Process to Determine if a VBT Update is Required


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## Process to Determine if a VBT Update is Required

## Section 1: Introduction

The VBT Analysis Subgroup ("Subgroup") of the Joint Academy Life Experience Committee and SOA Preferred Mortality Oversight Group ("Joint Committee") recommends the approach documented in this report for determining when there is sufficient change in the emerging actual mortality experience relative to the current VBT to warrant changes to the table.

In VM-20, Section 9.C, companies are required to use an Industry Mortality Table for determining their prudent best estimate mortality. The industry mortality table is currently based on the 2015 VBT and corresponding RR Tables, advanced forward to the valuation date using prescribed mortality improvement factors published by the Society of Actuaries (SOA) and the American Academy of Actuaries (AAA). The experience data underlying the 2015 VBT is from 2002-2009 and was collected on a voluntary basis for the SOA for individual life experience studies. With the introduction of PBR, there has been a significant increase in the number of contributing companies and amount of exposures and claims via the mandatory data collection within VM-51. This mandatory collection of experience data will continue to occur on an annual basis. This cadence will also provide the ability to review experience data every year or couple of years to monitor changes in the emerging experience.

The approach described in the report defines a set of metrics to be monitored as additional experience becomes available. It also defines a "threshold" that, if exceeded, would set into action a process to complete a deeper analysis to determine whether areas of the 2015 VBT need to be refreshed and, if so, to complete updates to the VBT to bring the metrics back within the threshold. Finally, it describes the software tools that can be used to complete the recommended process.

## Section 2: New Calculated Fields

New calculated fields were added to the individual life experience data to enable the calculation of the variance and skewness of $A / E s$. These fields were added by the data vendor because these calculations had to be added at the policy level in order to enable the appropriate aggregation of the data at higher levels and only the data vendor has access to the policy level data. They were added in addition to the expected deaths or $\sum_{i=1}^{n} f q_{i}^{s}$ (by count) and the expected claims or $\sum_{i=1}^{n} f b_{i} q_{i}^{s}$ (by amount). They are calculated for each of two policy year segments (one before the policy anniversary and one after the policy anniversary) in an observed calendar year. In one policy, the $f$ for the segment after the policy anniversary in year $y$ plus the $f$ for the segment before the policy anniversary in year $y+1$ will equal one, unless the policy terminates in the policy year for a reason other than death.

The new fields added to the database are shown in Table 1 below.

Table 1
NEW CALCULATED FIELDS

| Variable Name | Description | Formula |
| :---: | :---: | :---: |
| ExpDeathQx2015VBTwMI_byPol | Expected Deaths - Count - w/MI | $\sum_{i=1}^{n} f\left(\left(q_{i}^{S}\right)\left(1-M I_{x}\right)^{(y-2015)}\right)$ |
| ExpDeathQx2015VBTwMI_byAmt | Expected Deaths - Amount - w/MI | $\sum_{i=1}^{n} f b_{i}\left(\left(q_{i}^{s}\right)\left(1-M I_{x}\right)^{(y-2015)}\right)$ |
| 2CenMomP1wMI_byAmt | 2nd Central Moment - Amount - w/MI - Part 1 | $\sum_{i=1}^{n} f b_{i}^{2}\left(\left(q_{i}^{S}\right)\left(1-M I_{x}\right)^{(y-2015)}\right)$ |
| 2CenMomP2wMI_byAmt | 2nd Central Moment - Amount - w/MI - Part 2 | $\sum_{i=1}^{n} f b_{i}^{2}\left(\left(q_{i}^{S}\right)\left(1-M I_{x}\right)^{(y-2015)}\right)^{2}$ |
| 3CenMomP1wMI_byAmt | 3rd Central Moment - Amount - w/MI - Part 1 | $\sum_{i=1}^{n} f b_{i}^{3}\left(\left(q_{i}^{S}\right)\left(1-M I_{x}\right)^{(y-2015)}\right)$ |
| 3CenMomP2wMI_byAmt | 3rd Central Moment - Amount - w/MI - Part 2 | $\sum_{i=1}^{n-1} f b_{i}^{3}\left(\left(q_{i}^{S}\right)\left(1-M I_{x}\right)^{(y-2015)}\right)^{2}$ |
| 3CenMomP3wMI_byAmt | 3rd Central Moment - Amount - w/MI - Part 3 | $\sum_{i=1}^{n} f b_{i}^{3}\left(\left(q_{i}^{S}\right)\left(1-M I_{x}\right)^{(y-2015)}\right)^{3}$ |
| 2CenMomP2wMI_byPol | 2nd Central Moment - Count - w/MI - Part 2 | $\sum_{i=1}^{n} f\left(\left(q_{i}^{s}\right)\left(1-M I_{x}\right)^{(y-2015)}\right)^{2}$ |
| 3CenMomP3wMI_byPol | 3rd Central Moment - Count - w/MI - Part 3 | $\sum_{i=1}^{n-1} f\left(\left(q_{i}^{S}\right)\left(1-M I_{x}\right)^{(y-2015)}\right)^{3}$ |
| 2CenMomP1_byAmt | 2nd Central Moment - Amount - Part 1 | $\sum_{i=1}^{n} f b_{i}^{2}\left(q_{i}^{S}\right)$ |
| 2CenMomP2_byAmt | 2nd Central Moment - Amount - Part 2 | $\sum_{i=1}^{n} f b_{i}^{2}\left(q_{i}^{s}\right)^{2}$ |
| 3CenMomP1_byAmt | 3rd Central Moment - Amount - Part 1 | $\sum_{i=1}^{n} f b_{i}^{3}\left(q_{i}^{s}\right)$ |
| 3CenMomP2_byAmt | 3rd Central Moment - Amount - Part 2 | $\sum_{i=1}^{n} f b_{i}^{3}\left(q_{i}^{s}\right)^{2}$ |
| 3CenMomP3_byAmt | 3rd Central Moment - Amount - Part 3 | $\sum_{i=1}^{n} f b_{i}^{3}\left(q_{i}^{s}\right)^{3}$ |
| 2CenMomP2_byPol | 2nd Central Moment - Count - Part 2 | $\sum_{i=1}^{n} f\left(q_{i}^{S}\right)^{2}$ |
| 3CenMomP3_byPol | 3rd Central Moment - Count - Part 3 | $\sum_{i=1}^{n} f\left(q_{i}^{s}\right)^{3}$ |

Where,
$n$ : the number of records ( $i$ ). For each policy in a calendar year, there will be one or two records, one each for the periods before and after the anniversary date, as necessary.
$x$ : age of the policyholder in policy year
$y$ : calendar year of observation
$f$ : fraction of the calendar year for which the life was credited in the period covered by the record.
$q_{i}^{s}$ : the mortality rate from the standard table (2015 VBT)
$b_{i}$ : the amount insured for life $i$
MI: 2019 Mortality Improvement scale for AG38 and VM20, published by the SOA and AAA.
A comment regarding the new fields:
The variables in Table 1 above have been developed to be used in formulas to calculate confidence intervals and levels of credibility such as the ones in the paper "Credibility Theory Practices" (https://www.soa.org/globalassets/assets/files/research/projects/research-cred-theory-pract.pdf).

Some readers may think that there is an error in the formulas for second order and third order variables. For example, the variable 2CenMomP2_byAmt (2nd Central Moment - Amount - Part 2) has a formula of $\sum_{i=1}^{n} f_{i} b_{i}^{2}\left(q_{i}^{S}\right)^{2}$. This variable is used to calculate the part of the following Klugman formula after the subtraction sign.
$\sigma_{d}^{2}=\frac{\sum_{i=1}^{n} b_{i}^{2} f_{i} q_{i}\left(1-f_{i} q_{i}^{S}\right)}{E_{d}^{2}}$
In the Klugman formula, $q_{i}$ is the true mortality rate and equal to $\widehat{m} q_{i}^{S}$ and $q_{i}^{S}$ is the same as defined in Table 1.

The reader may question why the variable $f_{i}$ is squared in the Klugman formula but the $f$ in Table 1 is not. The reason is as follows. In the Klugman formula, it is assumed that the data being summarized contains one record per policy per policy year. In general, $f_{i}$ will have a value of 1 unless the policy lapses before the end of the policy year, in which case the expected mortality rate would be the product of $f_{i}$ and $q_{i}^{s}$.

The data that we are using was collected on a calendar year basis. Each record was then split into beforeanniversary and after-anniversary sub-records so that the correct duration could be assigned to each. If a policy is in force for a full policy year, there will be records in two consecutive calendar years with fractional periods that sum to 1 . This will result in the same value as the Klugman formula in which $f_{i}$ squared would have a value of 1 .

