Abstract

In this paper, we consider the optimal investment and benefit payment problem for a target benefit pension plan (TBP) with default risk and model uncertainty. The pension fund is invested in a risk-free asset, a stock and a defaultable bond. The objective is to maximize the wealth and benefit excess from the target value or minimize the wealth and benefit gap from the target value with CARA utility. Applying stochastic control approach, we establish the Hamilton-Jacobi-Bellman equations for both the post-default case and the pre-default case, respectively. Robust optimal investment strategies and benefit payment adjustment strategies are derived explicitly for the two cases. We also consider the no-ambiguity model for degenerate case and compare the results under two cases. Numerical analyses are provided to illustrate the effects of parameters on the optimal strategies and demonstrate properties of the strategies.

Optimal control problem with model uncertainty

Dynamics of pension fund under probability measure P (reference model):
\[
\begin{align*}
&dX(t) = \pi_1(t)\frac{dS(t)}{S(t)} + \pi_2(t)\frac{dB(t)}{B(t)} + (X(t) - \pi_1(t) - \pi_2(t))dS_0(t) - \pi_1(t)dB(t)dt, \\
&X(0) = x_0.
\end{align*}
\]

Dynamics of pension fund under probability measure Q (a family of probability measures equivalent to P):
\[
\begin{align*}
&dX^Q(t) = \left[rX(t) + \mu \pi_1(t) + (1 - Z(t))\pi_2(t) + C_1(t)\right]dt \\
&\quad - I(t)\left(f(t)L(t) - \phi_2(t)\pi_1(t)\right)dt \\
&\quad + \pi_1(t)\sigma dW^Q(t) - \pi_2(t)(1 - Z(t-))dZ(t), \\
&X(0) = x_0.
\end{align*}
\]

Solution

\[
\begin{align*}
&\sup_{\pi_1,\pi_2} \inf_{s_1,s_2,\xi(t,\theta)} \left\{ \int V_t + \int \left[ X(t)\pi_1(t) + \sigma^2(t)\pi_2(t)^2 + C_1(t)\right]dt + \int \sigma(t)dW^Q(t) + \int \phi(t)dt \right\} \\
&\quad \geq \int V_0 + \int \left[ X(t)\pi_1(t) + \sigma^2(t)\pi_2(t)^2 + C_1(t)\right]dt + \int \sigma(t)dW^Q(t) + \int \phi(t)dt + \int f^*(t)dt,
\end{align*}
\]

Financial Market

Defeatable bond
\[
P(t,T_1) = \int_{T_1}^{T} \exp(-\int_{T_1}^{T} r(t)dt)dt.
\]

Objective function

Maximize the wealth and benefit excess from the target value or minimize the wealth and benefit gap from the target value with CARA utility.

Table

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<th>Proportion</th>
<th>t = 0</th>
<th>t = 2</th>
<th>t = 4</th>
<th>t = 6</th>
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<td>Defaultable Bond</td>
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<td>0.3305</td>
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<td>0.6757</td>
<td>0.6667</td>
<td>0.9998</td>
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</tr>
</tbody>
</table>

Conclusion

This paper considers the optimal investment and benefit payment problem for a target benefit pension plan (TBP) with model uncertainty and default risk. We find that: 1. The optimal amount invested in the defeatable bond is much larger than that invested in the stock. 2. A higher model uncertainty aversion level results in the less investment in financial market. 3. The parameters of stock and defeatable bond have effects on the optimal benefit payment adjustment strategy.