When to Commence Income Annuities

Jeffrey K. Dellinger, FSA, MAAA

ABSTRACT

The question arises as to the appropriate time to deploy income annuities—fixed or variable—to generate regularly scheduled income. Intuitive judgments based on the psychological makeup of many humans often lead to decisions that run counter to objective decisions that rely more on logic and less on emotion—and this phenomenon may particularly apply here. This paper looks at the issue of timing the commencement of income annuities from the perspective of a real-life client scenario of a RISE insurance company client.

This analysis quashes the misconceptions that one should take withdrawals from mutual funds or deferred annuities for a number of years and then purchase an income annuity later or purchase income annuities on a staggered basis merely because a given amount of premium translates into higher periodic income with advancing age. To the extent one's objective is to maximize retirement income with the potential to keep pace with inflation while minimizing the probability of outliving that income,[1,2] delaying income annuity purchase can be suboptimal.

One generally considers commencing an income annuity at points in life when one has a need to supplement income, such as scaling back to working half-time and again when scaling back to complete retirement.[3] The decision arises whether to commence an income annuity at such points or to postpone income annuity purchase until a more advanced age and consume non-life-contingent assets in the interim. The findings are that an individual desirous of maximizing retirement income should commence an income annuity at some point and that the optimal time to do so is determinable.

The ages at which such supplemental income needs arise are inconsequential in terms of value derived from income annuities—that is, there are no issue ages that provide a "better" or "worse" deal for the consumer—because income annuities are, at all ages, equivalent to the net premium applied on an expected present value basis. As will be shown, the optimal timing decision depends in large part on the ability of the benefit of survivorship present in the income annuity to overpower any product expense differential between the income annuity product and the alternative withdrawal program product.

---

[1] Income annuities—also known as immediate annuities—efficiently generate lifetime income from a given sum of money, which forms the premium. Other financial instruments are better at providing liquidity and at wealth transfer. This paper is directed at that portion of an individual's wealth for which the financial objective is maximizing retirement income on an inexhaustible basis (i.e., guaranteed to last a lifetime). This can be achieved where the individual retains investment risks and rewards (variable income annuity) or where the individual transfers investment risks to an insurer (fixed income annuity). Under both forms of income annuity, the individual transfers mortality risks to the insurer, removing concern about spending down retirement assets too slowly or too rapidly—a risk otherwise present in the face of uncertain longevity.

[2] For a comprehensive look at income annuities, see *The Handbook of Variable Income Annuities* by Jeffrey K. Dellinger, John Wiley & Sons, Hoboken, NJ, 2006, 776 pages.

[3] Starting an income annuity too early can result in income generation, taxation, and then the need to reinvest a portion of the proceeds.

1. CLIENT SCENARIO AND INTRODUCTION

A financial advisor has a 62-year-old client who is about to retire. She received an inheritance of $1.3 million in October 2006, which will be followed by another $600,000 in April 2007 and another $700,000 in April 2008. She will use $700,000 to buy a house. The advisor is considering appropriate placement of the remaining $600,000 of the original inheritance. The client needs at least 5 percent income from all of the funds she will receive from this tripartite inheritance. An income annuity is explored for a portion of the funds. The advisor wonders whether given the age and gender of the client if (i) she should elect an income annuity now or (ii) defer income annuity purchase for 10 years under the thinking that the premium will translate into a higher amount of monthly income at that point because of her more advanced age.

This paper aims to address the question of optimal income annuity commencement timing. An income annuity can generate lifetime income superior in magnitude to a withdrawal program where the underlying investment portfolios are identical. This has implications for individual retirement security, as “financial ruin” may be defined as inadequate income to maintain one’s standard of living; that is, “financial ruin” may not necessarily be a total absence of wealth or income.

The paper is organized as follows. Section 2 presents an analysis of immediate annuitization versus delayed annuitization in pure form, unclouded by the effects of commercially available retail financial product expense differences. This allows for a transparent look at a comparison of retirement income generation based on (i) immediate income annuity commencement and (ii) withdrawals from an asset accumulation vehicle for a temporary time period followed by delayed income annuity commencement. Section 3 summarizes decision making with respect to income annuity commencement timing in such a scenario where product fees are the same for the income annuity and the withdrawal program product. Section 4 shows the methodology employed in the analysis described in Section 2 and summarized in Section 3. Section 5 extends the earlier analysis to introduce product load and fee differences. Section 6 concludes. A glossary of terms often encountered relating to this subject matter follows at the end of the paper.

2. ANALYSIS

The question arises as to whether income annuities are more effective in this scenario at age 62 or whether purchase of an income annuity should be deferred to a later age when mortality credits are higher. Mortality credits are increments to annuity income that result from participating in an annuitant pool where wealth released by decedents who no longer require income is reallocated to surviving annuitants who do. Mortality credits, unavailable in non-life-contingent assets, will be defined more precisely later in this paper.

It's important to understand that the client will be entitled to those same, higher mortality credits at older ages whether she purchases an income annuity now or whether she purchases an income annuity later. The only difference in the latter situation is that she

unnecessarily forfeits income attributable to mortality credits associated with the period she defers the purchase.

Let's examine the case for a female, age 62 with $600,000 who (i) purchases a lifetime variable income annuity (VIA) now or (ii) takes withdrawals now and delays purchase of a lifetime VIA for 10 years. For purposes of this Section 2, we assume no loads and identical annual fees for both the income annuity and the alternative investment product from which withdrawals are made. We relax these assumptions in Section 5.

To ensure a valid comparison, we assume that in both cases the money is invested in identical underlying investment portfolios. We can assume, for instance, that the withdrawal program product is a variable deferred annuity and the individual invests in the same subaccount, the performance of which determines VIA benefits. For simplicity, the illustration assumes a constant 7 percent investment subaccount rate of return. It assumes a 3.50 percent AIR and annual end-of-year income annuity benefits using the Annuity 2000 Mortality Table.

If the client purchases a lifetime VIA now, she would be entitled to annual end-of-year income payments as shown in column (2) of Table 1.

Now suppose instead that the client chooses to simply make withdrawals of identical amount from an active deferred variable annuity (VA) account for the first 10 years and then convert to a VIA. Income for the first 10 years will be identical. Income *in all future years* thereafter will be lower because she opted not to take advantage of the mortality credits available to her during the first 10 years of her income program. If the client takes withdrawals now and delays purchase of a lifetime VIA for 10 years, she would be entitled to annual end-of-year income payments as shown in column (3) of Table 1.

Column (4) of Table 1 shows the year-by-year difference in income between the two approaches. Figure 1 illustrates this year-by-year income difference.

Column (5) of Table 1 shows the year-by-year income under the first approach as a percentage of income under the second approach. Under the second approach, the client forfeits an additional 11.6 percent of income *in all future years* beyond the 10-year period if she initially takes income in the form of withdrawals rather than income annuity benefits.

Column (6) of Table 1 shows the cumulative effect of the two approaches. Depending on how long she survives, a decision to take withdrawals for 10 years and then purchase an income annuity can cumulatively reduce her income by tens of thousands or hundreds of thousands of dollars. Figure 2 illustrates this cumulative income difference. (Note: Scale of Figure 2 differs from scale of Figure 1.)

Income Comparison
Current VIA Purchase vs. Delayed VIA purchase

(1) Age	(2) VIA Benefit	(3) Withdrawals then VIA Benefit	(4) Year by Year Difference	(5) (2) / (3)	(6) Cumulative Difference
62	$39,137	$39,137	$0	100.0%	$0
63	40,461	40,461	0	100.0%	0
64	41,829	41,829	0	100.0%	0
65	43,243	43,243	0	100.0%	0
66	44,706	44,706	0	100.0%	0
67	46,218	46,218	0	100.0%	0
68	47,781	47,781	0	100.0%	0
69	49,396	49,396	0	100.0%	0
70	51,067	51,067	0	100.0%	0
71	52,794	52,794	0	100.0%	0
72	54,579	48,884	5,695	111.6%	5,695
73	56,425	50,537	5,887	111.6%	11,582
74	58,333	52,246	6,086	111.6%	17,668
75	60,305	54,013	6,292	111.6%	23,960
76	62,345	55,840	6,505	111.6%	30,465
77	64,453	57,728	6,725	111.6%	37,190
78	66,632	59,680	6,952	111.6%	44,142
79	68,886	61,698	7,187	111.6%	51,330
80	71,215	63,785	7,430	111.6%	58,760
81	73,623	65,942	7,682	111.6%	66,442
82	76,113	68,172	7,941	111.6%	74,383
83	78,687	70,477	8,210	111.6%	82,593
84	81,348	72,860	8,488	111.6%	91,081
85	84,099	75,324	8,775	111.6%	99,856
86	86,943	77,871	9,071	111.6%	108,927
87	89,883	80,505	9,378	111.6%	118,305
88	92,922	83,227	9,695	111.6%	128,001
89	96,065	86,041	10,023	111.6%	138,024
90	99,313	88,951	10,362	111.6%	148,386
91	102,671	91,959	10,713	111.6%	159,099
92	106,143	95,069	11,075	111.6%	170,173
93	109,733	98,284	11,449	111.6%	181,623
94	113,444	101,607	11,836	111.6%	193,459
95	117,280	105,043	12,237	111.6%	205,696
96	121,246	108,595	12,651	111.6%	218,347
97	125,346	112,268	13,078	111.6%	231,425
98	129,585	116,064	13,521	111.6%	244,945
99	133,967	119,989	13,978	111.6%	258,923
100	138,497	124,047	14,451	111.6%	273,374
101	143,181	128,241	14,939	111.6%	288,313
102	148,022	132,578	15,444	111.6%	303,757
103	153,028	137,061	15,967	111.6%	319,724
104	158,203	141,696	16,507	111.6%	336,231
105	163,553	146,488	17,065	111.6%	353,295
106	169,084	151,442	17,642	111.6%	370,937

107	174,801	156,563	18,238	111.6%	389,176
108	180,712	161,857	18,855	111.6%	408,031
109	186,824	167,331	19,493	111.6%	427,524
110	193,141	172,989	20,152	111.6%	447,676
111	199,673	178,839	20,833	111.6%	468,509
112	206,425	184,887	21,538	111.6%	490,047
113	213,405	191,139	22,266	111.6%	512,313
114	220,622	197,603	23,019	111.6%	535,333
115	228,083	204,285	23,798	111.6%	559,130

Table 1. Income Comparison: Current VIA Purchase vs. Delayed VIA Purchase

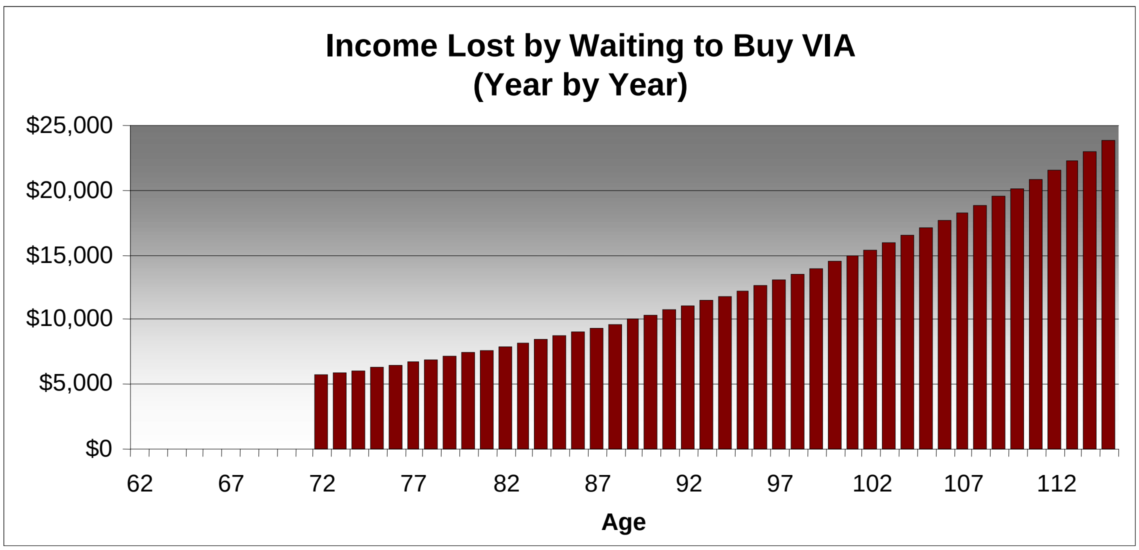

Figure 1. Year-by-Year Income Lost by Delaying VIA Purchase

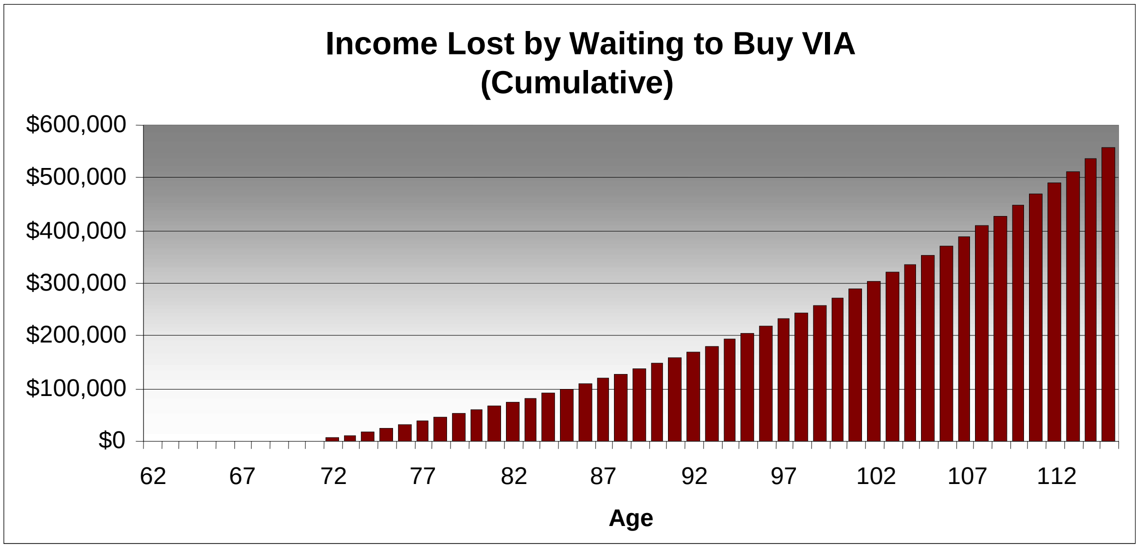

Figure 2. Cumulative Income Lost by Delaying VIA Purchase

Sometimes people believe they should wait to purchase an income annuity until a later date because the mortality credits are higher at higher ages. While it is true that mortality credits are higher at higher ages, it is equally true that an income annuity purchaser is entitled to these mortality credits whether the purchase is made at a younger age or at an older age; that is, the purchase of an income annuity at an earlier date does not somehow negate entitlement to the higher mortality credits at more advanced ages. To the contrary, purchase of an income annuity at a later date negates income attributable to mortality credits the purchaser could have enjoyed had he or she purchased the income annuity at an earlier date. The income annuity purchaser is entitled to mortality credits at all ages for which his or her annuity income is subject to mortality risk.

Income annuities are really a long series of pure endowments; that is, each future payment is discounted for both interest and survivorship and the sum of all those present values equals the net premium. If this client waits until age 72 to elect an income annuity, her income payments will be smaller because she doesn't receive the benefit of discounting for mortality for those 10 years.

In short, if she needed the income and if she were going to take withdrawals of approximately the same level as her income annuity benefits would be from age 62 to age 72 and then elect an income annuity, she might as well make this election at age 62 and enjoy the 10 years of mortality credits and resultant higher income *in all future years* that will provide for her.

Based on her age of 62, there's greater than a 50 percent probability that she will live to an age where the decision to postpone income annuity purchase will cumulatively cost her more than $100,000 in income. There's greater than a 20 percent probability that she'll live to an age where the decision to postpone will cumulatively cost her more than

$200,000 in forgone income. And there's greater than a 10 percent probability that she'll live to an age where the decision to postpone will cumulatively cost her more than one-quarter million dollars in forgone income.

Even for lifetime income with certain periods, the mortality rates during the certain period come into play because all lifetime income payments following the certain period are discounted through the certain period to the time of purchase, not only for interest but also for survivorship. If she postpones annuitization, she misses out on mortality credits for the years she defers. She's still going to get the higher mortality credits at ages 72 and up whether she elects an income annuity at age 62 or at age 72; it's just that in the latter case she misses out on the value of 10 years of mortality credits.

To view this same situation in a slightly different way, electing to postpone the income annuity purchase by 10 years has an effect equivalent to throwing away about 1½ years of investment gains. That is, if instead of assuming an investment return of 7 percent in each year for the income annuity elected at age 62, the return were only 3.75 percent in the ninth year and 0 percent in the tenth year—and if benefits under the 10-year withdrawal program are equal to those under the income annuity elected at age 62—then the income annuity elected at age 62 would produce in *all* years a benefit equal to that under the withdrawal program, plus delayed income annuity purchase where the investment return was 7 percent in *every* year under that program. In short, delaying income annuity purchase 10 years has an effect equivalent to throwing away about 1½ years of investment gains.