Our formulas will produce a different result than the Klugman formula for policies that lapse before the end of the policy year. In that case, under the Klugman formula, the value of $q_{i}^{S}$ squared would be multiplied by $f_{i}$ squared whereas, under our formula, the value of $q_{i}^{s}$ squared would be multiplied by $f$.

To apply the Klugman formula as defined in our data, we would need to combine each policy's afteranniversary record for a calendar year with its before-anniversary record from the subsequent year. It has been determined that this cannot be accomplished on a practical basis.

## Section 3: Metrics

Several metrics will be utilized to determine whether the 2015 VBT should be updated and to enable the Joint Committee to assess which areas of the 2015 VBT to potentially update. These metrics are actual-to-expected ratios $(A / E)$, confidence intervals (CI) of the $A / E$, and the absolute value of actual values minus expected values. The expected basis ("E") in the A/Es will be the 2015 VBT with and without a mortality improvement adjustment. The mortality improvement adjustment will 'improve' the expected VBT mortality to the observation year of the actual experience data. These two versions of $\mathrm{A} / \mathrm{Es}$, with and without improvement, will be useful in determining the extent to which mortality improvement may be a factor resulting in a breach of the threshold. The actual and expected values will be based on amount as opposed to count because the 2015 VBT was developed using amountbased mortality. A $95 \% \mathrm{Cl}$ will be assumed, but the percentage may be adjusted if more or less precision is warranted.

The Subgroup recommends the Cls have a skewness component in order to adequately address the appropriateness of the 2015 VBT, especially at older ages. However, hardware limitations (see Section 6) will defer implementation of Cl s with skewness until a later point in time. Until the limitations are overcome, the Cls implemented in this VBT analysis process will assume a normal distribution. If skewness were to be taken into account, the Cls would be shifted to the right if skewness was positive or to the left if negative. In our analysis, we calculated the confidence intervals using both the Standard Normal distribution and the Translated Gamma Distribution. Only one of the 102 cells, which has an A/E close to the 97.5 th percentile, moved from being outside the Standard Normal confidence interval to being inside the Gamma confidence interval. However, this could be a greater issue if smaller cells were to be analyzed in the course of determining the efficacy of the 2015 VBT.

### 3.1 CONFIDENCE INTERVALS

The Cl formula, which assumes a normal distribution, requires the mean and standard deviation of the metric that is being tested as input. The upper and lower bounds of the Cl interval are $u \pm z \sigma$, where $u$ is the mean, $\sigma$ is the standard deviation, and $z$ is the value from the Standard Normal distribution for the selected confidence level (e.g., for a $95 \%$ confidence level, $z=1.96)$. For the analysis of the $2015 \mathrm{VBT}, u=m=A / E$. The standard deviation of $A / E(\sigma)$ is defined in Appendix A.

To take skewness into account, we developed confidence intervals using a Translated Gamma Distribution ${ }^{1}$ to approximate the distribution of aggregate claims, $S$, then substitute $A / E$ for $S$.

Confidence Interval for $S:\left[G^{-1}(0.025\right.$, Alpha, Beta $)+X_{0}, G^{-1}(0.975$, Alpha, Beta $\left.)+X_{0}\right]$
Where:

- $G^{-1}$ is the inverse of the Gamma Distribution function. Note: if Microsoft Excel's Gamma.Inv function is used, replace Beta with 1/Beta
- $X_{0}=E[S]-2 \times[\operatorname{Var}(S)]^{2} \div\left(E\left[(S-E[S])^{3}\right]\right)$
- Alpha $=4 \times[\operatorname{Var}(S)]^{3} \div E\left[(S-E[S])^{3}\right]^{2}$
- $\operatorname{Beta}=2 \times[\operatorname{Var}(S)] \div E\left[(S-E[S])^{3}\right]$

[^1]Using the available fields in the individual life experience data and the formulas in Appendices A and B, calculate the following input parameters for the inverse Gamma function.

- Mean: $E[S]=m=\left(\sum_{i=1}^{n}\right.$ Death Claim Amount $) \div\left(\sum_{i=1}^{n}\right.$ ExpDeathQx2015VBTwMI_byAmt $)$
- Variance: $\operatorname{Var}(S)=\frac{m}{E^{2}} \times 2$ CenMomP1wMI $I_{\text {byAMT }}-\frac{m^{2}}{E^{2}} \times 2$ CenMomP $2 w$ MIbyAmt
- Skewness: $\gamma_{i}=E\left[(S-E[S])^{3}\right]=\frac{m}{E^{3}} \times 3$ CenMomP1wMIbyAmt $-3 \times \frac{m^{2}}{E^{3}} \times$

3CenMomP 2 wMIbyAmt $+\frac{m^{3}}{E[S]^{3}} \times 3$ CenMomP3wMIbyAmt

- Where the expected claims equal $E=\left(\sum_{i=1}^{n}\right.$ ExpDeathQx2015VBTwMI_byAmt $)$

If the upper and lower Cl boundaries, as defined above, straddle the $100 \%$ point for any segment of the experience, the experience in that segment is considered to be consistent with the 2015 VBT . If the upper Cl boundary is below the $100 \%$ point, the $A / E$ for that segment is low, and the 2015 VBT is too high for that segment. If the lower Cl boundary is above the $100 \%$ point, the $A / E$ for that segment is high, and the 2015 VBT is too low for that segment.

Three other distributions, which include skewness, were considered: the transformed (not Translated) Gamma, Burr, and generalized Pareto. These distributions are all anchored at zero and spread probability from zero to infinity, which makes it hard to fit a distribution that is tightly spread around its mean and is also skewed. When these three distributions were applied, none of them appeared to fit as well as the Translated Gamma.

For the Gamma Distribution, the larger the alpha parameter, the more symmetric the distribution is. This is because the skewness coefficient is $2 /$ alpha^2 and so goes to zero as the alpha parameter increases. Many of the cells in our analysis had large values of alpha; for these cells, the confidence intervals calculated based on the Standard Normal and Translated Gamma Distribution are very similar. However, it should be noted that this is less likely to be true when analyzing smaller subsets of the data.

## Section 4: Determining Credible Data Cells

To test whether a threshold for adjusting the 2015 VBT or creating a new VBT is breached, the experience study data is grouped into fully credible sized cells using the following method.

1. The experience data is initially summarized by gender, smoker status, 10-year attained age bands and duration.
2. The data groups from \#1 are combined to form fully credible cells. Data groups with consecutive durations within a gender/smoker status/age-band cohort are accumulated until full credibility is attained, as measured by a Z factor of 1 , as calculated thusly:

- The mean $(u=m=A / E)$ and the variance $\left(\sigma^{2}\right)$ of the $A / E$ as described in Section 2 and the standard deviation, $=\sqrt{\sigma^{2}}$, are determined for each group in \#1.
- A limited fluctuation credibility $Z$ factor ${ }^{2}$, assuming a normal distribution, is determined for each group in \#1 as:
o $Z=\min \left\{1, \frac{r m}{z \sigma}\right\}$, where
o $\quad r$ is the error term $=0.05$
$0 \quad z$ is 1.96 (based on a $95 \%$ probability)
o $\quad m$ and $\sigma$ are defined in Section 2.

In a few instances, the data for all durations within a gender/smoker status/age band were not fully credible.

[^2]
## Section 5: Trigger for Action

One possible trigger for action could be as follows. Confidence intervals for each cell from Section 3 are calculated and used to determine if the 'threshold' for further action on the 2015 VBT has been met. The $95 \% \mathrm{Cl}$ boundaries are determined around the A/Es for each cell. The cells where $100 \%$ falls within their $95 \% \mathrm{Cl}$ boundaries pass the Cl test. If the number of cells that pass the Cl test is greater than $95 \%$ of the total number of cells tested, no further action is needed. If the number of cells that pass the Cl test is less than $95 \%$ of the total number of cells tested, the 'threshold' has been breached and the process to perform corrective actions on the 2015 VBT is initiated.
Alternatively, the threshold for action could be based on a metric other than the count of cells; for example, the threshold could be based on whether the total exposed face amount (or expected death claims or actual death claims) of the cells that pass the Cl test exceeds $95 \%$ of the total. Also, even if the count of cells (or other chosen metric) exceeds the $95 \%$ threshold, the cells that failed the Cl test should be examined to ensure that there are no clusters, which may indicate a significant weakness in some areas of the table.

## Section 6: VBT Corrective Actions

If the Cl tests performed, as described in Section 4, indicate corrective action on the 2015 VBT is needed, the Joint Committee could begin a process to determine what areas of the VBT may need to be adjusted. Potential adjustments on the 2015 VBT may include slope adjustments using multiplicative factors and adjustments to mortality improvement factors. These adjustments may be made to a limited set of subsections of the table that warrant an update. The Joint Committee may use $|A-E|$ values to prioritize areas of the 2015 VBT to update. If too many partitions have failed the Cl tests in Section 4, the Joint Committee may decide to create a completely new table in place of making adjustments to the existing 2015 VBT. The Joint Committee may alternatively decide to hold off on making any adjustments to the 2015 VBT if the number of cells that failed the Cl test was small and degree of failure was minimal. Other considerations, such as potential industry disruption, should be taken into account before making any adjustments to the 2015 VBT.

