# Mortality Improvement Select Birth Cohorts and Their Effect on Pricing of Survivor Bonds

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Presented at
Re-Envisioning Retirement in the 21st Century
Society of Actuaries

May 3–4, 2006

## Abstract

This work investigates the effect of select birth cohorts on pricing of mortality-based securities, such as survivor bonds, life annuities in general or portfolios of life annuities. Existence of select birth cohorts in the United Kingdom has been indicated by works of Willets, and also shown in other countries in the recent work of the present authors. In this paper we compare and contrast the cohort effect to the influence of interest rates, and discuss their relative importance in pricing of mortality-based securities.

## 1. Introduction

The 20th century has witnessed unprecedented improvement in longevity in the United States and worldwide, commonly called among actuaries "mortality improvement" (despite inherent contradiction of this term and the actual nature of the process, which we believe, should be termed *longevity improvement*). While this is quite an achievement for public health and public policy in general, it is also a source of risk for both governments and private companies providing retirement benefits. Some argue that the annuities market in the United States is underdeveloped due to uncertainty of mortality improvement and resulting conservatism in pricing of annuities. To illustrate this, let us note that the annuity market in the United States is approximately $2 billion per year, while the United Kingdom annuity market is approximately £6 billion per year, in terms of premiums collected.

These authors believe, as it has been argued, e.g., by Blake and Burrows (2001), Dowd, Blake, Cairns and Dawson (2005) and Lin and Cox (2004), that a market instrument: *a survivor bond,* can help in pricing and development of the annuities market. This is especially important in the current period preceding the retirement of the baby boom generation, an event that can greatly affect the national economy and public finances. We also discuss the process of issuance of such securities, and argue that there may be a role for the public sector in it.

A *survivor bond* (Blake and Burrows, 2001) has a coupon that is proportional to the number of survivors in a chosen population cohort. Such bonds could be used as market instruments providing a pricing signal for retirement cost.

The survivor bond instrument would, of course, not be without risk. Such risk is three-fold:

- The standard interest rate risk.
- The uncertainty of the magnitude of mortality improvement (i.e., the magnitude of the overall trend in mortality changes).
- The uncertainty of the fluctuation in the process of mortality improvement (i.e., temporary deviations around the overall trend).

There has been much work done on the first two areas. Interest rate risk is one of the key risks faced by financial intermediaries, and has been studied in a wide variety of investment and actuarial literature (Ostaszewski, 2002, provides a review of asset-liability management methodologies utilized by insurance firms). The uncertainty of the magnitude of mortality improvement has been increasingly a topic of concern for retirement policy makers and pricing actuaries in insurance firms providing annuity products. One example of research in this area is the work of Carter and Lee (1992). But the last topic has been a relatively new concern, and it is a key issue in this work.

A *select birth cohort* with respect to *mortality improvement* is defined as an age cohort or generation characterized by greater rates of mortality improvement than previous and subsequent generations. It is generally expected, at least in the last 100 years, that subsequent generations live longer than the previous ones. But among the birth cohorts, or subsequent generations, there are some whose improvement in mortality or longevity is especially high. Willets (2004) calls this the *cohort effect*, describing it as a "wave of rapid improvements, rippling upwards through mortality rates in the United Kingdom." For example, in his estimation, for the past four decades, people born between 1925 and 1945 have benefited from faster mortality improvements than those born in adjacent generations. The implication is that when a select birth cohort reaches retirement, this cohort's lower mortality will be quite a shock to the retirement system. Willets (2004) documents such a phenomenon for the generation born in Great Britain in the 1930s, with emphasis on 1931. This generation is now in their seventies and if they continue experiencing the same mortality improvements that they did in younger ages, they will be a strong example of the nature of this problem.