The example we have been tracking—the results of which appear in Table 1 and in Figures 1 and 2—assumes a fairly young female. While the decision to defer income annuity purchase in this example is profound, it is relatively minor compared to males of the same age or older females, both of whom have higher mortality rates and therefore are entitled to higher mortality credits. The benefits of current VIA purchase over delayed VIA purchase for them are even more dramatic.

Table 2 illustrates the percentage of additional retirement income a female can enjoy by electing a VIA at a given starting date rather than postponing purchase of the VIA. Values shown are based on the same process as given earlier in the female age 62 example with a 10-year delayed VIA purchase. Here we simply show the results for a variety of starting ages and delay periods. Table 3 illustrates the results for males.

Additional Retirement Income Resulting From Immediate VIA Purchase Over Delayed VIA Purchase[4]

[4] Income from (1) the VIA and (2) the withdrawal program is identical during the delay period. Thereafter, income from the immediately purchased VIA exceeds income from the delayed purchase VIA by the percentages shown.

Female Age	VIA Purchase Delay Period			
	5 Years	10 Years	15 Years	20 Years
55	1.8%	5.4%	13.2%	32.5%
60	2.9%	9.3%	24.9%	79.8%
65	4.8%	16.9%	56.6%	530.1%
70	8.5%	35.7%	220.1%	∞[5]
75	17.1%	102.0%	∞	∞
80	38.4%	3,416.8%	∞	∞

Table 2. Income Comparison: Current VIA Purchase vs. Delayed VIA Purchase (Females)

[5] Withdrawals during the delay period fully exhaust account value. There are no funds remaining at the end of the delay period with which to purchase a VIA, resulting in zero income for the delayed VIA purchase.

Male Age	Additional Retirement Income Resulting From Immediate VIA Purchase Over Delayed VIA Purchase[6]			
	VIA Purchase Delay Period			
	5 Years	10 Years	15 Years	20 Years
55	3.2%	9.6%	24.8%	71.7%
60	4.8%	16.3%	50.1%	255.9%
65	8.2%	32.0%	140.3%	∞[7]
70	15.3%	73.2%	∞	∞
75	29.2%	262.3%	∞	∞
80	61.2%	∞	∞	∞

Table 3. Income Comparison: Current VIA Purchase vs. Delayed VIA Purchase (Males)

Figures 3 and 4 graphically illustrate selected information contained in Tables 2 and 3, respectively. That is, the surfaces shown depict the additional retirement income resulting from immediate VIA purchase over that from delayed VIA purchase. Income from the immediately purchased VIA exceeds income from the delayed VIA purchase in all post-delay-period years by the percentages shown. (Note: Scale of Figure 4 differs from scale of Figure 3.)

As expected, the surface increases in steepness for a given age as the delay period increases. Also as expected, the surface increases in steepness for a given delay period as the age increases.

Because mortality rates—and therefore mortality credits—are higher at higher ages, there is a bigger price to pay for missing out on them. This is why there is so much more additional income to be had for a 65-year-old who elects an income annuity now rather than delaying 10 years than there is for a 55-year-old. This is obvious in the Figure 3 and Figure 4 "wireframe" graphs, as the slope of the surface in the direction of additional years of delay increases more rapidly at age 65 than at age 55.

[6] Income from (i) the VIA and (ii) the withdrawal program is identical during the delay period. Thereafter, income from the immediately purchased VIA exceeds income from the delayed purchase VIA by the percentages shown.

[7] Withdrawals during the delay period fully exhaust account value. There are no funds remaining at the end of the delay period with which to purchase a VIA, resulting in zero income for the delayed VIA purchase.

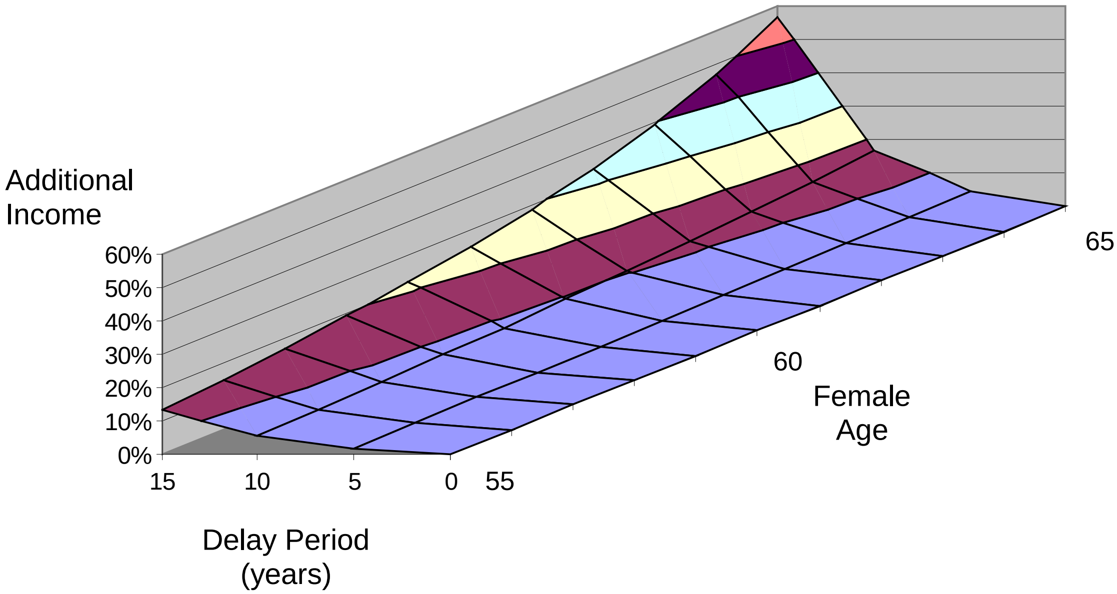

Figure 3. Additional Retirement Income Resulting From Immediate VIA Purchase Over Delayed VIA Purchase by Age and Delay Period (Females)

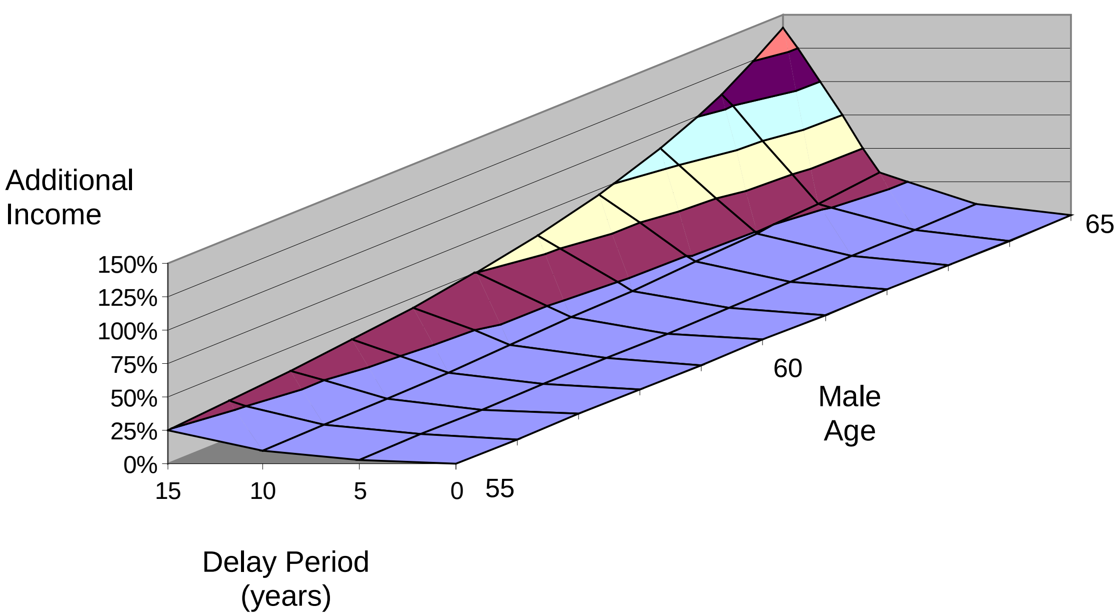

Figure 4. Additional Retirement Income Resulting From Immediate VIA Purchase Over Delayed VIA Purchase by Age and Delay Period (Males)

It's imperative to recognize that the client's decision to delay VIA purchase in this example shortchanges her income *in all future years* relative to what she could have had. It's as if she said, "I'm willing to throw away 1½ years of investment gains to take the same amount of income via withdrawals for the first 10 years than via an income annuity."

One could argue that she might experience adverse health or an accident during the first 10 years and die. That's true of anyone. But that misses the point. The objective here is not to maximize bequests; there is a different set of financial instruments to achieve that objective more tax efficiently. The objective here is to achieve retirement income having specific, highly desirable characteristics: One buys a VIA to derive the maximum income with the lowest (zero) probability of outliving it and having the potential to keep pace with or outpace inflation.[8]

It's also important to recognize that in this client scenario, 77 percent of the inheritance is being deployed outside of the initial income annuity. The facts and circumstances of this case are silent as to net worth prior to the inheritance; if positive, then something in excess of 77 percent of net worth is being deployed outside of the initial income annuity. Those assets can potentially be used to provide liquidity, to achieve bequests, to insure against catastrophic health-related expenses, and to achieve other financial objectives, freeing the remaining assets to be deployed in the efficient generation of lifetime retirement income.

Also noteworthy is the positive correlation between wealth and longevity. Income annuity experience studies confirm that higher income annuity benefits are associated with lower mortality rates.

Regarding postponement for purchasing an income annuity, this makes sense if one doesn't have a current need for income—but clearly the need for income exists here. In this example, the client expressed a current need for income as a percentage of all of the funds she will inherit.

It is sometimes thought that a younger retiree should take withdrawals from a mutual fund or deferred annuity for a number of years and then purchase an income annuity later. The rationale offered is that (i) the younger retiree might die during the period where the withdrawal program is employed and, if she does, the corpus of the fund goes to her beneficiary, and (ii) the younger retiree doesn't give up a lot of income under this approach because mortality rates—and hence mortality credits—are lower at younger ages.

---

[8] In contrast, a retirement income program that provides income for a period of time equal to life expectancy as of retirement income program inception leaves such individuals vulnerable to inadequate income, as over 50 percent of such individuals will still be alive beyond their life expectancy as determined at program inception. "Are We There Yet? The Embryonic State of Retirement Income Knowledge," Jeffrey K. Dellinger, *The RISE Report*, Volume 1, Number 1.

Indeed, it may have been this very line of thinking that created the real-world inquiry spawning this analysis. The 62-year-old female could take withdrawals for 10 years and then elect an income annuity. If she died during this 10-year period, her beneficiary would inherit the account value. And if she didn't, she'd "only" have to give up an extra 11.6 percent of income she could have had every year thereafter for the rest of her life.

The fact of the matter is that precisely because mortality rates are lower at younger ages, (i) the probability that she will die during the 10-year delay period is lower, and (ii) the amount of future income forgone in each year after the delay period is lower. That is, the same factor that causes her to only have to give up a smaller amount of income every year in the future is responsible for the fact that there's also only a smaller possibility that she'll die during the delay period and thereby bequeath account value to a beneficiary.

Equivalently, at higher ages, (i) the probability that she will die during the 10-year delay period is higher, and (ii) the amount of income forgone in each future year after the delay period is higher. Yet now, perhaps at age 72, the client in our example is willing to apply a mutual fund or deferred annuity account value to purchase a VIA. At this age, she actually has a much higher likelihood of dying during the next 10 years than she did during the 10-year period beginning at age 62—when she wasn't willing to purchase an income annuity because she might die during that 10-year period and not be able to bequeath the account value to a beneficiary.

So, she actually ends up electing an income annuity at age 72—when the chance of dying and being unable to bequeath an account value is higher—whereas she may have been unwilling to elect an income annuity at age 62—when the chance of dying and being unable to bequeath an account value is lower. This type of situation can occur when (i) a client's objectives aren't clear, (ii) the client's objectives are clear but the client or advisor tries to use one financial instrument to achieve multiple objectives—including maximizing lifetime income and bequests, or (iii) there is a failure to accurately perceive the effects of mortality rates on alternative approaches. As an example of the third point, a person may feel that she can have a long (e.g., 10-year) delay period at age 62 without having to give up "too much" future income without recognizing that the "cost" in forgone income is low precisely because the probability that she'll die and bequeath an account value is low.

An emotional approach driven by "loss aversion" (the desire to avoid losing an account value followed by the desire to avoid losing higher income) can drive suboptimal decision making. That is, the notion that delayed annuitization holds psychological appeal to some people in the absence of genuine economic advantage suggests that human aversion to loss is a driver of decision making. For example, at age 62, loss of principal from early death may be the driver, whereas at age 72, loss of income from failing to annuitize may be the driver. An advisor can help a client replace suboptimal, emotion-based decisions with optimal, logic-based decisions.

At least that's how our human wiring may work. While one hypothesis is that at age 62 "lost principal" is the driver and at age 72 "lost income" is the driver, the reality is that the *relative* balance between "principal preservation" by delaying annuitization and "income maximization" by not delaying annuitization is the same at both ages. For example, at age 72, the higher probability of dying serves to both (i) increase mortality credits and therefore increase income and (ii) simultaneously increase the risk of loss of principal due to death. So, interestingly, the risk-reward ratio is similar at both ages, yet we behave differently. Such decisions are thus psychological ones, not economic ones. But that still gives us clues about how to proceed in advising and encouraging people to act in their own best economic self-interest.

The case for electing annuitization at the point one needs supplemental income rather than taking withdrawals and delaying annuitization may be further enhanced in the non-tax-qualified market when the income tax effect is considered. Withdrawals from deferred annuities (applicable to deposits made after August 13, 1982) are taxed on an "appreciation out first, followed by principal" basis, making early withdrawals of any gains fully taxable. In contrast, a portion of each income annuity benefit is a nontaxable return of principal until the cost basis is fully recovered.

"Serial annuitization" programs, whereby assets in accumulation vehicles such as mutual funds or deferred annuities are used to purchase income annuities on a regular, periodic, staggered basis, can similarly result in consumers having to pay more for their retirement income benefits because of missing out on mortality credits in early years.[9] Multiple income annuity purchases coinciding with points of increased need for supplemental income—such as scaling back on work hours and thus on employment income—are appropriate. In contrast, multiple income annuity purchases as part of a scheduled serial annuitization program—where withdrawals from asset accumulation vehicles must be taken in the interim to generate sufficient income—result in that income being more expensive to the consumer as it stems from only two driving forces—principal and appreciation—rather than the three driving forces—principal, appreciation and mortality credits—available under income annuities.

For example, we see from Table 3 that a 65-year-old male who decides to wait five years to purchase an income annuity with part of his assets earmarked for retirement income generation and then again waits 10 years and 15 years to purchase income annuities with the rest of his assets so earmarked—and who instead takes comparable withdrawals from his asset accumulation vehicles—forgoes an additional 8 percent income due to the first delay, an additional 32 percent income due to the second delay, and an additional 140 percent income due to the third delay. The consequences of such a serial annuitization program are nontrivial.

The "preservation of principal" compulsion carried over from years of conditioning on asset accumulation rules can color asset liquidation decision making. Inadequate weight is often given to economic ruin—defined either as total absence of wealth or alternatively

---

[9] Sometimes serial annuitizations—where money is left in stock mutual funds for a number of years for growth and is then applied to the purchase of a fixed SPIA—occur because consumers or advisors are simply unaware of IVAs and use the sequential purchase of a stock mutual fund and a fixed SPIA so that equity market growth potential can be enjoyed for a period of time before they feel it must be abandoned to get immediate annuity income.

as inadequate income to sustain one's standard of living—stemming from uncertain longevity. As a result, the process of supplemental income "layering" via sequential income annuity purchases can seem attractive because it leaves a larger asset base to bequeath in the event of early death. Yet (i) all of the assets not earmarked for retirement income generation that remain at death will be bequeathed, (ii) delaying income annuity purchases carries a cost, potentially leaving fewer assets to bequeath, and (iii) one may desire to separate wealth transfer and retirement income objectives—and to use distinct financial instruments that achieve those objectives most efficiently.

There may be underappreciation of the potential for economic ruin, especially for more longevous individuals. Those who do reach advanced ages may wish they could turn back the hands of time and deploy that portion of their assets earmarked for retirement income generation in an optimal fashion—as they now find themselves in an unrecoverable position. Avoiding inefficiencies—such as drawing down non-life-contingent assets under the thinking that delaying income annuity purchases will help produce more retirement income because mortality credits are higher and therefore a given premium translates into higher periodic income at higher ages—can help safeguard against economic ruin.

More generally, there exist certain decisions in life for which one of the possible outcomes is so onerous that a strategy to minimize the probability of that outcome is sought and highly valued.[10] Exhaustion of adequate income at a stage of life where replacement of such income through one's labor may be physically or mentally unfeasible—and the concomitant change in living standard dreadful—is an example of one such outcome. Inefficient generation of retirement income not only creates an economic and emotional toll for longevous individuals, but it also creates externalities—an adverse economic effect of decisions by one set of parties on others in society who now must fund the first set of parties.[11]

The length of one's future lifetime is unknown. It's a random variable. The distribution of one's potential future lifetime is not concentrated around a single value. Rather, it is enormously wide. For example, Figure 5 reveals that a 60-year-old woman might need income for 10, 20, 30, 40, or even 50 years. The future lifetime distribution is so spread out that for a group of 60-year-old women the probability that any particular original member dies in any given year—and therefore requires income for exactly that many years—is at most about 4 percent. Those women whose deaths fall in the right-hand tail of the distribution particularly need to be concerned with efficient generation of lifetime income.