## Section 7: Software Tools

### 7.1 EXCEL WORKBOOK FOR DETERMINING DATA CELLS

An Excel workbook has been developed to create the data cells described in Section 3. Documentation on the functionality of the workbook is contained in the workbook. The workbook is currently set up with the 2009-2017 experience data. The workbook will need to be updated each time a review of the need to update the VBT takes place.

The analysis performed in the Excel workbook primarily uses Cls without a skewness adjustment based on the Standard Normal distribution. It also includes Cls with a skewness adjustment based on a Translated Gamma Distribution.

When we recalculated the confidence intervals using the Translated Gamma Distribution, only one cell - barely changed "position" from the "In Cl" group to the "Below" group. Under Standard Normal, the confidence interval for this cell was [100.0\%, 109.6\%], which brackets $100 \%$; under Gamma, the CI was [100.2\%,109.9\%].

The 102 cells identified by this process are listed in Appendix C.

### 7.2 TABLEAU WORKBOOK FOR VBT ANALYSIS

Several Tableau views have been developed to help identify the subsections of the 2015 VBT that may need to be adjusted as described in Section 5 . These views include the capability to see the Cl around various segments of the data. The workbook currently contains the 116 cells developed in the Excel workbook, described in Section 7.1, from the 2009-2017 data. These 116 data cells and their Cls can be viewed in the published Tableau views.

As of this writing, the published Tableau views show Cls without a skewness adjustment. To implement Cls with a skewness adjustment, an R subroutine needs to be called from Tableau and this capability and the skewness adjustment will be added by the SOA at a later date.

## Section 8: Initial Findings

Our analysis was principally concerned with mortality experience based on face amount with mortality improvement assumed. The data provided by MIB included 33.8 million records. We excluded records with any of the following characteristics:

- PLT indicator equals " $Y$ "
- Duration greater than 36
- Smoker Status equals "Unknown"
- Issue Age less than 18

After these exclusions, the number of remaining records was 31.8 million. These records had a mortality ratio of $90.7 \%$ based on face amount and $112.2 \%$ based on policy count. As discussed in Section 4, we summarized the records into 116 cells. Of the 116 cells, 104 were fully credible. We calculated mortality ratios and confidence intervals - assuming a Standard Normal distribution - for each of the cells. The confidence intervals were centered around the actual to expected ratio. All of the calculations were based on amount, not count.


- The 104 fully credible cells accounted for $96.7 \%$ of the expected death claims and had an A/E of 90.4\%.
- Thirty-two of the cells had confidence intervals that included $100 \%$. Those cells contained $20 \%$ of the expected death claims and had an A/E of $100 \%$. They are shown as green blocks in the above chart.
- For 42 of the cells, $100 \%$ were above the upper limit of their confidence interval, indicating the tabular mortality rates were too high. Those cells contained $65 \%$ of the expected death claims and had an A/E of $85 \%$. They are shown as yellow blocks.
- For 30 of the cells, $100 \%$ were below the lower limit of their confidence interval, indicating the tabular mortality rates were too low. Those cells contained $12 \%$ of the expected death claims and had an $A / E$ of $109 \%$. They are shown as red blocks.
- The 12 cells that were not full credible accounted for $3 \%$ of the expected death claims and had an A/E of $102 \%$. Of those cells, $100 \%$ were above the upper limit of the confidence interval for nine cells, $100 \%$ were below the lower limit of the confidence interval for two cells, and $100 \%$ were within the confidence interval for one cell. The pattern of the cells where the table was too high or too low was generally consistent with the pattern found in the fully credible cells. They are shown as grey blocks.

Additional detail about the cells can be found in Appendix C. One of the notable findings was that cells with low $A / E$ values occurred in the early durations, whereas cells with high A/E values occurred in the later durations. This indicates that the mortality rates of the current table are too high in the early durations and too low in the later durations. We also noted that both male and female insureds have low $A / E$ ratios of $91 \%$. However, smokers of both genders have $A / E$ ratios of $102 \%$, and non-smokers of both genders have $A / E$ ratios of $90 \%$.

| Smoker Status | Gender | Actual Number | Exposed Face | Actual Face | Expected Face w MI | A/E Face w MI |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NonSmoker | Female | 508,689 | 30,269,870,244,043 | 61,801,047,713 | 69,016,454,689 | 89.5\% |
| Smoker | Female | 147,483 | 1,041,696,120,501 | 6,995,458,975 | 6,863,158,975 | 101.9\% |
| NonSmoker | Male | 737,501 | 58,485,607,547,753 | 130,576,882,205 | 145,683,514,706 | 89.6\% |
| Smoker | Male | 196,748 | 2,470,774,043,884 | 14,897,472,200 | 14,611,681,156 | 102.0\% |
|  |  |  |  |  |  |  |
| NonSmoker | All | 1,246,190 | 88,755,477,791,796 | 192,377,929,918 | 214,699,969,395 | 89.6\% |
| Smoker | All | 344,231 | 3,512,470,164,385 | 21,892,931,175 | 21,474,840,131 | 101.9\% |
|  |  |  |  |  |  |  |
| All | Female | 656,172 | 31,311,566,364,544 | 68,796,506,688 | 75,879,613,664 | 90.7\% |
| All | Male | 934,249 | 60,956,381,591,637 | 145,474,354,405 | 160,295,195,862 | 90.8\% |
|  |  |  |  |  |  |  |
| All | All | 1,590,421 | 92,267,947,956,181 | 214,270,861,093 | 236,174,809,526 | 90.7\% |

The MS-Excel workbook includes summaries by face amount band and other criteria.

## Appendix A: Derivation of Variance of A/E

Assume:
$n$ : the number of policies
$q_{i}$ : the true mortality rate for life $i$
$q_{i}^{S}$ : the mortality rate from the standard table (2015 VBT)
$b_{i}$ : the amount insured for life $i$
$d_{i}$ : is 0 if the life doesn't die, is 1 if it does
$A=\sum_{i=1}^{n} b_{i} d_{i}$ is the actual claims
$E=\sum_{i=1}^{n} b_{i} q_{i}^{s}$ is the expected claims
$m=\frac{A}{E}=\frac{\sum_{i=1}^{n} b_{i} d_{i}}{E}$ is also known as the mortality ratio
$q_{i} \approx m q_{i}^{S}$ is an estimate of the true mortality since the true mortality is unknown
For each life, $u=\frac{b_{i} m q_{i}^{S}}{E}$ and $X=\frac{b_{i} d_{i}}{E}$ and

$$
\begin{aligned}
& \operatorname{Var}(m)=\sigma_{i}^{2}=E\left[(X-u)^{2}\right] \\
& \sigma_{i}^{2}=\left[\left(\frac{b_{i}}{E}-\frac{b_{i} m q_{i}^{s}}{E}\right)^{2} \times m q_{i}^{s}\right]+\left[\left(0-\frac{b_{i} m q_{i}^{S}}{E}\right)^{2} \times\left(1-m q_{i}^{S}\right)\right] \\
& \sigma_{i}^{2}=\left(\frac{b_{i}}{E}\right)^{2} m q_{i}^{S}-2\left(\frac{b_{i}}{E}\right)\left(\frac{b_{i} m q_{i}^{S}}{E}\right) m q_{i}^{s}+\left(\frac{b_{i} m q_{i}^{s}}{E}\right)^{2} m q_{i}^{S}+\left(\frac{b_{i} m q_{i}^{S}}{E}\right)^{2}-\left(\frac{b_{i} m q_{i}^{s}}{E}\right)^{2} m q_{i}^{s} \\
& \sigma_{i}^{2}=\left(\frac{b_{i}}{E}\right)^{2} m q_{i}^{S}-2\left(\frac{b_{i}}{E}\right)\left(\frac{b_{i} m q_{i}^{S}}{E}\right) m q_{i}^{S}+\left(\frac{b_{i} m q_{i}^{s}}{E}\right)^{2} \\
& \sigma_{i}^{2}=\left(\frac{b_{i}}{E}\right)^{2} m q_{i}^{S}-2\left(\frac{b_{i}}{E}\right)^{2}\left(m q_{i}^{s}\right)^{2}+\left(\frac{b_{i} m q_{i}^{s}}{E}\right)^{2} \\
& \sigma_{i}^{2}=\left(\frac{b_{i}}{E}\right)^{2} m q_{i}^{S}-\left(\frac{b_{i}}{E}\right)^{2}\left(m q_{i}^{S}\right)^{2} \\
& \sigma_{i}^{2}=\left(\frac{b_{i}}{E}\right)^{2}\left[m q_{i}^{S}\left(1-m q_{i}^{S}\right)\right]
\end{aligned}
$$