*Mortality improvement* between two birth cohorts, as measured for the cohort born in year $z$, is defined as

$$ {}^{z}i_x = -\left( \frac{{}^{z}m_x - {}^{z-1}m_x}{{}^{z-1}m_x} \right) = 1 - \frac{{}^{z}m_x}{{}^{z-1}m_x}, $$

where ${}^{z}m_x$ is the central death rate at age $x$ for the generation born in year $z$. This notion does not provide a natural definition of a select birth cohort, and there has been only limited effort to define it. In our work, we propose that a *select birth cohort* with respect to *mortality improvement* is a birth cohort whose mortality improvement exceeds that of the birth cohort just before it and just after it at the majority of individual ages. This is a form of a local definition (with respect to age), and we also study more global measures. A *select birth cohort* with respect to *longevity improvement* is a birth cohort whose relative longevity improvement exceeds that of the birth cohort just before it and just after it at the majority of individual ages. One more global approach considers relative mortality improvement at each age for each birth cohort. Such specific value can be compared to $90^{th}$ percentile of all values for the same age.

The purpose of our study of mortality improvement is to investigate the effect of select birth cohorts on the pricing of survivor bonds (and, by implication, life annuities and all retirement products), and estimate the effect of the cohort effect risk on the development of retirement products markets. In particular, we want to establish whether the cohort effect risk should be considered as significant as the interest rate risk and the general mortality improvement risk.

We also investigate the opportunities for the use of survivor bonds in the investment policy of the Social Security Trust Fund. In summary, we submit that the Trust Fund would be best served by an investment policy which manages the assets and liabilities of the system in an integrated fashion. Such an investment policy is better accomplished with the use of survivor bonds.

We study the cohort effect and pricing, as well as other financial parameters, of survivor bonds based on data from the online Human Mortality Database, using data from a variety of countries and investigating possible global issuance of survivor bonds.

## 2. Effect of Interest Rates and Mortality

When comparing the effect of interest rates and mortality on the prices of annuities, it is common to start with the observation that the price of a unit pure endowment, issued at age *x* for *n* years, as a function of the force of interest (i.e., continuously compounded forward interest rate) $\delta_t$ and the force of mortality $\mu_x(t)$ (representing the force of mortality at age $x+t$ for a person underwritten for life insurance or annuity at age $x$, also denoted often by $\mu(x+t)$, and defined as $\mu(u) = -s'(u)/s(u)$, where $s(u)$ is the probability that a newborn is still alive at age *u)* has the actuarial present value

$$ {}_nE_x = A_{x:\overline{n|}}^{\;\;1} = e^{-\int_0^n \delta_s ds} \cdot e^{-\int_0^n \mu_x(s) ds} = e^{-\int_0^n (\delta_s + \mu_x(s)) ds}. $$

A unit discrete life annuity immediate paid from age *x* for life has the actuarial present value

$$ a_x = \sum_{n=1}^{\omega} e^{-\int_0^n (\delta_s + \mu_x(s)) ds} = \sum_{n=1}^{\omega} {}_nE_x, $$

where $\omega$ is the *limiting age,* i.e., age at which a given cohort of newborns terminates its existence. A unit continuous life annuity from age *x* has the actuarial present value

$$ \overline{a}_x = \int_0^{\omega} e^{-\int_0^t (\delta_s + \mu_x(s)) ds} dt = \int_0^{\omega} {}_tE_x dt. $$

We see, therefore, that the effect of interest rate and mortality changes on the prices of annuities can be studied by analyzing the effect of these variables on the price of a unit pure endowment. The standard measures of sensitivity of a financial instrument to interest rates are duration and convexity. Following the approach of Gajek, Ostaszewski and Zwiesler (2005) we note that duration of a financial asset with price $P = P(i)$ expressed as a function of the interest rate is

$$D = -\frac{dP}{di} \cdot \frac{1}{P},$$

and the duration with respect to the force of interest, i.e., the *Macaulay duration,* is

$$D_M = -\frac{dP}{d\delta} \cdot \frac{1}{P},$$

where the interest rate $i$ as well as the force of interest $\delta = \ln(1+i)$ are the same regardless of maturity (this is commonly referred to as the *flat yield curve assumption*). If we make the flat yield curve assumption for pricing of an endowment, we obtain:

$$\frac{d}{d\delta}{}_nE_x = \frac{d}{d\delta}A_{\substack{1\\x:\overline{n|}}} = \frac{d}{d\delta}\left(e^{-\delta n} \cdot e^{-\int_0^n \mu_x(s)ds}\right) = -ne^{-\delta n} \cdot e^{-\int_0^n \mu_x(s)ds} = -n \cdot {}_nE_x.$$