---

[10] This bears similarities to a *minimax* decision rule of selecting the choice that minimizes the maximum expected loss. Such decision making in the face of uncertainty is associated with the realms of decision theory and game theory.

[11] Dellinger, Jeffrey K., "Efficient Deployment of Retirement Assets to Increase Financial Security of Seniors and to Minimize Welfare Burden on State," Retirement Income Solutions Enterprise, Inc. (RISE) White Paper, September 2007.

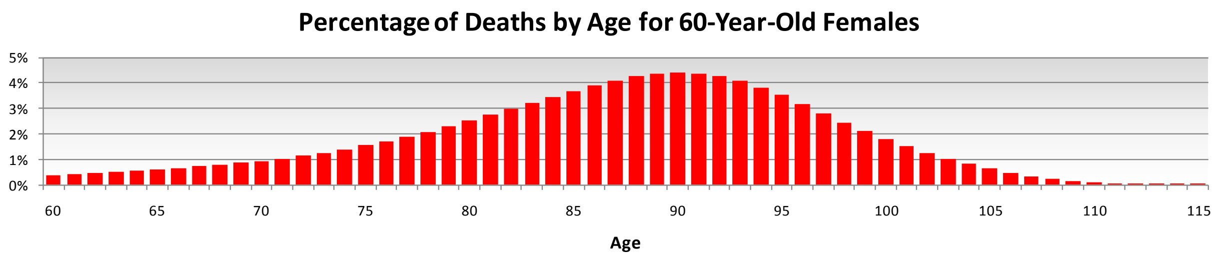

Figure 5. Percentage of Deaths by Age for 60-Year-Old Females.[12]

There exists the possibility that at the end of the 10-year deferral period of this example, immediate annuitant mortality may prove lower than that assumed at the start of the 10-year period, even though future mortality improvement is projected by immediate-annuity-writing insurers at both points in time.[13] Indeed, a recent annuitant mortality experience study suggests mortality rates lower than those found in the Annuity 2000 Basic Mortality Table.[14,15] Such an event in mortality evolution would raise the price of an immediate annuity per unit of income benefit, raising the cost of postponing annuitization.

3. Summary of Decision Making with Respect to Income Annuity Commencement Timing in the Absence of Loads and where Product Fees are Identical for the Income Annuity and the Withdrawal Program Product

- If our 62-year-old female takes the *same* level of income from the two programs during the first 10 years, then she'll receive 11.6 percent higher income every year thereafter if she purchases the VIA today rather than 10 years from now. That is, income *in all future years* after the first 10 years will be higher because the client took advantage of the mortality credits available to her during the first 10 years of her income program.

---

[12] Based on Annuity 2000 Mortality Table.

[13] Gender-based and age-based mortality improvement factors are used to project mortality rates assumed to exist for the population under study as of a given year to future years in which the annuitant will reach each attained age.

[14] "With expected mortality based on the Annuity 2000 Basic table, the overall income-weighted A/E ratios (combining all companies, study years, contract years, attained ages, benefit classes, annuity types, income groups, underwriting classes, tax classes, contract types) are 0.929 for males and 0.977 for females, which suggests that annuitants are living longer than would be implied by the Annuity 2000 Basic table."
– III. Principal Observations, Society of Actuaries 2000-04 Individual Payout Annuity Experience Report.

[15] The Annuity 2000 Basic Mortality Table was not based on a Year 2000 intercompany annuitant mortality experience study. Rather, it was based on a much earlier annuitant mortality experience study of 1971-1976, with mortality rates projected to Year 2000.

- This analysis compares post-age-71 income when the income under the two programs between ages 62 and 71 is *identical*. It's equally true that even if she takes some degree of *lower* income from the withdrawal program during the first 10 years, she suffers lower income in *both* the first 10 years and all years thereafter. This is because the effect of the mortality credits she enjoys by currently purchasing a VIA overwhelms the effect of taking lower withdrawals during the first 10 years.

- Depending on how long she survives, a decision to take withdrawals for 10 years and then purchase an income annuity can cumulatively reduce her income by tens of thousands or hundreds of thousands of dollars.

- If one needs no supplement to current income, one should delay VIA purchase.

- If one needs a supplement to lifetime income, one should purchase a VIA currently rather than embark on a withdrawal program plus delayed VIA purchase approach.

- Income annuities are more efficient lifetime income delivery instruments than comparably invested systematic withdrawal programs.

This analysis may serve to rectify some nebulous and "mushy" thinking on the issue of current income annuity purchase versus delayed income annuity purchase with respect to the line of reasoning that delaying income annuity commencement might be beneficial on the grounds that a given amount of premium will translate into higher periodic income benefits at a more advanced age; therefore, non-life-contingent assets should be consumed in the meantime. One should not delay annuitization merely because conversion rates are higher at more advanced ages.

Again, the ages at which such supplemental income needs arise are inconsequential in terms of value derived from income annuities; that is, there are no issue ages that provide a "better" or "worse" deal for the consumer because income annuities are, at all ages, equivalent to the net premium applied on an expected present value basis.

4. METHODOLOGY

- The analysis presented in Section 2, the findings of which are summarized in Section 3, uses the Annuity 2000 Mortality Table. This table is slightly conservative (i.e., assumes lower mortality rates than were projected to exist as of Year 2000) because it is a valuation table used to construct income annuity reserves and is concerned with solvency. The Annuity 2000 Basic Mortality Table would be slightly liberal (i.e., assumes higher mortality rates than are appropriate because it fails to recognize mortality improvement between Year 2000 and the future year the client will reach

each attained age).[16] A more exact analysis would use the Annuity 2000 Basic Mortality Table with dynamic mortality improvement projection, recognizing mortality improvement between Year 2000 and the year in which the client attains each future age. The choice of mortality table for this analysis over one precisely geared to this particular situation doesn't materially change the findings.

- The analysis uses annual, end-of-year benefit payments. The same analysis could be done monthly. It is easier to perform (and the relative magnitudes of the outcomes are perhaps easier to understand) on an annual basis. End-of-year benefit payments are consistent with the objective of looking at the effect of mortality credits. For example, both the withdrawal program and the income annuity provide benefits at identical points in time and both have identical subaccount performance. By using end-of-year payments, every payment during the delay period, including the first-year payment, reflects mortality credits for the income annuity and does not reflect mortality credits for the withdrawal program.

- The analysis is based on no product loadings and the same annual fees for the deferred VA withdrawal program and for the variable income annuity. For example, M&E and investment management charges reduce net investment performance for both approaches comparably.

- The analysis assumes a 3.50 percent AIR and constant 7 percent investment performance. If investment performance insurance is desired, this can be added to (i) a VIA in the way of a guaranteed payout annuity floor (GPAF) based on three factors driving income (principal, appreciation, mortality credits) or (ii) a deferred VA withdrawal program in the way of a guaranteed lifetime withdrawal benefit (GLWB) based on two factors driving income (principal, appreciation). Both approaches place a floor under income benefits.

- Retirement income benefits in this variable income annuity example increase 3.38 percent per year (1.07/1.035 – 1 = 0.0338), helping to cope with inflation.

5. EXTENSION OF INCOME ANNUITY COMMENCEMENT TIMING ANALYSIS TO INCLUDE PRODUCT LOADS AND FEE DIFFERENCES

A main point of this article is to communicate that delaying income annuity commencement under the notion that delay is beneficial because a single premium will translate into higher periodic income benefits at a more advanced age can be suboptimal if one requires additional income now. In essence, spending down non-life-contingent assets in the interim—before the delayed income annuity commences—can significantly reduce retirement income.

[16] The Annuity 2000 Basic Mortality Table is suitable for that subset of the U.S. population that deems itself in sufficiently good (not necessarily perfect) health to purchase an income annuity.

The client scenario presented in this paper facilitates illustration of this concept. As demonstrated so far, in the absence of product loads and under the condition that annual product fees for the non-life-contingent asset (e.g., deferred annuity, mutual fund) and the life-contingent asset (e.g., income annuity) are comparable, one needn't perform complex analyses to determine the best age to deploy income annuities. Rather, one can trigger income from these instruments as the need for supplemental income arises.

For example, if the client in the real-life situation were to place a portion of the $600,000 initial inheritance remaining after her house purchase in either (i) a no-load total domestic stock market mutual fund with 0.73 percent in annual expenses or (ii) a VIA with 0.73 percent in annual expenses[17] and no embedded load, then the analysis presented in Section 2 of this article accurately portrays the income annuity commencement timing decision.[18]

If the no-load mutual fund had a higher annual expense than the no-load VIA, then the decision to delay annuitization would result in even more lost retirement income than shown in Section 2. If the no-load mutual fund had a lower annual expense than the no-load VIA, then the decision to delay annuitization would result in less lost retirement income than shown in Section 2.

If the mutual fund and VIA have comparable front-end sales loads and comparable annual expenses, then the decision to delay annuitization would result in even more lost retirement income than shown in Section 2. This is because the client incurs the sales load on the mutual fund and then again on the delayed VIA purchase, whereas the sales load is only incurred once on the immediate VIA purchase.[19,20]

---

[17] For example, the Vanguard Lifetime Income Program—a VIA underwritten by American General Life Insurance Company—has a 0.52 percent mortality and expense (M&E) risk charge and its Total Stock Market Index Portfolio has a portfolio operating expense of 0.21 percent, for a total of 0.73 percent.

[18] It is also assumed that the mortality rates on which the variable income annuity is based are close to those that will emerge from the annuitant pool; that is, the majority of the product expenses consist of the mortality and expense (M&E) risk charge, administrative (A) charge, and investment subaccount expenses—and any margins embedded in the mortality assumptions employed in payout rate determination are minimal. Said another way, it is assumed that the insurer seeks to cover expenses, taxes, and profit primarily through the M&E&A charge and the investment subaccount fees rather than also through a margin introduced into the mortality rate assumptions themselves that are used to create the conversion factors that convert a single premium into regularly scheduled income benefits for life.

[19] It is conceivable that an individual with sufficiently large funds could self-establish an investment portfolio with a diversified asset collection, thereby reducing expenses. Here we are only concerned with "packaged products," such as mutual funds and income annuities.

[20] When talking about VIA purchase, what is meant is purchase of a single premium immediate variable annuity (SPIVA). A SPIVA can be synthetically created by purchasing a deferred variable annuity and immediately annuitizing it. If guaranteed payout rates in such a deferred annuity contract were sufficiently outdated (e.g., they failed to adequately project mortality improvement via age setback or otherwise), then such a purchase could be considered to capitalize on this underpricing. Or an individual might already be entitled to such favorable conversion factors, such as by virtue of having been sold a deferred annuity in the employer-sponsored 403(b) market at a very young age—such as in his or her 20s—where mortality improvement over the multi-decade accumulation period exceeds that expected when the contract was sold. In general, though, retail deferred annuities carry higher sales-related expenses than immediate annuities,

In this example, a variable income annuity is used. This allows for a comparable underlying investment portfolio to the mutual fund(s). This also allows for whatever portion of the underlying investment portfolio the contract owner wishes to be invested in common stock—the asset class with the best long-run track record relative to inflation. A VIA also allows for future adaptations to changing economic environments through the ability of the contract owner to make investment subaccount reallocations.[21] This ability is absent in traditional fixed income annuities.[22] A variable income annuity also avoids the potential problem of a fixed income annuity that is purchased in a low interest rate environment—such as exists today—and which therefore produces low fixed income benefits relative to historic norms. A variable income annuity potentially avoids the problem of a fixed income annuity that leaves the annuitant susceptible to purchasing power erosion if a level series of fixed income annuity benefits is elected, especially if inflation rises substantially during the potentially multidecade payout period.

The analysis to this point has focused on comparison of non-life-contingent and life-contingent registered securities products (e.g., mutual funds and VIAs, respectively). A similar comparison can be made between non-life-contingent, non-registered products (e.g., fixed deferred annuities, CDs) and life-contingent, non-registered products (e.g., fixed immediate annuities).

For example, fixed deferred annuities and fixed immediate annuities contain a “spread” between the earned rate on underlying assets (e.g., bonds and other fixed-income securities in the insurer’s general account portfolio segment backing fixed immediate annuity obligations) and the credited rate on the liability (e.g., fixed immediate annuity reserve). This spread, in essence, parallels its expense charge counterpart on registered products.[23] In this sense, the analysis presented for registered products carries over to nonregistered products.

---

and the degree of conversion factor underpricing would need to be sufficiently high to overcome these expenses on a new, synthetically created SPIVA.

[21] Ideally, these reallocations could include automatic rebalancing to a specified asset allocation or even automatic rebalancing to a progressively changing asset allocation based on age, gender, and customized financial objective(s).

[22] Inflation-indexed fixed immediate annuities (i.e., those with income benefits that fluctuate based on changes in a specific consumer price index) are a limited exception, as they allow for automatic adjustments. They do not, however, allow the degree of freedom of variable immediate annuities; for example, the opportunity to alter the mix of domestic versus international securities, the collective performance of which governs the magnitude of income benefits.

[23] The “spread” for a nonregistered product would be expected to exceed its expense charge counterpart for an otherwise comparable registered product. This is because the product manufacturer bears the asset depreciation (C-1) risk (e.g., default on principal or interest payments) for the nonregistered product, whereas the consumer bears this risk for the registered product. Also, the product manufacturer bears the interest rate (C-3) risk (e.g., disintermediation risk in rising interest rate environment, paying contractually guaranteed minimum interest rate in very low interest rate environment) for the nonregistered product, which increases the spread. Additionally, the spread on the nonregistered product may fluctuate over time and is generally not disclosed, whereas the expense charges on the registered product tend to be fixed in amount and disclosed.

To see this, reconsider the same information as in Table 1 in a different light. Let Column (2) values represent a fixed immediate annuity purchased at age 62 where income benefits automatically increase 3.38 percent per year. Let Column (3) represent withdrawals from a fixed deferred annuity earning 7 percent per year, where the withdrawals during the first 10 years equal the amounts available under the fixed immediate annuity of Column (2). At the end of 10 years, the remaining fixed deferred annuity account value is used to purchase—at age 72—a fixed immediate annuity where income benefits automatically increase 3.38 percent per year. The same result holds in this nonregistered product example as in the registered product example.[24]

We now address the annuitization timing decision in the face of disparate expense levels between the withdrawal program product and the income annuity product. We proceed to crystallize the decision-making process via formula-based quantitative analysis.

Consider a group of individuals aged x and let l_x represent the number of such lives. Figure 6 illustrates l_x values for females. Let i be the assumed investment rate (AIR). Let $v = 1 / (1 + i)$ be the factor that discounts a payment of 1, one year hence at interest rate i.

---

[24] This example assumes the same earned rate and spread for the fixed deferred annuity and fixed immediate annuity. One can assume, say, an 8.25 percent earned rate and a 1.25 percent spread to arrive at a 7 percent net credited rate on both the fixed deferred annuity and the fixed immediate annuity, to account for product charges relating to administration and risks assumed by the insurer. There may be less interest rate risk for the insurer associated with the fixed immediate annuity because of its irrevocable nature, whereas rising interest rates may spawn increased policyholder lapse behavior ("disintermediation risk" or "C-3 risk") on the fixed deferred annuity—or falling interest rates may trigger spread compression should crediting rates bump up against minimum contractual guarantees. This assumes an issue age for the fixed immediate annuity sufficiently high that the insurer can create a fixed-income securities portfolio that replicates its immediate annuity payout stream obligations without reinvestment rate risk for the bulk of its annuitants. Additionally, precisely because of the irrevocable nature of the fixed immediate annuity, the insurer can invest in more illiquid instruments, picking up incremental yield in the way of an illiquidity premium. The duration of the lifetime fixed immediate annuity liability may exceed the duration of the fixed deferred annuity liability, allowing the insurer to have longer asset duration, thereby also picking up incremental yield.