For all lives,

$$
\begin{aligned}
& \sigma^{2}=\frac{m}{E^{2}} \sum_{i=1}^{n} b_{i}^{2} q_{i}^{s}-\frac{m^{2}}{E^{2}} \sum_{i=1}^{n} b_{i}^{2}\left(q_{i}^{S}\right)^{2} \\
& \sigma=\sqrt{\sigma}=\sqrt{\operatorname{Var}(m)}
\end{aligned}
$$

## Appendix B: Derivation of Skewness of $A / E$

Assume:
$n$ : the number of policies
$q_{i}$ : the true mortality rate for life $i$
$q_{i}^{S}$ : the mortality rate from the standard table (2015 VBT)
$b_{i}$ : the amount insured for life $i$
$d_{i}$ : is 0 if the life doesn't die, is 1 if it does
$A=\sum_{i=1}^{n} b_{i} d_{i}$ is the actual claims
$E=\sum_{i=1}^{n} b_{i} q_{i}^{s}$ is the expected claims
$m=\frac{A}{E}=\frac{\sum_{i=1}^{n} b_{i} d_{i}}{E}$ is also known as the mortality ratio
$q_{i} \approx m q_{i}^{S}$ is an estimate of the true mortality since the true mortality is unknown
For each life, $u=\frac{b_{i} m q_{i}^{S}}{E}$ and $X=\frac{b_{i} d_{i}}{E}$ and

$$
\begin{aligned}
& \gamma_{i}=\operatorname{Skewness}(m)=E\left[(X-u)^{3}\right] \\
& \gamma_{i}=\left[\left(\frac{b_{i}}{E}-\frac{b_{i} m q_{i}^{S}}{E}\right)^{3} \times m q_{i}^{S}\right]+\left[\left(0-\frac{b_{i} m q_{i}^{S}}{E}\right)^{3} \times\left(1-m q_{i}^{S}\right)\right] \\
& \gamma_{i}=\left(\frac{b_{i}}{E}\right)^{3}\left(1-m q_{i}^{S}\right)^{3} \times m q_{i}^{S}+\left(\frac{b_{i}}{E}\right)^{3}\left(-m q_{i}^{S}\right)^{3} \times\left(1-m q_{i}^{S}\right) \\
& \gamma_{i}=\left(\frac{b_{i}}{E}\right)^{3}\left(1-m q_{i}^{S}\right) \times m q_{i}^{S} \times\left[\left(1-m q_{i}^{S}\right)^{2}-\left(m q_{i}^{S}\right)^{2}\right] \\
& \gamma_{i}=\left(\frac{b_{i}}{E}\right)^{3}\left(1-m q_{i}^{S}\right) \times m q_{i}^{S} \times\left[1-2 m q_{i}^{S}+\left(m q_{i}^{S}\right)^{2}-\left(m q_{i}^{S}\right)^{2}\right] \\
& \gamma_{i}=\left(\frac{b_{i}}{E}\right)^{3}\left(1-m q_{i}^{s}\right) \times m q_{i}^{s} \times\left[1-2 m q_{i}^{s}\right] \\
& \gamma_{i}=\left(\frac{b_{i}}{E}\right)^{3}\left[m q_{i}^{s}-\left(m q_{i}^{S}\right)^{2}-2\left(m q_{i}^{S}\right)^{2}+\left(2 m q_{i}^{S}\right)^{3}\right] \\
& \gamma_{i}=\left(\frac{b_{i}}{E}\right)^{3}\left[m q_{i}^{s}-3\left(m q_{i}^{s}\right)^{2}+\left(2 m q_{i}^{s}\right)^{3}\right]
\end{aligned}
$$

For all lives,

$$
\gamma=\frac{m}{E^{3}} \sum_{i=1}^{n} b_{i}^{3} q_{i}^{S}-3 \frac{m^{2}}{E^{3}} \sum_{i=1}^{n} b_{i}^{3}\left(q_{i}^{S}\right)^{2}+2 \frac{m^{3}}{E^{3}} \sum_{i=1}^{n} b_{i}^{3}\left(q_{i}^{S}\right)^{3}
$$

## Appendix C: Credible Data Cells

This section lists the 102 data cells identified by the MS-Excel workbook. In this table, green rows had confidence intervals that included $100 \%$. For yellow rows, $100 \%$ were above the upper limit of their confidence interval, indicating the tabular mortality rates were too high. For red rows, $100 \%$ were below the lower limit of their confidence interval, indicating the tabular mortality rates were too low. Blue rows were not fully credible.

