**Errata for** *Problems and Solutions in Mathematical Finance* by Chin et. al. page 138 (problem 13) change

$$\mathbb{E}\left[\left(\int_{0}^{t} \frac{1}{1-s} dW_{s}\right)^{2}\right] = \mathbb{E}\left(\int_{0}^{t} \frac{1}{(1-s)^{2}} ds\right) = \frac{1}{1-t}$$
  
to  $\mathbb{E}\left[\left(\int_{0}^{t} \frac{1}{1-s} dW_{s}\right)^{2}\right] = \mathbb{E}\left(\int_{0}^{t} \frac{1}{(1-s)^{2}} ds\right) = \frac{t}{1-t}$   
 $Var(X_{t}|X_{0}=x) = \frac{1}{1-t}$  to  $Var(X_{t}|X_{0}=x) = t(1-t)$   
 $\int_{0}^{t} \frac{1}{1-s} dW_{s} \sim N\left(yt + x(1-t), \frac{1}{1-t}\right)$  to  $\int_{0}^{t} \frac{1}{1-s} dW_{s} \sim N\left(yt + x(1-t), \frac{t}{1-t}\right)$ 

page 146 (problem 19) change  $G_t = e^{\frac{1}{t} \int_0^t S_u du}$  to  $G_t = e^{\frac{1}{t} \int_0^t \log(S_u) du}$ page 222 to 225 change  $Z_s = e^{-\int_0^s \lambda_u du - \frac{1}{2} \int_0^s \lambda_u^2 dW_u}$  to  $Z_s = e^{-\int_0^s \lambda_u dW_u - \frac{1}{2} \int_0^s \lambda_u^2 du}$ page 239 and 240 change  $Z_s = e^{-\int_0^s \lambda_u du - \frac{1}{2} \int_0^s \lambda_u^2 dW_u^x}$  to  $Z_s = e^{-\int_0^s \lambda_u dW_u - \frac{1}{2} \int_0^s \lambda_u^2 du}$ 

# Clarification notes

- 1. Definition (n-dimensional standard correlated Wiener Process) Let  $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and  $\{W_t^{(i)} : t \ge 0\}, i = 1, 2, ..., n$  be a sequence of standard Wiener processes. The vector  $\mathbf{W}_t = (W_t^{(1)}, W_t^{(2)}, ..., W_t^{(n)})^T$  is an n-dimensional standard correlated Wiener process if:
  - $W_0^{(i)} = 0$  and  $W_t^{(i)}$  has continuous sample paths for i = 1, 2, ..., n.
  - for each t > 0 and s > 0,  $\mathbf{W}_{t+s} \mathbf{W}_t \sim N_n(\mathbf{0}, \Sigma)$  where  $\Sigma$  is an  $n \times n$  matrix with  $\Sigma = (\rho_{ij}s), \rho_{ii} = 1$ , and,  $\rho_{ij} \in (-1, 1)$  for  $i \neq j$ .
  - for each t > 0 and s > 0,  $\mathbf{W}_{t+s} \mathbf{W}_t \perp \mathbf{W}_t$ .

Instead of using the above definition, authors use the following compact notation to define an n-dimensional standard correlated Wiener Process: If a sequence of standard Wiener processes  $\{W_t^{(i)} : t \ge 0\}, i = 1, 2, ..., n$  satisfies the condition  $(dW_t^{(i)})(dW_t^{(j)}) = \rho_{ij}dt$  with  $\rho_{ii} = 1$ , and,  $\rho_{ij} \in (-1, 1)$  for  $i \ne j$ , then the vector  $\mathbf{W}_t = (W_t^{(1)}, W_t^{(2)}, ..., W_t^{(n)})^T$  is an n-dimensional standard correlated Wiener Process.

2. In a few of places authors state that "sum of two normal distributions is also a normal distribution" or "a linear combination of normal variates is also normal". However, this result is true only if the joint distribution is bivariate normal or multivariate normal. Candidates may assume that for cases considered in the text multivariate normality is valid.

## Errata List for Problems and Solutions in Mathematical Finance, Vol 1: Stochastic Calculus

# Problem 1.2.1.7

" $\emptyset \cap \Omega = \emptyset$  and  $\emptyset \cup \Omega = \Omega$ " should be

 $\mathbb{P}(\emptyset \cup \Omega) = \mathbb{P}(\Omega) = 1 \text{ and } \mathbb{P}(\emptyset \cap \Omega) = 0$ 

# Problem 2.2.1.7

For part (b),  $B_{t+s} - B_t = W_{\tilde{t}+s} - W_{\tilde{t}+u}$  should be  $B_{t+s} - B_t = W_{\tilde{t}+s} - W_{\tilde{t}}$ 

# Problem 2.2.3.3

Under Solution,  $e^{X_t} \sim \log -N\left(-\frac{1}{2}\lambda^2 t, \lambda^2 t\right)$  should be  $X_t \sim \log -N\left(-\frac{1}{2}\lambda^2 t, \lambda^2 t\right)$ 