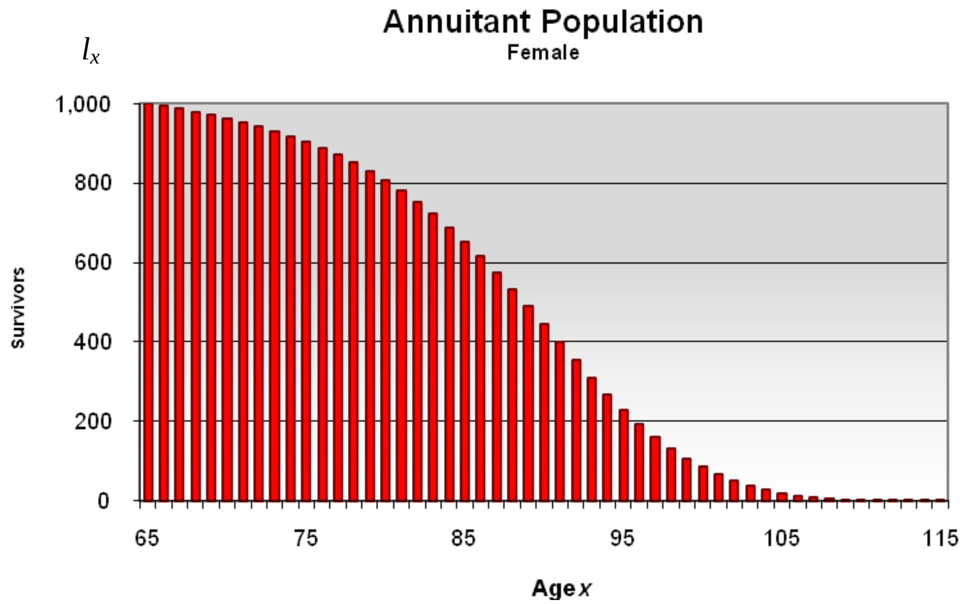

Figure 6. Female l_x Values.[25,26]

Then the net present value (NPV), the amount we need at program inception to generate a series of future year-end annuity benefit payments of 1 to all annuity program participants initially age 65, is:

$$NPV^{All} = 1 \times l_{66} \times v \; + \; 1 \times l_{67} \times v^2 \; + 1 \times l_{68} \times v^3 \; + \; \ldots$$

If we want to know the NPV per program participant, we need to divide by l_{65}, the number of lives starting the program:

$$NPV^{Participant} \; = \; \frac{l_{66}}{l_{65}} \times v \; + \; \frac{l_{67}}{l_{65}} \times v^2 \; + \frac{l_{68}}{l_{65}} \times v^3 \; + \; \ldots$$

$$= \; {}_1p_{65}\, v \; + {}_2p_{65}\, v^2 \; + {}_3p_{65}\, v^3 \; + \; \ldots,$$

where ${}_n p_x$ = probability life age x survives n years[27]

a_x is international actuarial notation (i.e., a shorthand or condensed way of saying the same thing) for the present value of a life annuity with annual payments of 1 to an

[25] Based on Annuity 2000 Mortality Table.

[26] Figure 5 represents a graph of d_x values, where d_x is the number of lives exactly age x dying before age $x+1$. Figure 6 represents a graph of l_x values, the number of lives beginning year of age x. l_x and d_x values are associated by the equation $l_{x+1} = l_x - d_x$.

[27] When n = 1, ${}_1 p_x$ is customarily written p_x; i.e., the "1" is implied.

individual age x, where payments are made at the *end* of each period.[28] Hereafter, we shall refer to $NPV^{Participant}$ as a_x.

That's all there is to calculating the net present value necessary to fund a lifetime immediate annuity—fixed or variable—to an individual aged x.[29,30,31] For purposes of illustration, we shall assume a variable immediate annuity.

With a variable immediate annuity, the income benefits paid to the annuitant fluctuate in accordance with performance of the underlying investment subaccount(s) chosen by the contract owner, who is often also the annuitant. The annuitant receives an income benefit equal to a fixed number of *annuity units* at the end of each year the annuitant survives. We may assume an initial annuity unit value of $1.00. Equation (i), known as a *recursion formula*, is used to determine the next annuity unit value from the last one. The

---

[28] A life annuity with payments made at the end of each period, denoted a_x, is referred to as an *annuity-immediate*. In contrast, a life annuity with payments made at the beginning of each period, denoted $\ddot{a}_x$ (i.e., with a diaeresis above the *a*), is referred to as an *annuity-due*.

[29] This paper focuses on a single "life only" annuity payout option, also known as a "straight life" or "non-refund" annuity. While various refund features (e.g., certain period, cash refund, installment refund, unit refund) can be added, these reduce the income available from a given premium—or, equivalently, increase the premium required to produce a given initial amount of income—thereby retarding the ability of the financial instrument to achieve the stated objective of maximizing income while minimizing (to zero) the probability of outliving that income. The paper concerns itself with retirement income, whereas these refund features introduce a bequest motive, as they create the potential for a beneficiary to inherit value from the immediate annuity.

For example, a "life with 10 years certain" annuity is a combination of a non-life-contingent, 10-year fixed-period immediate annuity and a "life only" annuity with first payment deferred 10 years. With respect to the 10-year fixed period annuity, there is no mortality risk, so there are no mortality credits and the optimal income annuity commencement timing protocol described in this paper doesn't apply. For a 10-year fixed-period immediate annuity, access may be relinquished without the beneficial tradeoff of obtaining mortality credits. And in the tax-qualified market where 100 percent of each income benefit is taxable at ordinary income rates, no favorable immediate annuity tax treatment is achieved.

[30] Note also that the immediate annuity under discussion involves a level series of annual year-end annuity benefit payments of 1 in the case of a fixed immediate annuity. In contrast, fixed immediate annuities can provide payments that increase linearly over time (arithmetic progression) or payments that increase exponentially over time (geometric progression). Immediate annuities can also pay benefits with frequency modes other than annually. See *The Handbook of Variable Income Annuities* for present value formulas for increasing annuities and annuities with various benefit frequencies.

[31] The simplifying assumption is made here that one uniform interest rate, i, applies for all time periods. When pricing fixed immediate annuities in actual practice, a vector of interest rates is used. In a traditional (i.e., positive sloping) yield curve environment, the interest rate used to discount any annuity benefit payment is time-dependent, with progressively higher interest rates associated with annuity payments due further into the future. There is, of course, one uniform interest rate that would produce the same present value, a_x. As we shall see, the use of one uniform interest rate, i, is appropriate for variable income annuities.

annuity unit value increases by the actual fund performance and then the AIR is backed out.[32]

$$\text{Annuity unit value}_{t+1} = \text{Annuity unit value}_{t} \times \frac{1 + \text{subaccount performance}}{1 + \text{AIR}} \qquad \text{(i)}$$

Example: Subaccount performs at 10 percent during 1st year and AIR is 3.5 percent

$$\text{Annuity unit value}_{1} = \$1.00 \times \frac{1.10}{1.035}$$

$$= \$1.0628$$

It's clear from Equation (i) that actual subaccount performance during a benefit payment frequency period (e.g., one year) equal to the assumed subaccount performance (AIR) will result in the annuity unit value being unchanged. Because the annuitant receives an income benefit equal to a fixed number of annuity units at the end of each year the annuitant survives, the benefit payment also will be unchanged. Similarly, if actual subaccount performance exceeds the AIR, then the benefit will increase. If actual subaccount performance is lower than the AIR, the benefit will decrease.[33]

The AIR elected should be sufficiently low that investment performance (net of fees) for the subaccounts chosen can exceed the AIR. This causes income benefits to rise and allows the VIA to do its job of providing rising income over time. For instance, if the AIR were 3.50 percent, and if the collection of subaccounts were expected to produce a mean return of 7 percent after fees, then this would allow for purchasing power retention even in the face of 3.38 percent inflation; that is, the annual benefit would increase by 3.38 percent because 1.07 ÷ 1.035 = 1.0338.[34]

---

[32] This is because the annuitant was already given credit for annual subaccount performance equal to the AIR. Annual subaccount performance equal to the AIR was assumed in the calculation of a_x, which was used to determine the number of annuity units payable on each payment date. Fund performance equal to the AIR thus must be backed out of the annuity unit value to avoid giving credit for it twice—once in the way of additional annuity units payable on each payment date and once in the way of a higher annuity unit value.

[33] For a more complete understanding of the mechanics of immediate variable annuities, including numerical examples and graphical illustrations, see "The Mechanics of Variable Annuitization," by Jeffrey K. Dellinger, FSA, MAAA, The VARDS Report, Executive Series, 1994.

[34] A goal of a variable immediate annuity or variable annuitization can be to provide a stream of retirement income payments to the annuitant that can keep pace with or outpace inflation as measured by a version of the Consumer Price Index (CPI) appropriate for the annuitant population, recognizing that inflation for goods and services on which seniors tend to spend more heavily (e.g., health care) may differ from inflation as measured by the basket of goods and services of the U.S. population in total. It is recognized that VIA benefits will not move in lock-step with inflation. The higher the AIR, the harder it is to achieve this goal because the "hurdle rate" at which the underlying fund must perform is higher. As a result, the NAIC Model Variable Annuity Regulation stipulates that the AIR "shall not exceed 5% except with the approval of the Commissioner."

The number of annuity units to be paid on each income benefit date is determined by dividing the net premium by a_x, then dividing the result by the initial annuity unit value. Suppose that our individual age x has P dollars of initial wealth earmarked for lifetime retirement income generation. This is sufficient to fund annual year-end income benefits of P / a_x annuity units, assuming an initial annuity unit value of $1.00. For example, if P = $120,000 and $a_x = 12$, then the value of 10,000 annuity units will be paid on each future income benefit date.

The present value of a life annuity at age $x + 1$, a_{x+1}, can be expressed in terms of the present value of a life annuity at age x, a_x, as shown by Equation (iv).

$$a_x = \frac{l_{x+1}}{l_x} v + \frac{l_{x+2}}{l_x} v^2 + \frac{l_{x+3}}{l_x} v^3 + \ldots$$

$$= \frac{v}{l_x} (l_{x+1} + l_{x+2} v + l_{x+3} v^2 + \ldots)$$

$$= \frac{v}{l_x} (l_{x+2} v + l_{x+3} v^2 + \ldots) + v \frac{l_{x+1}}{l_x}$$

$$\left(a_x - v \frac{l_{x+1}}{l_x}\right) \frac{l_x}{v} = l_{x+2} v + l_{x+3} v^2 + \ldots \qquad \text{(ii)}$$

$$a_{x+1} = \frac{l_{x+2}}{l_{x+1}} v + \frac{l_{x+3}}{l_{x+1}} v^2 + \ldots$$

$$= \frac{1}{l_{x+1}} (l_{x+2} v + l_{x+3} v^2 + \ldots) \qquad \text{(iii)}$$

$$= \frac{1}{l_{x+1}} \left(a_x - v \frac{l_{x+1}}{l_x}\right) \frac{l_x}{v} \qquad \text{Substituting (ii) into (iii)}$$

$$= a_x \frac{(1 + i)}{p_x} - 1 \qquad \text{(iv)}$$

Equation (iv) makes intuitive sense. If one starts with sufficient funds to pay a life annuity to an individual age x, a_x, then if this amount is accumulated one year for interest and survivorship, and the benefit payment of 1 is then made, then the remaining funds on hand should be precisely sufficient to pay a life annuity to an individual age $x+1$, a_{x+1}. Equation (iv) therefore shows the net premium necessary to fund a life annuity if the individual age x delays income annuity commencement for one year.[35] For a variable income annuity, the term i in Equation (iv) represents the AIR.

[35] The difference between the gross premium and the net premium is usually defined as follows:

$$\text{GP} \cdot (1 - \text{Load}) = \text{NP} \qquad \text{or} \qquad \text{GP} = \text{NP} / (1 - \text{Load})$$

Table 4 and Figure 7 show the progression of a_x values based on the Annuity 2000 Mortality Table and 3.5 percent interest.

	a_x	
Age Nearest Birthday	Male	Female
60	15.288	16.564
61	14.926	16.210
62	14.556	15.849
63	14.179	15.481
64	13.797	15.105
65	13.410	14.723
66	13.019	14.334
67	12.624	13.939
68	12.228	13.537
69	11.831	13.127
70	11.435	12.712
71	11.040	12.290
72	10.646	11.863
73	10.255	11.432
74	9.866	10.999
75	9.480	10.564
76	9.097	10.129
77	8.719	9.696
78	8.346	9.264
79	7.979	8.836
80	7.618	8.412

Table 4. Progression of a_x Values.

The load, when it exists, is typically a "percentage of premium" charge used to cover acquisition expenses such as wholesaler compensation, financial advisor sales compensation, policy issuance, record set-up and other policy acquisition expenses as well as state premium tax, if applicable.

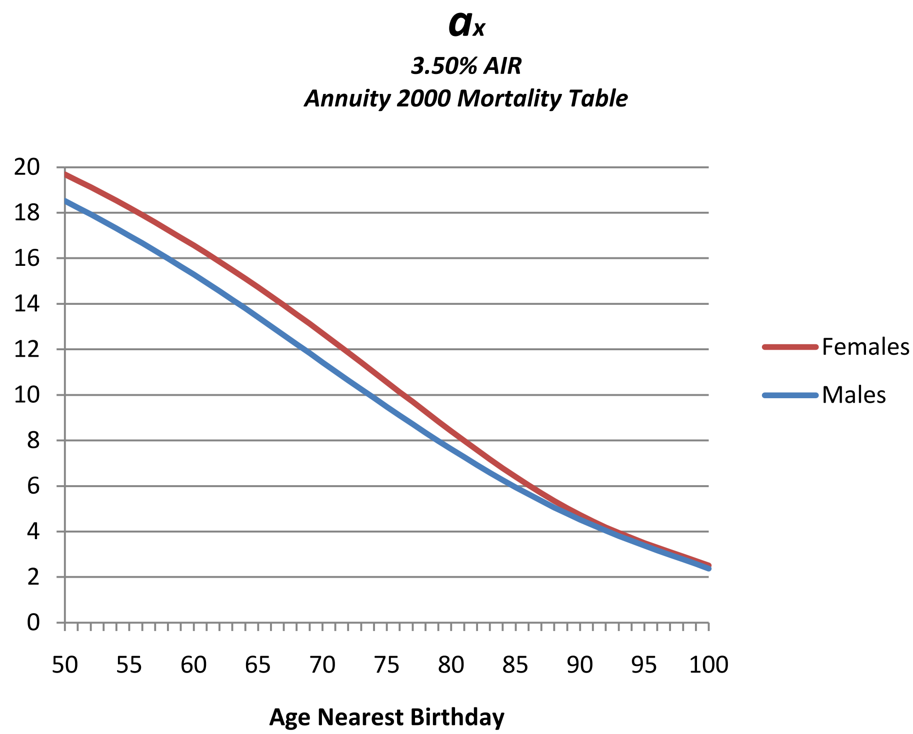

Figure 7. Progression of a_x Values.

The Equation (iv) expression of a_{x+1} in terms of a_x with modifying terms gives a clue as to the general nature of the conditions for decision making with respect to annuitizing now or delaying annuitization. If one starts with a_x, then after one year this quantity increases with benefit of interest i and with benefit of survivorship $1/p_x$ and decreases by the annuity payment of 1. The result is a_{x+1}. For delaying annuitization to be the right decision, the withdrawal program alternative would need to make the identical payment at the end of the first year and leave a quantity greater than a_{x+1}.[36]

If the underlying fund were identical for both the VIA and the withdrawal program alternative, then the gross investment return i would be the same. The decision to delay annuitization thus hinges on whether the benefit of survivorship (present in the VIA but absent in the withdrawal program) has the stronger positive effect or whether lower fees

[36] Here we are assuming that immediate annuity commencement either occurs immediately (age x) or is delayed one full year (age $x+1$). In lieu of this discrete-time model, one could look at a continuous-time model, where $\bar{a}_x$ represents a continuously payable life annuity. Alternatively, one can use the discrete-time model with interpolation if optimal ages to less than the nearest integral year of age are desired. Note, however, that insurers offering retail immediate annuities commonly consider only integral years of age in determining annuity benefits associated with a given premium.

$$\bar{a}_x = \int_0^\infty v^t \, {}_t p_x \, dt$$

in the withdrawal program product, giving it a higher net investment return, has the stronger positive effect.[37]

For instance, if the probability of one-year survival for a male age 60, p_{60}, is 0.993572, then the benefit of survivorship, $1/p_{60}$, is 1.006470.[38] Suppose the incremental fees attributable to the income annuity product are 0.0055.[39] Then the benefit of survivorship in the VIA product has a stronger positive effect than the lower fees in the withdrawal program product.

To see this clearly, suppose this 60-year-old male starts with $90,000. a_{60} = 15.288.[40] He will receive an annual benefit equal to $90,000 / a_{60} = 5,886.970 annuity units. Suppose the initial annuity unit value is $1.00 and the underlying investment subaccount performs at exactly the 3.50 percent AIR in the first year, so the annuity unit value remains $1.00 at year-end. He will receive an annuity benefit of $5,886.97. (If we assume a 0.52 percent M&E risk charge and an annual fee of 0.21 percent for the investment subaccount, then a gross investment return of 4.23 percent produces net VIA subaccount performance of 3.50 percent.)

The year-end VIA value (i.e., reserve held by the insurer to support future annuity benefits associated with this annuitant) can be derived from Equation (iv): ($90,000 × 1.035 / .993572) - $5,886.97 = $87,865.67.

In the withdrawal program product alternative—here, a similarly invested mutual fund—the male age 60 again begins with $90,000. The net investment return is 4.05 percent (4.23 percent gross return minus 0.18 percent annual fee). Accumulated value at year-end is $90,000 × 1.0405 = $93,645.00. After the $5,886.97 withdrawal (the same as for the VIA), remaining value is $87,758.03.

Because the effect of the VIA benefit of survivorship was stronger than the effect of the lower withdrawal program product fees, the value remaining to generate future retirement income after one year is greater under the annuitize now decision ($87,865.67) than under the annuitize later decision ($87,758.03).