This implies that the Macaulay duration of a pure endowment, when priced under flat yield curve assumption, is the time length of the contract. If we now consider the rate form of a derivative under the assumption of constant force of mortality, $\mu_x(t) = \mu_x$, then

$$\frac{d}{d\mu}{}_nE_x = \frac{d}{d\delta}A_{\substack{1\\x:\overline{n|}}} = \frac{d}{d\mu_x}\left(e^{-\int_0^n \delta_s ds} \cdot e^{-\mu_x n}\right) = -ne^{-\mu_x n} \cdot e^{-\int_0^n \delta_s ds} = -n \cdot {}_nE_x.$$

We see a clear illustration of the fact that the effect of mortality and interest on pricing of pure endowments, and consequently pricing of life annuities, is the same, e.g., 1 percent change in the force of interest has the same effect on the price as 1 percent changes in the force of mortality, so that when the force of interest declines from 5 percent to 4 percent and the force of mortality declines from 4 percent to 3 percent, the effect on price of a pure endowment and a life annuity is the same.

However, there is no concept of sensitivity to changes in the force of mortality developed in actuarial science. This is partly due to the fact that while the assumption of a flat yield curve is not correct in practice, it may be considered a reasonable approximation. The assumption of constant force of mortality is used in some theoretical models, also in pricing of life insurance for accidents, where the age of the insured is not relevant for the level of risk, but the constant force assumption is clearly incorrect when applied to human mortality, as it implies the same probability of death within one year, regardless of age. Nevertheless, we believe it might be also practical to develop measures of sensitivity with respect to the force of interest, under the assumption of constant force. If a financial instrument has its price $P = P(\mu)$ expressed as a function of the force of interest, then we will define its *mortality duration* as:

$$_M D = -\frac{dP}{d\mu} \cdot \frac{1}{P}.$$

However, as we had noted already, the standard measure of changes in mortality among cohorts is *mortality improvement*

$$1 - \frac{^z m_x}{^{z-1}m_x}.$$

Under the constant force assumption, with difference force $^z\mu$ for each birth cohort, we have

$$^z m_x = \frac{d_x}{L_x} = \frac{l_x - l_{x+1}}{\int_0^1 l_{x+t}dt} = \frac{e^{-^z\mu x} - e^{-^z\mu(x+1)}}{\int_0^1 e^{-^z\mu(x+t)}dt} = \frac{1 - e^{-^z\mu}}{\left. -\frac{1}{^z\mu} e^{-^z\mu(x+t)} \right|_{t=0}^{t=1}} = {}^z\mu.$$

Therefore, mortality improvement in terms of the force of interest equals

$$1 - \frac{^z\mu}{^{z-1}\mu} = \frac{^{z-1}\mu - {}^z\mu}{^{z-1}\mu}.$$

Consider now a unit pure endowment $_nE_x$ as a function of constant force of mortality. Then if we denote by $\Delta^z\mu = {}^{z-1}\mu - {}^z\mu,$ we have $\Delta^z\mu = {}^z i_x \cdot {}^{z-1}\mu,$ and the change in the price of the endowment as a result of mortality improvement is

$$^z_n E_x - {}^{z-1}_n E_x \approx -\frac{\partial_n E_x}{\partial\mu} \cdot \Delta^z\mu = -\frac{\frac{\partial_n E_x}{\partial\mu}}{^z\mu} \cdot {}^z\mu \cdot \left({}^z i_x \cdot {}^{z-1}\mu\right) =$$
$$= {}_M D \cdot {}^z i_x \cdot {}^{z-1}\mu \cdot {}^z\mu = n \cdot {}^z i_x \cdot {}^{z-1}\mu \cdot {}^z\mu.$$