| Cell ${ }^{\text {c\| }}$ | Sex/ <br> Tobac. | Attained Age |  | Actual Number | Actual Face | Expected Face with MI | Cred Factor Before Capping | Std.Dev. | A/E Face w/MI | \# of std <br> devs <br> from <br> 100\% | Standard Normal Position | Std. Norm. <br> Confidence Interval |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | FNS | 18-29 | 1-12 | 1,281 | 184,275,463 | 212,227,145 | 42\% | 0.053 | 86.8\% | (2.5) | Not Cred. | 81.8\%-91.8\% |
| 2 | FNS | 30-39 | 1-22 | 6,313 | 1,343,665,084 | 1,791,732,137 | 103\% | 0.019 | 75.0\% | (13.4) | Above | 73.2\% - 76.8\% |
| 3 | FNS | 40-49 | 1-8 | 7,148 | 2,099,216,584 | 2,639,434,607 | 104\% | 0.019 | 79.5\% | (10.5) | Above | 77.7\% - 81.4\% |
| 4 | FNS | 40-49 | 9-13 | 5,671 | 1,466,871,706 | 1,955,732,988 | 110\% | 0.017 | 75.0\% | (14.3) | Above | 73.3\%-76.7\% |
| 5 | FNS | 40-49 | 14-19 | 5,355 | 773,981,929 | 929,738,050 | 101\% | 0.021 | 83.2\% | (7.9) | Above | 81.2\% - 85.3\% |
| 6 | FNS | 40-49 | 20-32 | 4,719 | 305,471,370 | 278,063,088 | 101\% | 0.028 | 109.9\% | 3.6 | Below | 107.2\%-112.5\% |
| 7 | FNS | 50-59 | 1-9 | 13,003 | 2,759,300,489 | 3,516,609,707 | 105\% | 0.019 | 78.5\% | (11.3) | Above | 76.7\%-80.3\% |
| 8 | FNS | 50-59 | 10-13 | 7,806 | 1,627,621,243 | 2,112,231,706 | 103\% | 0.019 | 77.1\% | (12.0) | Above | 75.2\% - 78.9\% |
| 9 | FNS | 50-59 | 14-17 | 7,877 | 1,269,868,861 | 1,615,220,318 | 108\% | 0.019 | 78.6\% | (11.5) | Above | 76.8\% - 80.4\% |
| 10 | FNS | 50-59 | 18-21 | 7,669 | 789,738,410 | 871,068,808 | 108\% | 0.021 | 90.7\% | (4.4) | Above | 88.6\% - 92.7\% |
| 11 | FNS | 50-59 | 22-24 | 5,384 | 428,896,728 | 434,640,716 | 109\% | 0.023 | 98.7\% | (0.6) | lnCl | 96.5\% - 100.9\% |
| 12 | FNS | 50-59 | 25-27 | 5,367 | 366,943,745 | 360,270,385 | 117\% | 0.022 | 101.9\% | 0.8 | In Cl | 99.7\% - 104\% |
| 13 | FNS | 50-59 | 28-36 | 5,789 | 331,779,703 | 313,084,720 | 127\% | 0.021 | 106.0\% | 2.8 | Below | 103.9\% - 108\% |
| 14 | FNS | 60-69 | 1-12 | 22,485 | 3,722,500,377 | 4,511,005,009 | 106\% | 0.020 | 82.5\% | (8.8) | Above | 80.6\% - 84.4\% |
| 15 | FNS | 60-69 | 13-18 | 15,642 | 1,768,503,538 | 2,175,504,186 | 103\% | 0.020 | 81.3\% | (9.3) | Above | 79.4\% - 83.2\% |
| 16 | FNS | 60-69 | 19-22 | 11,437 | 827,144,336 | 826,719,976 | 107\% | 0.024 | 100.1\% | 0.0 | lnCl | 97.8\% - 102.3\% |
| 17 | FNS | 60-69 | 23-25 | 9,889 | 608,321,566 | 585,848,617 | 118\% | 0.022 | 103.8\% | 1.7 | lnCl | 101.7\% - 106\% |
| 18 | FNS | 60-69 | 26-27 | 6,824 | 386,325,167 | 374,550,822 | 107\% | 0.025 | 103.1\% | 1.3 | $\operatorname{lnCl}$ | 100.8\% - 105.5\% |
| 19 | FNS | 60-69 | 28-29 | 5,651 | 313,642,632 | 310,145,114 | 101\% | 0.025 | 101.1\% | 0.4 | lnCl | 98.7\%-103.5\% |
| 20 | FNS | 60-69 | 30-36 | 7,647 | 396,377,687 | 389,554,278 | 133\% | 0.020 | 101.8\% | 0.9 | InCl | 99.9\%-103.6\% |
| 21 | FNS | 70-79 | 1-18 | 40,249 | 5,613,651,700 | 6,218,944,506 | 101\% | 0.023 | 90.3\% | (4.3) | Above | 88.1\% - 92.4\% |
| 22 | FNS | 70-79 | 19-24 | 28,417 | 1,290,349,084 | 1,309,461,804 | 110\% | 0.023 | 98.5\% | (0.6) | lnCl | 96.4\% - 100.7\% |
| 23 | FNS | 70-79 | 25-27 | 16,477 | 644,066,340 | 625,655,383 | 107\% | 0.025 | 102.9\% | 1.2 | In Cl | 100.6\% - 105.3\% |
| 24 | FNS | 70-79 | 28-30 | 12,929 | 520,520,864 | 477,486,305 | 111\% | 0.025 | 109.0\% | 3.6 | Below | 106.6\% - 111.4\% |
| 25 | FNS | 70-79 | 31-36 | 8,067 | 315,929,027 | 295,808,090 | 102\% | 0.027 | 106.8\% | 2.5 | Below | 104.3\% - 109.3\% |
| 26 | FNS | 80-89 | 1-11 | 16,229 | 9,100,103,033 | 10,018,033,288 | 101\% | 0.023 | 90.8\% | (4.0) | Above | 88.7\%-93\% |
| 27 | FNS | 80-89 | 12-20 | 45,553 | 6,453,801,042 | 6,899,735,886 | 100\% | 0.024 | 93.5\% | (2.7) | Above | 91.3\%-95.8\% |
| 28 | FNS | 80-89 | 21-23 | 29,359 | 1,542,427,217 | 1,429,712,780 | 108\% | 0.026 | 107.9\% | 3.1 | Below | 105.5\%-110.3\% |
| 29 | FNS | 80-89 | 24-26 | 31,492 | 1,289,161,140 | 1,248,160,587 | 114\% | 0.023 | 103.3\% | 1.4 | In Cl | 101.1\% - 105.5\% |
| 30 | FNS | 80-89 | 27-29 | 25,423 | 947,423,658 | 924,501,875 | 104\% | 0.025 | 102.5\% | 1.0 | In Cl | 100.1\% - 104.9\% |
| 31 | FNS | 80-89 | 30-36 | 19,877 | 738,932,033 | 671,127,161 | 105\% | 0.027 | 110.1\% | 3.8 | Below | 107.6\% - 112.6\% |
| 32 | FNS | 90 PLUS | 1-18 | 16,202 | 7,870,810,390 | 9,244,744,623 | 104\% | 0.021 | 85.1\% | (7.1) | Above | 83.2\% - 87.1\% |
| 33 | FNS | 90 PLUS | 19-23 | 17,774 | 1,754,117,912 | 1,659,448,203 | 101\% | 0.027 | 105.7\% | 2.1 | Below | 103.2\% - 108.2\% |
| 34 | FNS | 90 PLUS | 24-36 | 37,684 | 1,949,307,655 | 1,799,371,007 | 133\% | 0.021 | 108.3\% | 4.0 | Below | 106.4\% - 110.3\% |
| 35 | MNS | 18-29 | 1-12 | 2,971 | 450,504,232 | 460,645,382 | 51\% | 0.049 | 97.8\% | (0.4) | Not Cred. | 93.1\% - 102.5\% |
| 36 | MNS | 30-39 | 1-22 | 9,810 | 3,172,926,122 | 3,096,924,317 | 122\% | 0.022 | 102.5\% | 1.1 | InCl | 100.4\% - 104.5\% |
| 37 | MNS | 40-49 | 1-6 | 8,410 | 4,421,370,650 | 4,876,632,977 | 111\% | 0.021 | 90.7\% | (4.5) | Above | 88.7\% - 92.6\% |
| 38 | MNS | 40-49 | 7-9 | 5,200 | 2,827,203,881 | 3,352,132,485 | 103\% | 0.021 | 84.3\% | (7.5) | Above | 82.4\%-86.3\% |
| 39 | MNS | 40-49 | 10-12 | 4,997 | 2,285,545,928 | 2,742,175,257 | 106\% | 0.020 | 83.3\% | (8.3) | Above | 81.4\% - 85.3\% |
| 40 | MNS | 40-49 | 13-16 | 5,548 | 1,722,690,680 | 1,971,094,816 | 108\% | 0.021 | 87.4\% | (6.1) | Above | 85.4\%-89.4\% |
| 41 | MNS | 40-49 | 17-32 | 11,461 | 1,127,547,934 | 1,157,978,218 | 133\% | 0.019 | 97.4\% | (1.4) | InCl | 95.6\% - 99.1\% |
| 42 | MNS | 50-59 | 1-5 | 11,150 | 4,559,584,269 | 5,363,960,608 | 104\% | 0.021 | 85.0\% | (7.2) | Above | 83\%-87\% |
| 43 | MNS | 50-59 | 6-8 | 9,063 | 4,194,170,397 | 4,878,932,229 | 112\% | 0.020 | 86.0\% | (7.1) | Above | 84.1\%-87.8\% |
| 44 | MNS | 50-59 | 9-10 | 6,987 | 3,364,653,530 | 3,734,292,303 | 109\% | 0.021 | 90.1\% | (4.7) | Above | 88.1\% - 92.1\% |
| 45 | MNS | 50-59 | 11-12 | 6,345 | 2,506,650,133 | 3,128,153,571 | 105\% | 0.019 | 80.1\% | (10.2) | Above | 78.3\%-82\% |
| 46 | MNS | 50-59 | 13-14 | 6,815 | 2,375,976,856 | 3,043,878,908 | 109\% | 0.018 | 78.1\% | (12.0) | Above | 76.3\%-79.8\% |
| 47 | MNS | 50-59 | 15-16 | 6,390 | 1,932,524,975 | 2,362,934,168 | 105\% | 0.020 | 81.8\% | (9.2) | Above | 79.9\%-83.7\% |
| 48 | MNS | 50-59 | 17-19 | 9,304 | 1,943,084,241 | 2,341,886,542 | 120\% | 0.018 | 83.0\% | (9.7) | Above | 81.3\% - 84.6\% |
| 49 | MNS | 50-59 | 20-21 | 6,114 | 922,698,233 | 954,009,328 | 103\% | 0.024 | 96.7\% | (1.4) | InCl | 94.4\% - 99\% |
| 50 | MNS | 50-59 | 22-23 | 6,412 | 799,758,975 | 811,114,585 | 108\% | 0.023 | 98.6\% | (0.6) | InCl | 96.4\% - 100.8\% |
| 51 | MNS | 50-59 | 24-25 | 6,818 | 740,286,673 | 758,980,779 | 116\% | 0.021 | 97.5\% | (1.2) | In Cl | 95.5\% - 99.6\% |
| 52 | MNS | 50-59 | 26-27 | 6,810 | 645,451,870 | 650,115,860 | 122\% | 0.021 | 99.3\% | (0.3) | In Cl | 97.3\% - 101.3\% |
| 53 | MNS | 50-59 | 28-29 | 5,603 | 462,501,522 | 455,654,179 | 114\% | 0.023 | 101.5\% | 0.7 | In Cl | 99.3\% - 103.7\% |
| 54 | MNS | 50-59 | 30-36 | 6,321 | 459,270,842 | 413,015,264 | 123\% | 0.023 | 111.2\% | 4.9 | Below | 109\% - 113.4\% |
| 55 | MNS | 60-69 | 1-6 | 16,789 | 4,963,552,043 | 5,832,674,836 | 110\% | 0.020 | 85.1\% | (7.6) | Above | 83.2\%-87\% |
| 56 | MNS | 60-69 | 7-9 | 13,177 | 4,428,977,142 | 5,503,739,896 | 119\% | 0.017 | 80.5\% | (11.3) | Above | 78.8\%-82.1\% |
| 57 | MNS | 60-69 | 10-11 | 9,361 | 2,836,350,270 | 3,676,115,021 | 103\% | 0.019 | 77.2\% | (12.0) | Above | 75.3\%-79\% |
| 58 | MNS | 60-69 | 12-13 | 9,583 | 2,772,330,016 | 3,309,090,520 | 107\% | 0.020 | 83.8\% | (8.1) | Above | 81.9\%-85.7\% |