---

[37] Note that fees are not necessarily lower in the withdrawal program product. Clearly, a high-cost withdrawal program product (e.g., mutual fund) can have higher costs than a low-cost VIA (and vice versa).

[38] The probability of death within one year for a male age 60, q_{60}, is 0.006428 in the Annuity 2000 Mortality Table. Note 1: This is a valuation table, with conservatism introduced (i.e., lower q_x values and therefore higher $p_x = 1 - q_x$ values, which result in a higher immediate annuity reserve), as opposed to an experience table. Note 2: This is an age nearest birthday table.

[39] The Vanguard Lifetime Income Program, with a VIA underwritten by American General Life Insurance Company, has a 0.52 percent M&E risk charge. The Total Stock Market Index Fund subaccount has an annual fee of 0.21 percent. The Vanguard Total Stock Market Index Fund mutual fund has an annual fee of 0.18 percent (0.07 percent for account values of $100,000 or more). The excess of the VIA fees over the mutual fund fee is 0.52 percent + 0.21 percent - 0.18 percent = 0.55 percent.

[40] Assumptions: Annuity 2000 Mortality Table, 3.5 percent AIR.

The one-year benefit of survivorship, $1/p_x$, is simply illustrated. Suppose 10 individuals aged x each contribute an identical sum of money at the beginning of the year to a common investment pool. At year-end, the pool will be shared equally among survivors. Suppose $q_x = 0.1$, and suppose this mortality assumption bears out in real-world experience, with one individual dying during the year and nine surviving the year. At year-end, even with no investment gain, each individual's share of the pool has risen by 11.1 percent, because the decedent's stake (e.g., an annuitant no longer requiring income) is distributed equally among survivors (e.g., those annuitants still alive and still requiring income). Here, $1/p_x = 1 / 0.9 = 1.111$.

The one-year benefit-of-survivorship values based on the Annuity 2000 Mortality Table appear in Figure 8. These values are important to the "annuitize now" or "annuitize later" decision because this decision turns primarily on the ability of the benefit of survivorship—present in the income annuity and absent in the withdrawal program alternative—to overpower whatever incremental fees may be associated with the income annuity.

Mortality improvement—recognized in immediate annuity pricing by projecting progressively lower mortality rates (i.e., q_x values) for the same age for each year into the future—means that p_x values are increasing over time for any given static age x. Hence, $1/p_x$ benefit-of-survivorship values are anticipated to decrease over time, raising the optimal income annuity commencement age (barring mortality rate retrogression from obesity, influenza pandemics, war, natural disasters, new viruses, etc.).[41]

[41] To quantify mortality improvement, mortality improvement factors are used, specific to each age and sex. Call these MIFs. If, say, a 75-year-old male had a 3 percent probability of dying during the next year as of 2010, we would say $q_{75} = .03$. Yet statistical studies might suggest that male age 75 mortality is improving on a relative basis by 1 percent per year.

To reflect this, in 2011 we would expect a 75-year-old male to have a probability of dying equal to 99 percent of 3 percent, or 2.97 percent.

In general, $q_x^{\text{Year N+1}} = q_x^{\text{Year N}} \times (1 - \text{MIF})$. In our example, $q_{75}^{2011} = q_{75}^{2010} \times (1 - .01)$.

Repeating the process, in 2012 we might expect another 1 percent relative mortality improvement. We'd have $q_{75}^{2012} = q_{75}^{2011} \times (1 - .01) = q_{75}^{2010} \times (1 - .01)^2$.

In general, $q_x^{\text{Year N+k}} = q_x^{\text{Year N}} \times (1 - \text{MIF})^k$. This formula assumes the mortality rate for age x improves annually on a relative basis by the constant mortality improvement factor for a k-year projection period. That is, mortality is modeled as the product of the mortality rate for a base year and a reduction factor, which follows an exponential decay. Of course, the mortality improvement factor for a given age and sex combination needn't be one uniform value in all years. Rather a vector of mortality improvement factors could be employed, resulting in the formula $q_x^{\text{Year N+k}} = q_x^{\text{Year N}} \times (1 - \text{MIF}_{N}) \times (1 - \text{MIF}_{N+1}) \times (1 - \text{MIF}_{N+2}) \times \ldots \times (1 - \text{MIF}_{N+k-1})$.

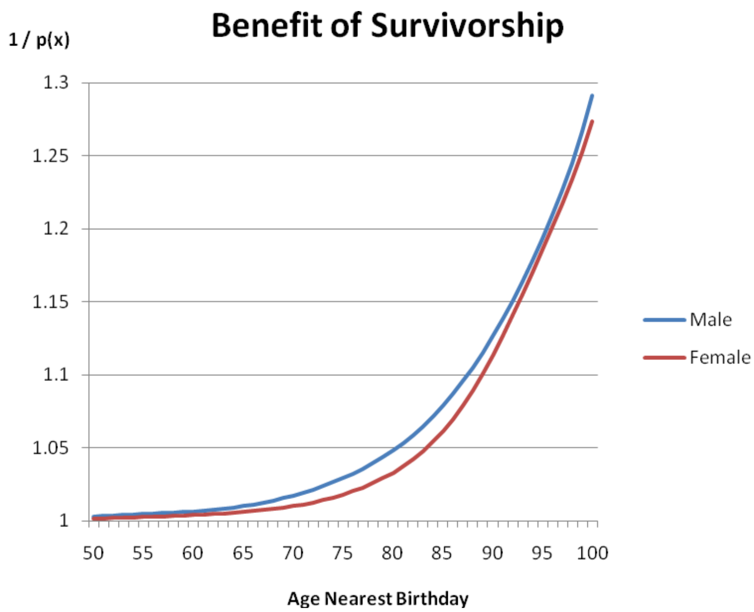

Figure 8. One-Year Benefit of Survivorship, $1/p_x$.[42]

The one-year benefit-of-survivorship factor, $1/p_x$, is related to the notion of *mortality credits*. Suppose again that our 10 individuals aged x each contribute $100 to a common investment pool to be shared equally among the survivors at year-end. Suppose the investment pool returns 6 percent. The collective $1,000 grows to $1,060 and is shared equally at year-end by the 9 survivors, each receiving $1,060 ÷ 9 = $117.78. Absent the pool, each would have $106 at year-end solely from the investment return. The mortality pooling approach yielded an additional $11.78, which can be viewed as a mortality credit. The benefit-of-survivorship factor, $1/p_x = 1/0.9 = 1.111$ is applied to both the $100 principal and the $6 interest, increasing both amounts by 11.1 percent for the survivors. Note that an 11th individual who decided not to participate in the mortality pool would have had to locate an investment that returned 17.78 percent for one year to do as well.

More generally, the one-year return to survivors is $[(1 + r) \times 1/p_x] - 1$, which clearly exceeds the pure investment return r.[43] The mortality credit, as a percentage of

---

[42] One-year benefit of survivorship can alternatively be defined as $1 + q_x/p_x$. How the factor q_x/p_x compares to incremental fees, if any, associated with an immediate annuity affects the annuitization timing decision. That q_x/p_x increases with age x while the amount of incremental fees remains static leads to a crossover point.

$$\frac{1}{p_x} = \frac{1}{1 - q_x} = \frac{1 - q_x + q_x}{1 - q_x} = 1 + \frac{q_x}{p_x}$$

[43] $$\frac{1 + r}{p_x} - 1 = \frac{1}{p_x} + \frac{r}{p_x} - 1 = \frac{q_x + r}{p_x} > r$$

beginning-of-year value, can be defined as $[(1 + r) \times 1/p_x] - 1 - r = q_x/p_x \times (1 + r)$.[44,45,46]

To make the comparison between the competing forces of benefit of survivorship and fees more general, we can express it formulaically. We further generalize the situation by allowing for different gross investment returns between the VIA subaccount(s) and the withdrawal program product(s) alternative; for example, the former might invest in a passively managed total domestic stock market index fund while the latter is an actively managed total domestic stock market mutual fund.[47]

Let P represent the initial principal available, x the age of the individual, i the AIR, r^a the gross annual return on the investment fund underlying the VIA, r^w the gross annual return on the withdrawal program product, f^a the total annual annuity fee[48], and f^w the total annual withdrawal program product fee. Immediate income annuity commencement produces a value one year later of

$$P \frac{(1 + r^a - f^a)}{p_x} - \frac{P}{a_x} \frac{(1 + r^a - f^a)}{1 + i} \qquad \text{(v)}$$

Under a withdrawal program alternative where the end-of-year withdrawal is the same amount as the end-of-year VIA income benefit, the value one year later—that is, the value available to be annuitized at age x+1—is

$$P (1 + r^w - f^w) - \frac{P}{a_x} \frac{(1 + r^a - f^a)}{1 + i} \qquad \text{(vi)}$$

---

44 $\frac{1 + r}{p_x} - 1 - r = \frac{q_x + r}{p_x} - r = \frac{q_x}{p_x} + \frac{r(1 - p_x)}{p_x} = \frac{q_x}{p_x}(1 + r)$

45 Because $q_x/p_x = 1/p_x - 1$, the mortality credit, as a percentage of beginning-of-year value, is also defined as $(1/p_x - 1) \times (1 + r)$. Thus, if the values on the vertical axis of Figure 8 are reduced by 1, then one-year mortality credits for any age equal the value shown on the vertical axis (minus one) multiplied by $1 + r$. For example, just shy of female age 90 the vertical axis value (minus one) is 11.11 percent. If the investment return r were 6 percent, we again have the $0.1111 \times (1 + 0.06) = 11.78$ percent mortality credit of our numerical example.

46 In a nonparticipating fixed immediate annuity, the mortality credits are fully guaranteed, even if lighter mortality experience ultimately develops than assumed by the insurer when pricing the annuity. In a participating fixed immediate annuity, the mortality credits depend on the actual mortality experience that develops. The terminology *participating* or *nonparticipating* relates to whether the contract owner-annuitant participates in the divisible surplus of the insurance company.

47 This investment assumption is for illustrative purposes only. Greater asset class diversification can be used to generate a more efficient portfolio in the Markowitz mean-variance sense. An IVA should offer a fairly complete set of asset classes to achieve these benefits of diversification generally, although it is recognized that in times of financial crises such benefits can wane as returns on different asset classes become increasingly positively correlated (financial contagion)—a phenomenon captured in *regime-switching* models of asset class performance.

48 For an immediate variable annuity, total annuity fee f^a is the sum of (i) the mortality and expense (M&E) risk charge, (ii) the annual administrative fee, and (iii) the weighted average fee for the investment subaccount(s) elected, the performance of which governs the magnitude of variable income annuity benefits.

Delaying annuitization from age x until age x+1 will be beneficial if the quantity given by Equation (vi) exceeds the quantity given by Equation (v).

$$P\,(1 + r^w - f^w) \;-\; \frac{P}{a_x}\,\frac{(1 + r^a - f^a)}{1 + i} \;>\; P\,\frac{(1 + r^a - f^a)}{p_x} \;-\; \frac{P}{a_x}\,\frac{(1 + r^a - f^a)}{1 + i} \qquad \text{(vii)}$$

Inequality (vii) reduces to the condition expressed by Inequality (viii).

$$1 + r^w - f^w \;>\; \frac{1 + r^a - f^a}{p_x} \qquad \text{(viii)}$$

We can rewrite Inequality (viii) in an equivalent form that expresses the left-hand side of the inequality as the difference in product fees. See Inequality (x).

$$(1 - q_x)\,(1 + r^w - f^w) \;>\; 1 + r^a - f^a \qquad \text{(ix)}$$

$$f^a - f^w \;>\; q_x + r^a - r^w + q_x\,(r^w - f^w) \qquad \text{(x)}$$

Δ fees > q_x + Δ gross investment performance + q_x (net withdrawal program performance)

If we make the simplifying (and perhaps appropriate for a true comparison[49]) assumption that gross investment performance of the investment fund(s) underlying the VIA and gross investment performance of the alternative withdrawal program investment fund(s) are identical, then Inequality (x) reduces to Inequality (xi).

$$f^a - f^w \;>\; q_x\,(1 + r^w - f^w) \qquad \text{(xi)}$$

Δ fees > q_x (1+ net withdrawal program performance) (xii)

At this stage, Inequality (xi) would serve as a decision-making basis for annuitizing immediately or delaying annuitization. If Inequality (xi) holds, then delaying is optimal. If Inequality (xi) does not hold, then immediate annuitization is optimal.[50]

A sex-distinct table of q_x values for a subpopulation that considers itself in sufficiently good (not perfect) health so as to seriously consider the purchase of a life-contingent income annuity will be helpful to this determination. Clearly, as the q_x value increases with age, at some point Inequality (x) fails to hold and delaying annuitization is no longer beneficial. Table 5 provides male and female mortality rates—that is, q_x values—on two bases. Figure 9 graphically represents p_x and q_x values on the second basis; that is, based

---

[49] This assumes a reasonably complete collection of asset classes available in the IVA.

[50] One rationale for illustrating the mechanics behind the "annuitize now" versus "annuitize later" decision by using registered securities products such as a VIA and a mutual fund is that disclosure of annual fees is mandatory. In contrast, one would generally need to "reverse engineer" a fixed immediate annuity product (e.g., SPIA) to estimate the "spread" counterpart to registered product annual fees.

on the Annuity 2000 Mortality Table.[51,52]

	1,000 q_x			
Age Nearest Birthday (x)	Annuity 2000 Basic Table		Annuity 2000 Mortality Table	
	Male	Female	Male	Female
55	5.077	2.746	4.534	2.457
56	5.465	3.003	4.876	2.689
57	5.861	3.280	5.228	2.942
58	6.265	3.578	5.593	3.218
59	6.694	3.907	5.988	3.523
60	7.170	4.277	6.428	3.863
61	7.714	4.699	6.933	4.242
62	8.348	5.181	7.520	4.668
63	9.093	5.732	8.207	5.144
64	9.968	6.347	9.008	5.671
65	10.993	7.017	9.940	6.250
66	12.188	7.734	11.016	6.878
67	13.572	8.491	12.251	7.555
68	15.160	9.288	13.657	8.287
69	16.946	10.163	15.233	9.102
70	18.920	11.165	16.979	10.034
71	21.071	12.339	18.891	11.117

[51] The Annuity 2000 Mortality Table shows mortality rates q_x increasing monotonically until some terminal age ω, where ω = 115. While observed mortality data at very advanced ages remains the scantest, current thinking is that mortality rates do not progress monotonically toward unity (i.e., $q_\omega = 1$) but rather reach a peak and level off at some *mortality plateau* at the highest ages. This has minuscule effect on the study at hand because the large amount of discounting for interest and for survivorship of payments at these advanced ages renders their effect on the present value of a life annuity, a_x, at issue ages under study here quite small.

[52] The intensity with which mortality is operating at each moment in time varies. The force of mortality, μ_x, is an instantaneous measure of the intensity with which mortality is operating at each moment in time, serving to decrease the number of lives l_x.

μ_x may be considered the elemental measure of mortality. q_x represents the annual effective rate of mortality resulting from the force of mortality μ operating over a one-year period.

In dual fashion, the intensity with which interest operates at a particular moment of time t is called the *force of interest,* δ_t. The accumulated value at any time n equals the initial principal brought forward n years on which the force of interest operates continuously. Just as an initial principal of 1 is subject to the incremental effect of the *force of interest* δ, an original cohort of l_x lives is subject to the decremental effect of the *force of mortality* μ.

Similarly, δ may be considered the elemental measure of interest. i represents the annual effective rate of interest resulting from the force of interest δ operating over a one-year period.

72	23.388	13.734	20.967	12.386
73	25.871	15.391	23.209	13.871
74	28.552	17.326	25.644	15.592
75	31.477	19.551	28.304	17.564

Table 5. Immediate Annuitant Mortality Rates.

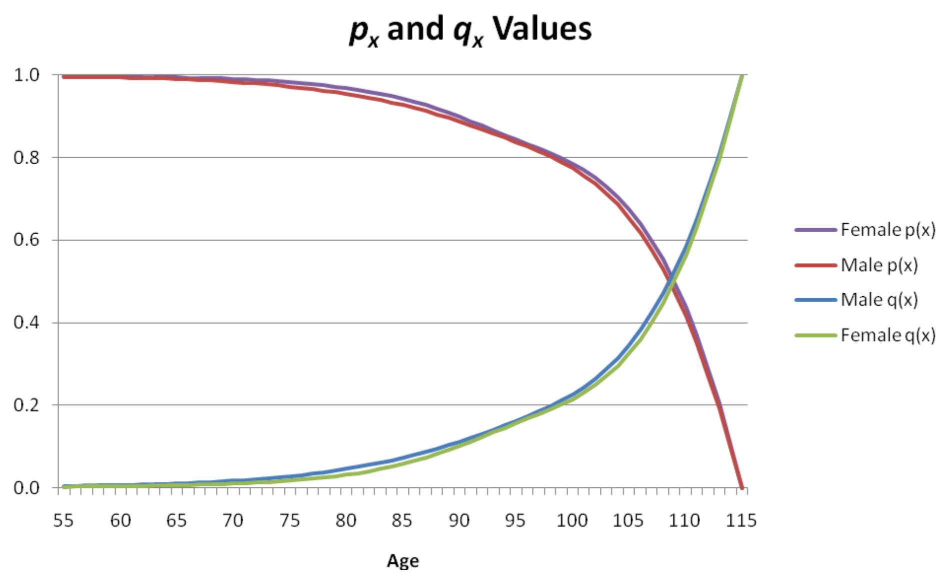

Figure 9. p_x and q_x Values Based on Annuity 2000 Mortality Table.