Similarly, for any security whose price $P = P(\mu)$ is dependent on the force of mortality,

$$P(\mu^z) - P({}^{z-1}\mu) \approx -\frac{\partial P}{\partial\mu} \cdot \Delta^z\mu = -\frac{\frac{\partial P}{\partial\mu}}{^z\mu} \cdot {}^z\mu \cdot \left({}^z i_x \cdot {}^{z-1}\mu\right) = {}_M D(P) \cdot {}^z i_x \cdot {}^{z-1}\mu \cdot {}^z\mu.$$

We are most interested in pricing of life annuities and their relative sensitivities to changes in interest rates and mortality, especially in relation to select mortality cohorts. We observe that under a constant force of mortality and constant force of interest regime, discrete life annuity price is

$$^z a_x = \frac{^z p}{^z q + i} = \frac{e^{-^z\mu}}{1 - e^{-^z\mu} + i} = \frac{e^{-^z\mu}}{e^{\delta} - e^{-^z\mu}} = \frac{1}{e^{\delta + ^z\mu} - 1}.$$

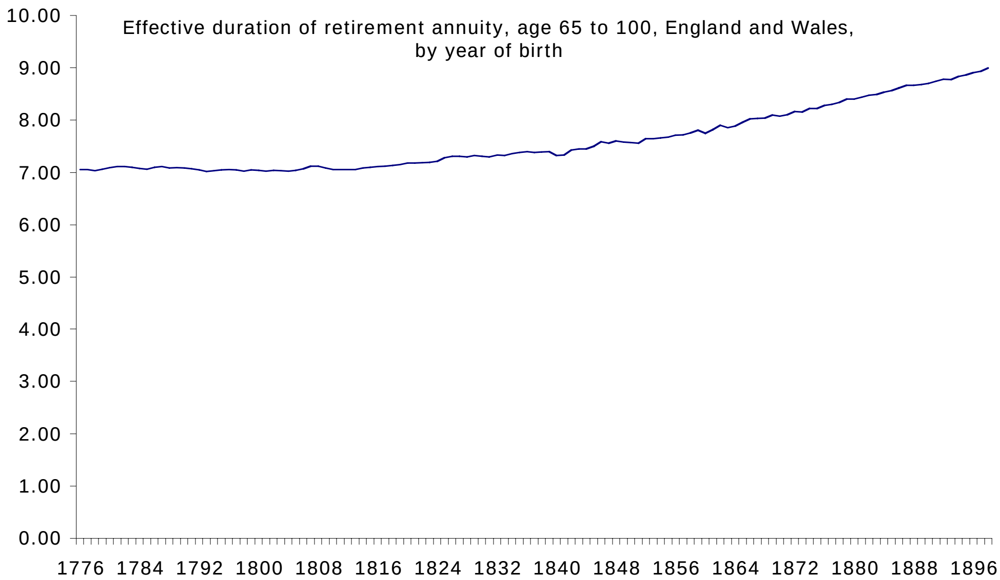
where ${}^{z}p$ is the probability of survival for one more year (same every age under constant force) for a cohort born in year $z$ and ${}^{z}q = 1 - {}^{z}p$. We have therefore