| Cell | Sex/ Tobac. | Attained Age |  | Actual Number | Actual Face | Expected Face with MI | Cred Factor Before Capping | Std.Dev. | A/E Face w/MI | $\#$ of std devs from $100 \%$ | Standard <br> Normal <br> Position | Std. Norm. Confidence Interval |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 60 | MNS | 60-69 | 16-18 | 13,163 | 2,690,437,404 | 3,301,104,414 | 109\% | 0.019 | 81.5\% | (9.7) | Above | 79.7\%-83.3\% |
| 61 | MNS | 60-69 | 19-20 | 9,741 | 1,466,591,854 | 1,571,587,665 | 103\% | 0.023 | 93.3\% | (2.9) | Above | 91.1\% - 95.5\% |
| 62 | MNS | 60-69 | 21-22 | 10,071 | 1,211,428,044 | 1,220,344,561 | 110\% | 0.023 | 99.3\% | (0.3) | In Cl | 97.1\% - 101.4\% |
| 63 | MNS | 60-69 | 23-24 | 11,850 | 1,345,986,901 | 1,322,329,890 | 126\% | 0.021 | 101.8\% | 0.9 | InCl | 99.8\% - 103.7\% |
| 64 | MNS | 60-69 | 25-25 | 6,717 | 735,870,508 | 689,288,114 | 100\% | 0.027 | 106.8\% | 2.5 | Below | 104.2\% - 109.3\% |
| 65 | MNS | 60-69 | 26-26 | 6,999 | 693,483,242 | 686,087,706 | 102\% | 0.025 | 101.1\% | 0.4 | InCl | 98.7\% - 103.5\% |
| 66 | MNS | 60-69 | 27-27 | 6,991 | 695,303,170 | 657,550,713 | 105\% | 0.026 | 105.7\% | 2.2 | Below | 103.3\% - 108.2\% |
| 67 | MNS | 60-69 | 28-28 | 6,791 | 673,024,577 | 597,949,021 | 108\% | 0.027 | 112.6\% | 4.7 | Below | 110\% - 115.1\% |
| 68 | MNS | 60-69 | 29-29 | 6,004 | 554,362,231 | 524,307,436 | 102\% | 0.026 | 105.7\% | 2.2 | Below | 103.2\% - 108.2\% |
| 69 | MNS | 60-69 | 30-31 | 9,775 | 870,693,559 | 788,159,850 | 134\% | 0.021 | 110.5\% | 5.0 | Below | 108.5\% - 112.5\% |
| 70 | MNS | 60-69 | 32-36 | 8,254 | 683,083,928 | 631,217,430 | 132\% | 0.021 | 108.2\% | 3.9 | Below | 106.2\% - 110.2\% |
| 71 | MNS | 70-79 | 1-10 | 25,555 | 6,478,922,231 | 7,521,250,470 | 108\% | 0.020 | 86.1\% | (6.8) | Above | 84.2\% - 88.1\% |
| 72 | MNS | 70-79 | 11-14 | 18,357 | 3,844,420,131 | 4,421,900,621 | 106\% | 0.021 | 86.9\% | (6.3) | Above | 85\%-88.9\% |
| 73 | MNS | 70-79 | 15-17 | 15,533 | 2,643,119,887 | 2,874,771,117 | 107\% | 0.022 | 91.9\% | (3.7) | Above | 89.9\% - 94\% |
| 74 | MNS | 70-79 | 18-20 | 18,684 | 2,228,982,156 | 2,435,065,674 | 114\% | 0.021 | 91.5\% | (4.1) | Above | 89.6\% - 93.5\% |
| 75 | MNS | 70-79 | 21-23 | 23,203 | 2,147,592,671 | 2,229,344,792 | 126\% | 0.020 | 96.3\% | (1.9) | InCl | 94.5\% - 98.2\% |
| 76 | MNS | 70-79 | 24-25 | 19,517 | 1,718,626,711 | 1,690,357,245 | 131\% | 0.020 | 101.7\% | 0.8 | lnCl | 99.8\%-103.6\% |
| 77 | MNS | 70-79 | 26-27 | 21,030 | 1,723,370,576 | 1,696,144,498 | 142\% | 0.018 | 101.6\% | 0.9 | InCl | 99.9\% - 103.3\% |
| 78 | MNS | 70-79 | 28-28 | 9,965 | 815,006,088 | 775,697,752 | 105\% | 0.026 | 105.1\% | 2.0 | Below | 102.6\% - 107.5\% |
| 79 | MNS | 70-79 | 29-29 | 9,036 | 784,179,836 | 697,030,532 | 104\% | 0.028 | 112.5\% | 4.5 | Below | 109.9\%-115.1\% |
| 80 | MNS | 70-79 | 30-31 | 14,735 | 1,191,683,664 | 1,100,487,658 | 128\% | 0.022 | 108.3\% | 3.8 | Below | 106.2\% - 110.3\% |
| 81 | MNS | 70-79 | 32-36 | 12,260 | 1,036,326,376 | 972,589,228 | 121\% | 0.022 | 106.6\% | 2.9 | Below | 104.4\% - 108.7\% |
| 82 | MNS | 80-89 | 1-12 | 16,783 | 9,971,189,030 | 12,000,802,482 | 104\% | 0.020 | 83.1\% | (8.3) | Above | 81.2\% - 85\% |
| 83 | MNS | 80-89 | 13-20 | 38,754 | 5,777,132,403 | 5,996,444,991 | 108\% | 0.023 | 96.3\% | (1.6) | In Cl | 94.2\% - 98.5\% |
| 84 | MNS | 80-89 | 21-23 | 30,673 | 2,661,373,250 | 2,584,452,972 | 117\% | 0.022 | 103.0\% | 1.3 | In Cl | 100.8\%-105.1\% |
| 85 | MNS | 80-89 | 24-25 | 26,278 | 2,126,478,692 | 2,033,243,774 | 127\% | 0.021 | 104.6\% | 2.2 | Below | 102.6\% - 106.6\% |
| 86 | MNS | 80-89 | 26-27 | 28,014 | 2,124,457,472 | 2,091,681,272 | 130\% | 0.020 | 101.6\% | 0.8 | In Cl | 99.7\% - 103.5\% |
| 87 | MNS | 80-89 | 28-29 | 25,120 | 2,002,932,806 | 1,885,408,728 | 129\% | 0.021 | 106.2\% | 3.0 | Below | 104.2\% - 108.2\% |
| 88 | MNS | 80-89 | 30-36 | 35,220 | 2,868,830,972 | 2,797,560,153 | 153\% | 0.017 | 102.5\% | 1.5 | InCl | 100.9\% - 104.2\% |
| 89 | MNS | 90 PLUS | 1-26 | 26,915 | 6,425,973,621 | 7,860,221,305 | 102\% | 0.020 | 81.8\% | (8.9) | Above | 79.8\%-83.7\% |
| 90 | MNS | 90 PLUS | 27-36 | 24,266 | 1,919,636,623 | 2,003,197,900 | 130\% | 0.019 | 95.8\% | (2.2) | Above | 94\% - 97.6\% |
| 91 | FSM | 18-29 | 1-12 | 240 | 23,132,562 | 15,068,076 | 13\% | 0.296 | 153.5\% | 1.8 | Not Cred. | 125.4\%-181.6\% |
| 92 | FSM | 30-39 | 1-22 | 1,044 | 95,350,574 | 112,816,607 | 33\% | 0.065 | 84.5\% | (2.4) | Not Cred. | 78.3\% - 90.7\% |
| 93 | FSM | 40-49 | 1-32 | 4,670 | 430,686,169 | 446,800,437 | 67\% | 0.036 | 96.4\% | (1.0) | Not Cred. | 92.9\% - 99.9\% |
| 94 | FSM | 50-59 | 1-36 | 15,301 | 1,075,713,152 | 1,123,145,492 | 114\% | 0.021 | 95.8\% | (2.0) | InCl | 93.7\%-97.8\% |
| 95 | FSM | 60-69 | 1-36 | 30,048 | 1,491,973,073 | 1,583,813,301 | 110\% | 0.022 | 94.2\% | (2.6) | Above | 92.1\% - 96.3\% |
| 96 | FSM | 70-79 | 1-36 | 43,047 | 1,683,151,040 | 1,494,353,805 | 88\% | 0.033 | 112.6\% | 3.9 | Not Cred. | 109.5\%-115.7\% |
| 97 | FSM | 80-89 | 1-36 | 42,308 | 1,682,475,349 | 1,541,358,207 | 85\% | 0.033 | 109.2\% | 2.8 | Not Cred. | 106\% - 112.3\% |
| 98 | FSM | 90 PLUS | 1-36 | 10,825 | 512,977,056 | 545,803,052 | 43\% | 0.056 | 94.0\% | (1.1) | Not Cred. | 88.7\% - 99.3\% |
| 99 | MSM | 18-29 | 1-12 | 1,034 | 110,811,072 | 67,250,423 | 26\% | 0.163 | 164.8\% | 4.0 | Not Cred. | 149.3\%-180.3\% |
| 100 | MSM | 30-39 | 1-22 | 2,937 | 419,258,297 | 404,549,169 | 53\% | 0.050 | 103.6\% | 0.7 | Not Cred. | 98.9\% - 108.4\% |
| 101 | MSM | 40-49 | 1-32 | 9,443 | 1,224,632,987 | 1,396,571,726 | 97\% | 0.023 | 87.7\% | (5.3) | Not Cred. | 85.5\%-89.9\% |
| 102 | MSM | 50-59 | 1-15 | 10,230 | 1,724,344,579 | 1,857,318,920 | 102\% | 0.023 | 92.8\% | (3.1) | Above | 90.6\% - 95\% |
| 103 | MSM | 50-59 | 16-22 | 7,963 | 752,957,590 | 718,483,104 | 108\% | 0.025 | 104.8\% | 1.9 | InCl | 102.4\% - 107.1\% |
| 104 | MSM | 50-59 | 23-26 | 5,262 | 355,435,043 | 313,782,774 | 100\% | 0.029 | 113.3\% | 4.6 | Below | 110.5\% - 116\% |
| 105 | MSM | 50-59 | 27-36 | 6,316 | 330,238,040 | 260,502,513 | 125\% | 0.026 | 126.8\% | 10.3 | Below | 124.3\%-129.2\% |
| 106 | MSM | 60-69 | 1-17 | 15,538 | 1,766,204,853 | 2,102,340,950 | 101\% | 0.021 | 84.0\% | (7.5) | Above | 82\% - 86\% |
| 107 | MSM | 60-69 | 18-23 | 12,709 | 964,200,867 | 943,437,883 | 108\% | 0.024 | 102.2\% | 0.9 | InCl | 99.9\% - 104.5\% |
| 108 | MSM | 60-69 | 24-26 | 8,918 | 601,818,864 | 541,359,393 | 111\% | 0.025 | 111.2\% | 4.4 | Below | 108.7\% - 113.6\% |
| 109 | MSM | 60-69 | 27-28 | 6,030 | 394,918,311 | 335,295,573 | 101\% | 0.030 | 117.8\% | 5.9 | Below | 114.9\% - 120.6\% |
| 110 | MSM | 60-69 | 29-30 | 5,038 | 313,945,127 | 270,923,777 | 100\% | 0.029 | 115.9\% | 5.4 | Below | 113.1\% - 118.7\% |
| 111 | MSM | 60-69 | 31-36 | 5,555 | 327,022,430 | 307,502,161 | 111\% | 0.024 | 106.3\% | 2.6 | Below | 104\% - 108.7\% |
| 112 | MSM | 70-79 | 1-24 | 26,807 | 1,732,159,837 | 1,663,550,873 | 101\% | 0.026 | 104.1\% | 1.6 | lnCl | 101.6\% - 106.6\% |
| 113 | MSM | 70-79 | 25-28 | 15,279 | 837,808,365 | 737,993,079 | 116\% | 0.025 | 113.5\% | 5.4 | Below | 111.2\% - 115.9\% |
| 114 | MSM | 70-79 | 29-36 | 14,461 | 838,973,493 | 742,375,021 | 132\% | 0.022 | 113.0\% | 6.0 | Below | 110.9\% - 115.1\% |
| 115 | MSM | 80-89 | 1-36 | 36,650 | 1,863,752,214 | 1,593,731,325 | 128\% | 0.023 | 116.9\% | 7.3 | Below | 114.7\% - 119.2\% |
| 116 | MSM | 90 PLUS | 1-36 | 6,578 | 338,990,231 | 354,712,490 | 68\% | 0.036 | 95.6\% | (1.2) | Not Cred. | 92.2\% - 99\% |


