 - y}|} = B_{r_1} \left[ \sum_{r_i \in D} v^{r_i - y} \;{}_{r_i - y}p_y^{(\tau)} \times \ddot{a}_{r_i} \times p(r_i) \right]. $$

The modified normal cost in dollars at age $x$ is therefore equal to this new calculated fraction ${}^{m}k$ times the salary at age $x$

$$ {}^{CS}MNC_x = {}^{m}k \times s_x. $$

Again, the modified normal cost will only be paid until age $r_1$, the first age at which partial retirement starts.

# 5 Numerical Illustrations

In this section, we will analyze a few Scenarios for the cumulative retirement function $R(x)$. The basic Scenario 0, defined in section 3, represents retirement at the normal retirement age. Under Scenario 1, gradual retirement starts earlier than under Scenario 2 (age 55 versus age 62).

Table 2: ${}^{CA}NC$ as a percentage of salary

| Age $x$ | Scenario 0 | Scenario 1 | Scenario 2 | Scenario 3 |
|---|---|---|---|---|
| 30 | 13.77% | 13.10% | 10.91% | 15.14% |
| 40 | 8.45% | 8.04% | 6.70% | 9.29% |
| 50 | 5.19% | 4.94% | 4.11% | 5.71% |
| 55 | 4.07% | 3.87% | 0 | 4.47% |
| 60 | 3.19% | 3.03% | 0 | 3.50% |
| 62 | 2.89% | 0 | 0 | 0 |
| 65 | 0 | 0 | 0 | 0 |

Table 3: ${}^{CS}NC$ as a percentage of salary

| Scenario 0 | Scenario 1 | Scenario 2 | Scenario 3 |
|---|---|---|---|
| 8.21% | 8.08 % | 7.30% | 9.33% |
| $30 \leq x < 65$ | $30 \leq x < 62$ | $30 \leq x < 55$ | $30 \leq x < 62$ |

Full early retirement at age 62 is defined in Scenario 3:

$$R(x) = \begin{cases} 0 & \text{if } x < 62 \\ 1 & \text{if } x \geq 62. \end{cases}$$

We will be able to compare Scenarios 1 and 2 involving two different retirement functions and also compare gradual retirement with respect to normal retirement (Scenarios 1 and 2 versus Scenario 0) or early retirement versus normal retirement (Scenario 3 versus Scenario 0).

Table 2 gives the calculated normal cost for all scenarios under the constant amount PBCM at various ages.

Table 3 is the corresponding table for the normal cost calculated under the constant percentage version of the PBCM.

Another interesting function to compute is the total income function for each scenario. Under Scenario 0,

$$TI(x) = \begin{cases} 0.525 \cdot FAE_{65} & \text{for } x \geq 65. \end{cases}$$

where $FAE_x$ represents the 5-year final average earnings caclculated at age $x$.

Under Scenario 1, $TI(x) = R(x)B_{r_{62}} + (1 - R(x))s_x$, so that

$$TI(x) = \begin{cases} .5B_{r_{62}} + .5s_{62} & \text{for } x = 62 \\ .5B_{r_{62}} + .5s_{63} & \text{for } x = 63 \\ .5B_{r_{62}} + .5s_{64} & \text{for } x = 64 \\ B_{r_{62}} & \text{for } x \geq 65. \end{cases}$$

The employee benefits from partial salary increase between ages 62 and 65.

Under Scenario 2, $TI(x) = R(x)B_{r_{50}} + (1 - R(x))s_x$, and

$$TI(x) = \begin{cases} 1/3B_{r_{55}} + 2/3s_x & \text{for } 55 \leq x < 60 \\ 2/3B_{r_{55}} + 1/3s_x & \text{for } 60 \leq x < 65 \\ B_{r_{55}} & \text{for } x \geq 65. \end{cases}$$

From age 50, the total income starts decreasing each year, until age 65, where it remains constant.