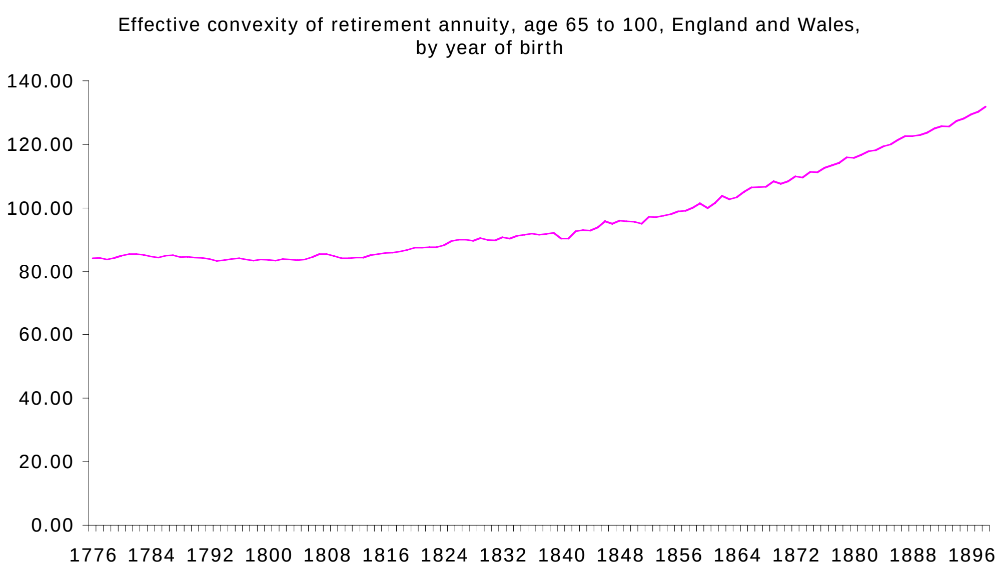
$$ {}^{z}a_x - {}^{z-1}a_x = \frac{1}{e^{\delta + {}^{z}\mu} - 1} - \frac{1}{e^{\delta + {}^{z-1}\mu} - 1} = \frac{e^{\delta + {}^{z-1}\mu} - e^{\delta + {}^{z}\mu}}{\left(e^{\delta + {}^{z}\mu} - 1\right)\left(e^{\delta + {}^{z-1}\mu} - 1\right)}. $$

On the other hand, we have

$$ \frac{\partial}{\partial \mu} \frac{1}{e^{\delta + \mu} - 1} = -\frac{e^{\delta + \mu}}{\left(e^{\delta + \mu} - 1\right)^2}. $$

These calculations allow us to provide an estimate, arguable only a rough approximation, but still possibly a good rule of thumb, for the effect of mortality improvement on the cost of life annuities. Let us assume that the force of interest is constant and equal to 3 percent. Suppose that for a cohort born in year $z$ we consider the life annuity from age 65 till death, and analyze it as a financial instrument. The figure below gives the duration of that security, increasing steadily from the range of 6.5 to about 8. The second figure shows the convexity increasing rather sharply from the range of about 85 to over 130. The rise in convexity, of course, represents the substantial increase in the dispersion of the cash flows, as a result of increasing life expectancy. Convexity increases also as a result of the increase in duration, but what we want to stress is that not only cash flows are paid over a longer period of time (giving greater duration), but they are also widely dispersed over time, as opposed to, e.g., a one-time payment of a zero-coupon bond. This is important for financial risk management, as these sharp increases in duration and convexity of the liabilities of institutions offering life annuities cannot be matched by financial instruments available currently in the market. Later we propose that survivor bonds may provide such a match, but no survivor bonds are currently available for trading.




Let us use the values of duration and convexity from the upper ranges in the above figures for estimation of the effect of changes in interest rates on the cost of life annuity from age 65 to 100. Taking the duration value to be 8.5, the upper range approached in the data set under consideration, and the upper range value of 130 for convexity, we conclude that to obtain a 1.3 percent increase in the price of a life annuity we must have

$$0.013 \approx -8.5 \cdot \Delta i + 65 \cdot (\Delta i)^2 .$$

This is equivalent to the following quadratic equation

$$65 \cdot (\Delta i)^2 - 8.5 \cdot \Delta i - 0.013 = 0,$$

which in turn solves to:

$$\Delta i \approx \frac{8.5 \pm \sqrt{8.5^2 + 260 \cdot 0.013}}{130} = \frac{8.5 \pm 8.69655104}{130} = \begin{cases} 0.132281162, \\ -0.00151193. \end{cases}$$