| Cell | Sex/ Tobac. | Attained Age | Dur | Actual Number | Actual Face | Expected Face with MI | Cred Factor Before Capping | Std.Dev. | A/E Face w/MI | $\#$ of std devs from $100 \%$ | Standard <br> Normal <br> Position | Std. Norm. Confidence Interval |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 59 | MNS | 60-69 | 14-15 | 9,807 | 2,550,697,060 | 3,198,286,390 | 105\% | 0.019 | 79.8\% | (10.4) | Above | 77.9\% - 81.6\% |
| 60 | MNS | 60-69 | 16-18 | 13,163 | 2,690,437,404 | 3,301,104,414 | 109\% | 0.019 | 81.5\% | (9.7) | Above | 79.7\%-83.3\% |
| 61 | MNS | 60-69 | 19-20 | 9,741 | 1,466,591,854 | 1,571,587,665 | 103\% | 0.023 | 93.3\% | (2.9) | Above | 91.1\% - 95.5\% |
| 62 | MNS | 60-69 | 21-22 | 10,071 | 1,211,428,044 | 1,220,344,561 | 110\% | 0.023 | 99.3\% | (0.3) | InCl | 97.1\% - 101.4\% |
| 63 | MNS | 60-69 | 23-24 | 11,850 | 1,345,986,901 | 1,322,329,890 | 126\% | 0.021 | 101.8\% | 0.9 | InCl | 99.8\% - 103.7\% |
| 64 | MNS | 60-69 | 25-25 | 6,717 | 735,870,508 | 689,288,114 | 100\% | 0.027 | 106.8\% | 2.5 | Below | 104.2\% - 109.3\% |
| 65 | MNS | 60-69 | 26-26 | 6,999 | 693,483,242 | 686,087,706 | 102\% | 0.025 | 101.1\% | 0.4 | In CI | 98.7\% - 103.5\% |
| 66 | MNS | 60-69 | 27-27 | 6,991 | 695,303,170 | 657,550,713 | 105\% | 0.026 | 105.7\% | 2.2 | Below | 103.3\% - 108.2\% |
| 67 | MNS | 60-69 | 28-28 | 6,791 | 673,024,577 | 597,949,021 | 108\% | 0.027 | 112.6\% | 4.7 | Below | 110\% - 115.1\% |
| 68 | MNS | 60-69 | 29-29 | 6,004 | 554,362,231 | 524,307,436 | 102\% | 0.026 | 105.7\% | 2.2 | Below | 103.2\% - 108.2\% |
| 69 | MNS | 60-69 | 30-31 | 9,775 | 870,693,559 | 788,159,850 | 134\% | 0.021 | 110.5\% | 5.0 | Below | 108.5\% - 112.5\% |
| 70 | MNS | 60-69 | 32-36 | 8,254 | 683,083,928 | 631,217,430 | 132\% | 0.021 | 108.2\% | 3.9 | Below | 106.2\% - 110.2\% |
| 71 | MNS | 70-79 | 1-10 | 25,555 | 6,478,922,231 | 7,521,250,470 | 108\% | 0.020 | 86.1\% | (6.8) | Above | 84.2\% - 88.1\% |
| 72 | MNS | 70-79 | 11-14 | 18,357 | 3,844,420,131 | 4,421,900,621 | 106\% | 0.021 | 86.9\% | (6.3) | Above | 85\%-88.9\% |
| 73 | MNS | 70-79 | 15-17 | 15,533 | 2,643,119,887 | 2,874,771,117 | 107\% | 0.022 | 91.9\% | (3.7) | Above | 89.9\% - 94\% |
| 74 | MNS | 70-79 | 18-20 | 18,684 | 2,228,982,156 | 2,435,065,674 | 114\% | 0.021 | 91.5\% | (4.1) | Above | 89.6\% - 93.5\% |
| 75 | MNS | 70-79 | 21-23 | 23,203 | 2,147,592,671 | 2,229,344,792 | 126\% | 0.020 | 96.3\% | (1.9) | InCl | 94.5\% - 98.2\% |
| 76 | MNS | 70-79 | 24-25 | 19,517 | 1,718,626,711 | 1,690,357,245 | 131\% | 0.020 | 101.7\% | 0.8 | InCl | 99.8\%-103.6\% |
| 77 | MNS | 70-79 | 26-27 | 21,030 | 1,723,370,576 | 1,696,144,498 | 142\% | 0.018 | 101.6\% | 0.9 | In Cl | 99.9\% - 103.3\% |
| 78 | MNS | 70-79 | 28-28 | 9,965 | 815,006,088 | 775,697,752 | 105\% | 0.026 | 105.1\% | 2.0 | Below | 102.6\% - 107.5\% |
| 79 | MNS | 70-79 | 29-29 | 9,036 | 784,179,836 | 697,030,532 | 104\% | 0.028 | 112.5\% | 4.5 | Below | 109.9\% - 115.1\% |
| 80 | MNS | 70-79 | 30-31 | 14,735 | 1,191,683,664 | 1,100,487,658 | 128\% | 0.022 | 108.3\% | 3.8 | Below | 106.2\% - 110.3\% |
| 81 | MNS | 70-79 | 32-36 | 12,260 | 1,036,326,376 | 972,589,228 | 121\% | 0.022 | 106.6\% | 2.9 | Below | 104.4\% - 108.7\% |
| 82 | MNS | 80-89 | 1-12 | 16,783 | 9,971,189,030 | 12,000,802,482 | 104\% | 0.020 | 83.1\% | (8.3) | Above | 81.2\% - 85\% |
| 83 | MNS | 80-89 | 13-20 | 38,754 | 5,777,132,403 | 5,996,444,991 | 108\% | 0.023 | 96.3\% | (1.6) | In Cl | 94.2\% - 98.5\% |
| 84 | MNS | 80-89 | 21-23 | 30,673 | 2,661,373,250 | 2,584,452,972 | 117\% | 0.022 | 103.0\% | 1.3 | In Cl | 100.8\% - 105.1\% |
| 85 | MNS | 80-89 | 24-25 | 26,278 | 2,126,478,692 | 2,033,243,774 | 127\% | 0.021 | 104.6\% | 2.2 | Below | 102.6\% - 106.6\% |
| 86 | MNS | 80-89 | 26-27 | 28,014 | 2,124,457,472 | 2,091,681,272 | 130\% | 0.020 | 101.6\% | 0.8 | In Cl | 99.7\% - 103.5\% |
| 87 | MNS | 80-89 | 28-29 | 25,120 | 2,002,932,806 | 1,885,408,728 | 129\% | 0.021 | 106.2\% | 3.0 | Below | 104.2\% - 108.2\% |