Consider the 0.55 percent fee differential and the 0.18 percent withdrawal program product fee mentioned earlier in this section. For our male age 60, let us assume q_{60}^{M} = 0.006428. Substituting these values in Inequality (xi), we have

0.0055 > 0.006428 (1+ net withdrawal program performance) (xiii)

0.0055 > 0.006428 (1+ gross withdrawal program performance – 0.0018)

0.8556 > .9982 + gross withdrawal program performance

-0.1426 > gross withdrawal program performance

For Inequality (xiii) to hold, gross withdrawal program performance would have to be less than a 14.26 percent loss. While certainly within the realm of possibility of all outcomes of the U.S. stock market total return random variable, based on this analysis, the 60-year-old male who fully retires and has first need for supplemental income would likely choose to annuitize immediately rather than to delay annuitization.

For a 60-year-old female, let us assume q_{60}^{F} = 0.003863. Repeating the exercise, for Inequality (xi) to hold, gross withdrawal program performance would only need to be in the interval (-∞, +42.56 percent). Given the distribution of outcomes of the U.S. stock market total return random variable, based on this analysis, the 60-year-old female would likely choose to delay annuitization rather than to annuitize immediately.

For a 63-year-old female, for Inequality (xi) to hold, gross withdrawal program performance must be in the range (-∞, +7.10 percent). For a 64-year-old female, gross withdrawal program performance must be in the range (-∞, -2.84 percent). Given the distribution of outcomes of the U.S. stock market total return random variable, based on this analysis, the 64-year-old female would likely choose to annuitize immediately rather than to delay annuitization.

Obviously, for any given age, the higher the fee differential in Inequalities (x) and (xi), the higher the net withdrawal program performance can be—thereby representing a greater percentage of all possible outcomes—and still have the inequality hold, which is associated with a decision to delay annuitization.

It is noteworthy that for some VIAs the mortality and expense (M&E) risk charge is fixed, whereas for some other VIAs the M&E risk charge may have both *current* and *maximum* levels.

To this point, the assumption is made that the mortality rates, AIR, M&E risk charge, fees for the investment subaccounts, and gross investment performance define VIA benefits. Similarly, the assumption is made that investment management fees and gross investment performance define the withdrawal program product benefits. Depending on product construction, the initial VIA payout rate per $1,000 of premium applied[53]—a typical quotation method—may involve a load factor to cover policy acquisition expenses. For instance, the initial benefit payment—payable immediately (annuity-due)—may be calculated by multiplying the gross premium by, say, 96 percent, and then dividing the result by $\ddot{a}_x$ to arrive at the dollar amount of the initial benefit payment. That dollar amount is then divided by the current annuity unit value to determine the number of annuity units payable on that first payment date and on all payment dates thereafter. In such an instance, there is thus a 4 percent load embedded in the construction of the variable income annuity payout rates per $1,000 of gross premium applied. Similarly, there might be a front-end load on the withdrawal program product (e.g., mutual fund), depending on product configuration.

These loads have not been accounted for in the analysis to this point, should they exist. This can affect the dynamics of income annuity commencement timing. For example, if there were a 4 percent front-end load on the mutual fund and also a 4 percent load embedded in the VIA payout rates, then applying fresh cash immediately to the VIA results in one instance of 4 percent of then-current retirement savings earmarked for lifetime income going toward product expenses and not being available to be invested to drive income. In contrast, purchasing the mutual fund first, making withdrawals for some finite period of time, and purchasing a VIA later, results in two such instances.

[53] The premium may be net of state premium tax, if applicable.

To account for a load (known or assumed) on the income annuity approach, we introduce a percentage of premium load factor L^a.[54,55,56] The gross premium (GP) and net premium (NP) for the income annuity bear the relationship: $GP \cdot (1 - L^a) = NP$; or, equivalently, $GP = NP / (1 - L^a)$. Similarly, to introduce a load on the withdrawal product approach, we introduce a load factor L^w.

An income annuity that commences immediately produces a value (i.e., reserve) one year later of

[54] To the extent there exists a flat dollar amount load, this can be handled by either (i) translating it into an equivalent percentage of premium load for the situation at hand or (ii) repeating the approach presented here to derive a modified formula that expressly recognizes a flat dollar amount load. Note: The flat dollar amount load, if any, tends to be quite small (e.g., $125) relative to the average income annuity premium.

[55] State premium tax may apply to an income annuity purchase. If the insurer offering the income annuity should have to pay state premium tax, it is in some instances deducted from the Gross Premium. In such an instance, it represents an explicit front-end load and is therefore part of L^a. Alternatively, an offering insurer may embed a loading factor into its pricing sufficient to cover state premium tax on an aggregate basis, without explicit recognition of state premium tax on a specific contract sold in a specific state. State premium tax as of Spring 2010 appears in Table 6.

State	Tax-Qualified	Non-tax-qualified
California	0.50%	2.35%
Maine	0.00	2.00
Nevada	0.00	3.50
South Dakota	0.00	1.25*
West Virginia	1.00	1.00
Wyoming	0.00	1.00

Table 6. State Premium Tax.

* South Dakota premium tax rate is 1.25 percent on first $500,000 of premium, 0.08 percent on amount over $500,000.

[56] Deferred annuity contracts may list the mortality table, load, and interest rate (for fixed annuitizations) and AIR (for variable annuitizations) on which minimum guaranteed fixed payout tables and variable payout tables in the back of the contract are premised.

For variable annuitizations of deferred annuity contracts where the conversion factors and AIR are known but the load, if any, is not, one can estimate the load by using a standard mortality table with projection, determining the present value that is the net premium (NP), and then determining the load that would equate to the gross premium (GP).

For fixed annuitizations of deferred annuity contracts where the conversion factors and interest rate are known, one can similarly estimate the load, if any, by using a standard mortality table with projection, determining the present value that is the net premium (NP), and then determining the load that would equate to the gross premium (GP). If non-life-contingent conversion factors are also provided—and if it is assumed that the same load applies to both non-life-contingent and life-contingent conversion factors—then the load is precisely determinable. Of course, these tables represent only the *minimum guaranteed* payout rates; actual payout rates may be higher if investment yields allow at point of annuitization.

$$P\,(1 - L^a)\,\frac{(1 + r^a - f^a)}{p_x} \;-\; \frac{P\,(1 - L^a)}{a_x}\,\frac{(1 + r^a - f^a)}{1 + i} \qquad \text{(xiv)}$$

Under a withdrawal program alternative where the end-of-year withdrawal is the same amount as the end-of-year VIA income benefit, the value one year later—the initial income annuity reserve for annuitization that occurs at age x+1—is

$$\left[P\,(1 - L^w)\,(1 + r^w - f^w) \;-\; \frac{P\,(1 - L^a)}{a_x}\,\frac{(1 + r^a - f^a)}{1 + i}\right](1 - L^a) \qquad \text{(xv)}$$

Delaying annuitization from age x until age x+1 will be beneficial if the quantity given by (xv) exceeds the quantity given by (xiv). This condition reduces to Inequality (xvi).

$$(1 - L^w)\,(1 + r^w - f^w) \;>\; (1 + r^a - f^a)\left[\frac{1}{p_x} - \frac{L^a}{a_x\,(1 + i)}\right] \qquad \text{(xvi)}$$

Inequality (xvi) indicates that the decision to delay annuitization is a function of gross investment performance, product loads, product fees, AIR, and mortality rates.

Suppose we again make the simplifying (and perhaps appropriate for a true comparison) assumption that gross investment performance of the investment fund(s) underlying the VIA and the alternative withdrawal program investment fund(s) are identical ($r^w \equiv r^a$). Suppose we also make the simplifying assumption that there is no load on the withdrawal program product ($L^w \equiv 0$); that is, a knowledgeable investor exists and seeks to accomplish his or her withdrawal and income annuity programs efficiently—making his or her own decisions, but still using packaged financial products. Inequality (xvi) then reduces to Inequality (xvii).

$$(1 - q_x)\,(1 + r^w - f^w) \;>\; 1 + r^a - f^a - \frac{(1 - q_x)\,L^a\,(1 + r^a - f^a)}{a_x\,(1 + i)} \qquad \text{(xvii)}$$

Note that when L^a is zero, we have the same result previously determined for the no-load case given by Inequality (ix).

Inequality (xvii) can equivalently be stated as a function of q_x, as shown by Inequality (xviii).

$$q_x \;<\; \frac{(f^a - f^w)\,a_x\,(1 + i) + L^a\,(1 + r^a - f^a)}{(1 + r^a - f^w)\,a_x\,(1 + i) + L^a\,(1 + r^a - f^a)} \qquad \text{(xviii)}$$

If Inequality (xviii) holds, the decision to delay annuitization is beneficial. If it does not hold, the decision to annuitize immediately is beneficial. The objective thus becomes to find the lowest age for which Inequality (xviii) fails to hold and immediately annuitize there. The quantities q_x, a_x, L^a, f^a, f^w, and i are known; the quantity r^a is not. Therefore, in order to ascertain whether Inequality (xviii) holds, one must establish a value for r^a. How one establishes a value for r^a will affect the optimal income annuity commencement age; however, optimal income annuity commencement age is not very sensitive to choice of r^a.

For example, one might set r^a as the median value of the rate of return random variable, with the clear (by definition) understanding that 50 percent of the possible rate of return outcomes for the ensuing year exceed this value and 50 percent are lower.

Alternatively, one might consider the distribution of potential outcomes for r^a based on actual discrete historical data or based on a continuous model deemed appropriate for the collection of asset classes that underlie the VIA subaccounts (and which constitute the withdrawal program products). One could establish a value for r^a such that the probability that the rate of return random variable R will be less than r^a is at most p; that is, $F_R(r^a) = P\{ R < r^a \} \leq p$. In other words, one could look at the cumulative distribution function (CDF) of the rate of return random variable and r^a is the data value where the CDF crosses p. See Figure 10.

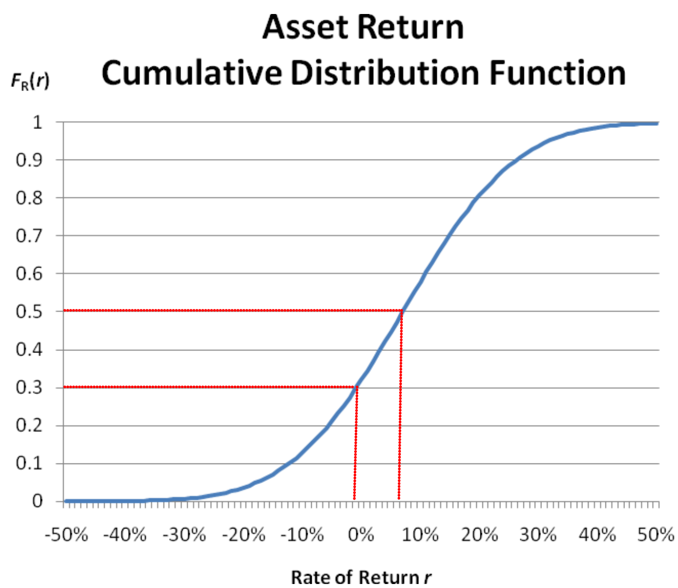

Figure 10. Asset Return CDF.

Because the decision to annuitize is generally irrevocable, an individual might elect to annuitize immediately only if called for by the decision-making Inequality (xviii) and where the r^a value chosen is reasonable in light of the asset classes employed. That is, the individual wants a reasonably high level of assurance that a decision to annuitize immediately is predicated on a rate of return assumption likely to be realized.

Note that the quantity on the right-hand side of Inequality (xviii) decreases as r^a increases. Thus as r^a increases, the value of q_x that satisfies Inequality (xviii) becomes smaller. This is the condition for delaying annuitization to be beneficial. So as r^a increases, a value of q_x previously associated with the decision to defer may now be too large and instead be associated with the decision to annuitize immediately. The higher the r^a value, the lower the age at which the decision to annuitize immediately occurs. An individual wishing to avoid an irrevocable annuitization election occurring "too soon"—

because he assumed a higher r^a value and a lower one actually materialized— could improve the likelihood of doing so by selecting an r^a value where, say, P{ R < r^a } ≤ 0.3.[57]

However determined, suppose that for an asset allocation an individual deems appropriate for himself and for the basis the individual selects to determine r^a (e.g., median return, n^{th} percentile), the resultant value of r^a is 7 percent. Now that we have a value for r^a, we can determine the first age for which Inequality (xviii) fails to hold, which is the condition associated with the decision to annuitize immediately.

Table 7 shows the optimal income annuity commencement age predicated on the Inequality (xviii) decision-making rule, a 3.50 percent AIR, and an r^a value of 7 percent.

Income Annuity Annual Fee f^a	Withdrawal Product Annual Fee f^w	Fee Difference $f^a - f^w$	Income Annuity Load (Known or Assumed)									
			0%		1%		2%		3%		4%	
			Male	Female	Male	Female	Male	Female	Male	Female	Male	Female
0.30%	0.30%	0.00%	Imm[58]	Imm	12	32	40	46	44	51	46	54
0.50%	0.30%	0.20%	46	53	48	55	50	58	52	60	54	62
0.70%	0.30%	0.40%	53	60	55	62	57	63	58	65	60	66
0.90%	0.30%	0.60%	59	64	60	66	61	67	63	68	64	69
1.10%	0.30%	0.80%	62	67	63	68	64	70	65	70	66	71
1.30%	0.30%	1.00%	65	70	66	71	66	71	67	72	68	73
1.50%	0.30%	1.20%	67	72	67	72	68	73	68	74	69	74
1.70%	0.30%	1.40%	68	73	69	74	69	74	70	75	70	75

Table 7. Optimal Income Annuity Commencement Age.[59,60]

---

[57] At customary immediate annuity commencement ages, the choice of r^a does not tend to change the optimal commencement age much. For example, assuming a 1 percent load, 0.80 percent in total annuity fees, and 0.30 percent in withdrawal product fees, a choice of r^a in the range (8.01 percent, 16.53 percent) is associated with an optimal income annuity commencement male age of 57, whereas a choice of r^a in the range (0.40 percent, 8.00 percent) is associated with an optimal income annuity commencement male age of 58.

That optimal income annuity commencement age is fairly insensitive to investment rate of return r^a is not surprising. This is because the same investment rate of return r^a—whether it is high or low—is assumed to apply to the (identically invested) withdrawal program product(s) and income annuity. Optimal income annuity commencement age remains predominantly driven by the ability of the benefit of survivorship to overpower any higher fees. Because the benefit of survivorship leverages both the beginning-of-year value (reserve) and investment appreciation during the year, a higher rate of return r^a—that produces more appreciation to be leveraged—favors the income annuity and potentially changes a "delay annuitization" decision to an "immediately annuitize" decision when both sides of Inequality (xviii) are reasonably close in value.

[58] Imm = Immediately annuitize. In other words, the optimal income annuity commencement age is zero. Annuity income commencement optimally occurs at the earliest point where both of two conditions are present: (i) a current need for supplemental lifetime income and (ii) attained age equal to at least the value shown in Table 7.

This is the scenario depicted in Section 2. Section 5 describes the general case, of which the no-load and equal fees scenario of Section 2 is a special case subset.

Using the 0.73 percent value for annual VIA expenses and the 0.18 percent value for annual index mutual fund expenses, the optimal income annuity commencement age for a 0 percent load is 57 years for males and 64 years for females. With a 1 percent load, it is 59 years for males and 65 years for females.

While the "Fee Difference" column in Table 7 is not strictly needed, as it can be derived easily from the first two columns, it turns out that it is a valuable column. It is the *difference* between the income annuity annual fee and the withdrawal product annual fee that is of primary importance, as opposed to their absolute magnitudes. If both the income annuity annual fee and the withdrawal product annual fee are increased in all years by 100 bps, the resultant optimal income annuity commencement ages shown in Table 7 are virtually unchanged.[61,62] Consequently, Table 7 can be used as a general guide even if the two left-most columns are removed; that is, one needn't reproduce Table 7 based on the income annuity annual fees and the withdrawal product annual fees specific to one's case each time one wishes to make an evaluation, but rather can generally rely on Table 7 as it stands with only a knowledge of the difference in annual fees (and a pick of income

---

[59] Based on Annuity 2000 Mortality Table, an assumption of f^w = 0.30 percent as stock and bond index funds can be had near this level, and an assumed gross rate of investment return of r^a = 7 percent.

[60] Higher loads are associated with higher optimal issue ages. This is because annuitization—and deduction of the load—at age *x* leaves less value to be increased by investment performance and by benefit of survivorship in the ensuing year. In contrast, under the withdrawal program the full value is available to be increased by investment performance in the ensuing year, with load deduction occurring upon annuitization at age *x+1*. The benefit of survivorship—present in the income annuity with annuitization occurring at age *x*—therefore has less value on which to exert its influence, delaying annuitization until a later age when the benefit of survivorship is more powerful.