The first solution of $\Delta i \approx 13.23\%$ is unrealistically large, but the other solution, a decline in the interest rate of approximately 15 basis points, is realistic. We see that under simplifying assumptions, a mortality improvement often viewed as typically occurring over a span of a decade, and actually commonly occurring in annual inter-cohort oscillations, is approximately equivalent to 15 basis points decline in interest rates. While this value of the parallel shift in the yield curve may not seem excessive or dangerous, we should be somewhat careful in disregarding it, especially when considering the business of private pension and annuity companies, whose profit margins may realistically be in the range of 25 basis points, and this variation may account for more than half of their profit margins. Furthermore, existence of oscillations in the costs of life annuities due to the cohort effect may result in a real opportunity of adverse selection, where the firms that identify birth cohorts with substantially higher mortality may be able to offer them annuities priced aggressively to bring them into their portfolio, while pricing select cohorts conservatively and driving them away. This pricing strategy can be achieved by appropriate adjustments to mortality tables used. Its results would not be substantially different than the results of finer pricing strategies already in existence in homeowners or auto insurance, and, indirectly, possibly in life insurance, through the common practice of backdating of life insurance (Carson and Ostaszewski, 2004), where consumers pay for coverage starting at a date earlier than the policy application date, in order to have the policy issued at an earlier date with lower premium due to resulting lower age.

An important consequence of our analysis is that inter-cohort oscillations in the pricing of life annuities are in the same range as decade-by-decade oscillations assumed to be common and then also in a range of more than half of profit margins common in the payout annuity practice. We believe that uncertainty so created contributed to the underdevelopment of payout annuities market in the United States and most developed economies. United Kingdom is an exception, with a large market for annuities in the payout status, but this exception is a result of a compulsory nature of payouts in the private retirement system in the U.K. We believe that the markets for payout annuities could be possibly helped in their development by existence of clear market signal of the risk of inter-cohort life annuity price oscillations.

In order to better understand the nature of the risk of the price changes in life annuities, we have also produced a summary of annual price changes in the cost of life annuities from age 65 to age 100 for a variety of countries, based on the data in the Human Mortality Database (2005). This table is reproduced below:

**Summary Statistics of Annual Oscillations of Life Annuity Costs for a Variety of Countries**

| | CA | CH | FI | FR | IT | JP | NL | NO | SE | UK | US |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Mean | 35.9 bps | 51.2 bps | 283 bps | 47 bps | 114 bps | 103 bps | 29.7 bps | 17.8 bps | 31.7 bps | 20.8 bps | 71.4 bps |
| StDev | 75.5 bps | 235 bps | 277 bps | 84.7 bps | 223 bps | 124 bps | 103 bps | 71 bps | 109 bps | 60.7 bps | 93.6 bps |
| n | 40 | 91 | 89 | 68 | 94 | 14 | 121 | 121 | 116 | 122 | 5 |
| Minimum | -170 bps | -1300 bps | 470 bps | 210 bps | -1300 bps | -90 bps | -320 bps | -210 bps | -380 bps | -140 bps | -23 bps |
| Median | 44.6 bps | 58.21 bps | 485 bps | 44.5 bps | 176.33 bps | 97.5 bps | 30.7 bps | 23.7 bps | 35.5 bps | 21.7 bps | 51.6 bps |
| Maximum | 175 bps | 1098.98 bps | 499 bps | 234 bps | 877.22 bps | 312 bps | 598 bps | 187 bps | 526 bps | 215 bps | 226 bps |

It should be noted that the basis point measure above does not refer to changes in market interest rates, but rather to changes in the value of a life annuity between two cohorts born in consecutive years. We see that a 1.3 percent change in the value of a life annuity, which was the basis for our estimate of 15 basis points equivalent interest rate variation, is within one standard deviation from the observed mean for all countries studied except for Norway and the United Kingdom. Furthermore, 1.3 percent change in the value of a life annuity is below the maximum observed in all countries.