| 88 | MNS | 80-89 | 30-36 | 35,220 | 2,868,830,972 | 2,797,560,153 | 153\% | 0.017 | 102.5\% | 1.5 | InCl | 100.9\% - 104.2\% |
| 89 | MNS | 90 PLUS | 1-26 | 26,915 | 6,425,973,621 | 7,860,221,305 | 102\% | 0.020 | 81.8\% | (8.9) | Above | 79.8\% - 83.7\% |
| 90 | MNS | 90 PLUS | 27-36 | 24,266 | 1,919,636,623 | 2,003,197,900 | 130\% | 0.019 | 95.8\% | (2.2) | Above | 94\% - 97.6\% |
| 91 | FSM | 18-29 | 1-12 | 240 | 23,132,562 | 15,068,076 | 13\% | 0.296 | 153.5\% | 1.8 | Not Cred. | 125.4\%-181.6\% |
| 92 | FSM | 30-39 | 1-22 | 1,044 | 95,350,574 | 112,816,607 | 33\% | 0.065 | 84.5\% | (2.4) | Not Cred. | 78.3\% - 90.7\% |
| 93 | FSM | 40-49 | 1-32 | 4,670 | 430,686,169 | 446,800,437 | 67\% | 0.036 | 96.4\% | (1.0) | Not Cred. | 92.9\%-99.9\% |
| 94 | FSM | 50-59 | 1-36 | 15,301 | 1,075,713,152 | 1,123,145,492 | 114\% | 0.021 | 95.8\% | (2.0) | InCl | 93.7\%-97.8\% |
| 95 | FSM | 60-69 | 1-36 | 30,048 | 1,491,973,073 | 1,583,813,301 | 110\% | 0.022 | 94.2\% | (2.6) | Above | 92.1\% - 96.3\% |
| 96 | FSM | 70-79 | 1-36 | 43,047 | 1,683,151,040 | 1,494,353,805 | 88\% | 0.033 | 112.6\% | 3.9 | Not Cred. | 109.5\% - 115.7\% |
| 97 | FSM | 80-89 | 1-36 | 42,308 | 1,682,475,349 | 1,541,358,207 | 85\% | 0.033 | 109.2\% | 2.8 | Not Cred. | 106\% - 112.3\% |
| 98 | FSM | 90 PLUS | 1-36 | 10,825 | 512,977,056 | 545,803,052 | 43\% | 0.056 | 94.0\% | (1.1) | Not Cred. | 88.7\% - 99.3\% |
| 99 | MSM | 18-29 | 1-12 | 1,034 | 110,811,072 | 67,250,423 | 26\% | 0.163 | 164.8\% | 4.0 | Not Cred. | 149.3\%-180.3\% |
| 100 | MSM | 30-39 | 1-22 | 2,937 | 419,258,297 | 404,549,169 | 53\% | 0.050 | 103.6\% | 0.7 | Not Cred. | 98.9\%-108.4\% |
| 101 | MSM | 40-49 | 1-32 | 9,443 | 1,224,632,987 | 1,396,571,726 | 97\% | 0.023 | 87.7\% | (5.3) | Not Cred. | 85.5\%-89.9\% |
| 102 | MSM | 50-59 | 1-15 | 10,230 | 1,724,344,579 | 1,857,318,920 | 102\% | 0.023 | 92.8\% | (3.1) | Above | 90.6\% - 95\% |
| 103 | MSM | 50-59 | 16-22 | 7,963 | 752,957,590 | 718,483,104 | 108\% | 0.025 | 104.8\% | 1.9 | InCl | 102.4\% - 107.1\% |
| 104 | MSM | 50-59 | 23-26 | 5,262 | 355,435,043 | 313,782,774 | 100\% | 0.029 | 113.3\% | 4.6 | Below | 110.5\% - 116\% |
| 105 | MSM | 50-59 | 27-36 | 6,316 | 330,238,040 | 260,502,513 | 125\% | 0.026 | 126.8\% | 10.3 | Below | 124.3\% - 129.2\% |
| 106 | MSM | 60-69 | 1-17 | 15,538 | 1,766,204,853 | 2,102,340,950 | 101\% | 0.021 | 84.0\% | (7.5) | Above | 82\% - 86\% |
| 107 | MSM | 60-69 | 18-23 | 12,709 | 964,200,867 | 943,437,883 | 108\% | 0.024 | 102.2\% | 0.9 | In Cl | 99.9\% - 104.5\% |
| 108 | MSM | 60-69 | 24-26 | 8,918 | 601,818,864 | 541,359,393 | 111\% | 0.025 | 111.2\% | 4.4 | Below | 108.7\% - 113.6\% |
| 109 | MSM | 60-69 | 27-28 | 6,030 | 394,918,311 | 335,295,573 | 101\% | 0.030 | 117.8\% | 5.9 | Below | 114.9\% - 120.6\% |
| 110 | MSM | 60-69 | 29-30 | 5,038 | 313,945,127 | 270,923,777 | 100\% | 0.029 | 115.9\% | 5.4 | Below | 113.1\% - 118.7\% |
| 111 | MSM | 60-69 | 31-36 | 5,555 | 327,022,430 | 307,502,161 | 111\% | 0.024 | 106.3\% | 2.6 | Below | 104\% - 108.7\% |
| 112 | MSM | 70-79 | 1-24 | 26,807 | 1,732,159,837 | 1,663,550,873 | 101\% | 0.026 | 104.1\% | 1.6 | InCl | 101.6\% - 106.6\% |
| 113 | MSM | 70-79 | 25-28 | 15,279 | 837,808,365 | 737,993,079 | 116\% | 0.025 | 113.5\% | 5.4 | Below | 111.2\% - 115.9\% |
| 114 | MSM | 70-79 | 29-36 | 14,461 | 838,973,493 | 742,375,021 | 132\% | 0.022 | 113.0\% | 6.0 | Below | 110.9\% - 115.1\% |
| 115 | MSM | 80-89 | 1-36 | 36,650 | 1,863,752,214 | 1,593,731,325 | 128\% | 0.023 | 116.9\% | 7.3 | Below | 114.7\% - 119.2\% |
| 116 | MSM | 90 PLUS | 1-36 | 6,578 | 338,990,231 | 354,712,490 | 68\% | 0.036 | 95.6\% | (1.2) | Not Cred. | 92.2\% - 99\% |

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[^1]:    ${ }^{1}$ Bowers Jr., Newton L., et al. (1997.) Society of Actuaries. Actuarial Mathematics. Pp. 387-389.

[^2]:    2 Klugman, Stuart, et al. (2009, December). "Credibility Theory Practices." https://www.soa.org/globalassets/assets/files/research/projects/research-cred-theory-pract.pdf.