[61] Optimal income annuity commencement ages where both the income annuity annual fee and the withdrawal product annual fee are increased by 100 bps in all years appear in Table 8. Unhighlighted ages are identical to Table 7. Values in red are one year higher. The higher ages occur where the quantity on the right-hand side of Inequality (xviii) is on the cusp of being higher than the quantity q_x on the left-hand side, and the 100 bps fee increase actually pushes the right-hand side higher, causing the inequality to hold, thereby changing the "annuitize immediately" decision to the "delay annuitization" decision.

Income Annuity Annual Fee f^a	Withdrawal Product Annual Fee f^w	Fee Difference $f^a - f^w$	Income Annuity Load (Known or Assumed)									
			0%		1%		2%		3%		4%	
			Male	Female	Male	Female	Male	Female	Male	Female	Male	Female
1.30%	1.30%	0.00%	Imm	Imm	12	32	40	46	44	51	46	54
1.50%	1.30%	0.20%	46	53	48	56	50	58	52	60	54	62
1.70%	1.30%	0.40%	53	60	55	62	57	63	58	65	60	66
1.90%	1.30%	0.60%	59	65	60	66	62	67	63	68	64	69
2.10%	1.30%	0.80%	63	68	64	69	64	70	65	71	66	71
2.30%	1.30%	1.00%	65	70	66	71	66	72	67	72	68	73
2.50%	1.30%	1.20%	67	72	67	72	68	73	69	74	69	74
2.70%	1.30%	1.40%	68	73	69	74	69	74	70	75	70	75

Table 8. Optimal Income Annuity Commencement Age.

[62] Lowering gross investment performance r^a from 7 percent to 6 percent and using the f^a and f^w values of Table 7 also produces the same optimal income annuity commencement ages shown in Table 8. Lowering gross investment performance by 1 percent or raising product fees by 1 percent both result in 1 percent lower net investment performance.

annuity load, known or assumed). On this basis, Table 7 values are given a visual representation in Figure 11 (males) and Figure 12 (females).

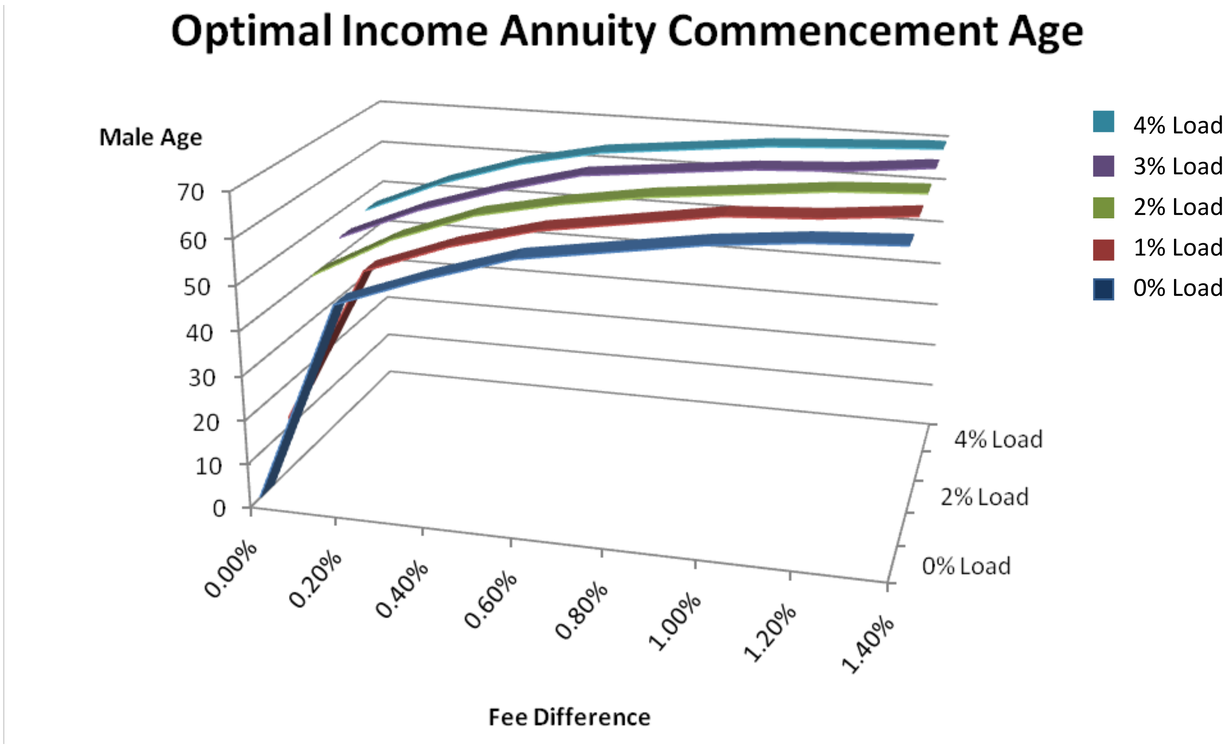

Figure 11. Optimal Income Annuity Commencement Age (Males).[63]

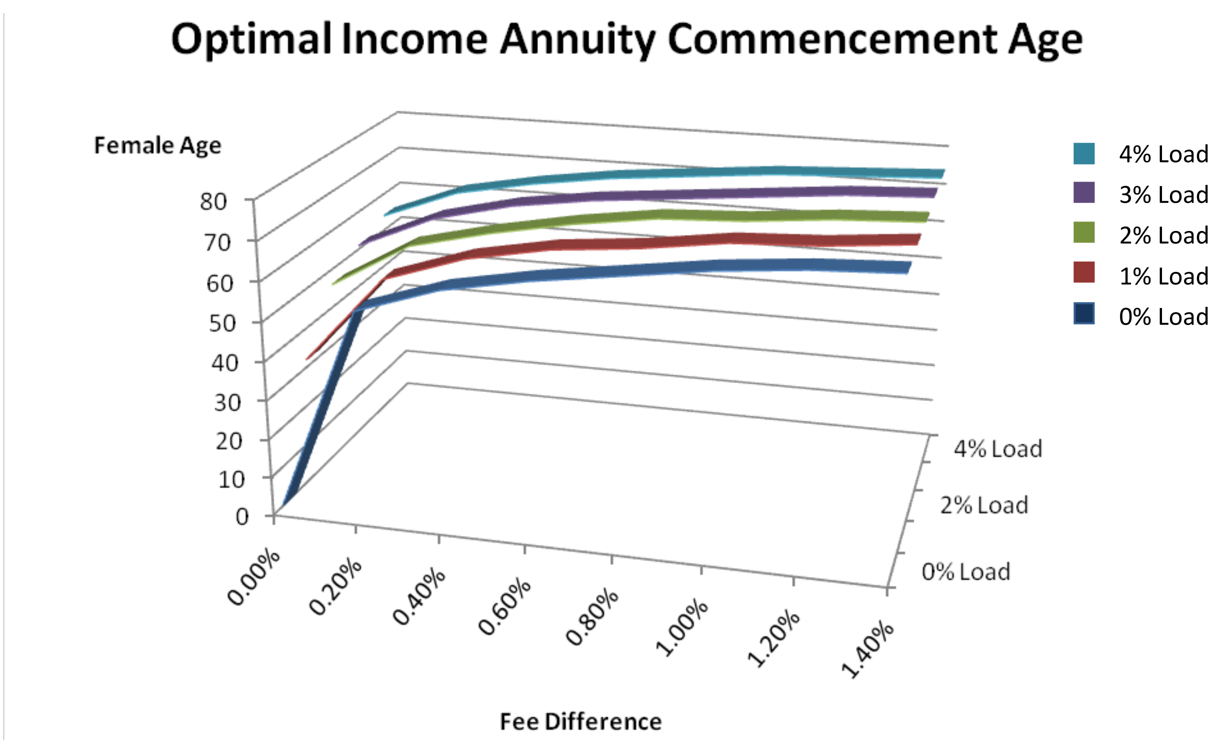

Figure 12. Optimal Income Annuity Commencement Age (Females).

[63] The optimal income annuity commencement age surface takes on a fairly stable pattern for fee differences of 0.20 percent and higher.

The loads (L^a and L^w), fees (f^a and f^w), mortality rates (q_x), annuity present values (a_x), and AIR (i) entering into the optimal income annuity commencement timing decision are objectively determinable.[64] The rate of return (r^a) factor entering into the optimal timing decision is subjective, but the resultant age is little affected by this factor.

Insurance companies offering immediate annuities may pay front-end sales commissions where such commissions are funded by (i) a load embedded in the conversion formula that translates a given amount of premium into a series of income benefits, (ii) the present value of a portion of the mortality and expense risk charge (for variable immediate annuities) or the present value of a portion of the spread (for fixed immediate annuities), or (iii) a combination thereof.

Ongoing trail commissions can be paid in lieu of or in addition to front-end commissions. Some insurers feel that trail commissions are necessary to overcome the reluctance of annuity salespeople to advocate income annuities, as an exclusively front-end commission on this irrevocable election would mark the end of commissionable opportunities on such assets—a condition that doesn't occur for withdrawal programs that tend to pay asset-based trail commissions.[65] Trail commissions can be (i) asset-based, payable as a percentage of the immediate annuity reserve, which tends to decline over time, and which forms a commission progression similar to that available under withdrawal programs or (ii) benefit-based, payable as a percentage of income benefits received by the annuitant, which might be level or might tend to rise over time. Trail commissions may align the interests of the annuitant and the salesperson with respect to variable immediate annuities, as both benefit from a judicious initial asset allocation decision and from judicious reallocation decisions during the payout period.

Annuitization of a deferred annuity contract may be a commissionable event, especially if the deferred annuity is well into or beyond any surrender charge period and acquisition expenses associated with the original deferred annuity sale have been mostly or fully recovered. This approach can overcome the salesperson's incentive to move the deferred

[64] Here it is assumed that mortality rates are those objectively determined from actual annuitant mortality experience, possibly projected, as opposed to subjectively determined based on one's own personal assessment that might include family history of longevity, wealth, etc.

[65] Salesperson compensation structures have heretofore played a role in the so-called "annuity puzzle," the phenomenon of relatively few retirees to date having voluntarily purchased life annuities in the U.S. market. Other factors for the dearth of income annuity usage include loss of access to an account value, lack of consumer understanding as to why an income annuity is more protective against longevity risk than comparably invested withdrawal programs, the feeling that one may not live sufficiently long and income annuity purchase will have been the "wrong choice," a perception that insurers offering immediate annuities—and who are familiar with mortality statistics—stand to benefit from the sales transaction at the expense of less informed contract owner-annuitants, the major benefits occur in the more distant future and humans may psychologically discount benefits that far out and place greater value on competing benefits (e.g., liquidity) that occur more near-term, a preference for bequests over personal consumption, other pre-annuitized wealth in the form of defined benefit pensions and Social Security, and the fact that the current generation of elderly has a higher percentage that participated in defined benefit pension plans and has a combination of pension plan benefits and Social Security benefits relative to their working period incomes that is higher than is likely for the next generation—which to some degree insulates the current generation of elderly from longevity risk, effectively hiding it.

annuity money to a single premium immediate annuity (SPIA) or single premium immediate variable annuity (SPIVA) of another insurer to gain a commission.

Table 7 has ramifications for product developers. As Table 7 indicates, loads and high income annuity fees (or spreads) increase the age at which income annuity purchase makes economic sense, decreasing market size—at least for informed consumers.[66] Yet even with reasonably high loads and fees, at some point income annuity commencement still makes sense, as Table 7 shows.

Front-end sales commissions on income annuities are often in the 3- to 4-percent range. There are income annuities for which no sales commissions exist.[67]

The analysis presented here is a pre-tax analysis. It does not recognize the effect of personal income taxation, which can affect income from the withdrawal program and from the income annuity dissimilarly. In the nontax-qualified market, long-term capital gains and dividends may be taxed at a more favorable (lower) tax rate than ordinary income should a portion of the withdrawal program consist of common stock. Also in the nontax-qualified market, the income annuity receives favorable Internal Revenue Code §72 tax treatment, where a portion of each income benefit is excludable from gross income as a return of the purchaser's investment in the annuity contract until the entire cost basis is recovered. This applies to both variable and fixed immediate annuities.

In addition to tax considerations, one might consider adjustments to the Table 7 values based on particular subpopulations to which one belongs. An aggregate mortality table intended to represent mortality rates of an annuity payout population is an amalgamation of mortality rates associated with finer distinctions of various annuity payout subpopulations. For example, immediate annuitants with high annuity income amounts tend to have lower mortality rates than those with low income amounts. Immediate annuitants electing a "life only" payout option tend to exhibit lower mortality rates than those electing a refund feature. These distinctions can meaningfully affect mortality rates. For example, in an annuitant mortality experience study, nontax-qualified, non-refund immediate annuities with annual income amounts of $7,500 and over exhibited an actual to expected (A/E) mortality ratio of 0.595, whereas those with annual income amounts of less than $2,500 exhibited a 1.446 A/E mortality ratio.[68,69] Because such distinctions

---

[66] And, of course, a higher M&E charge creates additional drag on investment subaccount performance, which retards retirement income.

[67] "AGL of Delaware will not pay any commission to entities that sell the Contracts. Payments may be made for services not directly related to the sale of the Contract, including the establishment of administrative arrangements, recruitment and training of personnel, the distribution and production of promotional literature, and similar services." May 3, 2010 Group Immediate Variable Annuity Prospectus, Issued by American General Life Insurance Company of Delaware Through Its Variable Account I (Vanguard Lifetime Income Program), p. 46.

[68] Table 2b3, Nonqualified Nonrefund and Refund Annuity Mortality Ratios by Annual Income, Contract-Year Group, and Contract Type, Based on Annuity 2000 Basic Table, for Experience Years 2000 through 2004. Society of Actuaries 2000-04 Individual Payout Annuity Experience Report. The A/E ratios shown in this paper (i.e., 0.595 and 1.446) are (i) for all contract years—as opposed to select and ultimate, and (ii) based on number of contracts—giving all contracts equal weight—as opposed to being weighted by

affect mortality rates, and because mortality rates affect optimal income annuity commencement ages, such distinctions affect optimal income annuity commencement ages.

The advent of more U.S. insurance companies writing "substandard" immediate annuities—that is, performing medical underwriting and offering higher annuity payouts for a given premium to individuals with health impairments that affect longevity—may affect optimal income annuity commencement ages.[70,71] For instance, if the market is bifurcated whereby those individuals with mild health impairments who still would have purchased immediate annuities now receive higher annuity benefits, then for the same aggregate payments to be made by insurers, the remainder of annuitants—those with superior health—must receive lower annuity benefits.[72,73] The lower q_x values associated with this latter "standard" class will serve to increase the optimal income annuity commencement age for them.[74,75]

---

amounts of annual income.

[69] The 0.595 and 1.446 A/E ratios are based on *immediate annuities*. *Annuitizations* of deferred annuities exhibit a similar but smaller effect. Non-tax-qualified, nonrefund annuitizations with annual income amounts of $7,500 and over exhibited an actual to expected (A/E) ratio of 0.880, whereas those with annual income amounts of less than $2,500 exhibited a 1.268 A/E ratio. A/E ratios in the Society of Actuaries 2000-04 Individual Payout Annuity Experience Report are based on fixed payout annuities; i.e., data for variable payout annuities was not requested as part of the study.

[70] Medically underwritten immediate annuities are more prominent in the United Kingdom than in the United States at this time.

[71] Such medically underwritten products are alternatively called "impaired life annuities," "substandard annuities," and "enhanced annuities."

[72] Dellinger, Jeffrey K., *The Handbook of Variable Income Annuities*, John Wiley & Sons, Hoboken, NJ, 2006. Chapter 13 contains a "Substandard Life Underwriting" section that addresses various types of immediate annuity underwriting and quantifies the effect of various degrees of medical impairment on immediate annuity payouts. See pp. 412-419.

[73] "Alan Lavine Interviews RISE CEO Jeff Dellinger," The RISE Report, Volume 1, Number 1, pp. 1-6. In this article where syndicated financial columnist Alan Lavine interviews RISE CEO Jeff Dellinger, the subject of medically underwritten annuities is discussed on p. 6.

[74] Even if a life insurance company doesn't offer substandard SPIAs, the fact that other insurance companies do may result in the first insurer experiencing lower q_x values, assuming individuals eligible for some degree of higher benefit due to medical underwriting would have purchased an immediate annuity on standard terms. Those individuals who otherwise would not have purchased an SPIA absent the higher benefit associated with medical underwriting (i.e., they would have remained outside the immediate annuity market) produce no effect on the immediate annuity mortality experience (i.e., q_x values) of the first insurer.