The long-term financial balance of private and social insurance systems that may be affected by interest rate, mortality improvement and cohort effect, is further complicated by the debate about the nature of future mortality improvements. This debate has been illustrated by the panel of experts at a conference June 9, 2005 at The Brookings Institution in Washington, D.C., presented in a special publication by the Center for Retirement Research at Boston College (December 2005). Among the experts present, Jim Oeppen and James Vaupel, by projecting mortality improvements specific to birth cohorts (and thus, possibly incorporating the cohort effect) argued that future mortality improvements can be significantly better than previously expected. Jay Olshansky, on the other hand, has argued sharply increased obesity rates in the United States have already contributed to decline in observed mortality improvement, and will have an even greater effect in the future. One significant development that is possible, and that we would like to point out as a significant danger in the long-term financial balance of the U.S. social insurance systems, is the unfortunate timing of cohorts with improved mortality and with worsening mortality. Let us elaborate. Willets (2004) proposes that the increased mortality improvements observed in generations that were not serving the military in World War II in Great Britain, because of being just too young at the time of hostilities, was due to decline in the level of smoking of cigarettes. These generations that did not go to war, and did not develop the smoking habit as a result, appear to show unusually high levels of mortality improvement. These are the generations that will begin retiring soon. On the other hand, at the time of their retirement, the unusually obese cohorts will enter the workforce. The United States has experienced especially high and troubling levels of obesity. This combination of non-smoking and healthy retirees and obese and unhealthy working population may result in increased expenditures and declining revenues of social insurance, a very undesirable combination of mortality developments. At this point, such concerns can only be raised by studying population models and alternative economic projections, especially within social insurance statutory modeling. There is, unfortunately, no market for this risk, and no market price signal for it. In addition to modeling by government experts, a market alternative may be worth considering, so that the price signals can allow us to better address these significant risks.

## 3. Survivor Bonds

A *survivor bond* (Blake and Burrows, 2001) has a coupon that is proportional to the number of survivors in a selected population cohort. Such bonds could be used as market instruments providing a pricing signal for retirement cost. For example, for retirement age of 65, letting the cohort be the number of individuals turning 65 in year $t$, the coupon in the year $t + 1$ would be proportional to the number in this age cohort who survive to $t + 1$. The market of survivor bonds is still only a theoretical consideration, as the only proposed such security, a 2004 issue by BNP Paribas, underwritten by European Investment Bank, for the British market, eventually did not materialize. Blake and Burrows (2001) argue that the creation of such bonds by the U.S. Government would, by dispersing an aggregate risk across the population as a whole, drive the associated risk premium to zero (this is an application of the Arrow-Lind, 1970, Theorem on Social Risk Bearing).

Private insurance companies could purchase survivor bonds as a hedge for their payout annuities liabilities portfolio. Market prices varying by birth cohorts would provide a signal for pricing of these liabilities. Thus there is a natural long position already in the market. The trouble is that the mortality improvement risk is feared somewhat, and the short side of the trade may fail to materialize. Blake and Burrows (2001) argue that governments would be natural issuers of those securities, providing an argument as to why such issuance may provide positive externalities. It should be noted, however, that *governments already are in the business of issuance of survivor bonds.* This is because nearly all developed economies provide either social insurance retirement systems or government-sponsored income support schemes for the elderly, effectively making life annuity payments to beneficiaries. Those payments are commonly indexed to inflation, at least partially. The World Bank (1994) points out that many of such payout annuities schemes are becoming increasingly costly to governments. It has been often argued that the main reason why those payout schemes are expensive is the aging of the population covered by them. But aging can be represented in the financing of a retirement scheme, and one can view the problem also as inadequate financing or inadequate pricing. A possibility that lack of market signals about the pricing of retirement is a contributing factor in the "old age crisis" should be considered.

