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# Expected Geometric Returns

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> The best estimate for the investment return assumption is ageometric return that includes a reduction for the volatility drag on the long-term expected return.

The expected long-term compounded geometric annual rate of return is calculated by taking the expected arithmetic annual return and adjusting it for variance drain or volatility drag, resulting in a reduction of about 50 percent of the variance.[1] Risk is measured using the standard deviation.

For example, suppose a portfolio with a mix of 50 percent domestic equities and 50 percent domestic fixed income has an expected arithmetic annual return of 7.50 percent and a standard deviation for the portfolio of 10 percent. The approximate expected annual compounded rate of return is $\mu p - 0.5\sigma_p^2$ , or $7.50\% - [.5\ (10\%)^2]$, which is 7.00 percent.

Just as the geometric mean is less than the arithmetic mean when the returns are not identical, the compounded return is lower than the expected annual return because of volatility. The reasons for the volatility drag are explained in an article by James D. MacBeth.[2] The variance drain would be about half the variance of the portfolio. For example, a sample portfolio of 100 percent equities with an annual expected rate of return of 10.0 percent and a standard deviation of 20 percent will have variance drain of about 200 basis points, and the compounded expected return is about 8.0 percent. Exact formulas for the compounded return after adjusting for the variance drain were developed by de La Grandville.[3]

To improve on the estimate for the variance drain, a factor of 0.46 could be used instead of 50 percent (Figure 1). For example, a sample portfolio of 100 percent equities with an annual expected rate of return of 10 percent and a standard deviation of 20 percent would have a variance drain of about 200 basis points, and the compounded expected return would be about 8.2 percent before expenses.

Figure 1: Formula for Rate of Return

**Annual Compounded Rate of Return =**

**Expected Annual Return – [0.46 (Variance)]**

$$= \mu_p - 0.46\ \sigma_p^2$$

Table 1 shows the expected geometric rates of return for sample portfolios of various equity and fixed-income mixes. These amounts precede the subtraction of expenses. For a portfolio with 75 percent in equities and 25 percent in fixed income, the arithmetic annual real return for the portfolio is 6.0 percent and the standard deviation is 15.0 percent. Using these assumptions, the expected compounded annual real return is 5.0 percent.

| Equity Percent | Fixed Income Percent | Arithmetic Annual Real Return | Standard Deviation | Expected Compounded Annual Real Return |
|---|---|---|---|---|
| 1% | 99% | 0.8% | 0.8% | 0.8% |
| 10% | 90% | 2.2% | 3.1% | 2.2% |
| 20% | 80% | 3.3% | 5.2% | 3.2% |
| 30% | 70% | 3.8% | 6.8% | 3.6% |
| 40% | 60% | 4.3% | 8.5% | 4.0% |
| 50% | 50% | 4.8% | 10.3% | 4.3% |
| 60% | 40% | 5.3% | 12.1% | 4.6% |
| 70% | 30% | 5.8% | 14.0% | 4.9% |
| 75% | 25% | 6.0% | 15.0% | 5.0% |
| 80% | 20% | 6.2% | 15.7% | 5.0% |
| 90% | 10% | 6.5% | 17.0% | 5.2% |
| 100% | 0% | 7.0% | 19.2% | 5.3% |

The best estimate for the investment return assumption is a geometric return that includes a reduction for the volatility drag on the long-term expected return. Based on the asset mix, the expected compounded return assumption before expenses can be developed by taking the expected annual return and subtracting about 50 percent, or 0.46, of the variance. ■

## ENDNOTES

1. The variance is the standard deviation squared ($\sigma^2$).
2. James D. MacBeth, "What's the Long-Term Expected Return to Your Portfolio?" *Financial Analysts Journal* 51, no. 5 (1995): 6–8.
3. Olivier de La Grandville, "The Long-Term Expected Rate of Return: Setting It Right," *Financial Analysts Journal* 54, no. 6 (1998): 75–80.

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