[75] Increased gradations in initial annuity benefit level for a given premium due to finer levels of medical underwriting can be expected to produce the effect of progressively moving objective mortality rates—those actually used by insurers in pricing—closer to subjective viewpoints of one's own mortality rates. A host of stratification factors beyond age, gender, and health status (e.g., annual income level, type of annuity payout option elected—refund versus nonrefund, tax-qualified versus non-tax-qualified plan, education level, marital status, geographic location, ancestral longevity records, source of payout annuity—immediate annuity, annuitization of deferred annuity, maturing deferred annuity, life insurance settlement option, structured settlement, etc.) affect individual annuitant mortality rates and would produce a similar

6. Conclusion

On an expected value basis, an income annuity will produce more retirement income than a comparably invested withdrawal program because there are three factors driving retirement income growth in the former case—principal, appreciation, and mortality credits—whereas there are only two factors driving retirement income growth in the latter case—principal and appreciation.[76] Similarly, on an expected value basis, a variable income annuity will produce more income than a fixed income annuity, because (i) the insurance company does not bear the investment risk in the former and thus does not have to introduce a margin for asset depreciation (C-1) risk, (ii) the insurance company does not bear interest rate risk in the former and thus does not have to introduce a margin for interest rate (C-3) risk, and (iii) the contract owner can choose to have his annuity income benefits based on a collection of assets with a higher mean return than that associated with the collection of typically high-quality, fixed-income securities held by the insurer to back fixed immediate annuity obligations.[77]

As a result of the above, one might expect greater use of variable income annuities in the years ahead—even though fixed income annuities dominate over variable income annuities today and withdrawal programs that retain full access to an account value but are less efficient retirement income generators dominate over both types of income annuities today. It must be remembered that the ability of a retirement income program to achieve its objectives (e.g., efficiently producing income that sustains one's standard of living by lasting a lifetime, by being sufficiently high, and by rising over time to keep

---

effect. For VIAs, the AIR elected among several available choices also might be expected to affect individual mortality rates, with those in less favorable health electing higher AIRs in order to receive "more income sooner."

Longevity-altering medical advancements such as migration toward engineered replacement parts (e.g., mechanical hearts) for humans will affect the immediate annuity underwriting landscape. Heretofore, the difference between subjective mortality rates and objective mortality rates (or, similarly, subjective force of mortality and objective force of mortality) has played a more prominent role in income annuity purchase decisions than one would expect it to play in the future, as increased underwriting more equitably addresses the question of where on the price-to-benefit-ratio spectrum the applicant should be placed.

[76] This distinction provides a competitive advantage for U.S. life insurance companies—that have the franchise to underwrite mortality risk—over other retirement asset aggregators such as banks and mutual funds—that do not have such a franchise. For more on the importance of this franchise to the U.S. life insurance industry, see the article "Efficacy and Prosperity" (The RISE Report, Volume 1, Number 1, pp. 9-10) and "IVA Business Value to Insurance Company" (The Handbook of Variable Income Annuities, Chapter 17, pp. 473-488).

[77] Insurers often target high credit quality in their investment policy statements for the segment of general account assets backing fixed immediate annuity obligations, as these must weather the inevitable economic downturns that can develop over a potentially multidecade holding period. Of course, higher credit quality is associated with lower yields, which serve to depress the amount of fixed immediate annuity income that can be generated by a given premium. Consequently, an insurer must balance its desire for a safer, high-credit-quality investment portfolio with its desire for attractive immediate annuity payout rates in order to garner sales. Because of the customarily irrevocable nature of a fixed immediate annuity purchase, prospective contract owners place significant value on insurance company claims-paying ratings. As a result, the relative balance may tilt toward safer, higher-quality, lower-yielding investments, which help achieve a stronger claims-paying rating.

pace with or outpace inflation, even if not moving in lock-step with it) can fail this mission by being too conservative as well as by being too aggressive. Especially given the shortfall in national retirement savings, the ability to stretch these finite savings into the maximum income possible takes on greater importance, hence the choice of a VIA to illustrate the income annuity commencement timing concepts found in this paper.

Given the unstoppable demographics of not only the United States but also other developed countries with significant aging populations—including Japan, Italy, Germany, Greece, Spain, France, and the United Kingdom[78]—and the underaccumulation of wealth necessary to sustain living standards in retirement, whether due to undersaving for retirement or low investment returns, it becomes increasingly important that finite nest eggs be transformed into lifetime income as efficiently as possible. Immediate fixed annuities have existed for centuries—even millennia.[79] Immediate variable annuities have existed since 1952, when the College Retirement Equities Fund (CREF) began issuing variable annuities for college teachers.[80] These immediate annuities are efficient generators of income. Indeed, investments inside an immediate annuity generate lifetime income superior to identical investments outside an immediate annuity—that is, superior to an identical investment portfolio that lacks the immediate annuity overlay.[81] Just as one wishes to employ instruments that efficiently generate income guaranteed to last a lifetime, it is also important to efficiency to trigger the use of these instruments at the optimal time.

This paper presents an analytic method to accomplish the optimal timing objective. It offers a table of income annuity commencement ages that can serve as a guidepost. The microeconomic approach to optimal income annuity commencement timing described in this paper studies how an individual can make an informed decision to allocate limited resources. The approach does not rely on utility-based arguments. It deals with that portion of one's wealth earmarked for retirement income generation, where the financial objective is to maximize income while simultaneously minimizing (to zero) the probability of outliving that income. The framework described in this paper in the context of fixed or variable immediate annuity purchase extends to similar situations such as annuitizations of existing fixed or variable deferred annuities.[82]

---

[78] These countries have an estimated Year 2010 age 65+ population of 16.2 to 22.2 percent. The United States has an estimated Year 2010 age 65+ population of 12.8 percent. Source: *The World Factbook* 2009, Washington, DC: Central Intelligence Agency, 2009.

[79] Dellinger, Jeffrey K., *The Handbook of Variable Income Annuities*, John Wiley & Sons, 2006, Hoboken, New Jersey, p. 59.

[80] Dellinger, Jeffrey K., *The Handbook of Variable Income Annuities*, John Wiley & Sons, 2006, Hoboken, New Jersey, p. 318.

[81] Dellinger, Jeffrey K., and R. H. "Rick" Carey, *Immediate Annuities: Structure, Mechanics, and Value*, in Retirement Income Redesigned, Master Plans for Distribution, edited by Harold Evensky and Deena B. Katz, Bloomberg Press, New York, 2006, p. 342.

[82] One must be careful, however, extending the framework to, say, employer-sponsored plans—such as 401(k) plans—with in-plan annuitization options. The latter require unisex mortality tables (i.e., gender-neutral pricing), whereas rolling the money to an individual IRA account outside the umbrella of an employer-sponsored plan allows for the use of sex-distinct mortality tables, which will likely result in

While the algorithm described in this paper may govern the "purely economic" optimal annuitization timing decision, this may be further adjusted by subjective preferences (e.g., utility associated with "I sleep better at night knowing I've arranged for lifetime income" or "I think insurers underestimate future mortality improvement so I'm going to annuitize now"). Similarly, the paper expressly avoids competing subjective preferences relating to consumption versus bequest motivations behind behavior by assuming that the individual is focusing exclusively on that portion of his or her wealth for which the financial objective is maximizing retirement income on an inexhaustible basis (i.e., guaranteed to last a lifetime).

Factors affecting the optimal income annuity commencement timing decision include age, gender, M&E risk charge and administrative charge in IVAs, withdrawal program product annual fees relative to IVA investment subaccount annual fees, withdrawal program product interest rate spread relative to interest rate spread in fixed SPIAs, product loads, mortality rate basis underlying immediate annuity payout rates, unisex versus sex-distinct mortality rates, investment performance, AIR, tax-qualified versus nontax-qualified market, long-term versus short-term capital gains in the withdrawal program product, individual income tax rates, and state premium tax.

Even without any bequest motive, the paper shows that a withdrawal product would be used for income generation prior to the optimal income annuity commencement date. This has implications for suitability of income annuity sales.

When the income annuity does not have a load and when there is no difference between the income annuity annual fee and the withdrawal program product annual fee, then the income annuity should commence immediately—assuming one needs incremental income at that point in life.[83] When the income annuity has a load or when the income annuity annual fee is higher than the withdrawal program product annual fee, then optimal income annuity commencement ages are higher for females than for males.[84] This is attributable to lower mortality rates for females than for identically aged males. These lower mortality rates reduce the one-year benefit of survivorship, and it is the ability of this one-year benefit of survivorship to offset the higher level of annuity fees that affects the optimal income annuity commencement age.

---

higher annuity benefits for males (except in the handful of states that require unisex rates). For a complete explanation, see "Unisex Rates" in Chapter 14 of *The Handbook of Variable Income Annuities*.

To the extent the employer-sponsored plan unisex rates are set equal to the female rates (under the assumption that 100 percent of males will roll their money out to achieve the more favorable sex-distinct rates), the framework described in this paper still applies to females in such instances.

[83] In the United States, Social Security and defined benefit (DB) pension plans may provide some level of pre-annuitized wealth. To the extent the sum of these amounts proves insufficient, this is the *incremental* amount being discussed. As DB pension plans wane, the expectation is for less pre-annuitized wealth and a greater need for individuals to make annuitization decisions—as to amount, type and timing.

[84] This assumes the same rating class—whether standard or substandard—for males and females.

Guaranteed minimum withdrawal benefits (GMWBs) and guaranteed lifetime withdrawal benefits (GLWBs) were invented in 1998 (Dellinger, et al., U.S. Patents 7,089,201 and 7,376,508).[85,86] While the GLWB features of variable and fixed deferred annuities also guarantee lifetime income, they are distinguishable from traditional immediate annuities in that they provide permanent access to an account value. The mortality credits available on traditional immediate annuities—where reserves released by deceased annuitants who no longer require retirement income help fund retirement income to surviving annuitants—are generally not available on GLWB features. Hence the protocol demonstrated in this paper is inapplicable to deferred annuities with GLWB features, even though they are annuity products offering lifetime income.[87]

The author thanks the Society of Actuaries' Committee on Post-Retirement Needs and Risks for helpful comments.

Jeffrey K. Dellinger, FSA, MAAA is CEO of Longevity Risk Management Corporation®, a retirement income consultancy located in Fort Wayne, Indiana. He can be reached at *JDellinger@Longevity.co*.

The viewpoints expressed herein are those of Longevity Risk Management Corporation®, and are not necessarily those of the Society of Actuaries or of the American Academy of Actuaries.

---

[85] Dellinger, et al. U.S. Patent 7,089,201, Method and Apparatus for Providing Retirement Income Benefits.

[86] Dellinger, et al. U.S. Patent 7,376,608, Method and System for Providing Retirement Income Benefits.

Along the topic of efficiency with respect to retirement income generation, this patent also describes an invention for controlling downside annuity income volatility through an IVA *benefit floor* that is less costly than if accomplished exclusively by the insurer hedging this risk in the derivatives market (or self-insuring it). This patent also describes an invention for controlling downside annuity income volatility through an IVA *benefit collar* consisting of a formulaic floor and ceiling; this approach is less costly than both (i) the benefit floor approach mentioned in the prior sentence and (ii) the insurer exclusively hedging this risk in the derivatives market (or self-insuring it).

[87] Similarly, *i4Life®*, a patented retirement income management approach (Dellinger, et al., U.S. Patent 7,376,608) that offers guaranteed lifetime income and features a liquidity period is distinct from traditional immediate variable annuities, so the optimal commencement timing protocol demonstrated in this paper is inapplicable to it. *i4Life®* is marketed by The Lincoln National Life Insurance Company. Over 50,000 individuals have collectively elected to have more than $10 billion managed under this system as of January 2011.

REFERENCES

Dellinger, Jeffrey K. (2006). *The Handbook of Variable Income Annuities,* John Wiley & Sons, Hoboken, New Jersey.

Dellinger, Jeffrey K., FSA, MAAA (1994). "The Mechanics of Variable Annuitization," The VARDS Report, Executive Series.

Dellinger, Jeffrey K. (2006). "Are We There Yet? The Embryonic State of Retirement Income Knowledge," The RISE Report, Volume 1, Number 1.

Dellinger, Jeffrey K. (2006). "Efficacy and Prosperity," The RISE Report, Volume 1, Number 1.

Dellinger, Jeffrey K. (2007). Efficient Deployment of Retirement Assets to Increase Financial Security of Seniors and to Minimize Welfare Burden on State, Retirement Income Solutions Enterprise, Inc. (RISE) White Paper, September 2007.

Dellinger, Jeffrey K., and R. H. "Rick" Carey (2006). "Immediate Annuities: Structure, Mechanics, and Value," in *Retirement Income Redesigned, Master Plans for Distribution,* edited by Harold Evensky and Deena B. Katz, Bloomberg Press, New York.

Dellinger, Jeffrey K., *et al*. U.S. Patent 7,089,201, Method and Apparatus for Providing Retirement Income Benefits.

Dellinger, Jeffrey K., *et al*. U.S. Patent 7,376,608, Method and System for Providing Retirement Income Benefits.

Johansen, Robert J. (1998). Annuity 2000 Mortality Tables. Transactions of the Society of Actuaries, 1995-96 Reports. ©1998 Society of Actuaries.

Lavine, Alan. (2006). "Alan Lavine Interviews RISE CEO Jeff Dellinger," The RISE Report, Volume 1, Number 1.

Model Variable Annuity Regulation, National Association of Insurance Commissioners, Model #250.

Society of Actuaries 2000-04 Individual Payout Annuity Experience Report, Society of Actuaries, April 2009.

The World Factbook 2009, Washington, DC: Central Intelligence Agency, 2009.

Vanguard Lifetime Income Program, May 3, 2010 Group Immediate Variable Annuity Prospectus, Issued by American General Life Insurance Company of Delaware Through Its Variable Account I.

GLOSSARY

Accumulation Unit – a measure used to calculate account value of a variable deferred annuity contract prior to any election to convert it into an immediate annuity contract

Accumulation Unit Value – the value of one accumulation unit on a valuation date, with valuation typically occurring once daily following the close of the market on each day major stock exchanges are open for business

Annuitant – a person on whose life annuity payouts are based

Annuitization – the process of converting a lump sum dollar amount into a series of periodic income benefits through the purchase of an immediate annuity

Annuity Payment Option – any of the optional forms of payment of annuity benefits defined in the contract or otherwise made available by the insurance company

Annuity Unit – a measure used to calculate the amount of periodic immediate variable annuity benefit payments

Annuity Unit Value – the value of one annuity unit on a valuation date, with valuation typically occurring once daily following the close of the market on each day major stock exchanges are open for business

Assumed Investment Rate (AIR) – the investment return assumed in calculating the initial annuity benefit; future annuity benefits fluctuate based on actual subaccount performance relative to the AIR; alternatively known as benchmark rate, hurdle rate, or target rate

Beneficiary – a person designated to receive residual benefits, if any, following death of the annuitant; the annuity payout option chosen defines whether such benefits shall be payable in a lump sum or in continuing installments

Conversion Rate – the factor that converts a lump sum dollar amount into an initial periodic income benefit based on factors such as age, sex, annuity payment option, benefit payment frequency, and AIR; often expressed as dollars of monthly income per $1,000 of net premium; alternatively known as annuity purchase rate

Deferred Annuity – an annuity contract purchased with a single premium, a schedule of periodic premiums, or flexible, ad hoc premiums used to accumulate retirement savings; ultimately, deferred annuity account value may be used to generate retirement income by conversion to an immediate annuity or via one or more withdrawals

Fixed Annuity – an annuity contract that guarantees a fixed rate of interest will be credited during the accumulation phase and a fixed amount of periodic income will be paid following conversion to an immediate annuity

Fund – any of the underlying investment options into which the variable annuity premium may be invested through the separate account

General Account – an account consisting of all assets owned by the insurance company other than those assets in separate accounts

Immediate Annuity – an annuity contract, generally purchased with a lump sum, that provides periodic income benefits as defined by the annuity payment option; an immediate annuity may be fixed, variable, or a combination

Mortality Credits – age-based increments to immediate annuity income that result from participating in an annuitant pool where wealth released by decedents who no longer require income is reallocated to surviving annuitants who do

Net Asset Value Per Share – the market value of a fund calculated each day by taking the closing market value of all securities owned, adding the value of all other assets including cash, subtracting all liabilities, and dividing the total net assets result by the number of shares outstanding

Nontax-Qualified Plan – a retirement plan other than a tax-qualified plan (defined below)

Owner – a person who has authority to exercise ownership rights within the variable annuity contract, such as investment allocation and reallocation, election of annuity payout option, and designation of beneficiary

Premium – the amount paid to purchase an annuity product, the terms of which are defined in writing in the annuity contract

Separate Account – a segregated investment account into which the insurance company invests the assets for variable immediate annuities (and possibly also for variable deferred annuities and variable life insurance products); such assets are segregated from the general account of the insurance company

Subaccount – the portion of the separate account that reflects investments in accumulation units and/or annuity units of a particular fund available under the contract

Tax-Qualified Plan – a retirement plan qualifying for special tax treatment under the Internal Revenue Code of 1986, as amended, including Sections 401, 403, 408, 408A, and 457

Valuation Date – a date on which (a) net asset value per share of funds underlying variable annuity subaccounts and (b) accumulation unit values and annuity unit values of variable annuity subaccounts are valued, which typically occurs once daily following the close of the market on each day major stock exchanges are open for business

Variable Annuity – an annuity contract that permits allocation of the premium to one or more variable subaccounts the performance of which govern account value for deferred annuities and periodic income levels for immediate annuities