## 4. Social Security Trust Fund investments

The Old-Age, Survivors and Disability Insurance (OASDI), known commonly as Social Security, is effectively the largest insurance scheme in the world. It is the social insurance system administered by the Board of Trustees appointed by the Federal Government of the United States. Funds received by OASDI are collected and placed in a trust fund, from which disbursements of benefits are made. The trust fund currently holds approximately $1.7 trillion, invested in special issue obligations of the U.S. Treasury. Those securities are puttable to the Treasury at par, issued only to the OASDI trust fund, and pay the rate of interest derived as the average of the market rate on long-term U.S. Government bonds. This investment policy has been created as an application of certain principles of trust fund investments that have been adopted. Kunkel (1999) provides an overview of general principles of investment operations of the trust fund:

- Principle 1: Non-intervention in the private economy.
- Principle 2: Security (investment assets should provide maximum degree of safety of principal and interest).
- Principle 3: Neutrality (Trust fund investments should not be materially different than those available to other investors).
- Principle 4: Minimal management of investment.

We should note that the bonds in the trust fund are assets for the liabilities of future annuity payouts for system beneficiaries. We submit that the trust fund would be best served by an investment policy which manages the assets and liabilities of the system in an integrated fashion. Such an investment policy is better accomplished with the use of survivor bonds. The liabilities of the system can be viewed as, either exactly or approximately, equal to the portfolio of the survivor bonds issued for the covered workers population. The liabilities are paid as life annuities, made additionally risky by their indexing to inflation. The liabilities are priced actuarially, but no market price is available, and long-term planning of the system may be enhanced by availability of such a market signal.

Many social insurance systems worldwide, notably Chile, United Kingdom and recently Poland, have transformed, either fully or partially, their social insurance defined benefit pension schemes into defined contribution systems of individual accounts. This form of transformation transfers risk of longevity from the government to individual investors, or private firms offering life annuities upon retirement, and this risk transfer is viewed as a negative by some, because the government may be better equipped to handle it than individuals or even life annuity insurance firms. But a vibrant survivor bonds market would provide assets that can be used to match the payout structure of liabilities. If the longevity risk were transferred to private accounts, but then accepted again by the government by offering survivor bonds for portfolios in the individual accounts, and investments of life annuity insurance firms, the longevity exposure risk would be unaffected, or at least not affected greatly, by the reform.

An interesting alternative has been proposed by Valdes-Prieto (2005), who points out that while payouts of a social insurance retirement system are tied to mortality, the receipts are tied to human capital (i.e., income from labor), and a social insurance system can securitize the tax receipts, by issuing securities paying tax receipts at a prescribed tax level. This could allow limited securitization of human capital in the national economy, and thus enhance efficiency of capital markets. That proposal, however, does not change the exposure of the social insurance system and private retirement systems to long-term mortality improvement risk. It retains the risk of divergence between the human capital securities and the longevity exposure of the liabilities. It should be noted that even with longevity bonds issued in the markets, and utilized as assets by entities with longevity liabilities, there will always remain some degree of *basis risk,* i.e., risk of divergence between the cash flows of survivor bonds in the asset portfolio, and the cash flows required by the liabilities. This basis risk would, however, be at the lowest level for a social insurance entity, covering nearly all of the population and paying them annuities structured similarly to the structure of survivor bonds' flows.

One possible scenario for creation of survivor bonds market is therefore for the asset portfolio of the Social Security Trust Fund to be invested in such bonds, and then for the trust fund to be authorized to trade them in a secondary market. Interestingly, a large portion of the U.S. long-term debt is currently purchased by foreign governments and financial institutions in countries with substantial long-term demographic risks, which may be more interested in longevity-indexed debt than the debt of the existing structure.

## 5. Conclusion

While the long-term trend of mortality improvement has received significant attention in actuarial and economic literature, the uneven structure of the trend and existence of select birth cohorts in mortality improvements is still a relatively unexplored subject. In our work we show that oscillations of prices of annuities due to the cohort effect can be in the same range as a decade-long mortality improvement, and comparable in its significance to more than half of a profit margin of an insurance company offering a life annuity product in the payout phase. Furthermore, we point out that the nature of this cohort effect risk can potentially be hedged with survivor bonds, which can become a unique mortality improvement risk management tool for all participants in the life annuities markets.

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