# Problem 2.2.3.4

 $\mathbb{E}(W_t^4) = 3t$  should be  $\mathbb{E}(W_t^4) = 3t^2$ 

## Problem 2.2.2.7

$$\mathbb{E}\left[f\left(\frac{S_t}{S_u}\cdot x\right)\right] = \int_{-\infty}^{\infty} f(w\cdot x) \frac{1}{\sigma w\sqrt{2\pi(t-u)}} e^{-\frac{1}{2}\left[\frac{(\log w-v(t-u))^2}{\sigma^2(t-u)}\right]} dw$$

Setting  $\tau = t - u$  and  $y = w \cdot x$ 

$$\mathbb{E}\left[f\left(\frac{S_t}{S_u}\cdot S_u\right)\right] = \int_{-\infty}^{\infty} f(y) \ \frac{1}{\sigma y\sqrt{2\pi\tau}} e^{-\frac{1}{2}\left[\frac{\left(\log\left(\frac{y}{x}\right)-v\tau\right)^2}{\sigma^2\tau}\right]} dy$$

#### Problem 3.2.1.4

$$f\left(W_{t_{i}}+\frac{W_{t_{i+1}}-W_{t_{i+1}}}{2},t_{i}\right)$$

should be

$$f\left(W_{t_i} + \frac{W_{t_{i+1}} - W_{t_i}}{2}, t_i\right)$$

#### Problem 3.2.2.11

 $Z_t = e^{\kappa Y_t}$  should be  $Z_t = e^{kt}Y_t$ 

Incorrect expression for  $Var(X_T|X_t = x)$ . It should be

$$\operatorname{Var}(X_T|X_t = x) = \left[ \exp\left\{ \frac{\sigma^2}{2\kappa} \left( 1 - e^{-2\kappa(T-t)} \right) \right\} - 1 \right] \times \dots$$

#### Problem 3.2.2.13

$$\mathbb{E}\left[\left(\int_0^t \frac{1}{1-s} \, dW_s\right)^2\right] = \dots = \frac{t}{1-t}$$

 $\operatorname{Var}(X_t | X_0 = x) = \frac{1}{1-t}$  should be  $\operatorname{Var}(X_t | X_0 = x) = t(1-t)$ .

# Problem 3.2.2.18

$$\int_t^T \frac{dX_u}{X_u^{\beta}} = \dots = \frac{1}{1-\beta} \cdot \left(X_T^{1-\beta} - X_t^{1-\beta}\right)$$

so that

$$X_{T} = \left[ X_{t}^{1-\beta} + \alpha(1-\beta) \int_{t}^{T} e^{-r(1-\beta)u} \, dW_{u} \right]^{1-\beta}.$$

#### Problem 3.2.2.19

Definition of  $G_t$  should be  $G_t = \exp\left(\frac{1}{t}\int_0^t \log S_u du\right)$ 

# Problems 4.2.3.1, 4.2.3.2

$$dW_t = d\widetilde{W}_t + \lambda_t dt$$
 should be  $dW_t = d\widetilde{W}_t - \lambda_t dt$ .

 $Z_s = e^{-\int_0^s \lambda_u du - \int_0^s \lambda_u^2 dW_u} \text{ should be } Z_s = e^{-\int_0^s \lambda_u dW_u - \frac{1}{2}\int_0^s \lambda_u^2 du}$ 

## **Problem 4.2.3.3**

 $Z_s = e^{-\int_0^s \lambda_u du - \int_0^s \lambda_u^2 dW_u} \text{ should be } Z_s = e^{-\int_0^s \lambda_u dW_u - \frac{1}{2}\int_0^s \lambda_u^2 du}$ 

# Problem 4.2.3.5

For part (b),  $\psi_t$  has a missing *n* from first term and  $\frac{1}{2}$  from second term.

# Problems 4.2.3.6, 4.2.3.8

 $dW_t = d\widetilde{W}_t + \lambda_t dt$  should be  $dW_t = d\widetilde{W}_t - \lambda_t dt$ .

# Problem 4.2.3.7

The original Arithmetic Brownian motion SDE should be  $dS_t = (\mu_t - D_t S_t)dt + \sigma dW_t$ .

... the trader will receive  $D_t S_t dt$  for every stock held...

 $dW_t = d\widetilde{W_t} + \lambda_t dt$  should be  $dW_t = d\widetilde{W_t} - \lambda_t dt$ .

# Problem 4.2.3.8

 $dW_t = d\widetilde{W_t} + \lambda_t dt$  should be  $dW_t = d\widetilde{W_t} - \lambda_t dt$ .

# Problems 4.2.3.15 and 4.2.3.16

 $dB_t^f = r_t^f B_t^f$  should be  $dB_t^f = r_t^f B_t^f dt$ .

# Problem 5.2.1.9

"and hence  $\mathbb{E}(|\widehat{N_t}|) \leq \mathbb{E}(N_t) + \lambda t = 2\lambda t < \infty$ ".