Under Scenario 3, $TI(x) = 0.48 \cdot FAE_{62}$ for $x \geq 62$. The total income would be lower than under Scenario 0 because of the smaller number of years of service (32 versus 35) and also the smaller 5-year final average salary ($s_{62}$ versus $s_{65}$).

# 6 Remarks and Conclusions

If the idea of gradual retirement appeals to both the employer and the employees, the plan sponsor could offer different gradual retirement scenarios, and the employee could choose the one that best meets his total income function objective for the future.

Another possibility would be that the employer would only define the age at which gradual retirement can start and the ages where increases in pension can occur. The employee would be free to choose the value of those jumps ($p(r_i)$ at age $r_i$). He could do so according to a desired total income function.

For example, an employee who joined the plan defined in section 1 at age 24, who would want to retire according to the following total income function in mind

$$TI(x) = \begin{cases} C \times FAE_{56} & \text{for } 56 \geq x < r_m \\ 0.48 \times FAE_{56} & \text{for } x \geq r_m, \end{cases}$$

where $C$ is a fraction, would calculate his retirement function $R(x)$ as follows.

The minimum percentage of pension ultimately desired (48% of $FAE_{56}$ in this example) defines the age $r_1$ at which pension service stops accruing. After 32 years of service ($32 \times 1.5\% = 48\%$), the participant who joined the plan at age 24 could retire, i.e. at age 56.

To start enjoying retirement at age 56 with a fraction $C$ of his $FA5_{56} \times s_{56}$ we will need to set $R(56)$ such that

$$\begin{aligned} TI(56) &= C \times FAE_{56} \\ &= R(56) \times 0.48 \times FAE_{56} + AE(56) \times s_{56} \\ &= R(56) \times 0.48 \times FAE_{56} + (1 - R(56)) \times s_{56}, \end{aligned}$$

so that

$$R(56) = \frac{(C \times FAE_{56}) - s_{56}}{(0.48 \times FAE_{56}) - s_{56}}, \quad \text{where } C > 0.48.$$

The employee can still define the age $r_m$ where full retirement is taken.

Note that with existing plan provisions, certain scenarios are not possible. An employee who wants a minimum ultimate pension could not retire before a certain age. Imposing constraints on the total income function at specified ages sets bounds on the retirement function at those ages.

In this simple plan, we have not considered any ancillary benefits such as death benefits or disability pension. The normal form of annuity was a straight life annuity. In practice, the annuity could have a 10 or 15-year garantee or be a last-survivor pension payable to a spouse. The actuary would have to evaluate each fraction of the pension payable from age $r_i$ with the remaining garantee if any.

If the employee is offered many possible retirement functions to choose from, the age $r_1$ of first gradual retirement is unknown at entry age. The actuary can not therefore compute the normal cost necessary to fund the pension from entry age to $r_1$. He could assume a certain percentage of employees would choose each retirement function. Or he could select the minimum possible age of retirement ($r_{min}$) among all scenarios and fund the the pensions for all employees from entry age to $r_{min}$. According to the actual retirement function selected by the employee, an actuarial gain or loss would emerge.

Another possibility would be to calculate the normal cost $NC_i$ payable from age $y$ to $r_i, i = 1, \ldots, m$, necessary to fund the increase in pension from age $r_i$. The total normal cost for an employee would be the sum

$$NC = \sum_i NC_i.$$

With the notation already defined, it would be possible to extend the method for calculating the normal cost to the aggregate version of the projected benefit cost method by using:
for the constant amount version

$$\sum_i R(x_i), \text{ the total number of retired employees at a certain time,}$$

$$\sum_i AE(x_i), \text{ the total number of active employees at a certain time,}$$

and for the constant percentage version

$$\sum_i AE(x_i) \cdot s_{x_i}, \text{ the total salary mass at a certain time.}$$

One other actuarial issue that comes up with gradual retirement versus total retirement pension plans is the cecrement table to be used. Currently, there are two types of decrement tables available: a service table for active employees who face death, disability and withdrawal, and a mortality table for fully retired employees. A participant partly active and partly retired would probably experience mortality close to that experienced by the employees who take a full early retirement.

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